On the Degree of Best Approximation of Unbounded Functions by Algebraic Polynomial

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Abstract

In this paper we introduce a new class of degree of best algebraic approximation polynomial $A_{k,n}(f,x)$ for unbounded functions in weighted space $L_{p,\alpha}(X)$, $(1 \leq p < \infty)$.We shall prove direct and converse theorems for best algebraic approximation in terms modulus of smoothness in weighted space.

Keyword: degree of best approximation, unbounded functions, weight space, modulus of smoothness.

1. Introduction

Many papers have recently appeared connected with best approximation to functions of one variable see [1,2,3] and the references there. In this paper we shall consider the degree of best algebraic approximation of unbounded functions. Indeed, in terms of the modulus of smoothness direct (Jackson – type) as well as inverse (Bernstein – type) theorems are established so that a constructive theory of unbounded functions may developed in weighted space $L_{p,\alpha}(X)$.

Let $X = (-\infty, \infty)$, by $\|.\|_p$ we denote the $L_p(X)$- norm, $(1 \leq p < \infty)$ and

$$\|f\|_p = \left(\int_X |f(x)|^p \, dx\right)^{\frac{1}{p}} < \infty \quad \ldots\ldots\ldots\ldots(1)$$

and define for a suitable set $W(\alpha, x)$ of all weight functions on open interval X, such that $|f(x)| \leq M\,\alpha(x)$, where M is positive real number and $\alpha: X \to \mathbb{R}^+$ weight function.

We shall denote by $L_{p,\alpha}(X)$ the space of all unbounded functions on X, which are equipped with the following norm $\|f\|_{p,\alpha} = \left(\int_X |f(x)\alpha(x)|^p dx\right)^{\frac{1}{p}} < \infty.$ ……..(2)

For $f \in L_{p,\alpha}(X)$ and $x \in X$, we define the local modulus of order k of f in point x as follows

$$\omega_k(f, x, \delta)_{p,\alpha} = \underbrace{sup}_{|h|<\delta} \left\{ \left|\Delta_h^k\right| : t, t + kh \in \left[x - \frac{k\delta}{2}, x + \frac{k\delta}{2}\right] \cap X \right\} \ldots\ldots\ldots(3)$$

where

$$\Delta_h^k f(x) = \begin{cases} \sum_{m=0}^{k} (-1)^{k+m} \binom{k}{m} f(x + mh) & if\ x, x + mh \in X \\ 0 & otherwise \end{cases} \ldots\ldots\ldots(4).$$

Also the modulus of smoothness of order k of function f is the following function of :

$$\omega_k(f, \delta)_{p,\alpha} = \underbrace{sup}_{|h|<\delta} \left\{ \left\|\Delta_h^k f(.)\right\|_{p,\alpha} \right\} \ldots\ldots\ldots\ldots(5).$$

Let $\mathbb{N}$ be the set of natural numbers and $\mathbb{P}_n$ the set of all algebraic polynomials of degree less than or equal to $\in \mathbb{N}$.

The degree of best algebraic approximation of order n of the function $f \in L_{p,\alpha}(X)$ is given by

$$E_n(f)_{p,\alpha} = inf\{\|f - p_n\|_{p,\alpha} \,; p_n \in \mathbb{P}_n\} \ldots\ldots\ldots\ldots(6).$$

The most essential consequence of the proved here is

Corollary :

For $0 < \beta < k$ and $1 \leq p < \infty$, we have

$\omega_k(f, \delta)_{p,\alpha} = O(\delta^{\beta})$ if and only if $E_n(f)_{p,\alpha} = O(\delta^{-\beta})$.

If k and i ,are positive integer and $k > \beta - i > 0$, then $E_n(f)_{p,\alpha} = O(\delta^{-\beta})$ equivalently $\omega_k(f,\delta)_{p,\alpha} = O(\delta^{\beta})$ is valid if and only if f has a derivative of order i satisfying $\|f^{(i)}\|_{p,\alpha} < \infty$ and $\omega_k(f^{(i)},\delta)_{p,\alpha} = O(\delta^{\beta-i})$.

Further details and elementary properties of $\omega_k(f,\delta)_{p,\alpha}$ see [4,5].

With the aid of these concepts one may now work out a constructive theory of functions for the weight space $L_{p,\alpha}(X)$, parallel to classical theory of Jackson-Bernstein for $L_{p,\alpha}(X)$, see sections 2 and 3 for detail.

