بعض النتائج حول المجموعات شبه قبل المفتوحة

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الخلاصة

إن أول من قدم تعريف المجموعة شبه قبل المفتوحة هو الرياضي اندرجفك، اذ عرفها بالشكل " اذا كان X) (τ , فضاء تبولوجي و A مجموعة جزئية من X فأن A تسمى مجموعة شبه قبل المفتوحة اذا كان $A \subseteq \overline{\overline{A}^{\circ}}$" .

لقد قمنا في هذا البحث بدراسة خواص المجموعات شبه قبل المفتوحة ولكن ليس عن طريق التعريف أعلاه انما بواساطة تعريف ثاني مكافئ لتعريفها وكذلك درسنا العلاقة بينها وبين أنواع أخرى من المجاميع المفتوحة الضعيفة.

Some Results on Semi-preopen Sets

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Abstract

The definition of semi-preopen sets were first introduced by "Andrijevic" as were is defined by :Let (X , τ) be a topological space, and let $A \subseteq X$, then Ais called semi-preopen set if $A \subseteq \overline{\overline{A}^{\circ}}$.

In this paper, we study the properties of semi-preopen sets but by another definition which is equivalent to the first definition and we also study the relationships among it and (open, α-open, preopen and semi-p-open)sets.

1.Preliminaries

Definition 1.1 (1)(4):

A subset A of a topological space (X , τ) is called α – open set if and only if A $\subseteq \overline{A}^{\circ}$ $\overline{\mathbf{A}^{\circ}}^{\circ}$

The family of all α – open sets is denoted by τ_α.

Definition 1.2 (1)(5) :

A subset A of a topological space (X , τ) is called a ***preopen set*** if A ⊆

The complement of a preopen set is called preclosed set .

The family of all preopen sets of X is denoted by PO(X).

The family of all preclosed sets of X is denoted by PC(X).

Theorem 1.3 (2) :

The union of any family of preopen sets is a preopen set.

Definition 1.4 (1) :

The intersection of all preclosed sets containing A is called the ***preclosure of A***, denoted by pre-cl A.

Definition 1.5 (1) :

A subset A of a topological space (X , τ) is said to be ***semi-p-open set***, if there exists a preopen set in X say U such that U⊆ A ⊆ pre-cl U.

The complement of a semi-p-open set is called semi-p-closed set.

The family of all semi-p-open sets of X is denoted by S-P(X). The family of all semi-p-closed sets of X is denoted by S-P C(X).

Proposition 1.6 (2):

For any subset A of a topological space (X, τ), pre-cl A ⊆ $\overline{A}$ and the converse is not true.

2. semi– preopen sets

Definition 2.1(6):

A subset A of a topological space (X , τ) is said to be ***semi-preopen set***, if and only if there exists a preopen set in X say U such that $U \subseteq A \subseteq \overline{U}$

The complement of semi-preopen set is called semi- preclosed set .

The family of all semi-preopen sets of X is denoted by SPO(X).

The family of all semi-preclosed sets of X is denoted by SPC(X).

Definition 2.2:

Let (X , τ) be a topological space and $A \subseteq X$, A is called ***semi-preneighborhood of a point x in X***, if there exists semi-preopen set U in X such that $x \in U \subseteq A$.

Theorem 2.3 :

The union of any family of semi- preopen sets is semi- preopen set.

Proof :

Let $\{ A_\alpha \}$; $\alpha \in \Lambda$ be any family of semi-preopen sets, we must prove $\bigcup_{\alpha\in\Lambda} \mathbf{A}_\alpha$ is semi-preopen set. This means we must prove there exists $U \in PO(X)$ such that $U \subseteq \bigcup_{\alpha\in\Lambda} \mathbf{A}_\alpha \subseteq \overline{\mathbf{U}}$.

Since for all $\alpha\in\Lambda$, A_α is semi- preopen , therefore there exists $U_\alpha \in PO(X)$ such that $\mathbf{U}_\alpha \subseteq \mathbf{A}_\alpha \subseteq \overline{\mathbf{U}}_\alpha$, and since $\bigcup_{\alpha\in\Lambda} \mathbf{U}_\alpha$ is a preopen set (by theorem 1.3), therefore let $U = \bigcup_{\alpha\in\Lambda} \mathbf{U}_\alpha$,then we get $U \subseteq \bigcup_{\alpha\in\Lambda} \mathbf{A}_\alpha$ ………… (1)

Now since $\mathbf{A}_\alpha \subseteq \overline{\mathbf{U}}_\alpha$ for all $\alpha\in\Lambda$, therefore $\bigcup_{\alpha\in\Lambda} \mathbf{A}_\alpha \subseteq \bigcup_{\alpha\in\Lambda} \overline{\mathbf{U}}_\alpha$ for all $\alpha\in\Lambda$, implies $\bigcup_{\alpha\in\Lambda} \mathbf{A}_\alpha \subseteq \overline{\bigcup_{\alpha\in\Lambda} \boldsymbol{U}_\alpha}$ for all $\alpha\in\Lambda$, thus $\bigcup_{\alpha\in\Lambda} \mathbf{A}_\alpha \subseteq \overline{\boldsymbol{U}} = \overline{\bigcup_{\alpha\in\Lambda} \boldsymbol{U}_\alpha}$ ….. (2) and from (1) and (2) we get, there exists $U \in PO(X)$ such that $U \subseteq \bigcup_{\alpha\in\Lambda} \mathbf{A}_\alpha \subseteq \overline{\mathbf{U}}$, therefore $\bigcup_{\alpha\in\Lambda} \mathbf{A}_\alpha$ is semi-preopen set. ■

Corollary 2.4 :

The intersection of any family of semi-preclosed sets is semi-preclosed set .

