Studying of Optical System Includes Elliptical Aperture Point Spread Function

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Abstract

In this work, optical system with elliptical aperture using point spread function was studied. This is due to its comparison with an optical system with a circular aperture.
The present work deals with the theoretical study of intensity distribution within the image. In this work, a special formula was derived which is called the point spread function (PSF) by using a pupil function technique. The work deals with the limited optical system diffraction only (ideal system), and the system with focal shift. Also a graphic relation was founded between eccentricity and the best of focal depth given to at least (80%) of intensity.

Theory

The complex amplitude at any point in the image plane [1] is given by:

$$F(u,v) = \frac{1}{A}\int_y\int_x f(x,y)e^{i2\pi(ux+vy)}dxdy \quad ...(1)$$

Where
(u,v)= image plane coordinates
(x,y)= xit pupil coordinates
A= exit pupil area
f(x,y) is the pupil function [2], which has the form:

$$f(x,y) = \tau(x,y).e^{ikw(x,y)}$$

$\tau(x,y)$ Is the real amplitude distribution across wave front which is equal to unity in most cases. (k) The wave number and equals to $(2\pi/\lambda)$.
$w(x,y)$ is wave aberration function [3]:

$$w(x,y) = \sum_{n=1}^{N} w_{2n}(x^2+y^2)^n \quad ..(2)$$

Point spread function (distribution of illuminance in image plane due to point source) G(u,v) is given by the squared modulus of complex amplitude [4]whichis:

$$G(u,v) = |F(u,v)|^2$$

$$G(u,v) = \left|\int_y\int_x f(x,y).e^{i2\pi(uv,vy)}dxdy\right|^2 \quad ..(3)$$

We have asymmetrical intensity distribution in image plane, so we can cancel one of the image plane coordinates (v = 0):

$$G(z) = N\left|\int_y\int_x f(x,y).e^{izx}dxdy\right|^2 \quad ..(4)$$

(z) is a displacement coordinate which equals to (z=2πu)
N is a normalizing constant, where G (0) =1
The pupil function condition using elliptical aperture equation [shown in fig(1)]

$$f(x,y) = \begin{cases} e^{ikw(x,y)} & \frac{x^2}{a^2}+\frac{y^2}{b^2} \le 1 \\ 0 & \frac{x^2}{a^2}+\frac{y^2}{b^2} > 1 \end{cases} \quad ..(5)$$

Where A= π, and ellipse area (A=abπ), so (ab=1)
Substitute eq (5) into eq (4):

$$G(z) = N\left|\int_{y=-b}^{y=b}\int_{x=-a\sqrt{1-a^2y^2}}^{x=a\sqrt{1-a^2y^2}} f(x,y).e^{izx}dxdy\right|^2$$

$$G(z) = N\left|\int_{y=-b}^{y=b}\int_{x=-a\sqrt{1-a^2y^2}}^{x=a\sqrt{1-a^2y^2}} e^{izx} \quad dxdy\right|^2 \quad ..(6)$$

for a diffraction – limited system (aberration free system), where w(x,y)=0

$$G(0) = 1 = N\left|\int_{y=-b}^{y=b}\int_{x=-a\sqrt{1-a^2y^2}}^{x=a\sqrt{1-a^2y^2}} dxdy\right|^2 \quad ...(7)$$

When we solve equation [7] by (Gauss quadrrature method):

$$N = \frac{1}{\pi^2}$$

We substitute N into eq (6):

$$G(z) = \frac{1}{\pi^2}\left|\int_{y=-b}^{y=b}\int_{x=-a\sqrt{1-a^2y^2}}^{x=a\sqrt{1-a^2y^2}} e^{izx} \quad dxdy\right|^2$$

$$G(z) = \frac{1}{\pi^2}\left[\int_{y=-b}^{y=b}\int_{x=-a\sqrt{1-a^2y^2}}^{x=a\sqrt{1-a^2y^2}} [\cos(zx)+i\sin(zx)] \quad dxdy\right]^2 \quad ...(8)$$

The term (isin (zx)) into eq (8) was canceled because (sin) is odd function.

$$G(z) = \frac{1}{\pi^2}\left[\int_{y=-b}^{y=b}\int_{x=-a\sqrt{1-a^2y^2}}^{x=a\sqrt{1-a^2y^2}} \cos(zx) \quad dxdy\right]^2 \quad ...(9)$$

