

## Evaluate the Rate of Pollution in Soil using Simulink Environment

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**Article history: Received 11 October 2018, Accepted 4 November 2018, Publish January 2019**

### Abstract

In this paper we design a Simulink model which can be evaluate the concentration of Copper, Lead, Zinc, Cadmium, Cobalt, Nickel, Crum and Iron. So, this model would be a method to determine the contamination levels of these metals with the potential for this contamination sources with their impact. The aim of using Simulink environment is to solve differential equations individually and as given data in parallel with analytical mathematics trends. In general, mathematical models of the spread heavy metals in soil are modeled and solve to predict the behavior of the system under different conditions.

**Keywords:** Simulink, Mathematical model, Partial differential equation, Soil layers, Heavy metals

### 1. Introduction

In this paper, a mathematical model [1] based on Simulink environment to estimate the concentration of heavy metals which can be used to determine the contamination in soil will be design, which may be used to estimate the concentration of heavy metals in soil before and after the rain. Simulink model is a block diagram environment for multi task simulation and model based design. It supports system design, simulation, code generation, and continuous testing [4]. Simulink environment can be used to describe and solve different applications of various fields especially contamination model. The new approach used for calculation the dispersion parameter by design suitable Simulink model. Then design a model equation based on Simulink environment that can estimate the effect of rainwaters in the soil which be contaminated by metals. There are many methods to solve PDE some of these gave analytic solution, others gave approximate solution for more details see [6-8].

### 2. Solving an PDE by Simulink

Simulink is a graphical setting for planning simulations of systems, after we have access to Simulink and MATLAB, we'll be able to begin MATLAB Associate in Nursing on the icon bar there's an icon that we just will click to launch Simulink. or else, we'll sort Simulink to cite the Simulink Library Browser as shown in **Figure 1**. Next, click the yellow and to bring up a replacement model. we have a tendency to build models by dragging and connecting the needed elements, or blocks, from teams like the continual, Math Operations, Sinks, or Sources. As Associate in Nursinging example, we'll use Simulink to unravel the PDE.

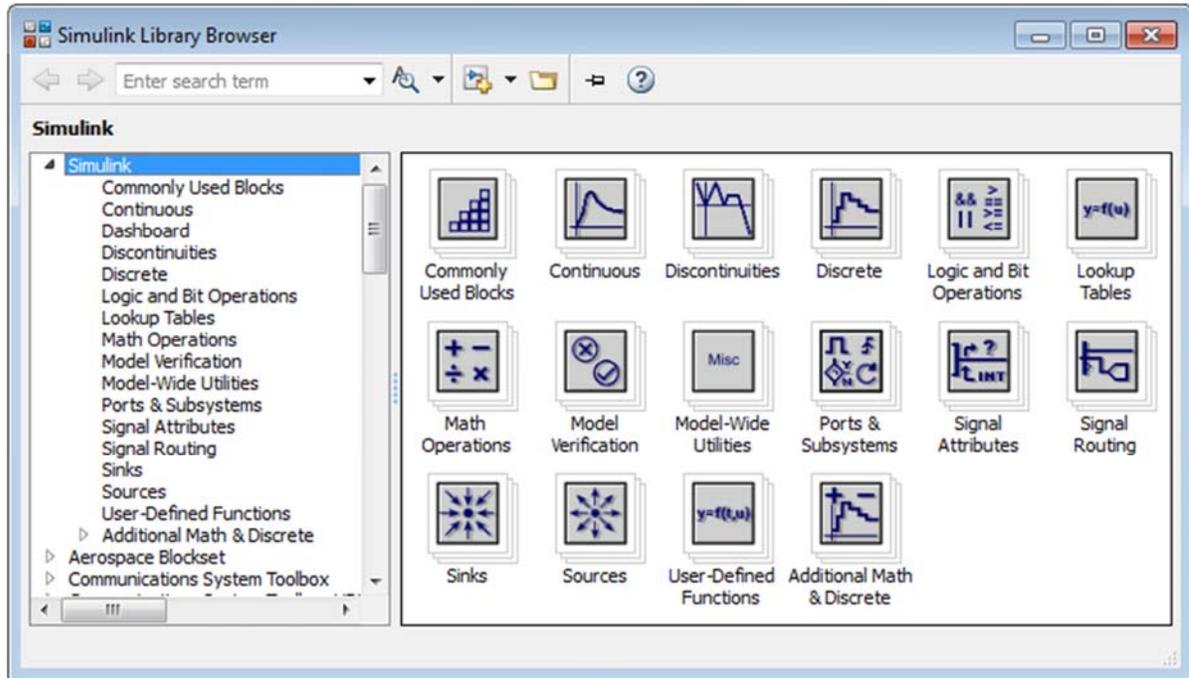


Figure 1. The Simulink Library Browser. This is where various blocks can be found for constructing models.

