# Analytic Solutions For Integro-Differential Inequalities Using Modified Adomian Decomposition Method

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## Abstract

This paper applies the Modified Adomian Decomposition Method (MADM) for solving Integro-Differential Inequality, this method is one of effective to construct analytic approximate solutions for linear and nonlinear integro-differential inequalities without solving many integrals and transformed or discretization. Several examples are presented, the analytic results show that this method is a promising and powerful for solving these problems.
**Keywords:** Modified Adomian Decomposition Method ,Linear and Nonlinear integro-Differential inequalities.

# Introduction

In the last years , the back of growing interest in the Integro Differential Inequalities (IDIs). IDIs performance a significant tool in various affiliates concerning nonlinear and linear efficacious analysis with applications in the theorem of mathematics, engineering, physics, chemistry, astronomy, biology, electrostatics, potential theory and economics[1]. The IDIs of high order appear in mathematical problems, engineering sciences and applied, astrophysics, solid state physics, astronomy, beam theory , fluid dynamics . To solve analytically so approximate solution is required to solve it easy and quickly because the analytic solutions are usually very difficult. The functional inequalities influence in real-life problems mathematical aspects, same partial or ordinary differential inequalities, stochastic inequalities, IDIs and integral. Many kinds of physical phenomena of mathematical formulation contain aspects IDIs, these inequalities appear from time to time in biological models, chemical kinetics and fluid dynamics[2]. The nonlinear essential problems are still difficult to solve either theoretically or numerically. Recently, the search for more efficient and better perform resolution ways for determining the solution, accurate or approximate , numerical or analytical, nonlinear problems[3,4,5], the analytical method called the Adomian decomposition method (ADM) known by Adomian. This method is a promising and powerful tool for solving this problems stochastic problems and nonlinear physical problems[6], the importance of this research is to give a comparatives study to find out the accurate result of the MADM in solving nonlinear and linear IDIs. This basic thing of this method can give us away for how to solve nonlinear and linear IDIs.

# Integro-Differential Inequalities

The IDIs of theory and application are very essential in important role .We can see them in many fields: engineering sciences. biological phenomena and physical in which it is very important to know how to deal with real life problems. The benefit of the IDIs advantage to know the fundamental problems and solve it in many methods.

# Remark

The IDIs is called ordinary if the derivative is taken with respect to one variable . Other IDIs**,** on the contrary**,** contain derivatives with respect to different variables are called partial integro- differential inequalities, which often occur in the mathematical physics, in the following sections the classification of the IDI is giving [4,7] **.**

# Ordinary Integro- Differential Inequalities(OIDIs):

The (OIDI) is an IDI such that the obscure function is based on one independent variable, the (OIDIs) is classified into nonlinear and linear.

## 1. Linear Ordinary Iintegro- Differential Inequalities:

The linear ordinary integro-differential inequality is an IDI where the unknown function depends on a single variable which has one of the general forms:

$$h(x)\frac{\partial f}{\partial y} \leq g(x) + \lambda \int_a^{b(x)} k(x,y)f(y)dy \qquad (1)$$

$$h(x)\frac{\partial f}{\partial y} \geq g(x) + \lambda \int_a^{b(x)} k(x,y)f(y)dy \qquad (2)$$

where k(x,y) known function namely kernel of IDI is given by ineqs, g and h are known function of x.(1,2) and the unknown function f must be determined and a and λ are known parameters. Next , we classified two types of the linear (OIDIs), called Fredholm and Voltera types.

### 1.1 Fredholm linear Ordinary Integro- Differential Inequalities:

The integral operator if the limit of in ineqs. (1,2) does not depend on x i.e. if b(x)=b then ineqs.(1,2) is called Fredholm linear OIDIs. In this case if h(x)=0 then ineqs.(1,2) minimize to the following inequalities:

$$g(x) \leq \lambda \int_a^b k(x,y)f(y)dy \qquad (3)$$

$$g(x) \geq \lambda \int_a^b k(x,y)f(y)dy \qquad (4)$$

which is called the Fredholm linear OIDIs of the first kind. If h(x)=1 in ineqs.(1,2) then ineqs.(1,2) becomes:

$$\frac{\partial f}{\partial y} \leq g(x) + \lambda \int_a^b k(x,y)f(y)dy \qquad (5)$$

$$\frac{\partial f}{\partial y} \geq g(x) + \lambda \int_a^b k(x,y)f(y)dy \qquad (6)$$

which is namely the Fredholm linear OIDIs of the second kind. If g(x)=0 then ineqs.(1) takes the form:

$$h(x)\frac{\partial f}{\partial y} \leq \lambda \int_a^b k(x,y)f(y)dy \qquad (7)$$

$$h(x)\frac{\partial f}{\partial y} \geq \lambda \int_a^b k(x,y)f(y)dy \qquad (8)$$

Which is namely the Fredholm linear OIDIs of the third type.

