Estimate the Parameters and Related Probability Functions for Data of the Patients of Lymph Glands Cancer via Birnbaum-Saunders Model

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Abstract

In this paper,we estimate the parameters and related probability functions, survival function, cumulative distribution function , hazard function(failure rate) and failure (death) probability function(pdf) for two parameters Birnbaum-Saunders distribution which is fitting the complete data for the patients of lymph glands cancer. Estimating the parameters (shape and scale) using (maximum likelihood , regression quantile and shrinkage) methods and then compute the value of mentioned related probability functions depending on sample from real data which describe the duration of survivor for patients who suffer from the lymph glands cancer based on diagnosis of disease or the inter of patients in a hospital for period of three years (start with 2010 to the end of 2012) .Calculating and estimating all previous probability functions , then comparing the numerical estimation by using statistical indicators mean squares error and mean absolute percentage error between the three considered estimation methods with respect to survival function. Concluding that, the survival function for the lymph glands cancer by using shrinkage method is the best.

Keywords : Birnbaum-Saunders Distribution , Lymph Glands Cancer Disease, Complete Real Data, Maximum Likelihood Estimator, Regression Quantile(Least Square) Method and Shrinkage Estimator, Mean Squares Error and Mean Absolute Percentage Error.

Introduction

1.1 The Procedures

The medical experiment is most important experiment which is related with human, thus the cancer disease is one of the diseases which hit the health of human and the lymph glands cancer disease is lethal cancer which killed human .This paper concerns with complete real data for failure (death) time for this disease which is fitting two parameters Birnbaum-Saunders model. Furthermore, estimating the parameters of the mentioned model, by using three methods (maximum likelihood estimator, regression quantile estimator and shrinkage estimator),depending upon iterative numerical method (Newton-Raphson method),then utilizing these estimated parameters to estimate the probability of failure(death) function , cumulative distribution function, survival function and hazard function .Finally, three considered estimators were compared, by using the mean squares error and mean absolute percentage error with respect to survival function to indicate the best estimator.

1.2 Description of Data

Cancer develops when cells in a part of the body begin to grow out of control. Survival analysis is concerned with studying the time between entry to a study and a subsequent event, such as death.

Lymphoma is a cancer that starts in the lymphoid cells in the immune system, and serves as a solid tumor of lymphoid cells. It can be treated with chemotherapy, and radiation therapy in some cases and / or bone marrow transplantation, and can be cured, depending on the tissue type, and stage of the disease. These malignant cells arise often in the lymph nodes, as a display of the node enlargement (tumor). Lymphoma is the most common form of hematologic malignancies, or "blood cancer", in the developed world. And classified some forms of cancer and lymph nodes (such as lymphoma, small lymphocytic) lazy, compatible with long life even without treatment, while other forms are aggressive (such as Burkitt's lymphoma), causing rapid deterioration and death. However, the most aggressive lymphoma responds well to treatment and cure. Speculate therefore depends on the correct classification of the disease, which was established by a pathologist after examination of a biopsy.

This paper depends on real data for the Lymph Glands cancer diseases, choosing this type of cancer because it is diffusion and deadly in current time in Iraq .To collect data for the Lymph Glands Cancer diseases returning Medical City Teaching Complex in Baghdad-Iraq (Baghdad hospital Teaching) ,Baghdad. The time (in days) of study point in this paper is determined for three years (started with2010 to the end of 2012), that means of the duration time of this study is constant. The number of patients in the experiment for the above duration time is (92) including (51) males and (41) females. All ninety two patients were dead during the time of study, that means the data became complete data.

1.3 Goodness of Fit

It is very important that we test whether the random variable T (the real time data of considered lymph glands cancer diseases) follows the Birnbaum –Saunders distribution. We use the software program (Easy Fit Professional) to fit the curve of demonstrating the good matching, of this data to the specific probability distribution. This program uses a variety of tests under consideration like Kolmukrove Samirnov test, Anderson-Darling test and Chi-Square test. We found from this program, the mentioned real time data follow Birnbaum-Saunders distribution.

1.4 The Model

Birnbaum and Saunders [11] proposed the two-parameter failure time distribution for fatigue failure caused under cyclic loading. It was also assumed that the failure is due to the

development and growth of a dominant crack. This distribution is the so-called two-parameters Birnbaum–Saunders distribution. A more general derivation was provided by Desmond [5] based on a biological model. Desmond also strengthened the physical justification for the use of this distribution by relaxing the assumptions made by Birnbaum and Saunders[11] . Desmond [6] investigated the relationship between the Birnbaum–Saunders distribution and the inverse Gaussian distribution. Artur and Silvia [4] introduce size and power properties of some tests in the Birnbaum-Saunders regression model.
The random variable T is said to follow a BS distribution with parameters α and β, denoted as BS (*α, β),* if its cumulative distribution function (CDF) is given by

$$F(t,\alpha,\beta)=\Phi\left(\frac{1}{\alpha}\left[\left(\frac{t}{\beta}\right)^{\frac{1}{2}}-\left(\frac{\beta}{t}\right)^{\frac{1}{2}}\right]\right) \quad , \quad t>0 \text{ and } \alpha,\beta>0 \quad \dots(1)$$

Where Φ (.) is the standard normal CDF, α and β are the shape and the scale parameters respectively. The parameter β is the median of the distribution: $FT(\beta)=\Phi(0)=0.5$

It is noteworthy that the reciprocal property holds for the BS distribution $T^{-1}\sim BS(\alpha,\beta^{-1})$. While, the probability density function (p.d.f) for the two parameters Birnbaum – Saunders distribution of the random variable T is given by:

$$f(t;\alpha,\beta)=\frac{t^{\frac{-3}{2}}(t+\beta)\exp\left[-\frac{1}{2\alpha^2}\left(\frac{t}{\beta}+\frac{\beta}{t}-2\right)\right]}{2\alpha\sqrt{\beta}\sqrt{2\pi}} \quad ,0<t<\infty \text{ and } \beta,\alpha>0 \quad \dots(2)$$

Estimation Methods:

In this section, we introduce three methods for the estimation of the parameters of the Birnbaum-Saunders distribution and the related probability functions as follows:

2.1 Maximum Likelihood Method(MLE)

The idea behind the maximum likelihood approach to fitting a statistical distribution to a data set is to find the parameters of the distribution that maximize the likelihood of having observed the data. Assuming the data are independent of each other, the likelihood of the data is the product of the likelihoods of each datum. See; [1]*and*[11]
Thus, the likelihood function of two-parameters Birnbaum-Saunders is:

$$L=\prod_{i=1}^{n} f(t_i,\alpha,\beta) \quad \dots(3)$$

$$L=\frac{\prod_{i=1}^{n} t^{\frac{-3}{2}}(t+\beta)}{2^n\alpha^n\beta^{\frac{n}{2}}(2\pi)^{\frac{n}{2}}}\exp-\left(\frac{1}{2\alpha^2}\sum_{i=1}^{n}\left(\frac{ti}{\beta}-2+\frac{\beta}{t_i}\right)\right) \quad \dots(4)$$

Taking the logarithm for the above likelihood function, so we get the following:

$$\operatorname{Ln} L=-n\operatorname{Ln}2-n\operatorname{Ln}\alpha-\frac{n}{2}\operatorname{Ln}\beta-\frac{n}{2}\operatorname{Ln}(2\pi)-\frac{3}{2}\operatorname{Ln}\sum_{i=0}^{n}t_i+\operatorname{Ln}\sum_{i=0}^{n}(t_i+\beta)$$
$$-\left(\frac{1}{\alpha^2}\sum_{i=0}^{n}\left(\frac{t_i}{\beta}\right)+\left(\frac{\beta}{t_i}\right)-2\right) \quad \dots(5)$$

The partial derivative for the log- likelihood function with respect to unknown parameters α and β are respectively as below:

$$\frac{\partial \text{Ln L}}{\partial \alpha} = \frac{-n}{\alpha} + \frac{1}{\alpha^3}\sum_{i=1}^{n}(\frac{t_i}{\beta} - 2 + \frac{\beta}{t_i}) \quad \dots (6)$$

Equating the partial derivative to zero and resolve this equation:

$$\frac{\partial \text{Ln L}(t_i)}{\partial \alpha} = 0 \quad \dots (7)$$

$$\hat{\alpha} = \left(\frac{s}{\hat{\beta}} + \frac{\hat{\beta}}{r} - 2\right)^{\frac{1}{2}} \quad \dots (8)$$

Where, $s=\left\{\frac{1}{n}\sum_{i=0}^{n} t_i\right\}$ is the arithmetic mean and $r=\left\{\frac{1}{n}\sum_{i=0}^{n}\frac{1}{t_i}\right\}$ is the harmonic mean.