We formulate and prove our Jackson-type theorem and in section 3, we formulate and prove our Bernstein-type theorem.

The corollary announced above follows from direct theorem (1) of section 2 combined with converse theorem (2) of section 3.

2. The Direct Algebraic Approximation Theorem:

Let $\psi \in L_{p,\alpha}(X)$,$1 \le p < \infty$, k natural number and $\delta > 0$. We define

$$A_{k\delta}(\psi,x) = \delta^{-k}\int_0^{\delta}\sum_i^k(-1)^{i+1}\binom{k}{i}\psi(x+t)dt \quad \ldots\ldots\ldots\ldots(7).$$

We set

$$f_n(x) = \begin{cases} f(x) & if \ |x| \le \sqrt{n} \\ 0 & if \ |x| > \sqrt{n} \end{cases}$$

and $\delta_n = \sqrt{n}$, $f_{kn}(x) = A_{k,\delta_n}(\psi,x)$.

It is clear that $f_{kn}(x) \in \mathbb{P}_n$.

Lemma 1 [4]:

We have for every $1 \le p < \infty$, $0 < h < 1$ and every positive function $g(x)$

$$|g(x)\,[\psi(x) - A_{k,\delta_n}(x)]| \le C(g)|g(x)\,\Delta_t^k\,\psi(x)|$$

and

$$|g(x)\ A^{(j)}{}_{k,\delta_n}(\psi,x)\,| \le C(k)h^{-j}\underbrace{sup}_{|t|\le h}|g(x)\,\Delta_t^j\,\psi(x)|\ , j = 1,2,\ldots,k\ .$$

Prove of this lemma see [4].

Lemma 2 :

For every $f \in L_{p,\alpha}(X)$ and $1 \le p < \infty$, we have

$\|f - f_{kn}\|_{p,\alpha} \le C(k)\omega_k(f, \sqrt{n})_{p,\alpha}$

and

$\left\|f^{(k)}\right\|_{p,\alpha} \le C(k)\, n^{\frac{k}{2}}\, \omega_k(f, \sqrt{n})_{p,\alpha}$.

Proof :

From theorem of Marchoud see [5], we have

$\omega_k(f_n, \delta)_{p,\alpha} \le C(k)\|f_n - f\|_{p,\alpha} + \omega_k(f, \delta)_{p,\alpha}$

since $f(x) = f_n(x)$ for $|x| \le \sqrt{n}$, we obtain

$|f(x) - f_n(x)| \le C(k)\, f(x)$ for $x \in (-\infty, \infty)$.

Consequently

$$\|f - f_n\|_{p,\alpha} = \left(\int_X \left|\left(f(x) - f_n(x)\right)\alpha(x)\right|^p dx\right)^{\frac{1}{p}} \le \left(\int_X |f(x)\,\alpha(x)|^p dx\right)^{\frac{1}{p}}$$

$$\le sup\left(\int_X \left|(\Delta_\delta^k f(x))\,\alpha(x)\right|^p dx\right)^{\frac{1}{p}} = sup\left\|\Delta_\delta^k f(.)\right\|_{p,\alpha} = \omega_k(f, \sqrt{n})_{p,\alpha}\,;\delta_n = \sqrt{n}.$$

Now

$$\|f - f_{kn}\|_{p,\alpha} = \left(\int_X \left|\left(f(x) - f_{kn}(x)\right)\alpha(x)\right|^p dx\right)^{\frac{1}{p}}$$

$$\le \left(\int_X \left|\left(f(x) - f_n(x)\right)\alpha(x)\right|^p dx\right)^{\frac{1}{p}} + \left(\int_X \left|\left(f_n(x) - f_{kn}(x)\right)\alpha(x)\right|^p dx\right)^{\frac{1}{p}}$$

$$\le \|f - f_n\|_{p,\alpha} + sup\left\|\Delta_\delta^k f(.)\right\|_{p,\alpha}$$

$$\le \|f - f_n\|_{p,\alpha} + C(k)\omega_k(f, \sqrt{n})_{p,\alpha}$$

since $\|f - f_n\|_{p,\alpha} \le \omega_k(f, \sqrt{n})_{p,\alpha}$

so

$\|f - f_{kn}\|_{p,\alpha} \le C(k)\omega_k(f, \sqrt{n})_{p,\alpha}$.