### 1.2 Volterra linear Ordinary Integro- Differential Inequalities:

If b(x)=x, then ineqs.(1,2) is called the Volterra linear OIDIs .i.e

$$\frac{\partial f}{\partial y} \leq g(x) + \lambda \int_a^x k(x,y)f(y)dy \qquad (9)$$

$$\frac{\partial f}{\partial y} \leq g(x) + \lambda \int_a^x k(x,y)f(y)dy \qquad (10)$$

and such the Fredholm linear OIDIs, the Volterra linear OIDIs can be divided into first, second and third kind.

### 3.2 Nonlinear Ordinary Integro- Differential Inequalities:

The nonlinear OIDIs take one of the general forms:

$$\frac{\partial f}{\partial y} \leq g(x) + \lambda \int_a^{b(x)} k(x,y,f(y))dy \qquad (11)$$

$$\frac{\partial f}{\partial y} \geq g(x) + \lambda \int_a^{b(x)} k\big(x,y,f(y)\big)dy \qquad (12)$$

and such as the linear OIDEs, the nonlinear OIDEs can be classified into Fredholm, Volterra of the first, second and third kind.

## Partial integro- differential inequality(PIDI):

The partial integro- differential inequality (PIDI) is an integro- differential inequality such that the unknown function depends on more than one independent variable like the OIDIs, the partial integro- differential inequalities (PIDIs) is divided into linear and nonlinear.

## 1. Linear partial integro- differential inequalities:

The linear partial integro-differential inequality is an integro-differential inequality where the unknown function depends on more than one variable which has one of the general forms:

$$h(x,y)\frac{\partial f(x,y)}{\partial x} \leq g(x,y) + \lambda \int_a^{b(x)} \int_c^{d(x)} k(x,y,z,m)f(z,m)dzdm \qquad (13)$$

$$h(x,y)\frac{\partial f(x,y)}{\partial x} \geq g(x,y) + \lambda \int_a^{b(x)} \int_c^{d(x)} k(x,y,z,m)f(z,m)dzdm \qquad (14)$$

Also if the upper limits of integral sign (b(x)&d(y))in ineqs.(3) do not depend on x and y respectively, then ineqs.(3) is called Fredholm linear PIDI. Moreover, if b(x)=x and d(y)=y then ineqs. (3) is called Volterra linear PIDI. On the other hand, the Fredholm or Volterra linear PIDIs may be of the first, second or third kind.

## 2 .Nonlinear partial integro- differential inequalities:

The nonlinear PIDIs has one of the general forms :

$$h(x,y)\frac{\partial f(x,y)}{\partial x} \leq g(x,y) + \lambda \int_a^{b(x)} \int_c^{d(x)} k(x,y,z,m,f(z,m))dzdm \qquad (15)$$

$$h(x,y)\frac{\partial f(x,y)}{\partial x} \geq g(x,y) + \lambda \int_a^{b(x)} \int_c^{d(x)} k(x,y,z,m,f(z,m))dzdm \qquad (16)$$

And like the linear PIDIs, the nonlinear PIDIs can be classified into Fredholm, Volterra of the first, second and third kind.

## Remark:

In this study, we attempt to solve the general form with less than or equal using the modified Adomian decomposition method while the general form with greater than or equal can be solved using the same arguments, also we shall solve OIDIs with the attempted method.

## Adomian Decomposition Method(ADM):

The ADM an important analytical method, applied in the vast fields of integro-differential equations.To illustrate procedure, consider the following Volterra integro - differential inequalities of the second type given by

$$L(q(v)) \leq u(v) + \lambda \int_a^v K(v,s)\,(R(q(s)) + N(q(s)))\,ds,\ \lambda \neq 0, \qquad (17)$$

where the kernel $K(v, s)$ and the function u(v) are given real valued functions, $\lambda$ is a parameter, R(q(v)) and $N$(q(v)) are linear and nonlinear operators of q(v) [8],the differential operator L(q(v)) is the highest order derivative in the inequality, respectively. Then, we assume that L is invertible via employing specified conditions and stratify $L^{-1}$ the operator inverse to jointly directions (17) ,we obtain inequality steps:

$$q(v) \leq \mu_0 + L^{-1} u(v) + L^{-1}\left(\lambda \int_a^v K(v,s)\,(R(q(s)) + N(q(s)))\,ds\right),\ \lambda \neq 0, \qquad (18)$$

where $\mu_0$ function appearing for integrating origin idiom from stratifying the specified conditions which are prescribed. And so on ADM permit entry decomposition of q to an infinite series from components [9]:

$$q(v) \leq \sum_{n=0}^{\infty} q_n(v) \qquad (19)$$

Moreover, the ADM identifies the nonlinear term (q(v)) by the decomposition series:

$$N(q)=\sum_{n=0}^{\infty} A_n(v) \tag{20}$$

where $A_n$ is the so-called Adomian polynomials, which can be evaluated by the following formula [10,11]:

$$A_n = \frac{1}{n!}\frac{d}{d\lambda^n}^{n} N\left(\sum_{i=0}^{n} \lambda^i q_i\right) \quad , \qquad n = 0, 1, 2, \ldots . \tag{21}$$

