التنبؤ من شهر الى شهر حتى
N من السنين لتخطيط مصنع انتاجي

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الخلاصة

هذا البحث يوفر طريقة تنبؤ شهري لتخطيط الانتاج، الخزين، القوى العاملة، المبيعات والاسعار حتى N من السنين. كل القرارات الشهرية سوف تعتمد على القرارات للشهر السابق مع الأخذ بالأعتبار تنبوءات الطالب المستقبلي. مدير المصنع يستطيع تشغيل البرنامج في أي شهر من السنة، هذه الطريقة انجزت بتقنية برمجة الحاسوب لتعظيم الارباح

Month – to – Month Until N Years Prediction for Planning a Productive Firm

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Abstract

This paper offers a monthly prediction method for planning production, inventory, work-force, sales and prices until N years. Each monthly decision will depend on last month, decisions and take in consideration the future forecasted demand. The manager can run the program in any month within a year. This method is executed by computer programming technique to maximize profits.

1. Introduction

In the field of production, inventory and man power control, H.M.M.S. in their text [1] developed a dynamic model to plan aggregate production rate of a firm and setting the size of its work-force which frequently both complex and difficult. The quality of these decisions can be of great importance to the profitability of an individual company, and when viewed on a national scale these decisions have a significant influence on the efficiency of the economy as a whole. They formalized a quadratic function cost as summation of the following costs:

a. a. Regular payroll cost = $c_1 W_t + c_{13}$ …(1.1)
b. Hiring and Layoff costs = $c_2(W_t - W_{t-1} - c_{11})^2$ …(1.2)
c. Over time and Idle time costs = $c_3 (P_t - c_4 W_t)^2 + c_5 P_t - c_6 W_t + c_{12} P_t W_t$ …(1.3)
d. Inventory related costs = $c_7 [I_t - (c_8 + c_9 S_t)]^2$ …(1.4)

the function subject to the following restriction

$I_t \equiv I_{t-1} + P_t - S_t$ …(1.5)

where

P_t = production rate required in period t.
I_t = level of inventory at the end of period t.
W_t = level of work-force required during period t.
S_t = shipment in month t.
c_1 - c_{13} numerical constants which must be evaluated from historical costs.

By using partial drivitive with this function they have got a linear decision rule for P_t and I_t.

2.Prediction Model

Recently, the model of H.M.M.S. was developed by introducing price variable to influence on the ordering pattern (see [2]), hopefully to move heavy demand away from peak periods and smoothing P_t, I_t and W_t and reducing costs. He used the following inverse price-demand.

$O_t = a - b_t p_t$ …(2.1)

where
O_t = the forecasted order.
a = maximum productive capacity.
b_t = the measure of change in demand per unit change in price.
a = optimal value of labour productivity x initial level of work-force x possible maximum shift ratio x v
i.e. $a = c_4\, W_0 \times N \times v$
where

$$N = \frac{\text{number of shifts possible Per day}}{\text{number of shifts worked Per day}}$$

v = a factor to compensete for unknown components in the productive capacity and for any large forecasted demands in the interval t = 1 to t = 12.
So equation (2.1) becomes:

$$O_t = c_4\, W_0 \times N \times v - b_t\, p_t \qquad \text{...(2.2)}$$

By substituting equation (2.2) in equation (1.4) above yield to

$$\text{Inventory connected costs} = c_7\, [\, I_t - c_8 - c_9\, (c_4 W_0 \times N \times v - b_t\, p_t)]^2 \qquad \text{...(2.3)}$$