The equation (9) refers to ideal system (free aberration system).
Now consider having a longitudinal focal shift (aberrated system):

$$w(x,y)=w_2(x^2+y^2)$$

Following the same procedures; the intensity is given by:

$$G(z)=\frac{1}{\pi^2}\left\{\left[\int_{y=-b}^{y=b}\int_{x=-a\sqrt{1-a^2y^2}}^{x=a\sqrt{1-a^2y^2}}\cos[zx+2\pi w_2(x^2+y^2)]\ dxdy\right]^2+\left[\int_{y=-b}^{y=b}\int_{x=-a\sqrt{1-a^2y^2}}^{x=a\sqrt{1-a^2y^2}}\sin[zx+2\pi w_2(x^2+y^2)]\ dxdy\right]^2\right\} \quad ..(9)$$

Results and Discussion

Twenty point Gauss quadrature were used [5] to evaluate the integral in eq (8). The results thus obtained for the normalized intensity are given in table (1), it shows the available values of parameter (a) [consider area of aperture (A) which always equals to (π)] and asymmetric intensity distribution for it, when (a=1) (circular aperture) and (1.5 , 2 , 2.5) for elliptical aperture.

Fig (2) shows how the diffraction pattern for a circular aperture varies with a pattern of elliptical aperture, so that the radius of the intensity distribution for elliptical aperture is smaller than for a circular aperture. Thus the central peak of the intensity is sharper for elliptical aperture, so the resolution of the elliptical aperture should be better than of the circular aperture.

Fig. (3) shows the focal shift for a circular aperture which varies with an elliptical aperture, that the depth of focus for a circular aperture is better than for the elliptical aperture.

Finally; fig (4) shows the relationship between eccentricity (e) for ellipse aperture and the best of focal depth which gives (80%) of intensity, that the tolerance of focal shift was better when decrease aperture flatting (approaches to a circular aperture), that means the eccentricity approaches to zero

References

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دراسة منظمة بصرية تحتوي فتحة بيضوية باستخدام دالة الانتشار النقطية

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الخلاصة

تمت دراسة منظومة بصرية تحتوي فتحة بيضوية باستخدام دالة الانتشار النقطية ، وتمت مقارنة النتائج مع منظومة بصرية ذي فتحة دائرية . يتعامل هذا البحث مع دراسة نظرية لتوزيع الشدة في مستوى الصورة ، واشتقاق صيغة خاصة للفتحة البيضوية باستخدام تقنية دالة البؤبؤ ، تعامل البحث مع منظومة بصرية محددة بالحيود فقط (النظام المثالي) ، ومنظومة ذو خطأ بؤري .

إن الاختلاف المركزي للفتحة البيضوية هو دالة لمقدار تفلطح الفتحة ، ومنها وجدت علاقة بيانية تربط عامل الاختلاف المركزي (e) وسماحية الخطأ البؤري التي تعطي (80%) من الشدة على الأقل.

Table (1): Intensity distribution in a circular and elliptical aperture (PSF)

z	a=1(circle)	a=1.5	a=2	a=2.5
-10	0.0000768	0.000752	0.000046	0.000097
-9	0.0029791	0.000032	0.000432	0.000015
-8	0.0034486	0.001381	0.00013	0.000046
-7	0.0000016	0.000223	0.000366	0.000345
-6	0.0084941	0.002979	0.001381	0.000752
-5	0.0171527	0.001306	0.000076	0.000697
-4	0.0010863	0.008494	0.003449	0.000076
-3	0.0511216	0.010533	0.008494	0.001306
-2	0.3326821	0.051122	0.001086	0.017153
-1	0.7746855	0.553501	0.332682	0.158195
0	1.0001215	1.000122	1.000122	1.000122
1	0.7746855	0.553501	0.332682	0.158195
2	0.3326821	0.051122	0.001086	0.017153
3	0.0511216	0.010533	0.008494	0.001306
4	0.0010863	0.008494	0.003449	0.000076
5	0.0171527	0.001306	0.000076	0.000697
6	0.0084941	0.002979	0.001381	0.000752
7	0.0000016	0.000223	0.000366	0.000345
8	0.0034486	0.001381	0.00013	0.000046
9	0.0029791	0.000032	0.000432	0.000015
10	0.0000768	0.000752	0.000046	0.000097

