Filter Bases and $j$-$\omega$-Perfect Mappings

G. S. Ashaea, Y. Y. Yousif

Department of Mathematics, College of Education for Pure Sciences, Ibn Al-Haitham, University of Baghdad

gaidaasadoon@gmail.com, yoyayousif@yahoo.com

Article history: Received 25 March 2019, Accepted 26 May 2019, Publish September 2019

Doi:10.30526/32.3.2289

Abstract

This paper consist some new generalizations of some definitions such: $j$-$\omega$-closure converge to a point, $j$-$\omega$-closure directed toward a set, almost $j$-$\omega$-converges to a set, almost $j$-$\omega$-cluster point, a set $j$-$\omega$-H-closed relative, $j$-$\omega$-closure continuous mappings, $j$-$\omega$-weakly continuous mappings, $j$-$\omega$-compact mappings, $j$-$\omega$-rigid a set, almost $j$-$\omega$-closed mappings and $j$-$\omega$-perfect mappings. Also, we prove several results concerning it, where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

Keywords: Filter base, $j$-$\omega$-closure converge, almost $j$-$\omega$-converges, almost $j$-$\omega$-cluster, $j$-$\omega$-rigid a set, $j$-$\omega$-perfect mappings.

Math Subject Classification 2010: 54C05, 54C08, 54C10.

1. Introduction

The notion "filter" first commence in Riesz [1] and the setting of convergence in terms of filters sketched by Cartan in [2, 3]. And was sophisticatedly by Bourbaki in [4]. Whyburn in [5]. Introduces the notion directed toward a set and the generalization of this notion studied in Section 2. Dickman and Porter in [6]. Introduce the notion almost convergence, Porter and Thomas in [7]. introduce the notion of quasi-H-closed and the analogues of this notions are studied in Section 3. Levine in [8]. Introduce the notion $\theta$-continuous functions, Andrew and Whittlesy in [9]. Introduce the notion weakly $\theta$-continuous functions, in Dickman [6]. Introduce the notions $\theta$-compact functions, $\theta$-rigid a set, almost closed functions and the analogues of this notions are studied in Section 4. In [5]. The researcher introduces the notion of $\theta$-perfect functions but the analogue of this notion studied in Section 5. The neighborhood denoted by nbd. The closure (resp. interior) of a subset $K$ of a space $G$ denoted by $\text{cl} (K)$ (resp., $\text{int}(K)$). A point $g$ in $G$ is said to be condensation point of $K \subseteq G$ if every $S$ in $\tau$ with $g \in S$, the set $K \cap S$ is uncountable [10]. In 1982 the $\omega$-closed set was first exhibiting by Hdeib in [10]. and he know it a subset $K \subseteq G$ is called $\omega$-closed if it incorporates each its condensation points and the $\omega$-open set is the complement of the $\omega$-closed set [12]. The $\omega$-interior of the set $K \subseteq G$ defined as the union of all $\omega$-open sets contain in $K$ and is denoted by $\text{int}_{\omega}(K)$. A point $g \in G$ is said to $\theta$-cluster points of $K \subseteq G$ if $\text{cl}(S) \cap K \neq \emptyset$ for each open set $S$. 165
of $G$ containment $g$. The set of each $\theta$-cluster points of $K$ is called the $\theta$-closure of $K$ and is denoted by $\text{cl}(K)$. A subset $K \subseteq G$ is said to be \theta-closed [11], if $K = \text{cl}(K)$. The complement of \theta-closed set said to be \theta-open. A point $g \in G$ said to $\theta$-o-cluster points of $K \subseteq G$ if $\omega \text{cl}(S) \cap K \neq \emptyset$ for each $\omega$-open set $S$ of $G$ containment $g$. The set of each $\theta$-o-cluster points of $K$ is called the $\theta$-o-closure of $K$ and is denoted by $\omega \text{cl}(K)$. A subset $K \subseteq G$ is said to be $\theta$-o-closed [11], if $K = \omega \text{cl}(K)$. The complement of $\theta$-o-closed set said to be $\theta$-o-open, $\delta$-closed [12], if $K = \text{cl}(K) = \{g \in G : \text{int}(\text{cl}(S)) \cap K \neq \emptyset, S \in \tau \text{ and } g \in S\}$. The complement of $\delta$-closed said $\delta$-open set, $\delta$-o-closed if $K = \omega \text{cl}(K) = \{g \in G : \text{int}(\text{cl}(S)) \cap K \neq \emptyset, S \in \tau \text{ and } g \in S\}$. The complement of $\delta$-o-closed said $\delta$-o-open.

2. Filter

In this section we introduce definition of filter, filter base, nbd filter, finer ultrafilter and some other related concepts.

**Definition 1** [4].

A nonempty family $\mathcal{J}$ of nonempty subsets of $G$ called filter if it satisfies the following conditions:

(a) If $M_1, M_2 \in \mathcal{J}$, then $M_1 \cap M_2 \in \mathcal{J}$.

(b) If $M \in \mathcal{J}$ and $M \subseteq M^* \subseteq G$, then $M^* \in \mathcal{J}$.

**Definition 2** [4].

A nonempty family $\mathcal{J}$ of nonempty subsets of $G$ is called filter base if $M_1, M_2 \in \mathcal{J}$ then $M_3 \subseteq M_1 \cap M_2$ for some $M_3 \in \mathcal{J}$.

The filter generated by a filter base $\mathcal{J}$ consists of all supersets of elements of $\mathcal{J}$. An open filter base on a space $G$ is a filter base with open members. The set $\mathcal{N}_g$ of all nbds of $g \in G$ is a filter on $G$, and any nbd base at $g$ is a filter base for $\mathcal{N}_g$. This filter called the nbd filter at $g$.

**Definition 3** [4].

Let $\mathcal{J}$ and $\mathcal{Y}$ be filter bases on $G$. Then $\mathcal{Y}$ is called finer than $\mathcal{J}$ (written as $\mathcal{J} < \mathcal{Y}$) if for all $M \in \mathcal{J}$, there is $G \in \mathcal{Y}$ such that $G \subseteq M$ and that $\mathcal{J}$ meets $G$ if $M \cap G \neq \emptyset$ for all $M \in \mathcal{J}$ and $G \in \mathcal{Y}$. Notice, $\mathcal{J} \rightarrow g$ iff $\mathcal{N}_g < \mathcal{J}$.

**Definition 4** [4].

A filter $\mathcal{J}$ is called an ultrafilter if there is no strictly finer filter $\mathcal{Y}$ than $\mathcal{J}$. The ultrafilter is the maximal filter.

**Definition 5** [13].

A subset $K$ of a space $G$ called:

(a) $\alpha$-o-open if $K \subseteq \text{int}_G(\text{int}_G(K))$.

(b) $\beta$-o-open if $K \subseteq \text{int}_G(\text{cl}(K))$.

(c) $\beta$-o-closed if $K \subseteq \text{cl}(\text{int}_G(K)) \cup \text{int}_G(\text{cl}(K))$.

(d) $\beta$-o-closed if $K \subseteq \text{cl}(\text{int}_G(\text{cl}(K)))$.

The complement of an (resp. $\alpha$-o-open, $\beta$-o-open, $\beta$-o-closed, $\beta$-o-closed) called (resp. $\alpha$-o-closed, $\beta$-o-closed, $\beta$-o-closed, $\beta$-o-closed).

The $j$-o-closure of $K \subseteq G$ is denoted by $\text{cl}_j(K)$ and defined by $\text{cl}_j(K) = \bigcap\{M \subseteq G; G$ is $j$-o-closed and $K \subseteq M\}$, where $j \in \{\emptyset, \delta, \alpha, \beta, \emptyset, \delta, \alpha, \beta\}$. Several characterizations of $\omega$-closed sets were provided in [11, 13-16]. Furthermore, we built some results about $\delta$-o-closed and $\delta$-o-open depending on the results in [17-19].
3. Filter Bases and $j$-$\omega$-Closure Directed toward a Set

In this section we defined filter bases and $j$-$\omega$-closure directed toward a set and the some theorems concerning of them.