As a result of using price variable (p_t) the manufacturer bears the following cost

$$\text{Opportunity cost} = Q \cdot P_c - \sum_{t=1}^{T} p_t\ \ (c_4\, W_0 \times N \times v - b_t\, p_t) \qquad \text{...(2.4)}$$

where
P_c = the (constant) selling price.
Q = the total quantity that would have been sold during the period t = 1 to t = T.
The total cost function is a summation of the equations (1.1), (1.2), (1.3), (2.3) and (2.4)

$$c_T = \sum_{t=1}^{T} \{(c_1 - c_6)W_t + c_{13} + c_2(W_t - W_{t-1} - c_{11})^2 + c_3\, (P_t - c_4\, W_t)^2 + c_5\, P_t + c_{12}\, P_t\, W_t + c_7$$
$$[I_t - c_8 - c_9\, (c_4\, W_0 \times N \times v - b_t\, p_t)]^2 - p_t\, (c_4\, W_0 \times N \times v - b_t\, p_t)\} + Q \cdot P_c \qquad \text{...(2.5)}$$

subject to the following restriction

$$I_t = I_{t-1} + P_t - c_4 W_0 \times N \times v + b_t\, p_t \qquad \text{...(2.6)}$$

By differentiating C_T with respect to W_r, I_r and p_r result a linear decision rules as follows:

$$P_t = g_1 - g_2\, W_{t-1} + g_3\, W_t - g_2\, W_{t+1} \qquad \text{...(2.7)}$$
$$I_t = C_{26(t)} + C_{27(t)}\, W_{t-1} - C_{28(t)}\, W_t + C_{29(t)}\, W_{t+1} - C_{30(t)}\, W_{t+2} \qquad \text{...(2.8)}$$
$$p_t = C_{36(t)} - C_{37(t)}\, W_{t-1} + C_{38(t)}\, W_t - C_{39(t)}\, W_{t+1} + C_{40(t)}\, W_{t+2} \qquad \text{...(2.9)}$$

By substituting the decision variables P_t, I_t and p_t above in equation (2.6) obtain for $t > 1$

$$C_{27(t)}\, W_{t-2} - C_{41(t)}\, W_{t-1} + C_{42(t)}\, W_t - C_{43(t)}\, W_{t+1} + C_{44(t)}\, W_{t+2} =$$
$$c_4 W_0 \times N \times v - C_{45(t)} \qquad \text{...(2.10)}$$

and for $t = 1$

$$C_{47(1)}\, W_1 - C_{48(1)}\, W_2 + C_{49(1)}\, W_3 = c_4 W_0 \times N \times v - I_0 + C_{46(1)}\, W_0 - C_{50(1)} \qquad \text{...(2.11)}$$

From equations 2.10 and 2.11, we have got 12-period of simultaneous linear equations to be solved for optimizing values of W_t and by adding two more unknowns in the end of 12-periods $W_{10} = W_{11} = W_{12}$ and applying the Gauss-Jordan method yields to obtain W_t, t = 1 to 14.
The researcher designed his program (pred.) to compute decision variables for one year from month 1 to 12 as well as cost and profit.
This program is very useful to manager or the planner for a short-time when use it in the end of a year for preparation of budgets for next year as well as offer an indications about the size of a decision variables rules.

3.Month-to-Month until N Years Prediction:

3.1 Long-Term Prediction

As there is an short-term prediction, there is a long term prediction. Harvey, M.Wagner (3,p.383) goes farther than that and says:

Unquestionably most, if not all, decision-making is part of an unending history of actions. Earlier choices have affected the present, current decisions will influence the future, and so on. In this light, all models must be viewed as imbedded in an unbounded horizon.
According to that the cost function (2.5) above becomes

$$\sum_{T=1}^{N}\sum_{t=1}^{12} C_t \qquad \text{...(3. 1.1)}$$

where N = number of years
Also, for the time-series decision quantitative variables P_t, I_t defined in equations (2.7) and [2.8] respectively which were applicable for any *t*, become

$$\sum_{T=1}^{N}\sum_{t=1}^{12} P_t \qquad \text{...(3. 1.2)}$$

$$\sum_{T=1}^{N}\sum_{t=1}^{12} I_t \qquad \text{...(3. 1.3)}$$

3.2 Month-to-Month until N Years Prediction:

The best prediction is when the present prediction is very close to immediately preceding period and predecessor periods.