Substituting (19) and (20) into both sides of (18) gives:

$$\sum_{n=0}^{\infty} q_n(v) \le \mu_{0+}L^{-1}\, u(v)+L^{-1}(\lambda\int_a^v K(v,s)\,[R\left(\sum_{n=0}^{\infty} q_n\,(s)\right) + \sum_{n=0}^{\infty} A_n\,(s)]\; ds), \tag{22}$$

The components various $q_n$ solution q can be facilely via employing recursive relation:

$$Q_0 \le \mu_{0+}\, L^{-1}\, u(v),$$

$$Q_{i+1} \le L^{-1}\, (\lambda\int_a^v K(v,s)\,[R\left(\sum_{n=0}^{\infty} q_i\,(s)\right) + \sum_{n=0}^{\infty} A_i\,(s)]\; ds), \text{ for } i \ge 0\,, \tag{23}$$

As a consequence, so few components can be written at first:

$$Q_0 \le \mu_{0+}\, L^{-1}\, u(v),$$

$$Q_1 \le L^{-1}\, (\int_a^x K(v,s)\,[R(\sum_{n=0}^{\infty} q_0\,(s)\,) + \sum_{n=0}^{\infty} A_0\,(s)]\; ds), \tag{24}$$

$$Q_2 \le L^{-1}\, (\lambda\int_a^x K(v,s)\,[R(\sum_{n=0}^{\infty}\, q_1\,(s)\,) + \sum_{n=0}^{\infty} A_1\,(s)]\; ds),$$

where the Adomian polynomial can be evaluated by (4), specify components $Q_n$ exists , $n \ge 0$, In series form the solution q follows instantly. As it is mentioned before, this series can be summarized to provide a solution in closed form .

## Modified Adomian Decomposition Method:

It clarifies how the modified on the assumption that the function T is possible to write as:

$$T \le \mu_{0+}\, E^{-1}\, u(v)\,, \tag{25}$$

components of $Q_n$ are specified via using the following relation:

$$Q_0 \le T\,, \tag{26}$$

$$Q_{i+1} \le E^{-1}\, (\int_a^x K(v,s)\,[R(\sum_{n=0}^{\infty} q_i\,(s)\,) + \sum_{n=0}^{\infty} A_i\,(s)]\; ds), \text{ for } i \ge 0 \tag{27}$$

About the above equations, note that the component $Q_0$ is particular by the Function T . The modified Adomian decomposition method will minimize the volume of calculations, we split function T into two parts, $T_0$ and $T_1$. As follows the function would be:

$$T \le T_0 + T_1 \tag{28}$$

Beneath this supposition ,we observe that incommodious different for components $Q_0$ and $Q_1$, where $T_0$ allocated to $Q_0$ and in (26) $T_1$ is combined with the other terms to allocate $Q_1$. As follows the modified recursive algorithm would be:

$$\left.\begin{aligned} q_0 &\le T_0\,, \\ q_1 &\le T_1 + E^{-1}(\lambda\int_a^v K(v,s)\,[R(\sum_{n=0}^{\infty}\, q_0\,(s)\,) + \sum_{n=0}^{\infty} A_0\,(s)]ds), \\ q_{i+1} &\le E^{-1}(\,\lambda\int_a^v K(v,s)\,[R(\sum_{n=0}^{\infty}\, q_i\,(s)\,) + \sum_{n=0}^{\infty} A_i\,(s)]ds), \end{aligned}\right\} \tag{29}$$

for $i \ge 1$.

However , the nonlinear term T(q) represents infinite series, It is called Adomian polynomials $A_n$ presented in the form:

$$T(q) \le \sum_{n=0}^{\infty} A\,n\,(q_0, q_1, q_2, \dots, q_n) \qquad (30)$$

Adomian polynomials of nonlinear operator T(q) are needed several rules to follow[6]:

$$A_0 \le T(q_0)\,,$$

$$A_1 \le q_1\, T'(q_0)\,, \qquad (31)$$

$$A_2 \le q_2\, T'(q_0) + \frac{1}{2!} q_1^2\, T''(q_0)\,,$$

and so on; then substituting (31) into (30) gives :

$$T(q) \le A_0 + A_1 + A_2 + \dots \qquad (32)$$

To illustrate the effectiveness of the method, we presented several examples in the next section.