Also, the partial derivative for log-likelihood w.r.t. β, is as follows:

$$\frac{\partial \text{Ln L}}{\partial \beta} = \frac{-n}{2\hat{\beta}} + \sum_{i=0}^{n}\frac{1}{t_i + \hat{\beta}} - \frac{1}{2\hat{\alpha}^2}\sum_{i=0}^{n}(\frac{1}{\hat{\beta}^2} - \frac{1}{t_i}) \quad \dots (9)$$

Equating the partial derivative to zero and resolve this equation:

$$\frac{\partial \text{Ln L}}{\partial \beta} = 0 \quad \dots (10)$$

$$\frac{-n}{2\hat{\beta}} + \sum_{i=0}^{n}\frac{1}{t_i + \hat{\beta}} - \frac{1}{2\hat{\alpha}^2}\sum_{i=0}^{n}(\frac{1}{\hat{\beta}^2} - \frac{1}{t_i}) = 0 \quad \dots (11)$$

Form equations (8) and (11) we can write the formula as:

$$\hat{\beta}^2 - \hat{\beta}\left[2r + k(\hat{\beta})\right] + r\left[s + k(\hat{\beta}\,)\,\right] = 0 \quad \dots (12)$$

Where, K $(\hat{\beta}) = \left\{\frac{1}{n}\sum_{i=0}^{n}\frac{1}{t_i+\hat{\beta}}\right\}^{-1}$; $\beta \geq 0$

Since (12) is a non-linear equation in $\hat{\beta}$, we shall use the Newton-Raphson method to find $\hat{\beta}$. As a consequence, the related estimation probability function using this method is as follows:

$$\hat{f}_{ML}(t) = \frac{t^{-\frac{3}{2}}(t + \hat{\beta}_{ML})\exp\left(\frac{-1}{2\hat{\alpha}_{ML}}\left(\frac{\hat{\beta}_{ML}}{t} + \frac{t}{\hat{\beta}_{ML}} - 2\right)\right)}{2\hat{\alpha}_{ML}\sqrt{\hat{\beta}_{ML}}\sqrt{2\pi}}, \qquad \hat{h}_{ML}(t) = \frac{\hat{f}_{ML}(t)}{\hat{S}_{ML}(t)}$$

$$\hat{F}_{ML}(t) = \Phi\left(\frac{1}{\hat{\alpha}_{ML}}\left[\left(\frac{t}{\hat{\beta}_{ML}}\right)^{\frac{1}{2}} - \left(\frac{\hat{\beta}_{ML}}{t}\right)^{\frac{1}{2}}\right]\right),$$

$$\hat{S}_{ML}(t) = 1 - \Phi\left(\frac{1}{\hat{\alpha}_{ML}}\left[\left(\frac{t}{\hat{\beta}_{ML}}\right)^{\frac{1}{2}} - \left(\frac{\hat{\beta}_{ML}}{t}\right)^{\frac{1}{2}}\right]\right)$$

2.2 Regression-Quantile (Least Square) Method (RQE):

The regression-quantile (least square) method [8], is based on the minimization of the quadratic measure of the difference between the empirical distribution function $F_n(t)$ and the theoretical cumulative distribution function

$$F(t) = \Phi(\frac{\lambda}{\sqrt{t}} - \mu\sqrt{t}\,) \quad \dots (13\ a)$$

Where, $\alpha=\frac{1}{\sqrt{\mu\lambda}}$, and, $\beta=\frac{\lambda}{\mu}$ and $\lambda, \mu > 0$ …(13 b)

If $T_{(1)} \leq T_{(2)} \leq K \leq T_{(n)}$ are order statistics of T_1, T_2, K, T_n, then by definition the empirical distribution function is given by $F_n(t_k) = k/n$,k=1, K, n.

Considering the following asymptotic equality

$\Phi^{-1}\left(1-\frac{k}{n}\right) \approx \frac{\lambda}{\sqrt{t_k}} - \mu\sqrt{t_k}$ k=1,…, n-1.

This can be used for the parameter estimation. Hence, the estimation of the parameters can be obtained by finding the minimum of the following function:

$$\mathbf{G}(\lambda,\mu) = \sum_{k=1}^{n}\left(\frac{\lambda}{\sqrt{t_k}} - \mu\sqrt{t_k} - y_k\right)^2 \qquad \dots(14)$$

Where $y_k = \Phi^{-1}\left(1-\frac{k}{n}\right)$ ...(13) ,for $k = 1, \dots, n-1$.

Since $\Phi^{-1}(0) = -\infty$, t_n is chosen by condition of further minimization of the function G.
The partial derivative for G w .r. t. to λ and μ respectively and equating the two equations with zero , we obtain the following:

$$\frac{\partial G}{\partial \mu} = 2\sum\left(\frac{\lambda}{\sqrt{t_k}} - \mu\sqrt{t_k} - y_k\right)(\sqrt{t_k}) \qquad \dots.(15)$$

$$\frac{\partial G}{\partial \mu} = 0 \qquad \dots.(16)$$

$$\Rightarrow 2\sum\left(\frac{\lambda}{\sqrt{t_k}} - \mu\sqrt{t_k} - y_k\right)(\sqrt{t_k}) = 0 \qquad \dots(17)$$

$$\frac{\partial G}{\partial \hat{\lambda}} = 2\sum\left(\frac{\hat{\lambda}}{\sqrt{t_{(k)}}} - \hat{\mu}\sqrt{t_{(k)}} - y_k\right)\left(\frac{1}{\sqrt{t_{(k)}}}\right) \qquad \dots(18)$$

$$\frac{\partial G}{\partial \hat{\lambda}} = 0 \qquad \dots(19)$$

$$\Rightarrow 2\sum\left(\frac{\hat{\lambda}}{\sqrt{t_{(k)}}} - \hat{\mu}\sqrt{t_{(k)}} - y_i\right)\left(\frac{1}{\sqrt{t_{(k)}}}\right) = 0 \qquad \dots(20)$$

Rewriting the statistics T_1 and T_2 are as follows :

$$T_1 = \frac{1}{n}\sum_{k=1}^{n} t_{(k)} \quad , \qquad T_2 = \frac{1}{n}\sum_{k=1}^{n}\frac{1}{t_{(k)}}$$

Furthermore,

$$T_3 = \sum_{K=1}^{n} y_{(k)}\sqrt{t_k} \quad , \qquad T_4 = \frac{1}{n}\sum_{k=1}^{n}\frac{y_{(k)}}{\sqrt{t_{(k)}}}$$

From equations (17) and (20) we get

$$\hat{\mu} = \frac{T_2 T_3 - T_4}{1 - T_1 T_2} \qquad \dots(21)$$

$$\hat{\lambda} = \frac{T_3 - T_1 T_4}{1 - T_1 T_2} \qquad \dots(22)$$

Thus, depending on equation (13 b),we can find the estimation of α and β as below:

$\hat{\alpha} = \frac{1}{\sqrt{\hat{\lambda}\hat{\mu}}}$ and, $\hat{\beta} = \frac{\hat{\lambda}}{\hat{\mu}}$.