Lemma 6 [15].

Let $\lambda : (G, \tau) \rightarrow (H, \sigma)$ be an injective mapping.

(a) If $\mathfrak{F} = \{M : M \subseteq G\}$ is a filter base in $G$, then $\lambda(\mathfrak{F}) = \{\lambda(M) : M \in \mathfrak{F}\}$ is a filter base in $H$.

(b) If $\varnothing = \{G ; G \subseteq \lambda(G)\}$ is a filter base in $\lambda(G)$, then $\lambda^{-1}(\varnothing) : G \in \varnothing$ is a filter base in $G$.

For each $\phi \neq K \subseteq G$ and any filter base $\varnothing$ in $\lambda(K)$, then $\{K \cap \lambda^{-1}(\varnothing) : G \in \varnothing\}$ is a filter base in $K$.

(c) If $\mathfrak{F} = \{M : M \subseteq G\}$ is a filter base in $G$, $\varnothing = \{\lambda(M) : M \in \mathfrak{F}\}$, $G^\ast$ is finer than $G$, and $\mathfrak{F}^\ast = \{\lambda^{-1}(G^\ast) : G^\ast \in \varnothing\}$, then the collection of sets $\mathfrak{F}^{\ast\ast} = \{M \cap M^\ast \text{ for all } M \in \mathfrak{F} \text{ and } M^\ast \in \mathfrak{F}^{\ast}\}$ is finer than both of $\mathfrak{F}$ and $\mathfrak{F}^{\ast}$.

Definition 7 [4].

Let $\mathfrak{F}$ be a filter base on a space $G$. We say that $\mathfrak{F}$ converges to $g \in G$ (written as $\mathfrak{F} \rightarrow g$) iff each open set $S$ about $g$ contains some element $M \in \mathfrak{F}$. We say $\mathfrak{F}$ has $g$ as a cluster point (or $\mathfrak{F}$ cluster at $g$) iff each open set $S$ about $g$ meets all element $M \in \mathfrak{F}$. Clear that if $\mathfrak{F} \rightarrow g$, then $\mathfrak{F}$ cluster at $g$.

Definition 8 [15].

Let $\mathfrak{F}$ be a filter base on a space $G$. We say that $\mathfrak{F}$ directed toward (shortly, dir-$\tau$) $G\cap h$ if and only if $\mathfrak{F}$ is finer than $\mathfrak{F}$ has a cluster point in $K$. (Note: Any filter base can't be dir-$\tau$ the empty set).

Now, we will generalizations Definitions 7 and 8 as follows.

Definition 9

Let $\mathfrak{F}$ be a filter base on a space $G$. We say that $\mathfrak{F}$ closure converges to $g \in G$ (written as $\mathfrak{F} \rightarrow g$) iff all open set $S$ about $g$ the $\text{cl}(S)$ contains some element $M \in \mathfrak{F}$. We say $\mathfrak{F}$ has $g$ as a closure cluster point (or $\mathfrak{F}$ closure cluster at $g$) iff all open set $S$ about $g$ the $\text{cl}(S)$ meets all element $M \in \mathfrak{F}$.

Clear that if $\mathfrak{F} \rightarrow g$, then $\mathfrak{F}$ closure cluster at $g$. $\text{cl}(\mathfrak{F}_g)$ used to denote the filter base $\{\text{cl}(S) : S \in \mathfrak{F}_g\}$. Notice, $\mathfrak{F} \rightarrow g$ if and only if $\text{cl}(\mathfrak{F}_g) \rightarrow \mathfrak{F}$. [10].

Definition 10

Let $\mathfrak{F}$ be a filter base on a space $G$. We say that $\mathfrak{F}$ closure directed toward (shortly, cl $\text{dir-} \tau$) $G\cap h$ if and only if $\mathfrak{F}$ has a closure cluster point in $K$.

Theorem 11

Let $\mathfrak{F}$ be a filter base on a space $G$. $\mathfrak{F} \rightarrow g \in G$ if and only if $\mathfrak{F}$ is cl $\text{dir-} \tau$ $g$.

Proof: ($\Rightarrow$) Assume $\mathfrak{F} \rightarrow g$, all open set $S$ about $g$, cl($S$) contains an element of $\mathfrak{F}$ and thus contains an element of every filter base $\mathfrak{F}^\ast < \mathfrak{F}$, therefore $\mathfrak{F}^\ast$ actually closure converges to $g$.

($\Leftarrow$) Assume $\mathfrak{F}$ is cl $\text{dir-} \tau$ $g$, it must $\mathfrak{F} \rightarrow g$. For if not, yond is an open set $S$ in $G$ about $g$ such that $\text{cl}(S)$ don't contains an element of $\mathfrak{F}$. Denote by $\mathfrak{F}^\ast$ the collection of sets $M^\ast = M \cap (G - \text{cl}(S))$ for $M \in \mathfrak{F}$, then the sets $M^\ast$ are nonempty. And $\mathfrak{F}^\ast$ is a filter base and indeed $\mathfrak{F}^\ast < \mathfrak{F}$, because result in $M_1^\ast = M_1 \cap (G - \text{cl}(S))$ and $M_2^\ast = M_2 \cap (G - \text{cl}(S))$, so there is an $M_3 \subseteq M_1 \cap M_2$ and this perform to

$$M_3^\ast = M_3 \cap (G - \text{cl}(S)) \subseteq M_1 \cap M_2 \cap (G - \text{cl}(S))$$

$$= M_1 \cap (G - \text{cl}(S)) \cap M_2 \cap (G - \text{cl}(S)).$$
By construction, \( g \) is not a closure cluster point of \( \mathfrak{Z}^* \). This contradiction crops that, \( \mathfrak{Z} \sim \sim g \).

**Theorem 12**

Let \( \lambda : (G, \tau) \to (H, \sigma) \) be an injective mapping and given \( L \subseteq H \). If for each filter base \( \wp \) in \( \lambda(G) \) cl dir. - tow \( a \) point \( h \in L \), the inverse filter \( \mathcal{M} = \{ \lambda^{-1}(G) : G \in \wp \} \) is cl dir. - tow \( \lambda^{-1}(h) \), then for any filter base \( \mathfrak{Z} \) in \( \lambda(G) \) cl dir. - tow a set \( L \), \( \mathcal{E} = \{ \lambda^{-1}(M) : M \in \mathfrak{Z} \} \) is cl dir. - tow \( K = \lambda^{-1}(L) \).

**Proof:** Suppose that the hypothesis is true and any \( h \in L \) is a closure cluster point of a filter base finer than \( \mathfrak{Z} \) must be in \( \lambda(G) \). Thus \( L \cap \lambda(G) \neq \emptyset \), and \( \mathfrak{Z} \) is cl dir. - tow \( L \cap \lambda(G) \). So we may assume \( L \subseteq \lambda(G) \). Let \( \mathcal{M} \) be a filter base finer than \( \mathcal{E} \). Then \( \wp = \{ (\lambda(m) : m \in \mathcal{M} \} \) finer than \( \mathfrak{Z} \) by Lemma (6, a). So \( \wp \) has a closure cluster point \( l \) in \( L \) and a filter base \( \wp^* \) finer than \( \wp \) closure converges to \( l \) and so is cl dir. - tow \( l \). By supposition \( \mathcal{M}^* = \{ \lambda^{-1}(G^*) : G^* \in \wp^* \} \) is cl dir. - tow \( \lambda^{-1}(l) \). In addition, by Lemma (6, c), \( \mathcal{M} \) and \( \mathcal{M}^* \) have a common filter base \( \mathcal{M}^* \) finer than \( \mathfrak{Z} \). So \( \mathcal{M}^* \) has a closure cluster point \( g \) in \( \lambda^{-1}(l) \). Since \( g \) is a closure cluster point of \( \mathcal{M} \) and \( g \in \lambda^{-1}(l) \subseteq K \), obtain result follows.