We have $f_{kn}(x) = \mathrm{A}_{k,\delta_n}(f_n, x)$, for $x \in X$ and by lemma 1,

$$\left\|f_{nk}^{(k)}\right\|_{p,\alpha} = \left(\int_X \left|f_{nk}^{(k)}(x)\,\alpha(x)\right|^p dx\right)^{\frac{1}{p}}$$

$$\le C(k)\sum_{i=0}^{k}\left(\int_X \left|\mathrm{A}_{k,\delta_n}^{(i)}(f_n, x)\,\alpha(x)\right|^p dx\right)^{\frac{1}{p}}$$

$\leq C(k)\sum_{i=0}^{k}\delta_n^{-i} sup\left\|\Delta_t^i f_n(x)\right\|_{p,\alpha}$.

As consequence of $f_n(x) = 0$ for $|x| > \sqrt{n}$,

hence $\Delta_t^i f_n(x) = 0$ for $|x| > \sqrt{n}$,

so $\left\|f_{nk}^{(k)}\right\|_{p,\alpha} \leq \delta^{-i}C(k)\ \omega_k(f_n,\sqrt{n})_{p,\alpha}$; $\delta_n = \sqrt{n}$.

■

Theorem 1 :

Let $f \in L_{p,\alpha}(X)$ and $1 \leq p < \infty$. For every natural k there exists constant $C(k)$ depending on , such that

$E_n(f)_{p,\alpha} \leq C(k)\ \omega_k(f_n,\sqrt{n})_{p,\alpha}$.

Proof :

From definition of degree of best algebraic approximation, we have

$E_n(f)_{p,\alpha} \leq \left\|f - A_{k,\delta_n}(f_n,.)\right\|_{p,\alpha}$, where $A_{k,\delta_n}(f_n) \in \mathbb{P}_n$.

Hence from lemma 2, we obtain

$E_n(f)_{p,\alpha} \leq C(k)\ \omega_k(f_n,\sqrt{n})_{p,\alpha}$

■

3. The Converse Approximation Theorem :

We need the following lemmas to prove converse theorem

Lemma 3 [6] :

We have for every $p_n \in \mathbb{P}_n$, $1 \leq p < \infty$ and n=1,2,…

$\left\|W_\beta p_n^{(j)}\right\|_p \leq C(j)n^{j/2}\left\|W_\beta p_n\right\|_p$.

Lemma 4 [7] :

We have for every $p_n \in \mathbb{P}_n$, $1 \leq p < \infty$ and n=1,2,…

$\omega_k(p_n)_{p,\alpha} \leq C(k)\delta^k\, n^{\frac{k}{2}}\, \|p_n\|_{p,\alpha}$.

Lemma 5 :

Let $f \in L_{p,\alpha}(X)$, $1 \leq p < \infty$, $p_n \in \mathbb{P}_n$ and $\|f - p_n\|_{p,\alpha} \leq \epsilon$.

We have

$$\omega_k(f_n, \sqrt{n})_{p,\alpha} \le C(k)\, \delta^k \; \{\|f\|_{p,\alpha} + \sum_{i=0}^{n}(i+1)^{\frac{k}{2}-1} E_i(f)_{p,\alpha} + n^{\frac{k}{2}}\epsilon\} .$$

Proof :

Let $p_n \in \mathbb{P}_n$ be a polynomial satisfying $\|f - p_n\|_{p,\alpha} \le 2\, E_n(f)_{p,\alpha}$.

By using lemma 4, we obtain

$$\omega_k(p_{2^r} - p_{2^{r-1}}, \delta)_{p,\alpha} \le C(k)\delta^k\, n^{\frac{k}{2}} \|p_{2^r} - p_{2^{r-1}}\|_{p,\alpha}$$

$$\le \; C(k)\delta^k\, n^{\frac{k}{2}} \left\{ \begin{matrix} \left(\int_X \left|\left(f(x) - p_{2^r}(x)\alpha(x)\right)\right|^p \; dx\right)^{\frac{1}{p}} + \\ \left(\int_X |(f(x) - p_{2^{r-1}}(x)\alpha(x)|^p \; dx\right)^{\frac{1}{p}} \end{matrix} \right\}$$

$$\le C(k)\delta^k\, n^{\frac{k}{2}} \{ E_{2^r}(f)_{p,\alpha} + \; E_{2^{r-1}}(f)_{p,\alpha}\}$$