The shorter the interval between successive reviews and the greater the detail, the more likely are forecasts made by judgment and intuition to be unduly influenced by recent events, [4].

a.The preceding period became real decisions which include inventory (I_{t-1}) and work-force size (W_{t-1}) and these variables would sharluded in present time (t) to predict the decision variables according to the equations (2.6), (2.7), (2.8), (2.9), (2.10), (2.11).

b.For the predecessor periods would share in present period (t) to predict the decision variables when the system of equations (2.10) and (2.11) requires values of forecasted demand for periods (months) t to t + 11, (values of b_t in equation (2.2)). So, to obtain values for the decision variables for one month we would need 12 monthly values of forecasted demand for: 12 month′s predicted values we would need 23 values of forecasted monthly demand: for 24 month′s predicted values we would need 35 values of forecasted demand, and so on.

This method will let the decision variables keep up with forecasted demand throughout planning horizon.

3.3Running the Program in any Month:

The planner knows the prediction is prediction and not always compatible with changes in the market such as actual sales greater than or less than predicted sales yield to actual inventory less than or greater than predicted inventory and influence the work-force size.
The I_t and W_t becomes initial inventory and initial work-force respectively for period t + 1.
So rerun the program from period t + 1 and provide it with new values for I_0 and W_0.
In this case the variable II represents the difference between t = 1 and the new period (month) for example the new period = 7 then II = 7 – 1 = 6.
This variable will be an input variable, in the normal case will be equal to zero, see table (3-3-1).

The program was written which was referred to as the (Pred.1) which was designed to execute the three cases (3-1, 3-2 and 3-3) above. Details of this program are given in section 5 below.

If we give the two programs Pred. and Pred.1 the same input data the results will be compatible in the first period (month) only.

4.Results and Comparisons with H.M.M.S. Model:

To execute the program Pred.1, we need data of H.M.M.S. paint factory which are available in [1] and [2]. But the factor v in equation (2.2) is not available, [2] specified the relationship between this factor and the decision variables as well as with costs and profit. He proved that the increase in value of v yields to the increase in the revenue and profits. So it is easy to let the methods used in this research better than the results of H.M.M.S. but it is not fair to do so. One of the main purposes of H.M.M.S., pred. and this model is to smooth out the time-series representing fluctuations in work-force, production, inventory levels. Work-force smoothing yields to smooth out the other decision variables according to the formulation of equations (2.7), (2.8), (2.9).

Therefore the value of v will be chosen after many running of the program until we get the best smooth for work-force and year after year until N years. Thus the preferred set of v for five years are (0.9,0.9,1,1,1). The output of program is as follows:

a. Three tables for each year, first table for decision variables (P_t,I_t,W_t,p_t) for each month and yearly total of P_t and I_t. Second table is for monthly basic costs and then total for each month and total for each of them in a year. This table is not important to be listed in this research while the second table in b below is a good breviary for the costs. The third table contains the sales, revenue, other cost and profit for each month and their total for a year. These tables will be repeated each year until N years, see tables from (4-1) to (4-10) below.

b. Final results in the end of N years will be three tables, first table is listing the yearly total of inventory, production and sales and their summation in the N years. Second table is listing the yearly total of each kind of cost and their summation in the end of N years. Third table is listing the yearly total of revenue, other cost and profit and their summation in the end of N years, see tables (4-11) to (4-13).
The other cost = production rate × O_C
where O_C = the other cost per unit of production.

c. Comparison with H.M.M.S. Model.
Table (4-14) shows the maximum and minimum for work-force, production rate, inventory rate and sales and also the variation for both models. It is clearly that variation in our model is considerably less than H.M.M.S. model. And this smooting is effective in increasing the profit and reducing costs.