**Theorem 13**

Let \( \lambda : G \to H \) be closed mapping and \( \lambda^{-1}(h) \) compact for every \( h \in H \) iff for every filter base \( \mathfrak{Z} \) in \( \lambda(G) \) cl dir. - tow a set \( L \subseteq H \), the collection \( \mathcal{E} = \{ \lambda^{-1}(M) : M \in \mathfrak{Z} \} \) is cl dir. - tow \( \lambda^{-1}(L) \).

**Proof:** \( (\Rightarrow) \) Suppose that \( \lambda \) is closed mapping and \( \lambda^{-1}(h) \) compact for every \( h \in H \). Then by Theorem 11 and 12 it suffices to prove that if \( \wp \) is a filter base in \( \lambda(G) \) j-\( \omega \)-closure converging to \( h \in L \), then \( \mathcal{M} = \{ \lambda^{-1}(G) : G \in \wp \} \) is cl dir. - tow \( \lambda^{-1}(h) \). In order to if not, yond is a filter base \( \mathcal{M}^* \) finer than \( \mathcal{M} \), no point of \( \lambda^{-1}(h) \) is a j-\( \omega \)-closure cluster point of \( \mathcal{M}^* \). For all \( g \in \lambda^{-1}(h) \), by supposition yond is an open set \( S_g \) about \( g \) and \( \mathcal{M}_g^* \subseteq \mathcal{M}^* \) with \( \mathcal{M}_g^* \cap S_g \neq \emptyset \). Since \( \lambda^{-1}(h) \) is compact, yond are a finite numbers of open sets \( S_g \) such that \( \lambda^{-1}(h) \subseteq S = \cup S_g \), suppose \( m^* \subseteq \mathcal{M}^* \) such that \( m^* \subseteq \cap m_e^* \) and let \( T = H - \lambda(G - S) \) be the open set. Then \( \lambda(m^*) \cap T = \emptyset \) because of \( m^* \subseteq G - \text{cl}(S) \). So since \( \lambda(m^*) \in \wp^* \), \( \wp^* \) cannot have \( h \) as a closure cluster point.

\( (\Leftarrow) \) Suppose that the hypothesis is true and \( \lambda \) is not closed. Let \( K \subseteq G \) be a closed set and for some \( h \in H - \lambda(K) \) is a closure cluster point of \( \lambda(K) \). Suppose \( \wp \) be a filter base of sets \( \lambda(K) \cap T \) for every open sets \( T \subseteq H \) such that \( h \in T \), then \( \wp \) is a filter base in \( \lambda(G) \) and \( \wp \sim \sim h \). Let \( \mathcal{M} = \{ \lambda^{-1}(G) : G \in \wp \} \) and \( \mathcal{M}^* = \{ K \cap m : m \in \mathcal{M} \} \). It apparent that \( \mathcal{M}^* \ll \mathcal{M} \).

Nevertheless, \( G - K \) is open and \( \lambda^{-1}(h) \subseteq G - K \), \( \mathcal{M}^* \) has no closure cluster point in \( \lambda^{-1}(h) \). The contradiction crops that \( \lambda \) is a closed mapping. Finally, to prove \( \lambda^{-1}(h) \) is compact, this is easy for \( h \in H - \lambda(G) \). And for \( h \in \lambda(G) \), \( \{ h \} \) is a filter base in \( \lambda(G) \) cl dir. - tow \( h \). By supposition, \( \{ \lambda^{-1}(h) \} \) cl dir. - tow \( \lambda^{-1}(h) \). This means that every filter base in \( \lambda^{-1}(h) \) has a closure cluster point in \( \lambda^{-1}(h) \), so that \( \lambda^{-1}(h) \) is compact.

**Corollary 14**

Let \( \lambda : G \to H \) be closed mapping and \( \lambda^{-1}(h) \) compact for every \( h \in H \) if and only if each filter base in \( \lambda(G) \sim h \in H \) has pre-image filter base cl dir. - tow \( \lambda^{-1}(h) \).
Corollary 15
Let \( \lambda : G \rightarrow H \) be closed mapping and \( \lambda^{-1}(h) \) compact for every \( h \in Y \), for every compact set \( W \subseteq H, \lambda^{-1}(W) \) is compact.

Proof. Let \( W \subseteq H \) be a compact set and \( \mathcal{F} \) is a filter base in \( \lambda^{-1}(W) \), \( \mathcal{W} = \{ \lambda(M) : M \in \mathcal{F} \} \), is a filter base in \( W \) and in \( \lambda(G) \) and is cl dir. tow \( W \). So \( \mathcal{F}^* = \{ \lambda^{-1}(G) : G \in \mathcal{W} \} \) is cl dir. tow \( \lambda^{-1}(W) \), so that \( \mathcal{F}^* \prec \mathcal{F} \) and \( \mathcal{F}^* \) has a closure cluster point in \( \lambda^{-1}(W) \).

4. Filter Bases and Almost \( j_{\omega} \)-Convergence
In this section, we defined filter bases, almost \( j_{\omega} \)-closure, and the some theorems about them. We now introduce the definition of almost \( j_{\omega} \)-closure, where \( j \in \{ \theta, \delta, \alpha, \pre, b, \beta \} \).

Definition 16
Let \( \mathcal{F} \) be a filter base on a space \( G \). We say \( \mathcal{F} \) almost \( j_{\omega} \)-converges to a subset \( K \subseteq G \) (written as \( \mathcal{F}_{j_{\omega}} \sim K \)) if for each cover \( \mathcal{K} \) of \( K \) by subsets open in \( G \), there is a finite subfamily \( \mathcal{L} \subseteq \mathcal{K} \) and \( M \in \mathcal{F} \) such that \( M \subseteq \cup \{ \text{cl}(L) : L \in \mathcal{L} \} \). We say \( \mathcal{F} \) almost \( j_{\omega} \)-converges to \( g \in G \) (written as \( \mathcal{F}_{j_{\omega}} \sim g \)) if \( \mathcal{F}_{j_{\omega}} \sim \{ g \} \). Now, \( \text{cl}(\mathcal{N}_g) \sim g \), while, \( j_{\omega} \text{ cl}(\mathcal{N}_g) j_{\omega} \sim g \), where \( j \in \{ \theta, \delta, \alpha, \pre, b, \beta \} \).

Also, we introduce the definitions of almost \( j_{\omega} \)-cluster point, and quasi \( j_{\omega} \)-H-closed set where \( j \in \{ \theta, \delta, \alpha, \pre, b, \beta \} \).

Definition 17
A point \( g \in G \) is called an almost \( j_{\omega} \)-cluster point of a filter base \( \mathcal{F} \) (written as \( g \in (\alpha j_{\omega}c_{(g)}\mathcal{F}) \)) if \( \mathcal{F} \) meets \( \text{cl} j_{\omega}(\mathcal{N}_g) \), where \( j \in \{ \theta, \delta, \alpha, \pre, b, \beta \} \).

For a set \( K \subseteq G \), the almost \( j_{\omega} \)-closure of \( K \), denoted as \( (\alpha j_{\omega}\text{ cl}(K)) \) is \( \alpha j_{\omega}c_{(g)} \{ K \} \) if \( K \neq \phi \) i.e. \( \{ g \in G : \text{every j}_{\omega}-\text{closed nbd of g meets K} \} \) and is \( \phi \) if \( K = \phi \); \( K \) is almost \( j_{\omega} \)-closed if \( K = (\alpha j_{\omega}\text{ cl}(K)) \). Correspondingly, the almost \( j_{\omega} \)-interior of \( K \), denoted as \( (\alpha j_{\omega}\text{-int}(K)) \), is \( \{ g \in G : \text{cl} j_{\omega}(S) \subseteq K \text{ for some open set S containing g} \} \); \( K \) is almost \( j_{\omega} \)-interior if \( K = (\alpha j_{\omega}\text{-int}(K)) \), where \( j \in \{ \theta, \delta, \alpha, \pre, b, \beta \} \).