### 3. Creation the Model

Creating an operating model with Simulink is easy; the model involves four basic steps as the Figure 2.:

1. Desired blocks must be select.
2. The block parameters must be configuring.
3. The block of inputs outputs must be connecting when we needed.
4. Simulation parameters must be configuring.

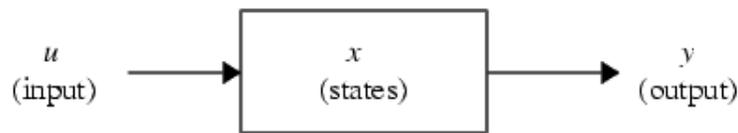


Figure 2. Schematic for a general system in which the block takes the input and produces an output.

### 4. Mathematical Description of the Model

The suggested model descriptions of the contamination of soils by heavy metals which are distributed throughout the soil system remain in the soil solution as iron, organic and inorganic complexes. Some of these heavy metals are mobile for uptake by plants. This mobility and availability depends on several factors including soil texture and PH [5]. Also, in the industrial regions, where some types of factories are active, several chemical and petrochemical processes would be also active. As a result, industrial water becomes contaminated with various substances which are harmful; these are sources of environmental contamination, which added in the

description of the model equation. Then, the model equation, can be written as the relation between the change of concentration  $C$  ( $\frac{mg}{L}$ ) of heavy metals proportionally with the change of time  $t$  ( $day^{-1}$ ) and the change of concentration of heavy metals proportionally with the change of space  $x$  ( $cm$ ) multiplicand with the average pore-water velocity  $V$  ( $\frac{cm}{hr}$ ). Then adding the fluid velocity in the reactor multiplicand with Dispersion parameter  $D$  ( $\frac{cm^2}{hr}$ ). So, the model equation can be written as:

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2} - V \frac{\partial C}{\partial x} \tag{1}$$

Which is a second order linear PDE, with initial - boundary conditions. Now, how to choose initial and boundary conditions. A variety of conditions may be specified depending on the type of soil. There are different types of soil depending on properties, structure, characteristic or layers. In this thesis we classify the soil depending on characteristic and properties of soil as follows: Sand (fine and coarse), Silt, Loam, Craggs and Clay [3]. If the soil is a loam land, then the conditions are:

IC:  $C(x, 0) = C_{x,0} = C_0 e^{-\frac{Vx}{D}}$   
 BC:  $C(0, t) = C_0$  and  $\frac{\partial C}{\partial x}(\infty, t) = 0$ .  
 Where;  $C_0$ : Initial concentration ( $\frac{mg}{L}$ ).

The amount of each element retained by each soil ( $\frac{mg}{kg}$ ) was calculated from the initial concentration in solution ( $\frac{mg}{L}$ ) and the final concentration  $C$  in solution ( $\frac{mg}{L}$ ). Equation (1), which can be represented as a mathematical model for spread of contamination through soil which can be used to determine the rate of contamination. The solution of model equation gives the concentration of the heavy metals in soil for any space and time. This model can be considered as an important model to give concentrations of heavy metals without spending in traditional laboratory inspecting. Thus, the comparison between the result of model equation (1) before and after the rain gives the effect of the rain of contaminated soil.

### 5. Solving the Model Equation

We will use the Adomian Decomposition Method (ADM) to solve Equation (1). First we consider the following an operator form of linear differential equation

$$L_t C = D L_{xx} C - V L_x C \tag{2}$$

Where  $L$ , is the symbol of derivative which can be invertible.