In the real life, the decision maker will choose value for v factor to spcify his productive copacity according to his experience and knowledge in the market, (5,p.400) say, *Predictions require skill, experience, and judgment, not all time series can be successfully predicted.*

5.Main Steps of the Program

The program (Pred.1) is written in general to accept any number of years by changing the input variable I R and provde the program monthly historical demand MSL = (I R + 1) ∗ 12. Execution time is 13 seconds for 5 years planning and consist of 435 programming instructions and statements.

1. variables declaration.
2. Read I R, Alpha, crival, forca.
3. MSL = (I R + 1) ∗ 12.
4. Declaration of dimensions.

5. Read I_0, W_0 and II.
6. From I = 1 to MSL read SL(I).
7. compute G_1, G_2 to G_6 and common terms.
8. K= 1, M = 12 and IYLO = 0.
9. print c_1 to c_{13}, I_0 and W_0.
10. Test for forecasting method to be used

    forca { = 1 12month moving average forecasting subroutine
    = 2 exponential weighted average subroutine
    = 3 forecosted sales equal to actual demands }

    see [6], [7], [8] and [9].
11. Read P_C and SHN(N in equation (2.2)).
12. Yearly loop IY = 1 to I R.
13. Read v.
14. Monthly computations, b_t must be > 0 from equation (2.2).
15. N = 14 and Jmax = N + 1 to book area in the memory for the matrix to build up simultaneous linear equations according to equations (2.10) and (2.11) and applying the Gauss-Jordan method to get W_t, t = 1 to 14 and we select W_t, W_{t+1} and W_{t+2} which are required in equations (2.7), (2.8) and (2.9). This step will be executed 60 times for 5 years, see [10], [11], and [12].
16. From I = K + II to M – 11 + II compute $c_{26} - c_{40}$ and for I > 1 + II compute P_t, I_t and p_t else for I = 1 + II compute P_t, I_t and p_t.
17. From I = K +II to m – 11 + II compute costs for t = 1 and t > 1.
18. From I = K + II to M – 11 + II compute sales, other cost, revenue and profit. Also compute check from equation (2.6) which must equal to zero otherwise there is an error in mathematical operations of this model or in programming of the model.
19. If the reminder of $\frac{K}{12}$ = zero step 20

    Otherwise
    K = K + 1
    M = M + 1
    Go back to step 14
20. print out monthly results within each year
    KS = K – 11 + II
    MS = M – 11
    From I = KS to MS print out the three tables explained in 4.a above.
21. K = K + 1
    M = M + 1
    II = 0
    Go back to step 12 to compute another year.
22. From IY = 1 to IR print table of yearly total for P_t, I_t and S_t.
    And the same for costs and another for revenue, other cost and profit.

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Table:(3-3-1)Decision Variables when Π= 6 and v = 0.9 and the same thing for cost table and profit table while the following years would be as calendar years

Month	Production	Inventory	Work-Force	Prices
7	453	296	83	94.85
8	453	307	85	91.55
9	456	311	86	92.17
10	459	312	87	91.87
11	461	313	88	90.84
12	463	315	88	88.91
	2751.6	1847.7		

$C_1 = 340.0$, $C_2 = 64.3$, $C_3 = 0.20$, $C_4 = 5.67$, $C_5 = 51.2$, $C_6 = 281.0$, $C_7 = 0.0825$,

$C_8 = 320.0$, $C_9 = C_{11} = C_{12} = 0$

$W_0 = 81$ men, $I_0 = 275$ units, $P_c = 100$.

Year 1

Table :(4-1)when v = 0.9

Month	Production	Inventory	Work-Force	Prices
1	452.	301	81	96.5
2	443.	314	81	95.6
3	440	319	81	96.1
4	438	319	81	96.6
5	437	320	80	94.9
6	436	319	80	94.9
7	435	319	80	94.4
8	434	321	80	92.7
9	435	320	80	94.2
10	435	318	80	94.4
11	436	318	81	93.6
12	437	320	81	91.9
Tot.	5256	3807		