Theorem 18
Let \( \mathcal{F} \) and \( \varnothing \) be filter bases on a space \( G, K \subseteq G \) and \( g \in G \).

(a) If \( \mathcal{F}_{j_{\omega}} \sim k \), then \( \text{cl} j_{\omega}(\mathcal{N}_k) < \mathcal{F} \).

(b) If \( \mathcal{F}_{j_{\omega}} \sim g \), iff \( \text{cl} j_{\omega}(\mathcal{N}_g) < \mathcal{F} \).

(c) If \( \mathcal{F} < \varnothing \), then \( (\alpha j_{\omega}c_{(g)}\varnothing) < (\alpha j_{\omega}c_{(g)}\mathcal{F}) \).

(d) If \( \mathcal{F} < \varnothing \) and \( \mathcal{F}_{j_{\omega}} \sim K \), then \( \varnothing j_{\omega} < K \).

(e) \( (\alpha j_{\omega}c_{(g)}\mathcal{F}) = \cap \{ \text{cl} j_{\omega}(M) : M \in \mathcal{F} \} \).

(f) If \( \mathcal{F}_{j_{\omega}} \sim g \) and \( g \in K \), then \( \mathcal{F}_{j_{\omega}} \sim K \).

(g) If \( \mathcal{F}_{j_{\omega}} \sim K \) iff \( \mathcal{F}_{j_{\omega}} \sim K \cap (\alpha j_{\omega}c_{(g)}\mathcal{F}) \).

(h) If \( \mathcal{F}_{j_{\omega}} \sim K \), then \( K \cap (\alpha j_{\omega}c_{(g)}\mathcal{F}) \neq \varnothing \).

(i) If \( S \subseteq G \) is open, then \( (\alpha j_{\omega}\text{cl}(S)) = \text{cl}(S) \).

(j) If \( \mathcal{F} \) is an open filter base, then \( (\alpha j_{\omega}\text{cl}\mathcal{F}) = (\alpha j_{\omega}c_{(g)}\mathcal{F}) \).

If \( S \) is an open ultrafilter on \( G \). Then \( S \sim g \) if and only if \( S_{j_{\omega}} \sim g \), where \( j \in \{ \theta, \delta, \alpha, \pre, b, \beta \} \).

Proof: The proof is easy, so it omitted.
Definition 19  
The subset $K$ of a space $G$ is said to be quasi $j$-$\omega$-H-closed relative to $G$ if every cover $\mathcal{K}$ of $K$ by open subsets of $G$ contains a finite subfamily $L \subseteq K$ such that $K \subseteq \cup \{j\text{-}\omega\text{-}(L) : L \in \mathcal{B}\}$. If $G$ is Hausdorff, we say that $K$ is $j$-$\omega$-H-closed relative to $G$. If $G$ is quasi- $j$-$\omega$-H-closed relative to itself, then $G$ is said to be quasi- $j$-$\omega$-H-closed (resp. $j$-$\omega$-H-closed), where $j \in \{\emptyset, \delta, \alpha, \text{pre}, b, \beta\}$.

Theorem 20  
The following are equivalent for a subset $K \subseteq G$:  
(a) $K$ is quasi-$j$-$\omega$-H-closed relative to $G$.  
(b) For every filter base $\mathfrak{F}$ on $K$, $\mathfrak{F}_{j\omega} \leadsto K$.  
(c) For all filter base $\mathfrak{F}$ on $K$, $(al$- $j$-$\omega$-$c_{\mathfrak{F}}) \cap K \neq \emptyset$. Where $j \in \{\emptyset, \delta, \alpha, \text{pre}, b, \beta\}$.  

Proof: Clearly (a) $\Rightarrow$ (b), and by Theorem (18, h), (b) $\Rightarrow$ (c). To show (c) $\Rightarrow$ (a), let $\mathcal{K}$ be a cover of $K$ by open subsets of $G$ such that the $j$-$\omega$-closed of the union of any finite subfamily of $\mathcal{K}$ is not cover $K$. Then $\mathfrak{F} = \{K - \text{cl}_{j\omega}(\cup \mathfrak{F}_{j\omega}) : k \text{ is finite subfamily of } \mathcal{K}\}$ is a filter base on $K$ and $(al$- $j$-$\omega$-$c_{\mathfrak{F}}) \cap K = \emptyset$. This contradiction crop s that $K$ is quasi- $j$-$\omega$-H-closed relative to $G$, where $j \in \{\emptyset, \delta, \alpha, \text{pre}, b, \beta\}$.
By concepts of closure directed toward a set, almost $j$-$\omega$-convergence characterized and related in the next result.

Theorem 21  
Let $\mathfrak{F}$ be a filter base on a space $G$ and $K \subseteq G$. Then:
(a) $\mathfrak{F}$ is cl-$\text{dir}$-$\text{tow}$ $K$ iff for each cover $\mathcal{K}$ of $K$ by open subsets of $G$, there is a finite subfamily $L \subseteq K$ and an $M \in \mathfrak{F}$ such that $M \subseteq \cup \{\text{cl}_{\text{j-}\omega\text{-}(L)} : L \in \mathcal{B}\}$, where $j \in \{\emptyset, \delta, \alpha, \text{pre}, b, \beta\}$.  
(b) For every filter base $\varnothing$, $\mathfrak{F} \prec \varnothing$ implies (al- $j$-$\omega$-$c_{\mathfrak{F}}) \cap K \neq \emptyset$ if $\mathfrak{F}_{j\omega} \leadsto K$, where $j \in \{\emptyset, \delta, \alpha, \text{pre}, b, \beta\}$.  

Proof: The proofs of the two facts are similar; so, we will only prove the fact (b):

($\Rightarrow$) Suppose for every filter base $\varnothing$, $\mathfrak{F} \prec \varnothing$ implies (al- $j$-$\omega$-$c_{\mathfrak{F}}) \cap K \neq \emptyset$. If $\mathfrak{F}_{j\omega} \leadsto g$ for some $g \in K$, then by Theorem (3.3, f), $\mathfrak{F}_{j\omega} \leadsto K$. So, assume that for each $g \in K$, $\mathfrak{F}$ does not $j\omega \leadsto g$. Let $\mathcal{K}$ be a cover of $K$ by subsets open in $G$. For every $g \in K$, yond is an open set $S_{g}$ containing $g$ and $T_{g} \in \mathcal{K}$ such that $S_{g} \subseteq T_{g}$ and $M - \text{cl}_{j\omega}(S_{g}) \neq \emptyset$ for every $M \in \mathfrak{F}$. So, $\varnothing = \{M - \text{cl}_{j\omega}(S_{g}) : M \in \mathfrak{F}\}$ is a filter base on $G$ and $\mathfrak{F} \prec \varnothing$. Now, $g \notin (al$- $j$-$\omega$-$c_{\mathfrak{F}})\varnothing$.

Assume that $\cup(A_{g} : g \in K)$ forms a filter sub base with $\varnothing$ denoting the generated filter. Then $\mathfrak{F} \prec \varnothing$ and (al- $j$-$\omega$-$c_{\mathfrak{F}}) \cap K = \emptyset$. This contradiction implies yond is a finite subset $L \subseteq K$ and $M_{g} \in \mathfrak{F}$ for $g \in L$ such that $\emptyset = \cap \{M_{g} - \text{cl}_{j\omega}(S_{g}) : g \in L\}$. There is $M \in \mathfrak{F}$ such that $M \subseteq \cap \{M_{g} : g \in L\}$. It easily follows that $\emptyset = \cap \{M - \text{cl}_{j\omega}(S_{g}) : g \in L\}$ and $M \subseteq \cup \{\text{cl}_{j\omega}(T_{g}) : g \in L\}$. Thus $\mathfrak{F}_{j\omega} \leadsto K$.  