Take the inverse operator  $L^{-1}$  to both sides of equation (2), and using the initial condition:

$$C(x, 0) = C_{x,0} = C_0 e^{-\frac{Vx}{D}}$$

We get:

$$C = C_0 e^{-\frac{Vx}{D}} + D L_t^{-1} L_{xx} C - V L_t^{-1} L_x C \tag{3}$$

Related to Adomian method, we define the solution  $C$  by an infinite series of components given by:

$$C = \sum_{n=0}^{\infty} C_{x,n}$$

Then the Equation (3) can be written as:

$$(\sum_{n=0}^{\infty} C_{x,n}) = C_0 e^{-\frac{Vx}{D}} + D L_t^{-1} L_{xx} (\sum_{n=0}^{\infty} C_{x,n}) - V L_t^{-1} L_x (\sum_{n=0}^{\infty} C_{x,n}) \quad (4)$$

Where  $c_{x,0}, c_{x,1}, c_{x,2}, \dots$  can be determined as far as we like.

As given in our model, the zeros component  $C_{x,0} = c_0 e^{-\frac{Vx}{D}}$ , then:

$$\begin{aligned} C_{x,1} &= L_t^{-1} (D L_{xx} c_0 e^{-\frac{Vx}{D}} - V L_x c_0 e^{-\frac{Vx}{D}}) \\ &= L_t^{-1} (c_0 \frac{V^2}{D} e^{-\frac{Vx}{D}} + c_0 \frac{V^2}{D} e^{-\frac{Vx}{D}}) \\ &= 2t c_0 \frac{V^2}{D} e^{-\frac{Vx}{D}} \end{aligned}$$

$$\begin{aligned} C_{x,2} &= L_t^{-1} (D L_{xx} (2t c_0 \frac{V^2}{D} e^{-\frac{Vx}{D}}) - V L_x (2t c_0 \frac{V^2}{D} e^{-\frac{Vx}{D}})) \\ &= L_t^{-1} (2t c_0 \frac{V^4}{D^2} e^{-\frac{Vx}{D}} + 2t c_0 \frac{V^4}{D^2} e^{-\frac{Vx}{D}}) \\ &= \frac{4t^2}{2} c_0 \frac{V^4}{D^2} e^{-\frac{Vx}{D}} \end{aligned}$$

$$\begin{aligned} C_{x,3} &= L_t^{-1} (D L_{xx} (\frac{4t^2}{2} c_0 \frac{V^4}{D^2} e^{-\frac{Vx}{D}}) - V L_x (\frac{4t^2}{2} c_0 \frac{V^4}{D^2} e^{-\frac{Vx}{D}})) \\ &= L_t^{-1} (2t^2 c_0 \frac{V^6}{D^3} e^{-\frac{Vx}{D}} + 2t^2 c_0 \frac{V^6}{D^3} e^{-\frac{Vx}{D}}) \\ &= 8 \frac{t^3}{3!} c_0 \frac{V^6}{D^3} e^{-\frac{Vx}{D}} \end{aligned}$$

And so on for other components. Consequently, the solution in a series form is given by:

$$\begin{aligned} C &= C_{x,0} + C_{x,1} + C_{x,2} + C_{x,3} + \dots \\ &= c_0 e^{-\frac{Vx}{D}} + 2t c_0 \frac{V^2}{D} e^{-\frac{Vx}{D}} + \frac{4t^2}{2} c_0 \frac{V^4}{D^2} e^{-\frac{Vx}{D}} + 8 \frac{t^3}{3!} c_0 \frac{V^6}{D^3} e^{-\frac{Vx}{D}} + \dots \end{aligned}$$

That is:

$$C = c_0 e^{-\frac{Vx}{D}} \left\{ 1 + \left(\frac{t}{D}\right)(2V^2) + \frac{1}{2!} \left(\frac{t}{D}\right)^2 (2V^2)^2 + \frac{1}{3!} \left(\frac{t}{D}\right)^3 (2V^2)^3 + \dots \right\}$$

i.e.,  $C = c_0 e^{-\frac{Vx}{D}} e^{\frac{2tV^2}{D}}$

Thus:

$$C = c_0 \exp\left(-\frac{Vx}{D} + \frac{2V^2}{D}t\right) = c_0 \exp\left\{\frac{V}{D}(-x + 2Vt)\right\} \quad (5)$$

Thus, equation (5) represents the closed form solution of the model equation.

Now, we choose  $V = 5.14 \times 10^{-6} \text{ ms}^{-1} = 44.4096 \times 10^{-2} \frac{m}{d}$ , depending on results of [1-2].