As a result, the related estimation probability function using this method is as follows:

$$\hat{f}_{RQ}(t) = \frac{t^{-\frac{3}{2}}(t+\hat{\beta}_{RQ})\exp\left(\frac{-1}{2\hat{\alpha}_{RQ}}\left(\frac{\hat{\beta}_{RQ}}{t}+\frac{t}{\hat{\beta}_{RQ}}-2\right)\right)}{2\hat{\alpha}_{RQ}\sqrt{\hat{\beta}_{RQ}}\sqrt{2\pi}} \quad , \quad \hat{h}_{RQ}(t) = \frac{\hat{f}_{RQ}(t)}{\hat{S}_{RQ}(t)}$$

$$\hat{F}_{RQ}(t) = \Phi\left(\frac{1}{\hat{\alpha}_{RQ}}\left[\left(\frac{t}{\hat{\beta}_{RQ}}\right)^{\frac{1}{2}} - \left(\frac{\hat{\beta}_{RQ}}{t}\right)^{\frac{1}{2}}\right]\right) \quad ; \quad \hat{S}_{RQ}(t)$$

$$= 1 - \Phi\left(\frac{1}{\hat{\alpha}_{RQ}}\left[\left(\frac{t}{\hat{\beta}_{RQ}}\right)^{\frac{1}{2}} - \left(\frac{\hat{\beta}_{RQ}}{t}\right)^{\frac{1}{2}}\right]\right)$$

2.3 Shrinkage Method (SHE)

The shrinkage estimation method is one of the Bayesian approach depended on prior information regarding the value of the specific parameter θ due to past experiences or from previous studies .However, in certain situations the prior information is available only in the form of an initial guess value (natural origin) $\theta_\circ$ of . In such a situation it is natural to start with an estimator $\hat{\theta}$ (e.g. MLE) of θ and modify it by moving it closer to $\theta_\circ$.Thompson [9] suggested the problem of shrinking an unbiased estimator $\hat{\theta}$ of the parameter θ toward prior information (a natural origin) $\theta_\circ$ by single stage shrinkage estimator $k\hat{\theta} + (1-k)\theta_\circ$, $0 \le k \le 1$, which is more efficient than $\hat{\theta}$ if $\theta_\circ$ is close to θ and less efficient than $\hat{\theta}$ otherwise.

According to Thompson [9] and AL − joboori[2,3], $\theta_\circ$ is a natural origin and as such may arise for anyone of a number of reasons, e.g., we are estimating θ and:
(i) We believe $\theta_\circ$ is closed to the true value of θ, or(ii) We fear that $\theta_\circ$ may be near the true value of θ ,i.e., something bad happens if $\theta_\circ = \theta$ and we do not know about it.(i.e.; something bad happens if $\theta_\circ \approx \theta$ and we doesn't use $\theta_\circ$).
Where, k is so called shrinkage weight factor; $0 \le k \le 1$ which represents the belief of $\hat{\theta}$,and (1-k) represents the belief of $\theta_\circ$. Thompson [9].
Noting that the shrinkage weight factor may be a function of $\hat{\theta}$ or may be constant and the choosing of k is ad hoc basis.

In this paper, we supposed $K = e^{\frac{-n}{10}}$, $0 \le k \le 1$, and $\theta = \theta_\circ$.
Where, θ may refer to α or β and n=92
Therefore, the shrinkage estimators of α and β respectively became as below:

$$\hat{\alpha}_{sh} = k\,\hat{\alpha}_{ML} + (1-K)\alpha_\circ \qquad ,\alpha = \alpha_\circ \qquad \text{....}(23)$$
$$\hat{\beta}_{sh} = K\,\hat{\beta}_{ML} + (1-K)\beta_\circ \qquad ,\beta = \beta_\circ \qquad \text{....}(24)$$

Hence, the estimation of the related probability functions using this method is as follows:

$$\hat{f}_{SH}(t) = \frac{t^{-\frac{3}{2}}(t+\hat{\beta}_{SH})\exp\left(\frac{-1}{2\hat{\alpha}_{SH}}\left(\frac{\hat{\beta}_{SH}}{t}+\frac{t}{\hat{\beta}_{SH}}-2\right)\right)}{2\hat{\alpha}_{SH}\sqrt{\hat{\beta}_{SH}}\sqrt{2\pi}} \quad , \quad \hat{h}_{SH}(t) = \frac{\hat{f}_{SH}(t)}{\hat{S}_{SH}(t)}$$

$$\hat{F}_{SH}(t) = \Phi\left(\frac{1}{\hat{\alpha}_{SH}}\left[\left(\frac{t}{\hat{\beta}_{SH}}\right)^{\frac{1}{2}} - \left(\frac{\hat{\beta}_{SH}}{t}\right)^{\frac{1}{2}}\right]\right) \quad , \quad \hat{S}_{SH}(t) = 1 - \Phi\left(\frac{1}{\hat{\alpha}_{SH}}\left[\left(\frac{t}{\hat{\beta}_{SH}}\right)^{\frac{1}{2}} - \left(\frac{\hat{\beta}_{SH}}{t}\right)^{\frac{1}{2}}\right]\right)$$

The Result of Estimation

In this section, we use the program MATLAB to obtain the estimation values for the shape and scale parameters as well as the related probability functions in Birnbaum-Saunders distribution under specific complete data. Newton -Raphson method [7] was used which is one of the numerical analysis methods.

3.1 Estimation the Parameters and Related Pr. Functions by MLE Method

To find the estimated values for the shape and scale parameters in Birnbaum-Saunders distribution, Newton -Raphson method was used which is one of the numerical analysis methods . Applying the maximum likelihood estimator method for estimating the two unknown parameters of Birnbaum−Saunders distribution under complete data by using MATLAB (2012a) program.

Estimating these unknown parameters using Newton-Raphson gave the program including small number of iteration and the smallest value of error. The results of estimation which satisfy above conditions are demonstrated in annexed table (1).

Now, these estimated values for two parameters in Birnbaum-Saunders distribution are used to find estimation values for probability, of death density function$\hat{f}_{ML}(t)$, cumulative distribution function $\hat{F}_{ML}(t)$, survival function$\hat{S}_{ML}(t)$ and hazard function $\hat{h}_{ML}(t)$.

The results for the estimation of these probability functions are shown in the table (2).

From the mentioned table, we display the following conclusions:

1. Noting that the values of death density function $\hat{f}_{ML}(t)$ were increasing slightly until (t= 23) then the values became decreasing slightly until the end of failure times. The patients who remain in a hospital for (t=1046) days had smallest probability of death with (0.00173947) value while the patients who remain in a hospital for (t=23) days had the largest probability of death with (0.571728762) value.

2. The values of cumulative death distribution function $\hat{F}_{ML}(t)$ is increasing with the increase of failure times because it collects the probability values for all previous observations step by step, that means there is a direct relationship between failure times and cumulative death distribution function.

3. The values of survival function $\hat{S}_{ML}(t)$ are decreasing gradually with the increase of the failure times for the Lymph Glands cancer patients in the hospital, that means there is an opposite relationship between failure times and survival function. Showing that the value of survival function for patients was high when the patients stay alive in the hospital for the Lymph Glands Cancer was low and vice versa, that means the patient who remains (t =11) days in a hospital had the greatest survival function with (0.98214277335) value ,but the patient who remains(t=1046) days in a hospital had a smallest value of survival function with (0.00623383239) value .

4. Noting that the values of hazard function $\hat{h}_{ML}(t)$ are increasing gradually with the increase the failure times of patients for the Lymph Glands Cancer patients in the hospital until (t=28) and decrease after that. That means there is concave and convex regions respectively when plot the failure times vs. hazard function. Showing that the patient who remains (t=1046) days in a hospital had the smallest value of hazard function for death with (0.27903817907) value but the patient who remains (t=28) days in a hospital had a largest value of hazard function for death with (0.65225072150) .