($\Leftarrow$) Suppose $\mathfrak{F}_{j\omega} \leadsto K$ and $\varnothing$ is a filter base such that $\mathfrak{F} \prec \varnothing$. By Theorem (18, d), $\varnothing_{j\omega} \leadsto K$, and Theorem (18, h), (al- $j$-$\omega$-$c_{\mathfrak{F}}) \cap K \neq \emptyset$.  

170
5. Filter Bases and $j$-$\omega$-Rigidity

In the section, we defined filter bases, $j$-$\omega$-rigidity, and the some theorems concerning of them.

**Definition 22**

A mapping $\lambda : G \rightarrow H$ is said to be $j$-$\omega$-closure continuous (resp. $j$-$\omega$-weakly continuous) if for every $g \in G$ and every nbd $T$ of $\lambda(g)$, there exists a nbd $S$ of $g$ in $G$ such that $\lambda(\text{cl} j$-$\omega$ $(S)) \subseteq \text{cl} j$-$\omega$ $(T)$ (resp. $\lambda(S) \subseteq \text{cl} j$-$\omega$ $(T)$).

Clearly, every continuous mapping is $j$-$\omega$-closure continuous, where $j \in \{ \theta, \delta, \alpha, \text{pre}, b, \beta \}$.

The notions of almost $j$-$\omega$-convergence and almost $j$-$\omega$-cluster can used to characterize $j$-$\omega$-closure continuous.

**Theorem 23**

Let $\lambda : G \rightarrow H$ be a mapping. The following are equivalent:

(a) $\lambda$ is $j$-$\omega$-closure continuous.

(b) For all filter base $\mathfrak{F}$ on $G$, $\mathfrak{F} \rightarrow g$ implies $\lambda (\mathfrak{F}) \rightarrow \lambda (g)$.

For all filter base $\mathfrak{F}$ on $G$, let $\lambda (\text{al} j$-$\omega$ $c \mathfrak{F}) \subseteq (\text{al} j$-$\omega$ $c \lambda (\mathfrak{F}))$. For all open $S \subseteq H$, $\lambda^{-1}(S) \subseteq (\text{al} j$-$\omega$ $\text{int} \lambda^{-1}(\text{al} j$-$\omega$ $\text{cl})(S))$, where $j \in \{ \theta, \delta, \alpha, \text{pre}, b, \beta \}$.

**Proof:** The proof of the equivalence of (a), (b) and (d) is straightforward.

(a) $\Rightarrow$ (c) Suppose $\mathfrak{F}$ is a filter base on $G$, $g \in (\text{al} j$-$\omega$ $c \mathfrak{F})$, $M \in \mathfrak{F}$ and $T$ is a nbd of $\lambda (g)$, yond is a nbd $S$ of $g$ such that $\lambda (\text{cl} j$-$\omega$ $(S)) \subseteq \text{cl} j$-$\omega$ $(T)$. Since $\text{cl} j$-$\omega$ $(S) \cap M \neq \emptyset$, then $\text{cl} j$-$\omega$ $(T) \cap \lambda (M) \neq \emptyset$. So, $\lambda (g) \in (\text{al} j$-$\omega$ $c \lambda (\mathfrak{F}))$. This shows that $\lambda (\text{al} j$-$\omega$ $c \mathfrak{F}) \subseteq (\text{al} j$-$\omega$ $c \lambda (\mathfrak{F}))$.

(c) $\Rightarrow$ (a) Let $S$ be an ultrafilter containing $\lambda (\text{cl} j$-$\omega$ $\mathcal{N}_g)$). Now, $\lambda^{-1}(S)$ is a filter base since $\lambda (G) \in S$ and $\lambda^{-1}(S)$ meets $\text{cl} j$-$\omega$ $\mathcal{N}_g)$. So, $\lambda^{-1}(S) \cup \text{cl} j$-$\omega$ $\mathcal{N}_g)$ contained in some ultrafilter $T$. Now $\lambda \lambda^{-1}(S)$ is an ultrafilter base that generates $S$. Since $\lambda \lambda^{-1}(S) < \lambda (T)$, then $\lambda (T)$ also generates $S$, hence $(\text{al} j$-$\omega$ $c \lambda (T)) = (\text{al} j$-$\omega$ $c S)$. Since $g \in (\text{al} j$-$\omega$ $c (T))$, then $\lambda (g) \in \lambda (\text{al} j$-$\omega$ $c T) \subseteq (\text{al} j$-$\omega$ $c \lambda (T)) = (\text{al} j$-$\omega$ $c S)$. So, $S$ meets $\text{cl} j$-$\omega$ $(\mathcal{N}_{\lambda(g)})$ and $\text{cl} j$-$\omega$ $(\mathcal{N}_{\lambda(g)}) \subseteq \bigcap \{ S : S \text{ ultrafilter}, S \supseteq \lambda (\text{cl} j$-$\omega$ $(\mathcal{N}_g)))$, (denote this intersection by $\varphi$). Nevertheless, $\varphi$ is the filter generated by $(\text{cl} j$-$\omega$ $\mathcal{N}_g)$ (see [4]. Proposition I.6.6), so $\lambda (\text{cl} j$-$\omega$ $(\mathcal{N}_{\lambda(g)}) < \lambda (\text{cl} j$-$\omega$ $(\mathcal{N}_g)$). Hence $\lambda$ is $j$-$\omega$-closure continuous, where $j \in \{ \theta, \delta, \alpha, \text{pre}, b, \beta \}$.

**Corollary 24**

If $\lambda : G \rightarrow H$ is $j$-$\omega$-closure continuous and $K \subseteq G$, then $\lambda (\text{al} j$-$\omega$ $\text{cl}(K)) \subseteq (\text{al} j$-$\omega$ $\text{cl}(\lambda(K)))$, where $j \in \{ \theta, \delta, \alpha, \text{pre}, b, \beta \}$.

Here are some similarly proven facts about $j$-$\omega$-weakly continuous mapping.

**Theorem 25**

Let $\lambda : G \rightarrow H$ be a mapping. The following are equivalent:

(a) $\lambda$ is $j$-$\omega$-weakly continuous.

(b) For all filter base $\mathfrak{F}$ on $G$, $\mathfrak{F} \rightarrow g$ implies $\lambda (\mathfrak{F}) \rightarrow \lambda (g)$.

(c) For all filter base $\mathfrak{F}$ on $G$, $\lambda (\text{al} j$-$\omega$ $c \mathfrak{F}) \subseteq (\text{al} j$-$\omega$ $c \lambda (\mathfrak{F}))$.

(d) For all open $S \subseteq H$, $\lambda^{-1}(S) \subseteq \text{int} \lambda^{-1}(\text{cl} j$-$\omega$ $(S))$. Where $j \in \{ \theta, \delta, \alpha, \text{pre}, b, \beta \}$.
Theorem 26
If \( \lambda : G \to H \) is \( j-\omega \)-weakly continuous mapping, then
(a) For all \( K \subseteq G \), \( \lambda(\text{cl } j-\omega(K)) \subseteq (\text{al- } j-\omega-\text{cl } \lambda(K)) \).
(b) For all \( L \subseteq H \), \( \lambda(\text{cl } j-\omega(\text{int}(\text{cl } j-\omega-\lambda^{-1}(L)))) \subseteq \text{cl } j-\omega(L) \).
(c) For all open \( S \subseteq H \), \( \lambda(\text{cl } j-\omega(S)) \subseteq \text{cl } j-\omega-\lambda(S) \). Where \( j \in \{ \theta, \delta, \alpha, \text{pre}, b, \beta \} \).

Now, We introduce the definitions of \( j-\omega \)-compact, \( j-\omega \)-rigid set, almost \( j-\omega \)-closed, and \( j-\omega \)-urysohn space as follows.