## Some Examples about Linear and Nonlinear Integro- Differential Inequalities:-

To demonstrate the accuracy and power of this method, we give several examples in this section, four examples for the integro-differential inequalities with Initial condition and two examples for the integro-differential inequalities with boundary condition.

**EXAMPLE 1 : -**

Consider the second order-linear IDI:

$$j''(z)+zj'-zj \le e^{z} - 2\sin(z)+\sin(z)\int_{-1}^{1} e^{-r} j(r)\,dr \qquad (33)$$

with initial condition : $j(0)\le 1$ , $j'(0)\le 1$ . (34)

Equation (33) can recast in operator form as follows:

$$Ej(z) \le e^{z} - zj' + zj - 2\sin(z)+\sin(z)\int_{-1}^{1} e^{-r} j(r)\,dr \qquad (35)$$

we obtain the following equation via operating with twofold integral operator $E^{-1}$ on (35) with the initial condition at z= 0 :

$$j(z) \le 1+z+E^{-1}(e^{z} - zj' + zj - 2\sin(z))+E^{-1}\left(\sin(z)\int_{-1}^{1} e^{-r} j(r)\,dr\right) \qquad (36)$$

the decomposition series replace in (19) for j(z) into (36) yields:

$$\sum_{n=0}^{\infty} j_n(z) \le 1+z+E^{-1}(e^{z} - zj' + zj - 2\sin(z))+E^{-1}\left(\sin(z)\int_{-1}^{1} e^{-r} j(r)\,dr\right) \qquad (37)$$

subsequently, we split the terms into two parts $j_0(z)$ and $j_1(z)$ which are assigned , that are not included under $E^{-1}$ in (37). the following repetition relation, we can obtain it:

$$j_0(z) \le e^{z},$$

$$j_1(z) \le 2\sin z - 2z + +L^{-1}\left(\sin(z)\int_{-1}^{1} e^{-r} j(r)\,dr\right). \qquad (38)$$

On the two-term approximant $\varphi_2$, we use the boundary conditions in (34) at z =0 where :

$$\varphi_2 \le \sum_{k=}^{1} j_k\,, \qquad (39)$$

To integrate these inequalities, we use Matlab which gives :

$$J(z) \leq e^{z}$$

**EXAMPLE 2 :-**

The second-nonlinear IDI:

$$j''(z) \leq \sinh(z)+z-\int_0^1 z(\cosh^2(r)-j^2(r))dr \qquad (40)$$

with initial condition : $j(0) \leq 0$ , $j'(0) \leq 1$ . (41)

Equation (40) can recast in operator form as follows:

$$Ej(z) \leq \sinh(z)+z-\int_0^1 z(\cosh^2(r)-j^2(r))dr \qquad (42)$$

we obtain the following equation via operating with twofold integral operator $E^{-1}$ on (42) with the initial condition at z= 0 :

$$j(z) \leq z+E^{-1}(\sinh(z)+z)-E^{-1}(\int_0^1 z(\cosh^2(r)-j^2(r))dr) \qquad (43)$$

the decomposition series replace in (19) for j(z) and the polynomials series (20) into (43) yields:

$$\sum_{n=0}^{\infty} j_n(z) \leq z+E^{-1}(\sinh(z)+z)-E^{-1}(\int_0^1 z(\cosh^2(r)-\sum_{n=0}^{\infty} A_n(r))dr) \qquad (44)$$

subsequently, we split the terms into two parts $j_0(z)$ and $j_1(z)$ which are assigned , that are not included under $E^{-1}$ in (44). the following repetition relation we can obtain it:

$$j_0(z) \leq \sinh(z),$$

$$j_1(z) \leq (\frac{z^3}{6}-E^{-1}(\int_0^1 z(\cosh^2(r)-A_0(r))dr) \qquad (45)$$

$$j_{k+1} \leq E^{-1}(j_k), \qquad \text{for } k \geq 1.$$

On the two-term approximant $\varphi_2$, we use the boundary conditions in (41) at z = 0 where:

$$\varphi_2 \leq \sum_{k=}^{1} j_k , \qquad (46)$$

To integrate these inequalities, we use Matlab which gives :

$$J(z) \leq \sinh(z).$$

**EXAMPLE 3 :-**

The third-order linear IDI:

$$j'''(z) \leq \sin(z)-z-\int_0^{\frac{\pi}{2}} zrj'(r))dr \qquad (47)$$

with initial condition : $j(0) \leq 1$ , $j'(0) \leq 0$, $j''(0) \leq -1$ . (48)