5. The mean squares error and mean absolute percentage error for survival function which estimate the parameters in Birnbaum-Saunders distribution by MLE method is $\mathrm{MSE}[\hat{S}(t)]_{MLE} = 0.000000000036821$ and $\mathrm{MAPE}[\hat{S}(t)]_{MEL} = 0.000025281310636$.

3.2 Estimation the Parameters and Related Function by RQE Method

To find the estimated values for the shape and scale parameters in Birnbaum- Saunders . Newton -Raphson method was used. After that ,applying the regression-quantile estimator method for estimating the two unknown parameters of Birnbaum-Saunders distribution under complete data by using program of MATLAB (2012 a) .The estimation values of the specific parameter using RQE method are carried out in attached table(3).

Now, these estimated values for two parameters in Birnbaum-Saunders distribution are used to find the estimation of the probability death (density) function $\hat{f}_{RQ}(t)$, cumulative distribution function $\hat{F}_{RQ}(t)$, survival function $\hat{S}_{RQ}(t)$ and hazard function $\hat{h}_{RQ}(t)$.
The results for these four probability functions are displayed in table (4).

From this table, we demonstrate the following conclusions:

1. The values of death density function $\hat{f}_{RQ}(t)$ were increasing slightly until (t= 22) then the values became decreasing slightly until the end of failure times. The patients who remain in a hospital for (t=1046) days had smallest probability of death with (0.00142356349) value while the patients who remain in a hospital for (t=22) days had largest probability of death with (0.593766620475) value.
2. The values of cumulative death distribution function $\hat{F}_{RQ}(t)$ are increasing with the increase of failure times because it collects the probability values for all previous observations step by step, that means there is a direct relationship between failure times and cumulative death distribution function .
3. The values of survival function $\hat{S}_{RQ}(t)$ are decreasing gradually with the increase of the failure times for the Lymph Glands cancer patients in the hospital, that means there is an opposite relationship between failure times and survival function. Showing that the value of survival function for patients was high when the patients stay alive in the hospital for the Lymph Glands Cancer was low and vice versa, that means the patient who remains (t =11) days in a hospital had a greatest survival function with (0.97970942845) value ,but the patient who remains(t=1046) days in a hospital had a smallest value of survival function with (0.00487351604) value .
4. Noting that the values of hazard function $\hat{h}_{RQ}(t)$ are increasing gradually with the increase of the failure times of patients for the Lymph Glands Cancer patients in the hospital until (t=27) and decreasing after that. That means there is concave and convex regions respectively when plot the failure times vs. hazard function. Showing that the patient who remains (t=1046) days in a hospital had a smallest value of hazard function for death with (0.29210194025) value but the patient who remains (t=27) days in a hospital had a largest value of hazard function for death with (0.67814630492) value .
5. The mean squares error and mean absolute percentage error for survival function which estimate the parameters in Birnbaum-Saunders distribution by RQE method is
$MSE[\hat{S}(t)]_{RQE} = 0.000157322008012$ and $MAPE[\hat{S}(t)]_{RQE} = 0.044260791532672$.

3.3. Estimation the Parameters and Related Function by Shrinkage Method:

To find the estimated values for the shape and scale parameters in Birnbaum- Saunders. Newton -Raphson method was used. Applying the shrinkage estimator method for estimating the two unknown parameters of Birnbaum-Saunders distribution under complete data using MATLAB (2012 a) program. The estimation values of the specific parameter using SH. method are performed in the annexed table (5).

Now , these estimated values for the two parameters in Birnbaum-Saunders distribution are used for estimating the probability death (density) function $\hat{f}_{SH}(t)$, cumulative distribution function , $\hat{F}_{SH}(t)$, survival function $\hat{S}_{SH}(t)$ and hazard function $\hat{h}_{SH}(t)$.
The results for these four probability functions are placed in the table (6).

From this table, we display the following conclusions:

1. The values of death (density) function are increasing slightly until (t= 23) then the values became decreasing slightly until the end of failure times. The patients who remain in a hospital for (t=1046) days had smallest probability of death with (0.00173911118) value while the patients who remain in a hospital for (t=23) days had largest probability of death with (0.57168907904) value.
2. The values of cumulative death distribution function $\hat{F}_{SH}(t)$ are increasing with the increase of failure times because it collects the probability values for all previous observations step by step, that means there is a direct relationship between failure times and cumulative death distribution function .
3. The values of survival function $\hat{S}_{SH}(t)$ are decreasing gradually with the increase of the failure times for the Lymph Glands cancer patients in the hospital , that means there is an opposite relationship between failure times and survival function . Showing that the value of survival function for patients was high when the patients stay alive in the hospital for the Lymph Glands Cancer was low and vice versa , that means the patient who remains (t =11) days in a hospital had a greatest survival function with (0.98214623898) value ,but the patient who remains(t=1046) days in a hospital had a smallest value of survival function with (0.00623244779) value .
4.The values of hazard function $\hat{h}_{SH}(t)$ are increasing gradually with the increase of the failure times of patients for the Lymph Glands Cancer patients in the hospital until (t=28) and decreasing after that. That means there is concave and convex regions respectively when plot the failure times vs. hazard function. Showing that the patient who remains (t=1046) days in a hospital had a smallest value of hazard function for death with (0.27904143697) value but the patient who remains (t=28) days in a hospital had a largest value of hazard function for death with (0.65220903452) value.
5. The mean squares error and mean absolute percentage error for survival function which estimate the parameters in Birnbaum-Saunders distribution by MLE method is $\text{MSE}[\hat{S}(t)]_{SH} = 0.00000000000$ and $\text{MAPE}[\hat{S}(t)]_{SH} = 0.000000002554411$.

Comparisons Between three Considered Estimation Methods

This section is related with comparisons between the three considered estimators using the two statistical indicators, "Mean Squared Error and Mean Absolute Percentage Error" with respect to the survival function.
i.e. ;

$$\text{MSE}[\hat{S}(t_i)] = \frac{\sum_{i=0}^{n}[\hat{S}(t_i) - S(t_i)]^2}{n} \quad , \quad \text{MAPE}[\hat{S}(t_i] = \frac{\sum_{i=0}^{n}\frac{|\hat{S}(t_i) - S(t_i)|}{S(t_i)}}{n} \qquad ..(25)$$

Where, $S(t_i)$ is the real survival function and $\hat{S}(t_i)$ is the estimated survival function
As a consequence, the computations of mentioned statistical indicators are carried out in annexed table (7):
That indicates , the mean squares error and mean absolute percentage using shrinkage method is less than that of MLE and RQE methods, so the shrinkage method is the best.

Conclusions and Results

We estimated the parameters of Birnbaum-Saunders distribution using MLE, RQE and shrinkage methods for complete data and the related probability functions, and we conclude the following results.
i. There is a direct relationship between the failure times and the estimated cumulative distribution function for each considered estimators .

ii. There is concave and convex regions respectively when ploting the failure times vs. hazard function, for each considered estimators.
iii. There is an opposite relationship between the failure times and survival function, for each considered estimators .
iv. There is a vibrate relationship between the failure times and the probability death (density function) for each considered estimators .
v. According to comparisons between MLE, RQE and SH Methods, we conclude that the shrinkage estimation method is the best in the sense of MSE and MAPE.