Table (2): Axial intensity for circular and elliptical aperture

w20	a=1(circle)	a=1.5	a=2	a=2.5
0	1.000122	1.000122	1.000122	1.000122
0.2	0.875232	0.636018	0.255324	0.161672
0.4	0.572814	0.200478	0.137527	0.085907
0.6	0.254552	0.125858	0.083003	0.05337
0.8	0.054673	0.080299	0.054111	0.036835
1	0	0.042107	0.039839	0.030017
1.2	0.024324	0.029903	0.030451	0.059813
1.4	0.046765	0.013408	0.032086	0.064329
1.6	0.035791	0.009179	0.021195	0.051075
1.8	0.010794	0.004254	0.009275	0.002354
2	0	0.00365	0.032252	0.001014

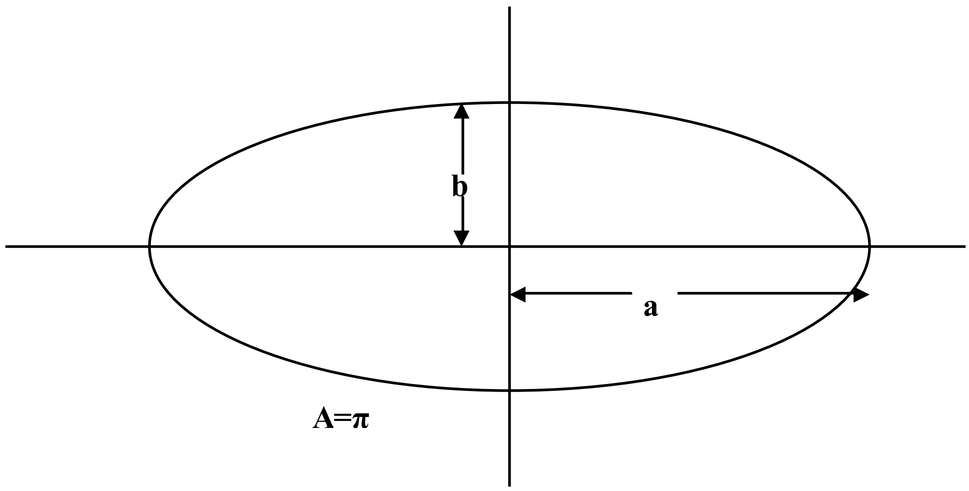

Fig.(1): Elliptical aperture

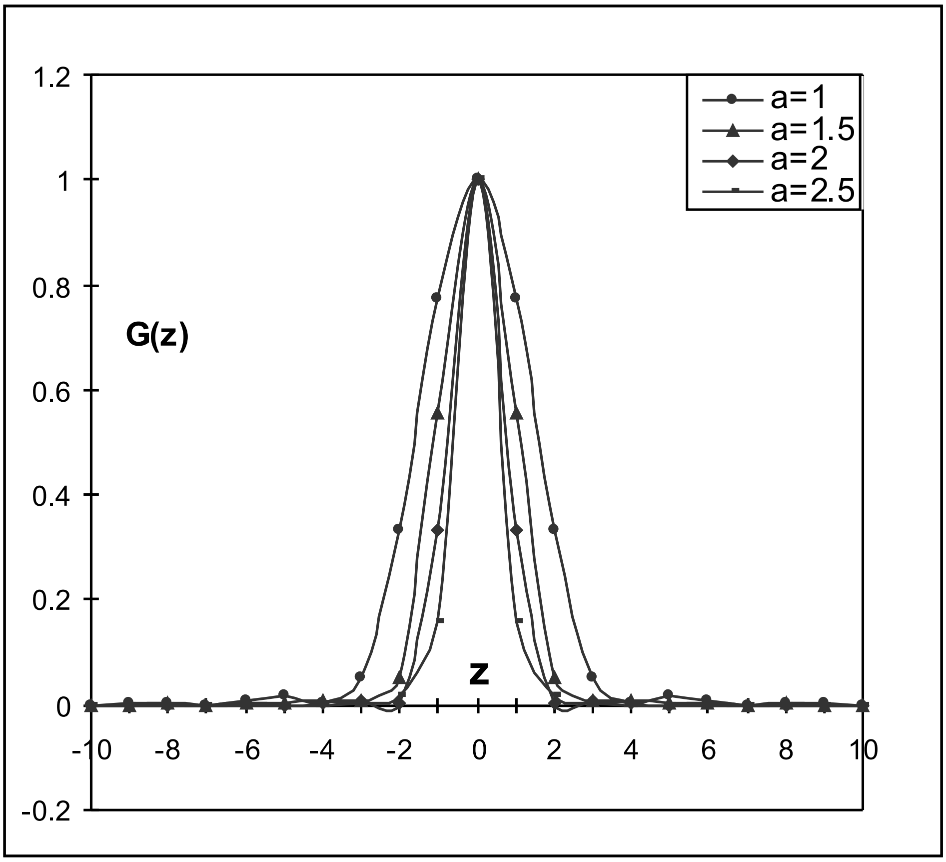

Fig. (2): Intensity distribution in a circular and elliptical aperture (PSF)