Equation (47) can recast in operator form as follows:

$$Ej(z) \leq \sin(z)-z-\int_0^{\frac{\pi}{2}} zrj'(r))dr \qquad (49)$$

we obtain the following equation via operating with threefold integral operator $E^{-1}$ on (49) with the initial condition at z= 0 :

$$j(z) \leq 1-\frac{z^2}{2}+E^{-1}(\sin(z)-z)-E^{-1}(\int_0^{\frac{\pi}{2}} zrj'(r))dr) \qquad (50)$$

subsequently, we split the terms into two parts $j_0(z)$ and $j_1(z)$ which are assigned , that are not included under $E^{-1}$ in (50). The following repetition relation we can obtain it:

$$j_0(z) \leq \cos(z),$$

$$j_1(z) \leq -\frac{z^4}{24}-E^{-1}(\int_0^{\frac{\pi}{2}} zrj'(r))dr) \qquad (51)$$

On the two-term approximant $\varphi_2$, we use the boundary conditions in (48) at z =0 where:

$$\varphi_2 \leq \sum_{k=}^{1} j_k, \qquad (52)$$

To integrate these inequalities, we using Matlab which gives :

$$J(z) \leq \cos(z).$$

**EXAMPLE 4 :-**

The fourth-order linear IDI:

$$j^{iv}(z) \leq z(1+e^z)+ 3e^z + j(z) - \int_0^z j(r)dr \qquad (53)$$

Boundary condition : $j(0)\leq 1$ , $j'(0)\leq 1$ , $j(1)\leq 1+e$ , $j'(1)\leq 2e$ (54)

Exact solution :- $J(z)\leq 1+ze^z$

Equation (53) can recast in operator form as follows:

$$Ej(z) \leq z(1+e^z)+ 3e^z + j(z) - \int_0^z j(r)dr \qquad (55)$$

we obtain the following equation via operating with fourfold integral operator $E^{-1}$ on (55) with the boundary condition at z= 0 :

$$j(z) \leq 1 + z + \frac{M}{2!}z^2 + \frac{N}{3!}z^3 + E^{-1}(z(1+e^z)+ 3e^z) + L^{-1}(j(z))E^{-1}\left(\int_0^z j(r)dr\right) \qquad (56)$$

subsequently, specify the constants:

$$j''(0) = M, \; j'''(0) = N.$$

replace the series decomposition (19) for j(z) into (56) yields:

$$\sum_{n=0}^{\infty} j_n(z) \leq 1 + z + \frac{M}{2!}z^2 + \frac{N}{3!}z^3 + E^{-1}(z(1+e^z)+3e^z)+L^{-1}(j(z)) - E^{-1}\left(\int_0^z j(r)dr\right) \qquad (57)$$

subsequently, we split the terms into two parts $j_0(z)$ and $j_1(z)$ which are assigned , that are not included under $E^{-1}$ in (57). the following repetition relation we can obtain it:

$$j_0(z) = (1+z),$$

$$j_1(z) \leq \frac{M}{2!}z^2 + \frac{N}{3!}z^3 + E^{-1}(z(1+e^z)+ 3e^z) + L^{-1}(j(z)) - E^{-1}\left(\int_0^z j(r)dr\right) \qquad (58)$$

On the two-term approximant $\varphi_2$,we use the boundary conditions in (54) at z =1 to determine the constants M and N , where:

$$\varphi_2 \leq \sum_k^1 j_k, \qquad (59)$$

The coefficients M and N , were obtained by using Matlab with boundary conditions at z=1 in (54) given:

$$M=1.981460647, \; N= 3.073642363. \qquad (60)$$

As follows we get the series solution:

$$J(z) \leq (z+0.008333333333z^4 \times (z+5) - 0.001388888889z^5(z+6)+e^z(z-1)+ 0.4907303236z^2+0.1789403938z^3+0.008333333333z^5+2).$$

**EXAMPLE 5 :-**

The fourth-order nonlinear IDI:

$$j^{iv}(z) \leq 1+\int_0^z e^{-z} j^2(r)dr \qquad (61)$$

Boundary condition: $j(0)\leq 1$ , $j'(0)\leq 1$ , $j(1)\leq e$ , $j'(1)\leq e$ (62)

exact solution :- $J(z)=e^z$

Equation (62) can recast in operator form as follows:

$$Ej(z) \leq 1+\int_0^z e^{-z} j^2(r)dr \qquad (63)$$

we obtain the following equation via operating with sevenfold integral operator $E^{-1}$ on (63) with the boundary condition at z= 0 :

$$j(z) \leq 1+z+\frac{A}{2!}z^2+\frac{B}{3!}z^3+E^{-1}(1)+E^{-1}\left(\int_0^z e^{-z} j^2(r)dr\right) \qquad (64)$$

subsequently, specify the constants:

$$j''(0) = M \quad j'''(0) = N\,.$$

Substituting the decomposition series (19) for j(z) and the series of polynomials (20) into (64) yields:

$$\sum_{n=0}^{\infty} j_n(z) \leq 1+z+\frac{M}{2!}z^2+\frac{N}{3!}z^3+L^{-1}(1)+E^{-1}\left(\int_0^z e^{-z}\sum_{n=0}^{\infty} A_n(r)\,dr\right) \qquad (65)$$