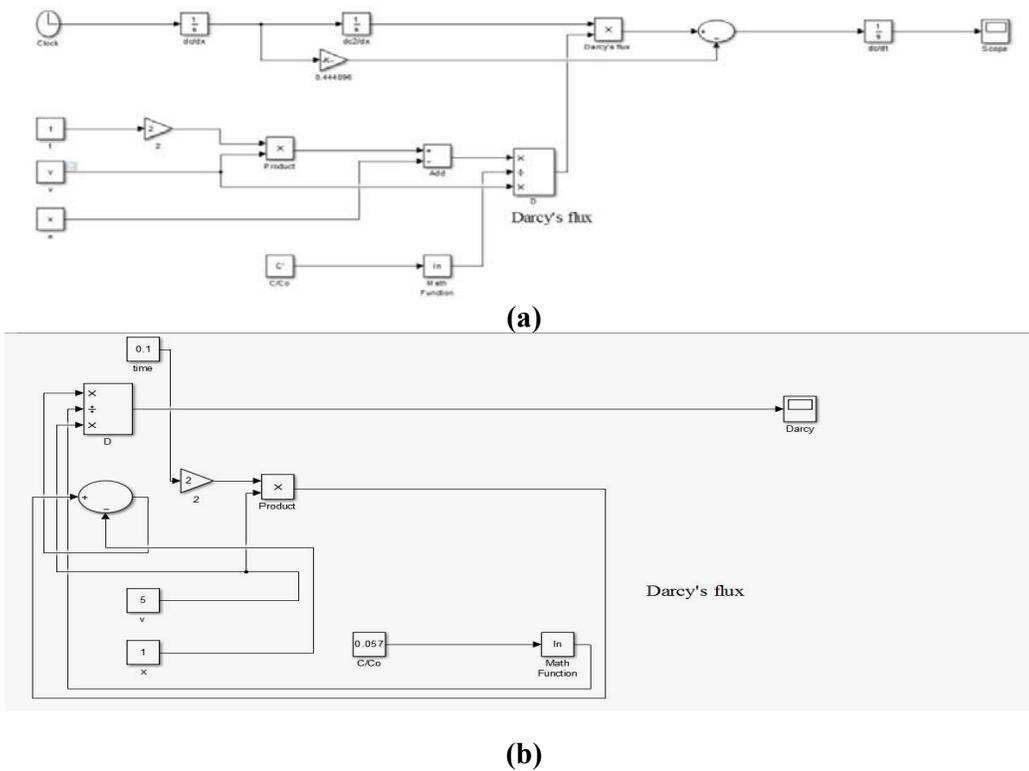
$$\text{Therefore, } C = c_0 \exp\left\{\frac{44.4096 \times 10^{-2}}{D} (-x + 2(44.4096 \times 10^{-2})t)\right\} \quad (6)$$

$$\text{So, } C = c_0 \exp\left\{\frac{44.4096 \times 10^{-2}}{D} (-x + 0.888192t)\right\}$$

$$\text{Then, } D = \frac{Ln \frac{C}{c_0}}{44.4096 \times 10^{-2} (-x + 0.888192t)} \quad (7)$$

Now, we build Simulink model to estimate the dispersion parameter D for the Equation (7), we firstly looking the terms in the bracket. We have 3 input signal lines: t, y, and x, also we see that

both  $t$  and  $y$  are multiplied by constants, as in the first case of the model; this can be done by using a Gain block in Simulink. The signals are then summed together as illustrated in **Figure 3**.



**Figure 3.** Simulink model to estimate the dispersion parameter

Suggested model consist both inputs:  $x$  dot and  $x$  with outputs of the model. So, a feedback loop must be needed which can be used for each of these signals. The model is must be including a Scope block for viewing the results after the running. Hence, that the model is complete, we are ready to run the simulation and look at the results. Note that Simulink model illustrated in **Figure 3**. can be used to estimate the dispersion parameter  $D$  for any type of soil in any time. Depending on this value we design Simulink model to estimate the concentration of heavy metals to measure the effect of rainwater. This model can be considered as an important model to give concentrations of heavy metals without spending in traditional laboratory inspecting. Thus, the comparison between the result of suggested model Equation (1) before and after the rain gives the effect of the rain for the contaminated soil. Depending on those, we design Simulink model illustrated in **Figure 4**.



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