Table:(4-2) when v = 0.9

Month	Sales	Revenue	Oth.cost	Profit	Check
1	414	39973.6	2873.435	18776.96	0
2	429	41075.4	2817.999	20654.84	0
3	436	41867.89	2796.725	21261.13	0
4	438	42333.13	2786.251	21546.68	0
5	435	41322.93	2776.146	21357.3	0
6	436	41379.92	2770.126	21434.87	0
7	435	41099.93	2764.673	21424.47	0
8	432	40062.01	2759.876	21277.02	0
9	436	41023.98	2763.275	21423.31	0
10	437	41206.88	2767.50	21491.12	0
11	437	40884.56	2771.607	21449.65	0
12	435	39935.58	2777.427	21314.12	0
Tot.	5200	492165.8	33425.04	253441.5	0

Year 2

Table :(4-3) when v = 0.9

Month	Production	Inventory	Work-Force	Prices
13	439	321	81	92.9
14	443	321	82	94.9
15	446	320	83	97.1
16	451	321	83	98.6
17	456	324	84	99.4
18	462	334	85	101.3
19	474	330	86	122.4
20	483	322	87	135.9
21	488	319	88	138.2
22	491	313	88	143.7
23	491	312	89	135.2
24	490	318	89	124.9
Tot.	5615	3856		

Table :(4-4) when v = 0.9

Month	Sales	Revenue	Oth.cost	Profit	Check
13	438	40690.25	2792.539	21359.43	0
14	443	42032.51	2814.231	21516.23	0
15	447	43379.31	2839.153	21768.89	0
16	450	44415.98	2866.012	21987.49	0
17	452	44896.22	2896.898	21953.22	0
18	452	45826.44	2940.874	21961.52	0
19	479	58572.75	3013.166	21504.37	0
20	490	66637.63	3071.028	40126.72	0
21	491	67926.71	3105.166	41447.71	0
22	498	71515.66	3125.387	45976.2	0
23	492	66518.3	3123.501	39683.23	0
24	484	60402.02	3114.966	32769.11	0
Tot.	5616	652813.8	35702.92	362054.1	0

Year 3

Table : (4-5)when v = 1

Month	Production	Inventory	Work-Force	Prices
25	509	320	91	118.4
26	516	319	92	123.7
27	520	319	93	125.7
28	524	317	93	128.6
29	525	315	94	127.8
30	524	317	94	122.6
31	524	313	94	125.6
32	521	311	94	118.6
33	516	312	94	110.3
34	512	314	94	104.9
35	509	318	94	100.8
36	508	323	94	98.6
Tot.	6208	3798		

Table : (4-6) when v = 1

Month	Sales	Revenue	Oth.cost	Profit	Check
25	507	60034.6	3235.868	30126.08	0
26	517	63931.66	3278.793	33668.24	0
27	521	65442.37	3309.639	35077.97	0
28	526	67627.47	3331.249	37360.88	0
29	527	67364.36	3339.015	37100.73	0
30	522	63999.07	3334.758	33667.88	0
31	529	66436.77	3331.434	36348.96	0
32	523	61970.68	3310.569	32129.32	0
33	515	56748.36	3281.667	28072.88	0
34	510	53520.22	3256.184	26179.1	0
35	506	50955.2	3237.153	24955.89	0
36	503	49538.37	3229.369	24388.34	0
Tot.	6203	727569.1	39475.7	379076.3	0

Year 4

Table: (4-7) when v = 1

Month	Production	Inventory	Work-Force	Prices
37	509	322	94	102.8
38	511	323	94	104.
39	513	326	94	105.1
40	516	326	94	110.1
41	519	324	94	115.8
42	521	317	94	121.5
43	520	318	94	114.4
44	519	314	94	115.6
45	516	318	94	107.8
46	515	319	94	107.2
47	514	319	94	107.6
48	513	324	94	103.9
Tot.	6186	3848		