Remark 2.5 :

The intersection of two semi-preopen sets need not to be semi-preopen set, as the following example shows:

Example 1 :

Let $X=\{1,2,3\}$, $\tau=\{X,\emptyset, \{1,2\}\}$

$PO(X) = \tau \cup \{\{1\},\{2\},\{1,3\},\{2,3\}\}$, SPO(X) = PO(X)

Let $A= \{1,3\}$ and $B=\{2,3\}$ are both semi-preopen sets, but $A \cap B=\{3\}$ is not semi-preopen set.

Remark 2.6 :

The union of two semi-preclosed sets need not to be semi-preclosed set, as the example 1, $\{1\}$ and $\{2\}$ are two semi-preclosed sets since $X-\{1\}=\{2,3\}$ and $X-\{2\}=\{1,3\}$ are two semi-preopen sets, but $\{1\}\cup\{2\}=\{1,2\}$ is not semi-preclosed set since $X-\{1,2\}=\{3\}$ is not semi-preopen set.

Definition 2.7 :

The union of all semi-preopen sets contained in A is called the ***semi-preinterior of A***, denoted by S-pre-int A .

Definition 2.8 :

The intersection of all semi-preclosed sets containing A is called the ***semi-preclosure of A***, denoted by S-pre-cl A .

Proposition 2.9:

1. If $A\subseteq B$, then S-pre-int A $\subseteq$ S-pre-int B.
2. S-pre-int A $\subseteq$ A .
3. S-pre-int A $\cup$ S-pre-int B $\subseteq$ S-pre-int (A $\cup$ B).
4. S-pre-int (A $\cap$ B) $\subseteq$ S-pre-int A $\cap$ S-pre-int B.

Proof :

The proof of (1) and (2) is direct by the definition of subsets and S-pre-int A .

3. Since A$\subseteq$ A$\cup$B, therefore S-pre-int A$\subseteq$ S-pre-int (A $\cup$ B) (by part 1), and since B$\subseteq$ A$\cup$B, therefore S-pre-int B$\subseteq$ S-pre-int (A $\cup$ B) (by part 1), implies S-pre-int A $\cup$ S-pre-int B $\subseteq$ S-pre-int (A $\cup$ B).

The converse is not true in general, as the following of example (1):

Let A={2}, B={3} and A$\cup$B={2,3}, then:

S-pre-int{2}={2}, S-pre-int{3}= $\varnothing$ and S-pre-int{2,3}={2,3}.

But S-pre-int (A $\cup$ B) ={2,3}$\not\subset$ {2}= S-pre-int A $\cup$ S-pre-int B.

4. A$\cap$B $\subseteq$ A, then this implies that S-pre-int (A $\cap$ B) $\subseteq$ S-pre-int A (by part 1), and A$\cap$B $\subseteq$ B, then S-pre-int (A $\cap$ B)$\subseteq$ S-pre-int B (by part 1), therefore S-pre-int (A $\cap$ B) $\subseteq$ S-pre-int A $\cap$ S-pre-int B.

But S-pre-int A $\cap$ S-pre-int B $\not\subset$ S-pre-int (A $\cap$ B), as the following example shows:

Example 2 :

Let X={1,2,3,4}, τ={X,$\varnothing$,{1},{2},{1,2}}

PO(X) = τ $\cup$ {{1,2,3},{1,2,4}}

SPO(X) = PO(X) $\cup$ {{1,3},{1,4},{1,3,4},{2,3},{2,4},{2,3,4}}

let A ={2,3,4}, B ={1,3,4} and A $\cap$ B= {3,4},then:

S-pre-int {2,3,4}={2,3,4}, S-pre-int {1,3,4}= {1,3,4} and S-pre-int (A $\cap$ B) = $\varnothing$ .But S-pre-int A $\cap$ S-pre-int B = {3,4} $\not\subset$ $\varnothing$ = S-pre-int (A $\cap$ B).

Proposition 2.10 :

1. If A$\subseteq$ B, then S-pre-cl A $\subseteq$ S-pre-cl B.
2. A $\subseteq$ S-pre-cl A.
3. S-pre-cl $\varnothing$ = $\varnothing$, S-pre-cl X = X.
4. S-pre-cl A $\cup$ S-pre-cl B $\subseteq$ S-pre-cl (A $\cup$ B).
5. S-pre-cl (A $\cap$ B) $\subseteq$ S-pre-cl A $\cap$ S-pre-cl B.

Proof :

The proof of (1) and (2) is clear by the definition of subsets and S-pre-cl A .