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Table (1): Estimated values for the parameters by MLE method

Estimated values **parameters**	**Number of iteration**	**Errors for all**
$\hat{\alpha} = 1.221842074333230$ $\hat{\beta} = 93.132544387531269$	3	0

Table (2): Estimated values for the functions $\hat{f}_{ML}(t)$, $\hat{F}_{ML}$(t), $\hat{S}_{ML}$(t), $\hat{h}_{ML}$(t)

Time/d	$\hat{f}_{ML}(t)$	$\hat{F}_{ML}(t)$	$\hat{S}_{ML}(t)$	$\hat{h}_{ML}(t)$
11	0.3685931194	0.017857226642	0.9821427733	0.3752948445
12	77601	194	57806	74820
13	0.4121714824	0.023501579574	0.9764984204	0.4220912945
14	08991	998	25002	55910
15	0.4490768172	0.029726613230	0.9702733867	0.4628353445
16	53728	974	69026	30202
17	0.4797176870	0.036438614961	0.9635613850	0.4978589786
18	03165	713	38287	30721
19	0.5046857390	0.043551277906	0.9564487220	0.5276662798
20	35166	672	93328	30023
22	0.5246353800	0.050987398684	0.9490126013	0.5528223538
23	59563	398	15602	15185
24	0.5402153359	0.058679240315	0.9413207596	0.5738908128
25	24527	717	84283	46531
27	0.5520326769	0.066568160454	0.9334318395	0.5914011645
28	77164	402	45599	94833
29	0.5606363933	0.074603873681	0.9253961263	0.6058339530
31	76021	091	18909	83585
34	0.5665123024	0.082743566169	0.9172564338	0.6176160575
36	12906	199	30801	36867
37	0.5717182777	0.099195580186	0.9008044198	0.6346752582
38	82102	543	13457	54723
39	0.5717287622	0.107451696186	0.8925483038	0.6405577825
40	67906	761	13239	03542
41	0.5703841597	0.115697883623	0.8843021163	0.6450105107
43	48949	097	76903	58342
45	0.5679131435	0.123916273489	0.8760837265	0.6482407175
53	00851	929	10071	43248
55	0.5603397932	0.140212961774	0.8597870382	0.6517192844
56	89702	250	25750	00955
58	0.5555421675	0.148268987329	0.8517310126	0.6522507215
60	39112	926	70074	01328
62	0.5502355921	0.156251941627	0.8437480583	0.6521325728
63	30225	824	72176	34339
65	0.5384811738	0.171973552575	0.8280264474	0.6503188099
70	82880	537	24463	33909
72	0.5190840331	0.194882051816	0.8051179481	0.6447304202
73	12647	985	83015	87006
77	0.5055829419	0.209676401395	0.7903235986	0.6397163679
77	81444	414	04586	20829
77	0.4987719380	0.216926674027	0.7830733259	0.6369415500
78	70614	007	72993	78818
80	0.4919565256	0.224078551823	0.7759214481	0.6340287754
85	70227	198	76802	98070
89	0.4851582124	0.231132131035	0.7688678689	0.6310033648
92	62618	407	64593	77197
93	0.4783949419	0.238087792018	0.7619122079	0.6278872250
94	75058	229	81771	15437

94	**0.4716816213**	**0.244946151684**	**0.7550538483**	**0.6246993143**
97	**44913**	**722**	**15278**	**56662**
104	**0.4584519012**	**0.258374380421**	**0.7416256195**	**0.6181716073**
109	**80311**	**123**	**78877**	**13993**
113	**0.4455430394**	**0.271425086811**	**0.7285749131**	**0.6115267371**
119	**34656**	**125**	**88875**	**54006**
123	**0.3978231778**	**0.320066732300**	**0.6799332676**	**0.5850915035**
123	**85650**	**223**	**99777**	**11059**
126	**0.3869315264**	**0.331395732295**	**0.6686042677**	**0.5787153105**
133	**49798**	**187**	**04813**	**94797**
139	**0.3816405074**	**0.336943789084**	**0.6630562109**	**0.5755779089**
140	**02063**	**826**	**15174**	**60855**
142	**0.3713623859**	**0.347814400866**	**0.6521855991**	**0.5694121220**
143	**71546**	**973**	**33027**	**48096**
155	**0.3614799977**	**0.358393971893**	**0.6416060281**	**0.5633986931**
157	**77416**	**161**	**06839**	**88747**
158	**0.3519804066**	**0.368693745816**	**0.6313062541**	**0.5575430376**
190	**13211**	**822**	**83178**	**00070**
206	**0.3473697885**	**0.373742116228**	**0.6262578837**	**0.5546753144**
218	**88976**	**991**	**71009**	**20461**
227	**0.3384177780**	**0.383642412584**	**0.6163575874**	**0.5490607805**
237	**36051**	**858**	**15142**	**36985**
241	**0.3175212225**	**0.407305598887**	**0.5926944011**	**0.5357250244**
242	**09823**	**983**	**12017**	**20828**
242	**0.3097170633**	**0.416360706873**	**0.5836392931**	**0.5306652018**
247	**02596**	**420**	**26580**	**63687**
255	**0.3059265522**	**0.420804837009**	**0.5791951629**	**0.5281925192**
261	**51481**	**332**	**90668**	**05155**
273	**0.2914679346**	**0.438049473559**	**0.5619505264**	**0.5186718775**
296	**40485**	**913**	**40087**	**52837**
342	**0.2914679346**	**0.438049473559**	**0.5619505264**	**0.5186718775**
385	**40485**	**913**	**40087**	**52837**
402	**0.2914679346**	**0.438049473559**	**0.5619505264**	**0.5186718775**
407	**40485**	**913**	**40087**	**52837**
414	**0.2880208145**	**0.442232618385**	**0.5577673816**	**0.5163816029**
463	**75601**	**842**	**14158**	**22135**
477	**0.2813164535**	**0.450451900790**	**0.5495480992**	**0.5119050615**
484	**34507**	**969**	**09031**	**21291**
499	**0.2655938181**	**0.470183524114**	**0.5298164758**	**0.5012939957**
565	**90546**	**201**	**85799**	**11441**
566	**0.2539926577**	**0.485182774410**	**0.5148172255**	**0.4933647226**
606	**31106**	**637**	**89363**	**74799**
736	**0.2458088750**	**0.496005169875**	**0.5039948301**	**0.4877210247**
1046	**11500**	**270**	**24730**	**38818**
	0.2431733069	**0.499534988144**	**0.5004650118**	**0.4858947202**
	38741	**090**	**55910**	**66187**
	0.2405821183	**0.503027076682**	**0.4969729233**	**0.4840950221**
	29847	**902**	**17098**	**67519**
	0.2405821183	**0.503027076682**	**0.4969729233**	**0.4840950221**
	29847	**902**	**17098**	**67519**
	0.2330646130	**0.513283222808**	**0.4867167771**	**0.4788505841**

	60989	966	91034	24229
	0.216896760822130	0.536005147740367	0.463994852259633	0.467455101637126
	0.206398845448705	0.551278713122001	0.448721286878000	0.459971147980822
	0.198560005552210	0.562969733166075	0.437030266833925	0.454339254328268
	0.187639113013728	0.579690234637021	0.420309765362979	0.446430534041204
	0.180864705731564	0.590329036093880	0.409670963906120	0.441487734466362
	0.180864705731564	0.590329036093880	0.409670963906120	0.441487734466362
	0.176026986025824	0.598057317649233	0.401942682350767	0.437940516782960
	0.165477811199037	0.615305792357647	0.384694207642353	0.430154153381171
	0.157178733130197	0.629276337208012	0.370723662791988	0.423978151128674
	0.155856575781007	0.631536061294135	0.368463938705865	0.422990038939531
	0.153261739049460	0.635998840815408	0.364001159184592	0.421047392796179
	0.151988468370971	0.638202366755164	0.361797633244836	0.420092489295300
	0.137859759748035	0.663281265386252	0.336718734613748	0.409421114943827
	0.135694363560949	0.667230618244363	0.332769381755637	0.407772983334727
	0.134630282847964	0.669182027344286	0.330817972655714	0.406961815790085
	0.106058973645866	0.724407354490493	0.275592645509507	0.384839636956884
	0.094907353192139	0.747586211130918	0.252413788869082	0.375999083161675
	0.087574170671342	0.763382253182597	0.236617746817403	0.370108209757078
	0.082568415451330	0.774432025774209	0.225567974225791	0.366046712680410
	0.077444183038708	0.785977799805039	0.214022200194961	0.361851167627291
	0.075512303197385	0.790394149388213	0.209605850611787	0.360258566146810
	0.075039259132030	0.791480948688078	0.208519051311922	0.359867641157540
	0.075039259132030	0.791480948688078	0.208519051311922	0.359867641157540
	0.072731396932255	0.796814026821620	0.203185973178380	0.357954812502747
	0.069227683151939	0.805010074179189	0.194989925820811	0.355032101584348