$$\le C(k)\delta^k\, n^{\frac{k}{2}} \; E_{2^{r-1}}(f)_{p,\alpha}$$

We have , from properties of modulus of continuity and lemma 4

$$\omega_k(p_1, \delta)_{p,\alpha} \le \omega_k(p_1 - p_0, \delta)_{p,\alpha} + \omega_k(p_0, \delta)_{p,\alpha}$$

$$\le C(k)\delta^k\, n^{\frac{k}{2}} \{\|p_1 - p_0\|_{p,\alpha} + \|p_0\|_{p,\alpha}\}$$

$$\le \; C(k)\delta^k\, n^{\frac{k}{2}} \left\{\left(\int_X |(f(x) - p_1(x))\alpha(x)|^p \; dx\right)^{\frac{1}{p}}\right\}$$

$$+\left(\int_X \; |(f(x) - p_0(x))\alpha(x)|^p \; dx\right)^{\frac{1}{p}} + \left(\int_X \; \left|\left(f(x) - p_0(x)\right)\alpha(x)\right|^p \; dx\right)^{\frac{1}{p}}$$

$$+\left(\int_X \; \left|\left(f(x)\alpha(x)\right)\right|^p \; dx\right)^{\frac{1}{p}}$$

$$\le C(k)\delta^k\, n^{\frac{k}{2}}\{\|f - p_1\|_{p,\alpha} + 2\|f - p_0\|_{p,\alpha} + \|f\|_{p,\alpha}\}$$

$$\le C(k)\delta^k\, n^{\frac{k}{2}}\{\, c(k)E_0(f)_{p,\alpha} + \|f\|_{p,\alpha}\}$$

so ,

$$\omega_k(p_1, \delta)_{p,\alpha} \le C(k)\delta^k\, n^{\frac{k}{2}}\{\, E_0(f)_{p,\alpha} + \|f\|_{p,\alpha}\}$$

also, we can prove

$$\omega_k(p_{2^m},\delta)_{p,\alpha} \le C(k)\delta^k\, n^{\frac{k}{2}}\{\, E_0(f)_{p,\alpha} + \sum_{i=0}^{m-1} 2^{ik/2}\; E_{2^i}(f)_{p,\alpha} + \|f\|_{p,\alpha}\}$$

$$\le C(k)\delta^k \,\{\textstyle\sum_{i=0}^{2m-1}(i+1)^{\frac{k}{2}-1}\; E_i(f)_{p,\alpha} + \|f\|_{p,\alpha}\} \;\ldots(8).$$

Let $2^m < n \le 2^{m+1}$

$$\omega_k(p_{2^m},\delta)_{p,\alpha} \le C(k)\delta^k\, n^{\frac{k}{2}}\|p_n - p_{2^{m+1}}\|_{p,\alpha}$$

$$\le C(k)\delta^k\, n^{\frac{k}{2}}\left\{\left(\int_X |(f(x)-p_n(x))\alpha(x)|^p\; dx\right)^{\frac{1}{p}} + \left(\int_X |(f(x)-p_{2^{m+1}}(x))\alpha(x)|^p\; dx\right)^{\frac{1}{p}}\right\}$$

$$\le C(k)\delta^k\, n^{\frac{k}{2}}\{\|f-p_n\|_{p,\alpha} + \|f-p_{2^{m+1}}\|_{p,\alpha}\}$$

$$\le C(k)\delta^k\, n^{\frac{k}{2}}\,\epsilon \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(9)$$

From (8) and (9), we get

$$\omega_k(f_n,\sqrt{n})_{p,\alpha} \le C(k)\;\delta^k\;\{\|f\|_{p,\alpha} + \textstyle\sum_{i=0}^{n}(i+1)^{\frac{k}{2}-1} E_i(f)_{p,\alpha} + n^{\frac{k}{2}}\epsilon\,\}\,.$$

■

Theorem 2 :

Let $f \in L_{p,\alpha}(X)$,$1 \le p < \infty$. For every natural number n there exist a constant $C(k)$ depending on k, such that

$$\omega_k(f_n,\tfrac{1}{\sqrt{n}})_{p,\alpha} \le C(k)\;\frac{k}{\sqrt{n}}\;\{\|f\|_{p,\alpha} + \textstyle\sum_{i=0}^{n}(i+1)^{\frac{k}{2}-1} E_i(f)_{p,\alpha}\,\}\,,\quad k = 1,2,\ldots\;.$$

Proof :