Definition 27
A mapping \( \lambda : G \to H \) is said to be \( j-\omega \)-compact if for every subset \( C \) quasi- \( j-\omega \)-H-closed relative to \( H \), \( \lambda^{-1}(C) \) is quasi- \( j-\omega \)-H-closed relative to \( G \), where \( j \in \{ \theta, \delta, \alpha, \text{pre}, b, \beta \} \).

Definition 28
A subset \( K \) of a space \( G \) is said to be \( j-\omega \)-rigid provided whenever \( \mathfrak{F} \) is a filter base on \( G \) and \( K \cap (\text{al- } j-\omega-c_\mathfrak{F}) = \phi \), there is an open \( S \) containing \( K \) and \( M \in \mathfrak{F} \) such that \( \text{cl } j-\omega(S) \cap M = \phi \), where \( j \in \{ \theta, \delta, \alpha, \text{pre}, b, \beta \} \).

Definition 29
A mapping \( \lambda : G \to H \) is said to be almost \( j-\omega \)-closed if for any set \( K \subseteq G \), \( \lambda(\text{al- } j-\omega-\text{cl}(K)) = (\text{al- } j-\omega-\text{cl } \lambda(K)) \), where \( j \in \{ \theta, \delta, \alpha, \text{pre}, b, \beta \} \).

Definition 30
A space \( G \) is said to be \( j-\omega \)-Urysohn if every pair of distinct points are contained in disjoint \( j-\omega \)-closed nbds, where \( j \in \{ \theta, \delta, \alpha, \text{pre}, b, \beta \} \).

Before characterizing \( j-\omega \)-rigidity, we can show that a \( j-\omega \)-closure continuous, \( j-\omega \)-compact mapping into a \( j-\omega \)-Urysohn space with a certain property (the “\( j-\omega \)-closure” and “quasi- \( j-\omega \)-H-closed relative” analogue of property \( \alpha \) in [15].) is almost \( j-\omega \)-closed.

Theorem 31
Suppose \( \lambda : G \to H \) is a \( j-\omega \)-closure continuous mapping and \( j-\omega \)-compact and \( H \) is \( j-\omega \)-Urysohn with this property: For each \( L \subseteq H \) and \( h \in (\text{al- } j-\omega-\text{cl}(L)) \), there is a subset \( C \) quasi-\( j-\omega \)-H-closed relative to \( H \) such that \( h \in (\text{al- } j-\omega-\text{cl}(C \cap L)) \). Then \( \lambda \) is almost \( j-\omega \)-closed, where \( j \in \{ \theta, \delta, \alpha, \text{pre}, b, \beta \} \).

Proof: Let \( K \subseteq H \). By corollary (24), \( \lambda(\text{al- } j-\omega-\text{cl } (K)) \subseteq (\text{al- } j-\omega-\text{cl } \lambda(K)) \). Suppose \( h \in (\text{al- } j-\omega-\text{cl } \lambda(K)) \). Yond is a subset \( C \) quasi- \( j-\omega \)-H-closed relative to \( H \) such that \( h \in (\text{al- } j-\omega-\text{cl}(C \cap \lambda(K)) \). Then \( \mathfrak{F} = \{ \text{cl } j-\omega(S) \cap C \cap \lambda(K) \mid S \in \mathcal{N}_h \} \), is a filter base on \( H \) such that \( \mathcal{F}_{j-\omega} \hookrightarrow h \). Now, \( \varnothing = \{ K \cap \lambda^{-1}(M) \mid M \in \mathcal{F} \} \) is a filter base on \( K \cap \lambda^{-1}(C) \). Since \( \lambda^{-1}(C) \) is quasi- \( j-\omega \)-H-closed relative to \( H \), then there is \( g \in (\text{al- } j-\omega-c_\mathfrak{F}(\varnothing) \cap \lambda^{-1}(C) \). By theorem 23, \( \lambda(g) \in (\text{al- } j-\omega-\text{cl } \lambda(\varnothing)) \subseteq (\text{al- } j-\omega-c_h \mathcal{F}_h) \). Since \( \mathcal{F}_{j-\omega} \hookrightarrow h \) and \( H \) is \( j-\omega \)-Urysohn, \( (\text{al- } j-\omega-c_h \mathcal{F}_h) = \{ h \} \). So, \( h \in \lambda(\text{al- } j-\omega-\text{cl } (K)) \), where \( j \in \{ \theta, \delta, \alpha, \text{pre}, b, \beta \} \).

Theorem 32
Let \( K \) be a subset of a space \( G \). The following are equivalent:
(a) \( K \) is \( j-\omega \)-rigid in \( G \).
(b) For all filter base $\mathcal{F}$ on $G$, if $K \cap (\text{al- } j\omega-c_{\mathcal{F}}) = \emptyset$, then for some $M \in \mathcal{F}$, $K \cap (\text{al- } j\omega-\text{cl}(M)) = \emptyset$.

(c) For all cover $\mathcal{K}$ of $K$ by open subsets of $G$, there is a finite subfamily $\mathcal{B} \subseteq \mathcal{K}$ such that $K \subseteq \text{int cl } j\omega-(\cup \mathcal{B})$. Where $j \in \{\theta, \delta, \alpha, \text{pre}, b, \beta\}$.

**Proof:** The proof that (a) $\implies$ (b) is straightforward. (b) $\implies$ (c) Let $\mathcal{K}$ be a cover of $K$ by open subsets of $G$ and $\mathcal{F} = \{\bigcap_{g \in \mathcal{B}}(G - \text{cl } j\omega-(S)) : \mathcal{B} \text{ is a finite subset of } \mathcal{K}\}$. If $\mathcal{F}$ is not a filter base, then for some finite subfamily $\mathcal{B} \subseteq \mathcal{K}$, $G \subseteq \cup\{\text{cl } j\omega-(S) : S \in \mathcal{B}\}$; thus, $K \subseteq G \subseteq \text{int cl } j\omega-(\cup \mathcal{B})$ which completes the proof in the case that $\mathcal{F}$ is not a filter base. So, suppose $\mathcal{F}$ is a filter base. Then $K \cap (\text{al- } j\omega-c\mathcal{F}) = \emptyset$ and there is an $M \in \mathcal{F}$ such that $K \cap (\text{al- } j\omega-\text{cl}(M)) = \emptyset$. For each $x \in K$, yond is open $T_g$ of $g$ such that $\text{cl } j\omega-(T_g) \cap M = \emptyset$. Let $T = \cup\{T_g : g \in K\}$. Now, $T \cap M = \emptyset$. Since $M \in \mathcal{F}$, then for some finite subfamily $\mathcal{B} \subseteq \mathcal{K}$, $M = \cap\{G - \text{cl } j\omega-(S) : S \in \mathcal{B}\}$. It follows that $T \subseteq \text{cl } j\omega-(\cup \mathcal{B})$ and hence, $K \subseteq \text{int cl } j\omega-(\cup \mathcal{B})$, where $j \in \{\theta, \delta, \alpha, \text{pre}, b, \beta\}$.

(c) $\implies$ (a) Let $\mathcal{F}$ be a filter base on $G$ such that $K \cap (\text{al- } j\omega-c\mathcal{F}) = \emptyset$. For all $g \in K$ yond is open $T_g$ of $g$ and $M_g \in \mathcal{F}$ such that $\text{cl } j\omega-(T_g) \cap M_g = \emptyset$. Now $\{T_g : g \in K\}$ is a cover of $K$ by open subsets of $G$; so, there is finite subset $L \subseteq K$ such that $K \subseteq \text{int cl } j\omega-(T_g : g \in T))$. Let $S = \text{int cl } j\omega-(\cup\{T_g : g \in L\})$. Yond is $M \in \mathcal{F}$ such that $M \subseteq \cap\{M_g : g \in L\}$. Since $\text{cl } j\omega-(S) = \cup\{\text{cl } j\omega-(T_g) : g \in L\}$, then $\text{cl } j\omega-(S) \cap M = \emptyset$. So $K$ is $j\omega$-rigid in $G$, where $j \in \{\theta, \delta, \alpha, \text{pre}, b, \beta\}$.