	0.066742275243201	0.810898480049882	0.189101519950118	0.352944150109454
	0.062105522735611	0.822053856208727	0.177946143791273	0.349013029517850
	0.054303814133360	0.841345145352402	0.158654854647598	0.342276410349866
	0.042030633641299	0.873121669746656	0.126878330253345	0.331267234975226
	0.033466298633487	0.896436792647003	0.103563207352997	0.323148534009930
	0.030659874930121	0.904300480200595	0.095699519799405	0.320376476228792
	0.029887288271794	0.906485736615124	0.093514263384876	0.319601386889906
	0.028843251477094	0.909453129445426	0.090546870554574	0.318544984497393
	0.022602956731835	0.927548397832354	0.072451602167646	0.311973180103514
	0.021113019845947	0.931964553739530	0.068035446260470	0.310323823924323
	0.020409756177097	0.934062545188482	0.065937454811518	0.309532059365022
	0.018989472144429	0.938326712573725	0.061673287426275	0.307904328387380
	0.013918053426184	0.953867154756614	0.046132845243386	0.301695101456579
	0.013853722454135	0.954067633698273	0.045932366301727	0.301611337920851
	0.011528614795413	0.961374461331578	0.038625538668422	0.298471301445917
	0.006458764293984	0.977764969648423	0.022235030351577	0.290476972230696
	0.001739477241512	0.993766167600120	0.006233832399880	0.279038179073465

Table (3): Estimated values for the parameters by RQE method

Estimated values
$\hat{\alpha} =$ 1.2153602433570740
$\hat{\beta} =$ 88.772096874537695

Table (4): Estimated values for the functions $\hat{f}_{RQ}(t)$,$\hat{F}_{RQ}(t)$ $\hat{S}_{RQ}(t)$,$\hat{h}_{RQ}(t)$

Time/d	$\hat{f}_{RQ}$(t)	$\hat{F}_{RQ}$(t)	$\hat{S}_{RQ}$(t)	$\hat{h}_{RQ}$(t)
11	0.402494561005	0.020290571542	0.979709428457	0.410830547624
12	788	049	951	012
13	0.446291183521	0.026470948532	0.973529051467	0.458426158775
14	062	308	692	882
15	0.482795666928	0.033234599492	0.966765400507	0.499392786165
16	308	214	786	830
17	0.512603764215	0.040479550370	0.959520449629	0.534229118736
18	710	120	880	801
19	0.536451696993	0.048113692616	0.951886307383	0.563566985712
20	639	156	844	840
22	0.555102053625	0.056055915564	0.943944084435	0.588066669179
23	336	683	317	253
24	0.569282648939	0.064235995922	0.935764004077	0.608361345872
25	855	270	730	594
27	0.579657239772	0.072593850141	0.927406149858	0.625030618850
28	054	754	246	926
29	0.586814649091	0.081078507461	0.918921492538	0.638590623742
31	109	071	930	810
34	0.591268045645	0.089647006272	0.910352993727	0.649493163333
36	722	594	406	047
37	0.593766620475	0.106897383501	0.893102616498	0.664835831299
38	155	642	358	176
39	0.592510650466	0.115524188287	0.884475811712	0.669900343932
40	222	951	049	888
41	0.589963499187	0.124123039135	0.875876960864	0.673568920691
43	723	859	141	400
45	0.586354932761	0.132676878168	0.867323121831	0.676051310062
53	882	648	352	845
55	0.576698215390	0.149596169436	0.850403830563	0.678146304924
56	601	021	979	498
58	0.570950778229	0.157941012146	0.842058987853	0.678041308822
60	795	435	565	280
62	0.564752063994	0.166198869510	0.833801130489	0.677322257482
63	758	686	314	829
65	0.551374165456	0.182431586275	0.817568413724	0.674407372155
70	842	950	050	335
72	0.529897171634	0.206020436453	0.793979563546	0.667393968262
73	387	837	163	481
77	0.515212037155	0.221218053978	0.778781946021	0.661561352041
77	289	386	614	141
77	0.507861736909	0.228656512939	0.771343487060	0.658411907831
78	483	034	967	850
80	0.500538972103	0.235988262527	0.764011737472	0.655145657525
85	515	312	688	201
89	0.493263042178	0.243213847954	0.756786152045	0.651786559314
92	722	152	848	367
93	0.486049834352	0.250334069576	0.749665930423	0.648355240150
94	110	972	028	554

94	0.478912344845	0.257349937176	0.742650062823	0.644869459816
97	657	512	488	479
104	0.464904629812	0.271073469995	0.728926530004	0.637793537037
109	337	237	763	675
113	0.451301206656	0.284395381562	0.715604618437	0.630657202355
119	691	332	668	662
123	0.401442249784	0.333923542234	0.666076457765	0.602696950333
123	192	105	895	120
126	0.390138252151	0.345433153440	0.654566846559	0.596025072462
133	925	419	581	043
139	0.384655432163	0.351066323461	0.648933676538	0.592749992904
140	274	972	028	295
142	0.374019634102	0.362097491876	0.637902508123	0.586327266847
143	899	931	069	398
155	0.363810828247	0.372825588461	0.627174411538	0.580079195761
157	633	233	767	553
158	0.354012576147	0.383262794665	0.616737205334	0.574008788647
190	784	948	052	727
206	0.349262070371	0.388375987221	0.611624012778	0.571040480874
218	523	178	822	358
227	0.340047459991	0.398398623849	0.601601376150	0.565237171109
237	089	064	936	421
241	0.318581101223	0.422329023691	0.577670976308	0.551492310137
242	266	879	121	008
242	0.310578549065	0.431477137074	0.568522862925	0.546290341723
247	239	386	614	469
255	0.306694336803	0.435965060503	0.564034939496	0.543750600055
261	935	104	896	908
273	0.291893540394	0.453368145799	0.546631854200	0.533985603201
296	270	717	283	459
342	0.291893540394	0.453368145799	0.546631854200	0.533985603201
385	270	717	283	459
402	0.291893540394	0.453368145799	0.546631854200	0.533985603201
407	270	717	283	459
414	0.288368272770	0.457586983222	0.542413016777	0.531639661754
463	391	852	148	040
477	0.281515551152	0.465873281960	0.534126718039	0.527057609448
484	268	599	401	217
499	0.265463031879	0.485748919965	0.514251080034	0.516212881579
565	940	104	896	025
566	0.253633858925	0.500841631063	0.499158368936	0.508123022090
606	785	979	021	999
736	0.245296704531	0.511722786783	0.488277213216	0.502371804154
1046	331	168	832	622
	0.242613021343	0.515270183823	0.484729816176	0.500511858868
	557	256	744	394
	0.239975124732	0.518778890506	0.481221109493	0.498679546675
	923	250	750	290
	0.239975124732	0.518778890506	0.481221109493	0.498679546675
	923	250	750	290
	0.232325411218	0.529079369164	0.470920630835	0.493343030664