We insert in lemma 5 $\epsilon = E_i(f)_{p,\alpha}$, $\delta = \frac{1}{\sqrt{n}}$ and take in consideration that by the monotony of $E_n(f)_{p,\alpha}$

$$n^{\frac{k}{2}} E_n(f)_{p,\alpha} \le C(k)\textstyle\sum_{i=0}^{n} i^{\frac{k}{2}-1}\, E_i(f)_{p,\alpha}$$

So,

$$\omega_k(f_n,\tfrac{1}{\sqrt{n}})_{p,\alpha} \le C(k)\;\frac{1}{\sqrt{n}}\;\{\|f\|_{p,\alpha} + \textstyle\sum_{i=0}^{n}(i+1)^{\frac{k}{2}-1} E_i(f)_{p,\alpha} + n^{\frac{k}{2}}\epsilon\}$$

$$\le C(k)\;\frac{1}{\sqrt{n}}\;\{\|f\|_{p,\alpha} + \textstyle\sum_{i=[\frac{n}{2}]}^{n}(i+1)^{\frac{k}{2}-1} E_i(f)_{p,\alpha}\}.$$

■

Theorem 3 :

Let $f \in L_{p,\alpha}(X)$,$1 \le p < \infty$ and $\sum_{i=0}^{n}(i+1)^{\frac{k}{2}-1} E_i(f)_{p,\alpha} < \infty$. Then f has derivative of order k,$f^{(k)} \in L_{p,\alpha}(X)$ and

$$E_i\left(f^{(k)}\right)_{p,\alpha} \le C(k)\,\{n^{\frac{k}{2}} E_n(f)_{p,\alpha} + \sum_{i=n+1}^{\infty}(i+1)^{\frac{k}{2}-1} E_i(f)_{p,\alpha}\}$$

Proof :

Let $p_n \in \mathbb{P}_n$ satisfy $\|f-p_n\|_{p,\alpha} \le 2E_n(f)_{p,\alpha}$, we have in consequence of lemma 3

$$\left\|p_{2^{i+1}}{}^{(k)} - p_{2^i}{}^{(k)}\right\|_{p,\alpha} \le C(k)n^{\frac{k}{2}} \left\|p_{2^{i+1}} - p_{2^i}\right\|_{p,\alpha}$$

$$\le C(k)n^{\frac{k}{2}}\{ \left\|f - p_{2^{i+1}}\right\|_{p,\alpha} + \left\|f - p_{2^i}\right\|_{p,\alpha}\}$$

$$\le C(k)\, E_{2^i}(f)_{p,\alpha}$$

the sequence $\{p_{2^i}^{(k)}\}$ is convergent in weighted space $L_{p,\alpha}(X)$, we denote it is limit by $f^{(k)} \in L_{p,\alpha}(X)$.

Since $\left\|f - p_{2^i}\right\|_{p,\alpha} \to 0$, $f^{(k)}$ is derivative of order k of f. Moreover , we have

$$\left\|f^{(k)} - p_{2^i}{}^{(k)}\right\|_{p,\alpha} \le C(k)\left\|f - p_{2^i}\right\|_{p,\alpha} \le C(k)\sum_{i=m}^{\infty} 2^{ik/2} E_{2^i}(f)_{p,\alpha}$$

$$\le C(k)\{2^{ik/2} E_{2^i}(f)_{p,\alpha} + \sum_{i=2^i}^{\infty}(i+1)^{\frac{k}{2}-1} E_i(f)_{p,\alpha}\} \quad \ldots\ldots(10)$$

Let $2^{m-1} < n < 2^m$, form lemma 3 we have

$$\left\|p_{2^m}{}^{(k)} - p_n{}^{(k)}\right\|_{p,\alpha} \le C(k)n^{\frac{k}{2}} \left\|p_{2^m} - p_n\right\|_{p,\alpha}$$

$$\le\ C(k)n^{\frac{k}{2}} E_n(f)_{p,\alpha} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(11)$$

From (10) and (11), we obtain

$$E_i\left(f^{(k)}\right)_{p,\alpha} \le \left\|f^{(k)} - p_n{}^{(k)}\right\|_{p,\alpha}$$

$$\le C(k)\,\{n^{\frac{k}{2}} E_n(f)_{p,\alpha} + \sum_{i=n+1}^{\infty}(i+1)^{\frac{k}{2}-1} E_i(f)_{p,\alpha}\}$$