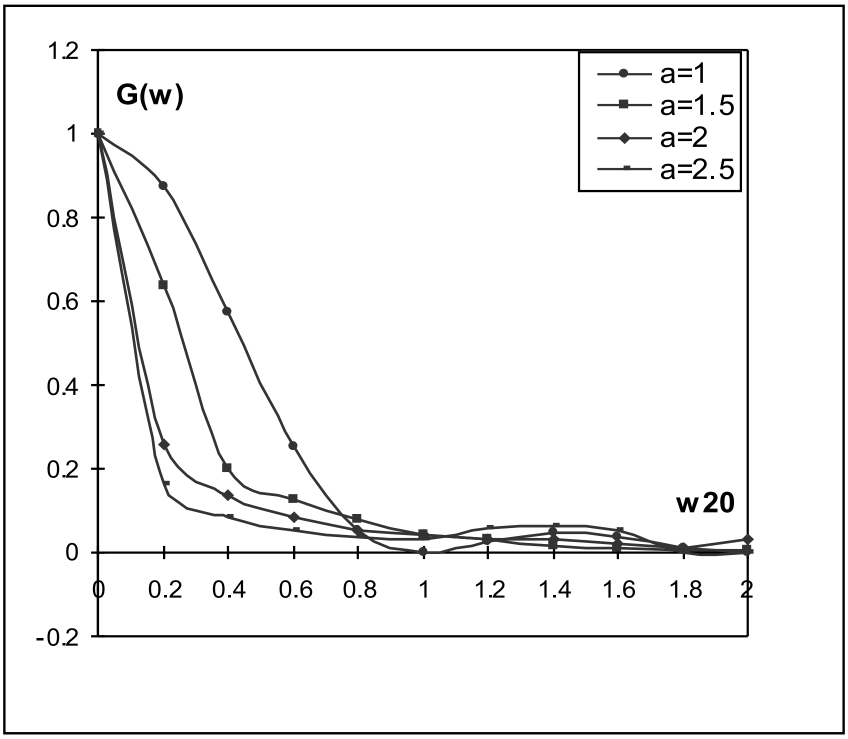

Fig. (3): Axial intensity for a circular and elliptical aperture

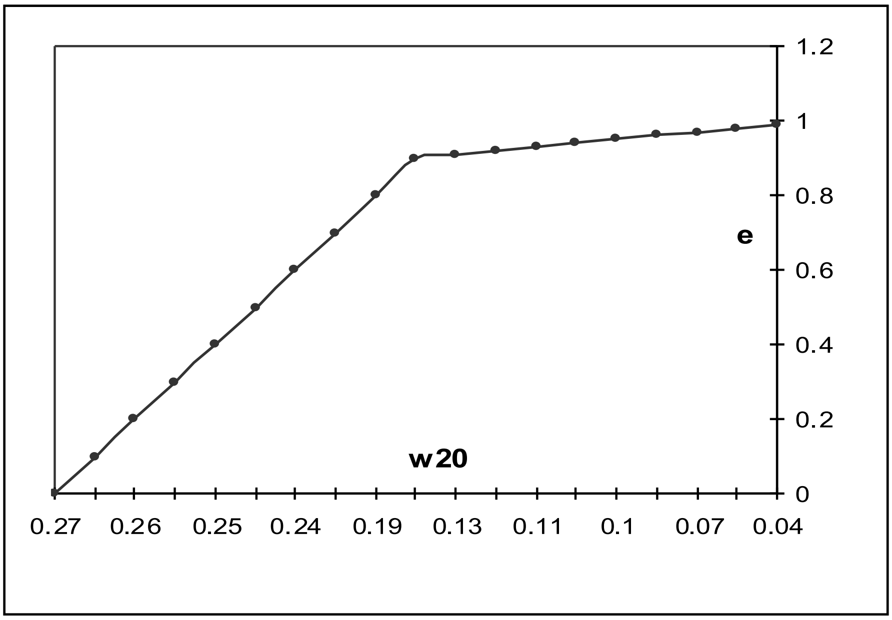

Fig. (4): Relationship between eccentricity (e) and the best focal depth which gives (80%) of intensity