6. Filter Bases and $j\omega$-Perfect Mappings

In the section, we defined filter bases, $j\omega$-perfect mappings, and the some theorems about them.

In Corollary 14, we show that a mapping $\lambda : G \to H$ is perfect (i.e. closed and $\lambda^{-1}(y)$ compact for each $h \in H$) iff for all filter base $\mathcal{F}$ on $\lambda(G)$, $\exists h \in H$, implies $\lambda^{-1}(\mathcal{F})$ is (cl-dir-tow) $\lambda^{-1}(y)$ and in Corollary 15, proved that a perfect mapping is compact (i.e. inverse image of compact sets are compact). In view Theorem 21, we say that a mapping $\lambda : G \to H$ is $j\omega$-perfect if for every filter base $\mathcal{F}$ on $\lambda(G)$, $\exists j\omega h \in H$ implies $\lambda^{-1}(\mathcal{F})$ $j\omega \lambda^{-1}(h)$, where $j \in \{\theta, \delta, \alpha, \text{pre}, b, \beta\}$.

**Theorem 33**

Let $\lambda : G \to H$ be a mapping. The following are equivalent:

(a) $\lambda$ is $j\omega$-perfect.

(b) For all filter base $\mathcal{F}$ on $G$, $(\text{al- } j\omega-(c \lambda(\mathcal{F}))) \subseteq (\lambda(\text{al- } j\omega-(c\mathcal{F})))$.

(c) For all filter base $\mathcal{F}$ on $\lambda(G)$, $\lambda^{-1}(L) \subseteq H$, implies $\lambda^{-1}(\mathcal{F})$ $j\omega \lambda^{-1}(L)$. Where $j \in \{\theta, \delta, \alpha, \text{pre}, b, \beta\}$.

**Proof:** (a) $\implies$ (b) Assume $\mathcal{F}$ is a filter base on $G$ and $h \in (\text{al- } j\omega-c \lambda(\mathcal{F}))$. For if not. Assume that $\lambda^{-1}(h) \cap (\text{al- } j\omega-c(\lambda(\mathcal{F}))) = \emptyset$. For each $g \in \lambda^{-1}(h)$, yond is open $S_g$ of $g$ and $M_g \in \mathcal{F}$ such that $\text{cl } j\omega-(S_g) \cap M_g = \emptyset$. Since $\lambda^{-1}(\text{cl } j\omega-(N_h)) \subseteq \lambda^{-1}(y)$ and $\{S_g : g \in \lambda^{-1}(h))\}$ is an open cover of $\lambda^{-1}(y)$, yond is a $V \in N_h$ and a finite subset $B \subseteq \lambda^{-1}(y)$ such that $\lambda^{-1}(\text{cl } j\omega-(T)) \subseteq \cup\{\text{cl } j\omega-(T_g) : g \in L\}$. Yond is an $M \in \mathcal{F}$ such that $M \subseteq \cap\{M_g : g \in L\}$. Thus, $M \cap \lambda^{-1}(\text{cl } j$-
ω(−T) = ϕ implying clj−ω−(T) ∩ λ(M) = ϕ, a contradiction as h ∈ (al− j−ω−c λ(3)). This shows that h ∈ λ(al− j−ω−c 3), Where j ∈ {pre , b , α , β}.

(b) ⇒ (c) Assume 3 is a filter base on λ(G) and 3_j−ω ∋ L ⊆ H. Let ϕ be a filter base on G such that λ_−1(3) < ϕ. Then 3 < λ(ϕ) and (al− j−ω−c λ(ϕ)) ∩ L ≠ ϕ. Therefore λ(al− j−ω−c ϕ) ∩ L ≠ ϕ and (al− j−ω−c ϕ) ∩ λ_−1(L) ≠ ϕ. By Theorem (3.6, b), λ_−1(3) j−ω ∋ λ_−1(L), where j ∈ {θ, δ, α, pre, b, β}.

(c) ⇒ (a) Clearly.

**Corollary 34**

If λ : G → H is j-ω-perfect mapping, then:

(a) For all K ⊆ G, (al− j−ω−cl λ(K)) ≤ λ(al− j−ω−cl (K)).

(b) For all almost j-ω-closed K ⊆ G, λ(K) is almost j-ω-closed.

(c) λ is j-ω-compact. Where j ∈ {θ, δ, α, pre, b, β}.

**Proof:** (a) is an immediate consequence of Theorem 33, and (b) follows easily from (a). To prove (c) Let C be quasi- j-ω-H-closed relative to H, and ϕ be a filter base on λ_−1(C), then λ(ϕ) is a filter base on C. By Theorem 20, (al− j−ω−c λ(ϕ)) ∩ C ≠ ϕ and by Theorem (33, b), (al− j−ω−c ϕ) ∩ λ_−1(C) ≠ ϕ. By Theorem 20, λ_−1(C) is quasi- j-ω-H-closed relative to G, where j ∈ {θ, δ, α, pre, b, β}.

**Theorem 35**

An j-ω-closure continuous mapping λ : G → H is j-ω-perfect if and only if

(a) λ is almost j-ω-closed, and

(b) λ_−1(γ) j-ω-rigid for each h ∈ H, where j ∈ {θ, δ, α, pre, b, β}.

**Proof:** (⇒) If λ is j-ω-closure continuous and j-ω-perfect mapping, then by Corollaries 34 and 24, λ is almost j-ω-closed. To show λ_−1(h), for h ∈ H, is j-ω-rigid, Let 3 be a filter base on G such that λ_−1(h) ∩ (al− j−ω−c 3) = ϕ. So, h ∈ λ(al− j−ω−c 3) and by Theorem (33, b), h ∉ (al− j−ω−c λ(3)). Yond is open S of h and M ∈ 3 such that cl j−ω−(S) ∩ λ(M) = ϕ. So, λ_−1(cl j−ω−(S)) ∩ M = ϕ. Since λ is j-ω-closure continuous, then for any g ∈ λ_−1(h), yond is open T of g such that cl j−ω−(T) ⊆ λ_−1(cl j−ω−(S)). So, λ_−1(h) ∩ cl j−ω−(M) = ϕ, where j ∈ {θ, δ, α, pre, b, β}.

(⇐) Assume that j-ω-closure continuous mapping λ satisfies (a) and (b). Let 3 be a filter base on λ(G) such that 3_j−ω ∋ h. Let ϕ be a filter base on G such that λ_−1(3) < ϕ. So, 3 < λ(ϕ) implying that h ∈ (al− j−ω−c λ(ϕ)). Therefore, for each G ∈ ϕ, h ∈ (al− j−ω−cl λ(ϕ)) ⊆ λ(al− j−ω−cl G). Hence, λ_−1(h) ∩ (al− j−ω−cl ϕ) ≠ ϕ for each G ∈ ϕ. By (b), λ_−1(h) ∩ (al− j−ω−c ϕ) ≠ ϕ. By Theorem 33, λ is j-ω-perfect mapping, where j ∈ {θ, δ, α, pre, b, β}.

Actually, in the proof of the converse of Theorem 35, we have shown that property (a) of Theorem 35 can reduced to this statement: For each K ⊆ G, al j−ω−cl λ(K) ≤ λ(al j−ω−cl K); in fact, we have shown the next corollary (the mapping is not necessarily j-ω-closure continuous).

**Corollary 36**

Let λ : G → H be a mapping if

(a) For all K ⊆ G, (al− j−ω−cl λ(K)) ⊆ λ(al− j−ω−cl K)
(b) $\lambda^{-1}(h)$ $j$-$\omega$-rigid for each $h \in H$, then $\lambda$ is $j$-$\omega$-perfect, where $j \in \{\theta, \delta, \alpha, \text{pre, } b, \beta\}$.

**Corollary 37**

Let $\lambda: G \to H$ be a mapping.