Table:(4-8) when v = 1

Month	Sales	Revenue	Oth.cost	Profit	Check
37	510	52418.7	3238.215	25750.64	0
38	510	53085.52	3247.866	26025.76	0
39	510	53589.44	3260.601	26182.69	0
40	516	56902.75	3281.733	28256.71	0
41	522	60372.86	3302.816	30857.41	0
42	529	64207.63	3316.141	34361.96	0
43	519	59354.84	3307.677	29900.3	0
44	523	60429.01	3300.306	30984.11	0
45	512	55260.47	3281.509	27081.95	0
46	513	54977.12	3271.919	27012.84	0
47	514	55325.92	3266.683	27316.17	0
48	508	52729.62	3260.28	25649.91	0
Tot.	6185	678653.9	39335.75	339380.5	0

Year 5

Table :(4-9)when v = 1

Month	Production	Inventory	Work-Force	Prices
49	514	323	94	108.6
50	515	319	94	112.1
51	514	320	94	109.2
52	514	312	94	109.1
53	513	318	94	108.9
54	511	320	93	106.3
55	511	320	93	107.
56	510	318	93	107.5
57	508	319	93	104.5
58	507	323	93	102.6
59	508	320	93	107.8
60	507	320	93	105.5
Tot.	6132	3841		

Table :(4-10) when v = 1

Month	Sales	Revenue	Oth.cost	Profit	Check
49	515	55889.1	3268.135	27671.69	0
50	519	58168.63	3274.49	29322.01	0
51	514	56083.59	3269.85	27721.54	0
52	514	56068.57	3265.72	27750.21	0
53	514	55920.2	3260.1	27698.5	0
54	509	54136.96	3251.245	26542.92	0
55	511	54668.93	3246.956	26933.41	0
56	512	55012.54	3241.982	27213.82	0
57	507	53011.87	3231.969	25986.69	0
58	503	51601.76	3225.632	25201.28	0
59	512	55159.48	3230.654	27379.03	0
60	507	53462.98	3225.837	2625.11	0
Tot.	6135	659184.6	38992.58	325672.2	0

Table :(4-11)Yearly Total of Inventory, Production and Sales

Year	Y.Inv.	Y.Prod.	Y. Sal.
1	3807	5256	5200
2	3856	5615	5616
3	3798	6208	6203
4	3848	6186	6185
5	3841	6132	6135
Tot.	19150	29397	29339

Table: (4-12)Yearly Total of Each Basic Cost (Regular Payroll, Hiring and Layoff, Overtime. Inventory Related and Opportunity Cost)

Year	Y.RPAC.	Y.HLC.	Y.OTC.	Y.INCC	Y.OPC.	Y.TOTC.
1	231873.3	24.76	376.40	34.33	- 27009.5	205299.3
2	246227.5	386.58	1429.44	36.23	6976.99	255056.7
3	268562.9	319.91	4259.14	24.21	35850.24	309017.1
4	270916.2	8.48	742.17	13.80	28257	299937.7
5	268669.9	10.11	229.04	2.37	25608.43	294519.8
Tot.	1286250	749.86	7036.2	110.94	69683.87	1363831

Table: (4-13)Yearly Total of Revenue, Other Cost and Profit

Year	Y.Rev.	Y.OTHC.	Y. Prof.
1	492165.8	33425.04	253441.5
2	652813.8	35702.92	362054.1
3	727569.1	39475.7	379076.3
4	678653.9	39335.75	339380.5
5	659184.6	38992.58	325672.2
Tot.	3210387	186932	1659624.6

Table :(4-14)Comparison with H.M.M.S. for 5 Years

	Work-force		Production		Inventory		Sales		Total Cost		Profit	
	H.M. M.S.	Pred .1	H.M. M.S.	Pred .1	H.M. M.S.	Pred .1	H.M. M.S.	Pred .1	H.M.M .S.	Pred.1	H.M.M.S.	Pred.1
Max .	111	94	736	525	492	334	841	529	2,290,8 50	1,550,763	1,039,573	1,659,624
Min.	66	80	359	434	117	301	289	414				
ar.	45	14	377	91	375	33	552	115				