Then, we split the terms into two parts which are assigned to $j_0(z)$ and $j_1(z)$ that are not included under $E^{-1}$ in (65). We can obtain the following recursive relation:

$$j_0(z) \leq 1\ ,$$

$$j_1(z) \leq z+\frac{M}{2!}z^2+\frac{N}{3!}z^3+E^{-1}(1)+L^{-1}\left(\int_0^z e^{-z}A_0(r)\,dr\right) \qquad (66)$$

$$j_{e+1} \leq E^{-1}(j_e), \qquad \text{for } e \geq 1.$$

To determine the constants M and N ,we use the boundary conditions in (62) at z =1 on the two-term approximant $\varphi_2$, where:

$$\varphi_2 \leq \sum_k^1 j_k\,, \qquad (67)$$

The coefficients M and N , were obtained by using Matlab with boundary conditions at z=1 in (62) given:

$$M= 0.9770418826547086\ ,\ N= 1.092182087646879\ . \qquad (68)$$

As follows we get the series solution:

$$J(z) \leq (4z+4/e^z+ z/ e^z -3455245347068909z^2/6755399441055744 + 4711175235158855z^3/13510798882111488 + z^4/24-3)\ .$$

## Some Applications about (IDI):

To illustrate our study we present the following three applications

### 1. The movement process (LIDI):

1. U(z,t) the density of a population is at position z and time t.

2. At rate G, individuals move to a new position s instantaneously.

3. H(u-v) is the proportion individuals moving from v to u.

$$\partial_e M \leq G\int_{\mathbb{R}} H(u-v)M(v,e)dv - GM$$

### 2 . Position jump process or ( kangaroo process):

1. An individual starts at position l and time v.

2. He waits time an exponentially-distributed( with parameter B).

3. … then jumps to a new location w that is governed by the distribution u(l-w).

$$\partial_v R \leq D \int_{\mathbb{R}} u(l-w)R(w,v)dw - BR$$

**3. Distributed infectives:**

$$\partial_z S \leq -\beta IS - DS + D\int_{\Omega} v(r-g)S(g,z)dg$$

$$\partial_z I \leq \beta IS - DI + D\int_{\Omega} v(r-g)I(g,z)dg$$

1. The infection rate is $\beta$.

2. The dispersal rate is D.

3. The kernel dispersal distribution k(u) .

4. Assumptions: K= S+I is the constant and $\Omega = \mathbb{R}$.

$$\partial_z I \leq \beta I(K-I) - DI + D\int_{\mathbb{R}} v(r-g)I(g,z)dg$$

## Conclusion

The main idea of this paper was to give simple method for solving the integro-differential inequalities(IDIs). We applied a reliable modification of adomian decomposition method for IDIs. The analytic results show that the present method provides highly accurate analytical solutions for solving these types of equations.

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**Table (1) Comparison between J(z) and MADM of EX(1)**

| z | J(z) | MADM | error MADM |
|---|---|---|---|
| 0 | 1.000000000000000 | 1.000000000000000 | 0 |
| 0.1 | 1.105170918075648 | 1.105170918075648 | 0 |
| 0.2 | 1.221402758160170 | 1.221402758160170 | 0 |
| 0.3 | 1.349858807576003 | 1.349858807576003 | 0 |
| 0.4 | 1.491824697641270 | 1.491824697641270 | 0 |
| 0.5 | 1.648721270700128 | 1.648721270700128 | 0 |
| 0.6 | 1.822118800390509 | 1.822118800390509 | 0 |
| 0.7 | 2.013752707470477 | 2.013752707470477 | 0 |
| 0.8 | 2.225540928492468 | 2.225540928492468 | 0 |
| 0.9 | 2.459603111156950 | 2.459603111156950 | 0 |
| 1 | 2.718281828459046 | 2.718281828459046 | 0 |

**Table (2) Comparison between J(z) and MADM of EX (2)**

| z | J(z) | MADM | error MADM |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0.1 | 0.100166750019844 | 0.100166750019844 | 0 |
| 0.2 | 0.201336002541094 | 0.201336002541094 | 0 |
| 0.3 | 0.304520293447143 | 0.304520293447143 | 0 |
| 0.4 | 0.410752325802816 | 0.410752325802816 | 0 |
| 0.5 | 0.521095305493747 | 0.521095305493747 | 0 |
| 0.6 | 0.636653582148241 | 0.636653582148241 | 0 |
| 0.7 | 0.758583701839533 | 0.758583701839533 | 0 |
| 0.8 | 0.888105982187623 | 0.888105982187623 | 0 |
| 0.9 | 1.026516725708175 | 1.026516725708175 | 0 |
| 1 | 1.175201193643801 | 1.175201193643801 | 0 |