(a) $\lambda$ is almost $j$-$\omega$ closed

(b) $\lambda^{-1}(h)$ $j$-$\omega$ rigid for each $h \in H$, then $\lambda^{-1}$ preserves $j$-$\omega$ rigidity, where $j \in \{\theta, \delta, \alpha, \text{pre, } b, \beta\}$.

**Proof.** Let $C \subseteq H$ be $j$-$\omega$ rigid and $\mathcal{F}$ be a filter base on $G$ such that all $j$-$\omega$ $c_\mathcal{F} \mathcal{F} \cap \lambda^{-1}(C) = \emptyset$.

By Corollary 36 and Theorem 33, $(\text{al-} j$-$\omega \text{ cl}(\mathcal{F})) \cap C = \emptyset$. So, there is $M \in \mathcal{F}$ such that $(\text{al-} j$-$\omega \text{ cl}(M)) \cap C = \emptyset$. Nevertheless $(\text{al-} j$-$\omega \text{ cl}(M)) = \lambda(\text{al-} j$-$\omega \text{ cl}(M))$. So, $(\text{al-} j$-$\omega \text{ cl}(M)) \cap \lambda^{-1}(C) = \emptyset$. So, by Theorem 32, $\lambda^{-1}(C)$ is $j$-$\omega$ rigid, where $j \in \{\theta, \delta, \alpha, \text{pre, } b, \beta\}$.

**Theorem 38**

Suppose $\lambda: G \to H$ has $j$-$\omega$ rigid point-inverses. Then:

(a) $\lambda$ is $j$-$\omega$ closure continuous iff for each $h \in H$ and open set $T$ containing $h$, there is an open set $S$ containing $\lambda^{-1}(h)$ such that $\lambda(\text{cl} j$-$\omega (S)) \subseteq \text{cl} j$-$\omega (T)$, where $j \in \{\theta, \delta, \alpha, \text{pre, } b, \beta\}$.

(b) If for each $h \in G$ and open set $S$ containing $\lambda^{-1}(h)$, there is an open set $T$ of $h$ such that $\lambda^{-1}(\text{cl} j$-$\omega (T)) \subseteq \text{cl} j$-$\omega (S)$, then for each $K \subseteq G$, $(\text{al-} j$-$\omega \text{ cl}(\lambda(K)) \subseteq \lambda(\text{al-} j$-$\omega \text{ cl}(K))$, where $j \in \{\theta, \delta, \alpha, \text{pre, } b, \beta\}$.

**Proof.** (a) ($\Rightarrow$) Is obvious.

($\Leftarrow$) Is straightforward using Theorem (32, c) (b) Let $\phi \neq K \subseteq G$ and $h \in \lambda(\text{al-} j$-$\omega \text{ cl}(K))$. Then $\lambda^{-1}(h) \cap (\text{al-} j$-$\omega \text{ cl}(K)) = \emptyset$. Now, $\mathcal{F} = \{K\}$ is a filter base and $(\text{al-} j$-$\omega \text{ cl}(\mathcal{F})) \cap \lambda^{-1}(h) = \emptyset$. So, yond is open set $S$ containing $\lambda^{-1}(h)$ such that $\text{cl} j$-$\omega (S) \cap K = \emptyset$, yond is open $T$ of $h$ such that $\lambda^{-1}(\text{cl} j$-$\omega (T)) \subseteq \text{cl} j$-$\omega (S)$. Therefore, $\text{cl} j$-$\omega (T) \cap \lambda(K) = \emptyset$. Hence $h \in (\text{al-} j$-$\omega \text{ cl}(\lambda(K))$, where $j \in \{\theta, \delta, \alpha, \text{pre, } b, \beta\}$.

The next result related to Theorem (38, b); the proof is straightforward.

**Theorem 39**

Let $\lambda: G \to H$. The following are equivalent:

(a) For all $j$-$\omega$-closed $K \subseteq G$, $\lambda(K)$ is $j$-$\omega$-closed, where $j \in \{\theta, \delta, \alpha, \text{pre, } b, \beta\}$.

(b) For all $L \subseteq H$ and $j$-$\omega$ open $S$ containing $\lambda^{-1}(L)$, there is $j$-$\omega$-open $T$ containing $L$ such that $\lambda^{-1}(T) \subseteq S$, where $j \in \{\theta, \delta, \alpha, \text{pre, } b, \beta\}$.

**Theorem 40**

If $\lambda: G \to H$ is $j$-$\omega$ closure continuous and $H$ is $j$-$\omega$ Urysohn, then $\lambda$ is $j$-$\omega$ perfect if and only if for all filter base $\mathcal{F}$ on $G$, if $\lambda(\mathcal{F})$ $j$-$\omega$ $\rightsquigarrow h \in H$, then $(\text{al-} j$-$\omega \text{ c}_\mathcal{F} \mathcal{F}) \neq \emptyset$, where $j \in \{\theta, \delta, \alpha, \text{pre, } b, \beta\}$.

**Proof.** ($\Rightarrow$) Assume that $\lambda$ is $j$-$\omega$ perfect and $\lambda(\mathcal{F})$ $j$-$\omega$ $\rightsquigarrow h$. Therefore, $\lambda^{-1}(\mathcal{F})$ $j$-$\omega$ $\rightsquigarrow \lambda^{-1}(h)$.

Since $\lambda^{-1}(\mathcal{F}) < \mathcal{F}$, then by Theorem (18, d), $\mathcal{F} j$-$\omega$ $\rightsquigarrow \lambda^{-1}(h)$, by Theorem (18, h), $(\text{al-} j$-$\omega \text{ c}_\mathcal{F} \mathcal{F}) \neq \emptyset$.

($\Leftarrow$) Assume that for each filter base $\mathcal{F}$ on $G$, if $\lambda(\mathcal{F})$ $j$-$\omega$ $\rightsquigarrow h \in G$, then $(\text{al-} j$-$\omega \text{ c}_\mathcal{F} \mathcal{F}) \neq \emptyset$.

Suppose $\mathcal{F}$ is a filter base on $\lambda(G)$ such that $\mathcal{F} j$-$\omega$ $\rightsquigarrow h \in H$, and assume $\mathcal{L}$ is a filter base on $G$ such that $\lambda^{-1}(\mathcal{F}) < L$. Then $\mathcal{F} = \lambda \lambda^{-1}(G) < \lambda(L)$. So, $\lambda(L)$ $j$-$\omega$ $\rightsquigarrow h$. Therefore, $(\text{al-} j$-$\omega \text{c}_\mathcal{F}$
(L) ≠ φ. Let i ∈ H – {h}. Because of H j-ω-Urysohn, are open sets S_i of i and S_h of h such that cl j-ω-(S_i) ∩ cl j-ω-(S_h) = φ. Yond is H ∈ L such that λ(H) ⊆ cl j-ω- (S_h). For every g ∈ λ^−1(i), there is open T_i of i such that λ (cl j-ω- (T_i)) ∋ cl j-ω- (S_i). So, cl j-ω- (T_g) ∩ H = φ. It follows that λ^−1(i) ∩ (al- j-ω-c_λ L) = φ for each i ∈ H – {h}. So, (al- j-ω-c_λ L) ∩ λ^−1(h) ≠ φ and λ is j-ω-perfect, where j∈{θ, δ, α, pre, b, β}.

**Corollary 41**

If λ : G → H be a mapping is j-ω-closure continuous, G is quasi- j-ω-H-closed, and H is j-ω-Urysohn, then λ is j-ω-perfect, where j∈{θ, δ, α, pre, b, β}.

**Proof.** Since G is quasi- j-ω-H-closed, then all filter base on G has non void almost j-ω-cluster; now, the corollary follows directly from Theorem 35, Where j∈{θ, δ, α, pre, b, β}.

7. **Conclusions**

The starting point for the application of abstract topological structures in j-ω-mapping is presented in this paper. We use filter base to introduce a new notion namely filter base and j-ω-perfect mapping. Finally, certain theorems and generalization concerning these concepts of studied; j∈{θ, δ, α, pre, b, β}.

**References**