	956	293	707	564
	0.215889401753699	0.551875323062390	0.448124676937610	0.481761913289493
	0.205228855740035	0.567179414654363	0.432820585345637	0.474166115680809
	0.197274352102100	0.578883013924087	0.421116986075913	0.468454986678068
	0.186200477803944	0.595604717864506	0.404395282135494	0.460441765840276
	0.179335948857441	0.606233665465295	0.393766334534705	0.455437484439479
	0.179335948857441	0.606233665465295	0.393766334534705	0.455437484439479
	0.174436125420611	0.613949448599778	0.386050551400222	0.451847885692491
	0.163758137623501	0.631153263288116	0.368846736711884	0.443973394161860
	0.155364254946148	0.645069980011463	0.354930019988537	0.437732077301225
	0.154027540336259	0.647319459281515	0.352680540718485	0.436733878264086
	0.151404579686175	0.651760718765011	0.348239281234989	0.434771686724245
	0.150117726975961	0.653952982318738	0.346047017681262	0.433807313184914
	0.135848344474172	0.678872907543202	0.321127092456798	0.423036074081628
	0.133663090453749	0.682791865987171	0.317208134012829	0.421373464680271
	0.132589425689769	0.684727697389330	0.315272302610670	0.420555261568611
	0.103809120904614	0.739345187029032	0.260654812970968	0.398262820169664
	0.092606297312991	0.762160734294009	0.237839265705991	0.389365048862318
	0.085251291041881	0.777666728049687	0.222333271950313	0.383439196005415
	0.080236876534289	0.788491309346302	0.211508690653698	0.379354986720900
	0.075109674805944	0.799780758774535	0.200219241225465	0.375137146391256
	0.073178361419439	0.804093083722847	0.195906916277153	0.373536385596067
	0.072705604700750	0.805153759854181	0.194846240145819	0.373143483016860
	0.072705604700750	0.805153759854181	0.194846240145819	0.373143483016860
	0.070400013387035	0.810355581642052	0.189644418357948	0.371221120012911
	0.066902642310478	0.818339680118627	0.181660319881373	0.368284292101691

	0.064423993959165	0.824067847844319	0.175932152155681	0.366186584827067
	0.059805325057983	0.834900387423554	0.165099612576446	0.362237827967591
	0.052052375727490	0.853568758152623	0.146431241847377	0.355473156348310
	0.039914963189190	0.884110914280642	0.115889085719358	0.344423833715020
	0.031503469231161	0.906317720116232	0.093682279883768	0.336279916225862
	0.028760852054407	0.913760586841992	0.086239413158008	0.333500090054084
	0.028007226463179	0.915824170728035	0.084175829271965	0.332722905202278
	0.026989836200336	0.918622881729545	0.081377118270455	0.331663700730172
	0.020936714043383	0.935594377148813	0.064405622851187	0.325075872517498
	0.019499663008001	0.939708461484028	0.060291538515972	0.323422879693739
	0.018822636315337	0.941658649665609	0.058341350334391	0.322629425055350
	0.017458015932046	0.945613375263535	0.054386624736465	0.320998333995547
	0.012619658529039	0.959909304070425	0.040090695929575	0.314777736739894
	0.012558685707587	0.960092368279538	0.039907631720462	0.314693835894757
	0.010363176375734	0.966736605492231	0.033263394507769	0.311548972348979
	0.005648027025552	0.981393135215350	0.018606864784650	0.303545336139154
	0.001423563491390	0.995126483959102	0.004873516040898	0.292101940250888

Table (5): Estimated values for the parameters by **SH. method**

Estimated values
$\hat{\alpha}_{sh} = 1.2218000042511650$ $\hat{\beta}_{sh} = 93.132999953965182$

Table (6): Estimated values for the functions $\hat{f}_{SH}(t)$, $\hat{F}_{SH}(t)$ $\hat{S}_{SH}(t)$, $\hat{h}_{SH}(t)$

Time/d	$\hat{f}_{SH}$(t)	$\hat{F}_{SH}$(t)	$\hat{S}_{SH}$(t)	$\hat{h}_{SH}$(t)
11	0.368528502756	0.0178537610105	0.982146238989	0.3752277289
12	334	75	425	53612
13	0.412106296853	0.0234974355259	0.976502564474	0.4220227491
14	886	67	033	93553
15	0.449012295381	0.0297218187250	0.970278181274	0.4627665591
16	207	02	998	64692
17	0.479654696498	0.0364332086988	0.963566791301	0.4977908130
18	151	81	119	79461
19	0.504624862538	0.0435453049845	0.956454695015	0.5275993365
20	179	01	499	58227
22	0.524576984899	0.0509809074208	0.949019092579	0.5527570404
23	259	04	196	02201
24	0.540159630724	0.0586722800015	0.941327719998	0.5738273921
25	547	09	491	49264
27	0.551979755748	0.0665607798137	0.933439220186	0.5913397935
28	962	46	254	42018
29	0.560586268999	0.0745961198553	0.925403880144	0.6057747120
31	816	96	604	23272
34	0.566464931630	0.0827354840798	0.917264515920	0.6175589721
36	261	02	198	37916
37	0.571676146994	0.0991869658021	0.900813034197	0.6346224191
38	305	90	810	83125
39	0.571689079049	0.1074428721858	0.892557127814	0.6405069896
40	214	04	196	74977
41	0.570346796679	0.1156888839722	0.884311116027	0.6449616954
43	929	23	777	28936
45	0.567877970453	0.1239071294673	0.876092870532	0.6481938040
53	185	38	662	51752
55	0.560308615012	0.1402036127466	0.859796387253	0.6516759355
56	909	08	392	11671
58	0.555512801562	0.1482595729983	0.851740427001	0.6522090345
60	951	65	635	27705
62	0.550207922061	0.1562424839407	0.843757516059	0.6520924692
63	019	87	213	09372
65	0.538456570773	0.1719640664046	0.828035933595	0.6502816471
70	605	83	317	20840
72	0.519063312082	0.1948726437056	0.805127356294	0.6446971501
73	699	49	351	15132
77	0.505564399764	0.2096671095801	0.790332890419	0.6396853856
77	076	30	870	04403
77	0.498754378844	0.2169174561309	0.783082543869	0.6369116292
78	813	07	093	39674
80	0.491939885043	0.2240694170294	0.775930582970	0.6339998652
85	910	96	504	46463
89	0.485142430563	0.2311230876350	0.768876912364	0.6309754172
92	614	49	951	11304
93	0.478379963145	0.2380788474860	0.761921152513	0.6278601946
94	039	39	961	75818
94	0.471667393845	0.2449373127490	0.755062687250	0.6246731586

97	321	12	988	78460
104	0.458439035522	0.2583657701897	0.741634229810	0.6181470826
109	173	58	242	11696
113	0.445531368810	0.2714167247276	0.728583275272	0.6115037003
119	168	37	363	06073
123	0.397815023226	0.3200594859792	0.679940514020	0.5850732748
123	526	02	797	28212
126	0.386924011492	0.3313887825325	0.668611217467	0.5786980555
133	134	99	401	87137
139	0.381633284605	0.3369369888852	0.663063011114	0.5755611129
140	747	52	748	08622
142	0.371355697901	0.3478079014399	0.652192098560	0.5693961928
143	391	72	028	10839
155	0.361473785080	0.3583877743784	0.641612225621	0.5633835682
157	020	04	596	13375
158	0.351974617095	0.3686878502646	0.631312149735	0.5575286603
190	968	23	377	36258
206	0.347364193060	0.3737363713849	0.626263628615	0.5546612914
218	876	91	009	90097
227	0.338412538765	0.3836369681105	0.616363031889	0.5490474302
237	347	90	410	58708
241	0.317516716220	0.4073008954763	0.592699104523	0.5357131701
242	483	98	602	34947
242	0.309712797545	0.4163562945914	0.583643705408	0.5306538812
247	150	94	506	55131
255	0.305922397148	0.4208005689543	0.579199431045	0.5281814531
261	662	87	613	41949
273	0.291464166087	0.4380457726123	0.561954227387	0.5186617554
296	352	89	611	99728
342	0.291464166087	0.4380457726123	0.561954227387	0.5186617554
385	352	89	611	99728
402	0.291464166087	0.4380457726123	0.561954227387	0.5186617554
407	352	89	611	99728
414	0.288017130151	0.4422290566113	0.557770943388	0.5163716998
463	923	25	675	27364
477	0.281312924222	0.4504486140647	0.549551385935	0.5118955777
484	667	37	263	79155
499	0.265590610292	0.4701809049627	0.529819095037	0.5012854628
565	439	46	254	68725
566	0.253989650927	0.4851806679372	0.514819332062	0.4933568634
606	330	34	766	83837
736	0.245805992672	0.4960034351772	0.503996564822	0.4877136270
1046	885	41	759	94518
	0.243170461732	0.4995333749337	0.500466625066	0.4858874689
	636	14	286	20487
	0.240579308260	0.5030255837412	0.496974416258	0.4840879135
	725	76	724	62698
	0.240579308260	0.5030255837412	0.496974416258	0.4840879135
	725	76	724	62698
	0.233061897408	0.5132820833751	0.486717916624	0.4788438835
	893	12	888	88763