**Table (3) Comparison between J(z) and MADM of EX(3)**

| z | J(z) | MADM | error MADM |
|---|---|---|---|
| 0 | 1 | 1 | 0 |
| 0.1 | 0.995004165278026 | 0.995004165278026 | 0 |
| 0.2 | 0.980066577841242 | 0.980066577841242 | 0 |
| 0.3 | 0.955336489125606 | 0.955336489125606 | 0 |
| 0.4 | 0.921060994002885 | 0.921060994002885 | 0 |
| 0.5 | 0.877582561890373 | 0.877582561890373 | 0 |
| 0.6 | 0.825335614909678 | 0.825335614909678 | 0 |
| 0.7 | 0.764842187284489 | 0.764842187284489 | 0 |
| 0.8 | 0.696706709347165 | 0.696706709347165 | 0 |
| 0.9 | 0.621609968270664 | 0.621609968270664 | 0 |
| 1 | 0.540302305868140 | 0.540302305868140 | 0 |

**Table (4) Comparison between J(z) and MADM of EX(4)**

| z | J(z) | MADM | error MADM |
|---|---|---|---|
| 0 | 1.000000000000000 | 1.000000000000000 | 0 |
| 0.1 | 1.110517091807565 | 1.110436665972639 | 8.0425834926E-5 |
| 0.2 | 1.244280551632034 | 1.244007774009804 | 2.72777622230E-4 |
| 0.3 | 1.404957642272801 | 1.404452691951032 | 5.04950321769E-4 |
| 0.4 | 1.596729879056508 | 1.596020529500754 | 7.09349555755E-4 |
| 0.5 | 1.824360635350064 | 1.823532376710967 | 8.28258639097E-4 |
| 0.6 | 2.093271280234305 | 2.092449721387156 | 8.21558847150E-4 |
| 0.7 | 2.409626895229334 | 2.408949949987269 | 6.76945242065E-4 |
| 0.8 | 2.780432742793974 | 2.780009947446942 | 4.22795347032E-4 |
| 0.9 | 3.213642800041255 | 3.213498935541315 | 1.43864499941E-4 |
| 1 | 3.718281828459046 | 3.718281828459046 | 0 |

**Table (5) Comparison between J(z) and MADM of EX(5)**

| z | J(z) | MADM | error MADM |
|---|---|---|---|
| 0 | 1.000000000000000 | 1.000000000000000 | 0 |
| 0.1 | 1.105170918075648 | 1.105071487041982 | 9.9431033665E-5 |
| 0.2 | 1.221402758160170 | 1.221066243364148 | 3.36514796022E-4 |
| 0.3 | 1.349858807576003 | 1.349237553045258 | 6.21254530745E-4 |
| 0.4 | 1.491824697641270 | 1.490954828770756 | 8.69868870514E-4 |
| 0.5 | 1.648721270700128 | 1.647709497531333 | 1.011773168795E-3 |
| 0.6 | 1.822118800390509 | 1.821119620065656 | 9.99180324853E-4 |
| 0.7 | 2.013752707470477 | 2.012933431747174 | 8.19275723302E-4 |
| 0.8 | 2.225540928492468 | 2.225031968358038 | 5.08960134430E-4 |
| 0.9 | 2.459603111156950 | 2.459430918853188 | 1.72192303762E-4 |
| 1 | 2.718281828459046 | 2.718281828459046 | 0 |