■

Theorem 4 :

We have for every natural k and m every $f^{(m)} \in L_{p,\alpha}(X)$

$$\omega_k(f^{(m)},\tfrac{1}{\sqrt{n}})_{p,\alpha} \le C(k,m)\ \{\tfrac{k}{\sqrt{n}}\textstyle\sum_{i=0}^{n}(i+1)^{\frac{km}{2}-1}E_i(f)_{p,\alpha} + \sum_{i=n+1}^{\infty}(i+1)^{\frac{m}{2}-1}E_i(f)_{p,\alpha}\}\ .$$

Proof :

From theorem 2 and theorem 3 , we have

$$\omega_k(f^{(m)},\tfrac{1}{\sqrt{n}})_{p,\alpha} \le C(k,m)\ \tfrac{k}{\sqrt{n}}\ \{\|f^{(m)}\|_{p,\alpha} + \textstyle\sum_{i=0}^{n}(i+1)^{\frac{k+m}{2}-1}E_i(f^{(m)})_{p,\alpha}\ \}$$

$$E_i(f^{(m)})_{p,\alpha} \le C(m)\ \{n^{\frac{m}{2}}E_n(f)_{p,\alpha} + \textstyle\sum_{i=n+1}^{\infty}(i+1)^{\frac{m}{2}-1}E_i(f)_{p,\alpha}\}\ ..(12)$$

As well as from (10) and take $n = 0$

$$\|f^{(m)}\|_{p,\alpha} \le \textstyle\sum_{i=0}^{\infty}(i+1)^{\frac{m}{2}-1}E_i(f)_{p,\alpha}\quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(13)$$

new, from (12) and (13), we obtain

$$\omega_k(f^{(m)},\tfrac{1}{\sqrt{n}})_{p,\alpha} \le\ C(k,m)\ \tfrac{k}{\sqrt{n}}\ \{\textstyle\sum_{i=0}^{\infty}(i+1)^{\frac{m}{2}-1}E_i(f)_{p,\alpha} +\ \sum_{i=0}^{n}(i+1)^{\frac{k}{2}-1}\ (E_n(f)_{p,\alpha} + \sum_{i=n+1}^{\infty}(i+1)^{\frac{m}{2}-1}E_i(f)_{p,\alpha})\ \}$$

$$\le C(k,m)\ \tfrac{k}{\sqrt{n}}\ \{\textstyle\sum_{i=0}^{\infty}(i+1)^{\frac{m}{2}-1}E_i(f)_{p,\alpha} + \sum_{i=0}^{n}(i+1)^{\frac{m}{2}-1}E_i(f)_{p,\alpha} + \sum_{i=0}^{n}(i+1)^{\frac{k}{2}-1}\left(\sum_{i=j}^{\infty}(j+1)^{\frac{m}{2}-1}E_j(f)_{p,\alpha}\right)\}$$

$$\le C(k,m)\ \tfrac{k}{\sqrt{n}}\ \{\textstyle\sum_{i=0}^{n}(i+1)^{\frac{k+m}{2}-1}E_i(f)_{p,\alpha} + \sum_{i=0}^{n}(\sum_{j=0}^{i}(j+1)^{\frac{k}{2}-1})\ (i+1)^{\frac{m}{2}-1}E_i(f)_{p,\alpha}\}$$

$$\le C(k,m)\ \tfrac{k}{\sqrt{n}}\ \{\textstyle\sum_{i=0}^{n}(i+1)^{\frac{k+m}{2}-1}E_i(f)_{p,\alpha} + \sum_{i=0}^{\infty}(i+1)^{\frac{m}{2}-1}E_i(f)_{p,\alpha}\}$$

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دراسة درجة أفضل تقريب للدوال الغير مقيدة بواسطة متعددات الحدود الجبرية

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الخلاصة

في هذا البحث عرضنا نوع جديد من درجة افضل تقريب للدوال الغير مقيدة بواسطة متعددات الحدود الجبرية في فضاء الوزن($L_{p,\alpha}(X)$) و($1 \leq p < \infty$) . سوف نبرهن المبرهنات المباشرة والمعكوسة لأفضل تقريب لهذه الدوال بواسطة متعددات الحدود الجبرية بشروط مقياس النعومة في فضاء الوزن.

الكلمات المفتاحية : درجة أفضل تقريب , الدوال الغير مقيدة , فضاء الوزن , مقياس النعومة.