	0.216894212061364	**0.536004791016775**	**0.463995208983225**	**0.467449249177927**
	0.206396380272672	**0.551278880149113**	**0.448721119850887**	**0.459965825413476**
	0.198557590946775	**0.562970298659147**	**0.437029701340853**	**0.454334317181601**
	0.187636753091748	**0.579691364814032**	**0.420308635185968**	**0.446426119722062**
	0.180862371096010	**0.590330521521439**	**0.409669478478561**	**0.441483636437112**
	0.180862371096010	**0.590330521521439**	**0.409669478478561**	**0.441483636437112**
	0.176024665658643	**0.598059058785998**	**0.401940941214002**	**0.437936640957716**
	0.165475511775786	**0.615308095852627**	**0.384691904147373**	**0.430150751788094**
	0.157176441241742	**0.629279086360109**	**0.370720913639891**	**0.423975112972504**
	0.155854284429027	**0.631538881602137**	**0.368461118397863**	**0.422987057920006**
	0.153259448252420	**0.636001800837971**	**0.363998199162029**	**0.421044523311497**
	0.151986177611919	**0.638205395353293**	**0.361794604646707**	**0.420089674251316**
	0.137857460192091	**0.663285053446012**	**0.336714946553988**	**0.409418891566749**
	0.135692061354129	**0.667234522049710**	**0.332765477950290**	**0.407770848676795**
	0.134627979231343	**0.669185987919548**	**0.330814012080452**	**0.406959724543355**
	0.106056616459391	**0.724412785978303**	**0.275587214021698**	**0.384838668353608**
	0.094904975594586	**0.747592164819958**	**0.252407835180042**	**0.375998532402494**
	0.087571783710047	**0.763388522723379**	**0.236611477276621**	**0.370107928482555**
	0.082566025118534	**0.774438494623076**	**0.225561505376924**	**0.366046613231110**
	0.077441792664393	**0.785984456096215**	**0.214015543903785**	**0.361851252725870**
	0.075509913882296	**0.790400871384858**	**0.209599128615142**	**0.360258720449858**
	0.075036870176194	**0.791487686324503**	**0.208512313675497**	**0.359867812377606**
	0.075036870176194	**0.791487686324503**	**0.208512313675497**	**0.359867812377606**
	0.072729010331219	**0.796820838094125**	**0.203179161905875**	**0.357955066105212**
	0.069225302209583	**0.805016988178678**	**0.194983011821322**	**0.355032479819419**
	0.066739900017	**0.8109054596641**	**0.189094540335**	**0.3529446164**

	208	64	836	79546
	0.062103162514256	0.822060940131385	0.177939059868615	0.349013659845741
	0.054301494632718	0.841352341767367	0.158647658232633	0.342277315893896
	0.042028432319558	0.873128828464618	0.126871171535382	0.331268575917870
	0.033464236799363	0.896443702475220	0.103556297524780	0.323150185930079
	0.030657872836342	0.904307254000355	0.095692745999645	0.320378232603499
	0.029885304076609	0.906492467333572	0.093507532666428	0.319603172326442
	0.028841292541267	0.909459797787758	0.090540202212242	0.318546809445581
	0.022601178331568	0.927554578639515	0.072445421360485	0.311975248499225
	0.021111293330748	0.931970584034548	0.068029415965452	0.310325952841767
	0.020408055530892	0.934068499077676	0.065931500922324	0.309534217261876
	0.018987826650738	0.938332500835942	0.061667499164058	0.307906545719049
	0.013916642492349	0.953872204897124	0.046127795102876	0.301697543992966
	0.013852314940001	0.954072672740238	0.045927327259762	0.301613783481302
	0.011527340063626	0.961379063287480	0.038620936712520	0.298473860161170
	0.006457860665309	0.977768298288213	0.022231701711787	0.290479817920796
	0.001739111189438	0.993767552202029	0.006232447797971	0.279041436978308

Table (7): showed MSE and MAPE for the estimation methods

Method	MSE	MAPE
MLE	0.000000000036821	0.000025281310636
RQE	0.000157322008120	0.044260791532672
SHI	0.000000000000000	0.000000002554411

تقدير معالم والدوال الاحتمالية ذات العلاقة لبيانات مرضى سرطان الغدد اللمفاوية عبر إنموذج بيرنبوم- سوندرز

عباس نجم سلمان
طه أنور طه
قسم الرياضيات / كلية التربية للعلوم الصرفة(ابن الهيثم)/جامعة بغداد

أستلم البحث في : 16 حزيران 2013 ، قبل في 26 آب 2013

المستخلص

في هذا البحث ، تقدير معالم والدوال الاحتمالية ذات الصلة ،دالة البقاء ، والدالة التجميعية التوزيعية، ودالة الخطورة(نسبة الفشل) ، ودالة الفشل(الموت) الاحتمالية لتوزيع بيرنبوم- سوندرز الذي يطابق البيانات الحقيقية لمرضى سرطان الغدد اللمفاوية.سيكون تقدير المعالم (الشكل،القياس) باستخدام ثلاث طرائق مقترحة (الامكان الأعظم ، والمربعات الصغرى، وطريقة التقلص) ثم بعد ذلك يتم تقدير الدوال الاحتمالية المتعلقة بها والتي ذكرت اعلاه بالاعتماد على عينة بيانات حقيقية تصف المدة التي يعاني منها مرضى سرطان العقد اللمفاوية بالاعتماد على وقت تشخيص المرض أو دخول المريض إلى المستشفى ولمدة ثلاث سنوات(ابتدآ" من 2010سنة الى نهاية سنة 2012) . في هذا البحث قمنا بتقدير وحساب قيم المعالم ومن ثم تقدير قيم الدوال الاحتمالية المذكورة اعلاه ثم بعد ذلك مقارنة النتائج العددية باستخدام مؤشرين إحصائيين (متوسط مربعات الخطأ ، و متوسط الخطأ النسبي المطلق) للمقارنة بين الطرائق الثلاثة المقترحة بالنسبة إلى دالة البقاء .استنتجنا من ذلك إن طريق التقلص المستخدمة في تقدير دالة البقاء بالنسبة إلى مرضى سرطان العقد اللمفاوية في المدة اعلاه هي الافضل .

كلمات مفتاحية : إنموذج بيرنبوم- سوندرز ، مرض سرطان الغدد اللمفاوية ، البيانات الحقيقية الكاملة ، مقدر الإمكان الأعظم ،مقدر المربعات الصغرى، مقدر التقلص ، متوسط مربعات الخطأ و متوسط الخطأ النسبي المطلق .