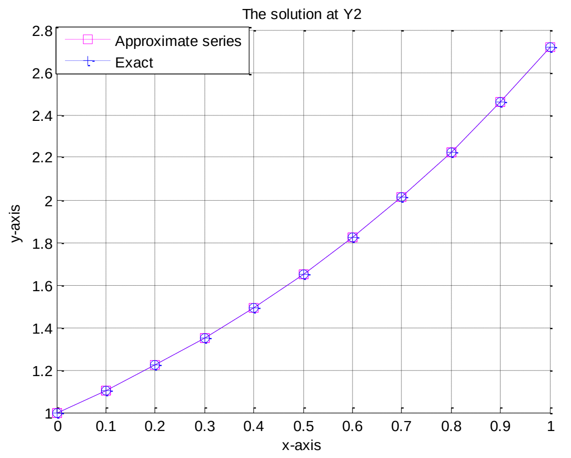


**Figure (1) Comparison between J(z) and MADM of EX(1)**

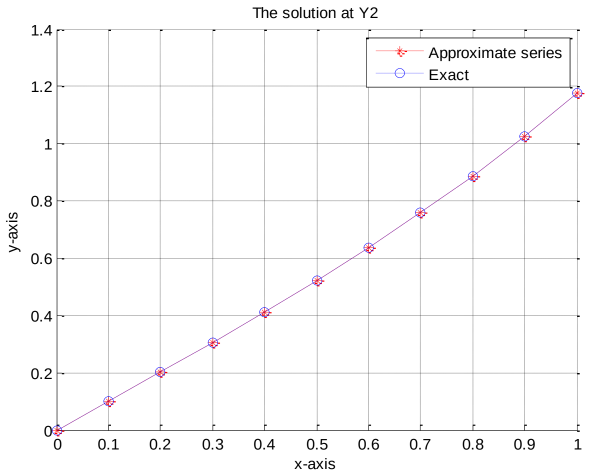


**Figure (2) Comparison between J(z) and MADM of EX(2)**

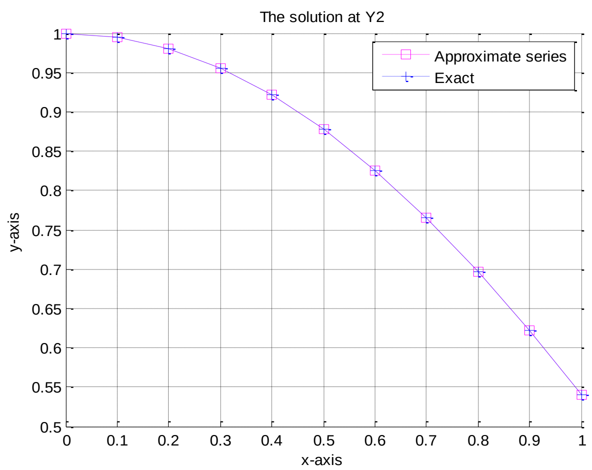


**Figure (3) Comparison between J(z) and MADM of EX(3**

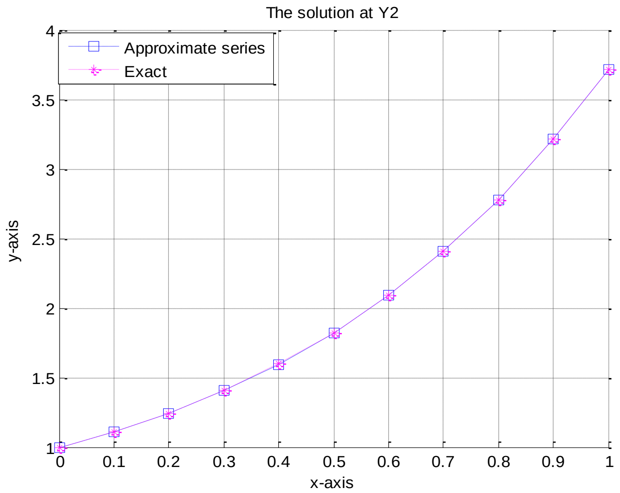


**Figure (4) Comparison between J(z) and MADM of EX(4).**

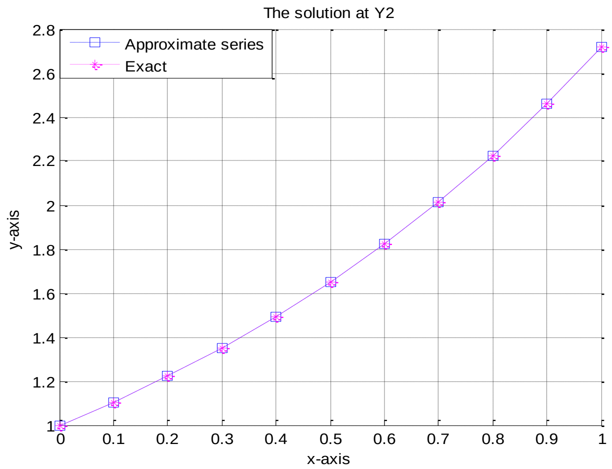


**Figure (5) Comparison between J(z) and MADM of EX(5)**

# الحلول التحليلية للمتراجحات التفاضلية التكاملية بطريقة ادومين التركيبية المطورة

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**استلم في:2/تشرين الثاني/2016 قبل في:18/كانون الأول/2016**

## الخلاصة

يطبق هذا البحث طريقة ادومين التركيبية المطورة لحل المتراجحات التفاضلية التكاملية وهي واحدة من الطرائق الفعالة لتكوين الحلول التقريبية التحليلية لحل المتراجحات التفاضلية التكاملية الخطية وغير الخطية دون حل الكثير من التكاملات والتحويلات.وقدمنا العديد من الامثلة والنتائج اثبتت دقة وكفاءة الطريقة وسهولة الاداء لحل هذه المسائل.

**الكلمات المفتاحية**: طريقة ادومين التركيبية المطورة ، المتراجحات التفاضلية التكاملية الخطية وغير الخطية.