3. $\varnothing$ and X are semi-preopen sets (by being open set), thus S-pre-cl $\varnothing$ = $\varnothing$ and S-pre-cl X = X.

4. The proof is similarly to the proof of part (3) in proposition 2.9 .

5. The proof is similarly to the proof of part (4) in proposition 2.9 .

But the converse of part (4) is not true to see this, let A={1},B={2} and A$\cup$ B={1,2} in the example 2, then:

SPC(X) ={X,$\varnothing$,{2,3,4},{1,3,4},{3,4},{4},{3},{2,4},{2,3},{2},{1,4},{1,3},{1}}

S-pre-cl {1}={1}, S-pre-cl {2}={2}and S-pre-cl {1}$\cup$ S-pre-cl {2}={1,2 but S-pre-cl ({1}$\cup${2}) = X, which shows S-pre-cl (A $\cup$ B) $\not\subset$ S-pre-cl A $\cup$ S-pre-cl B

And also, the converse of part (5) is not true to see this,

Let A={1,2,3}, B={1,3,4} and A $\cap$ B ={1,3} in the example 2, then:

S PC(X) ={X,$\varnothing$,{2,3,4},{1,3,4},{3,4},{4},{3},{2,4},{2,3},{2},{1,4},{1,3},{1}} S-pre-cl {1,2,3}=X, S-pre-cl {1,3,4}={1,3,4} and S-pre-cl {1,2,3}$\cap$ S-pre-cl{1,3,4}={1,3,4}

But S-pre-cl ({1,2,3}$\cap$ {1,3,4}) ={1,3} which shows S-pre-cl A $\cap$ S-pre-cl B $\not\subset$ S-pre-cl (A $\cap$ B).

Proposition 2.11 :

A is semi-preclosed set, if and only if A= S-pre-cl A.

Proof:

Necessity, clear.

Sufficiency. The proof is direct (by corollary 2.4).

Corollary 2.12 :

S-pre-cl (S-pre-cl A) = S-pre-cl A .

Now, we give the connection between semi-preopen sets and some other kinds of weakly open sets.

3. Relationship among open, α-open, preopen, semi –p- open and semi-preopen sets

Remark 3.1 (3) :

1. Every open set is a preopen set, but not conversely.
2. Every closed set is a preclosed set, but not conversely.

Remark 3.2 :

Every preopen set is semi-preopen set.

Proof:

Since A is a preopen set and $A \subseteq A$, and since for any subset A of X, $A \subseteq \overline{A}$, therefore there exists a preopen set A such that $A \subseteq A \subseteq \overline{A}$. Thus A is semi-preopen set.

But the converse need not to be true in general, as the following of example 2

$X=\{1,2,3,4\}$, $\tau=\{X,\emptyset,\{1\},\{2\},\{1,2\}\}$
$PO(X) = \tau \cup \{\{1,2,3\},\{1,2,4\}\}$
$SPO(X) = PO(X) \cup \{\{1,3\},\{1,4\},\{1,3,4\},\{2,3\},\{2,4\},\{2,3,4\}\}$
it is clear that $\{1,3\}$ is semi-preopen set, but it is not a preopen set.
From remark 3.1 and remark 3.2 we obtain the following:

Remark 3.3 :

Every open set is semi-preopen set .
But the converse may be false, as the example 2 in remark 3.2 .

Remark 3.4 :

Every α-open set is semi-preopen set.

Proof:

Since A is α-open set, therefore $A \subseteq \overline{\mathbf{A}^{\circ}}^{\circ}$, and since $\boldsymbol{A}^{\circ}$ is open set this implies $\mathbf{A}^{\circ}$ is a preopen set (by remark 3.1). And $\mathbf{A}^{\circ} \subseteq A$ so $\mathbf{A}^{\circ} \subseteq A \subseteq \overline{\mathbf{A}^{\circ}}^{\circ} \subseteq \overline{\mathbf{A}^{\circ}}$, hence $\mathbf{A}^{\circ} \subseteq A \subseteq \overline{\mathbf{A}^{\circ}}$.
Thus, A is semi-preopen set. ■
The converse of remark 3.4 is not true, as the following of example 2:
$X=\{1,2,3,4\}$, $\tau=\{X,\emptyset,\{1\},\{2\},\{1,2\}\}$
$\tau_{\alpha} = PO(X) = \tau \cup \{\{1,2,3\},\{1,2,4\}\}$
$SPO(X) = PO(X) \cup \{\{1,3\},\{1,4\},\{1,3,4\},\{2,3\},\{2,4\},\{2,3,4\}\}$.

Remark 3.5 :

Every semi-p-open set is semi-preopen set.

Proof:

Let A be any semi-p-open set, this means there exists a preopen set in X say U such that $U \subseteq A \subseteq$ pre-cl U, and since pre-cl $U \subseteq \overline{\mathbf{U}}$ (by proposition 1.6), therefore $U \subseteq A \subseteq \overline{\mathbf{U}}$. Thus A is semi-preopen set. ■
But the converse is not true, as the following example show:

Example 3 :

Let $X= \{1,2,3,4\}$, $\tau=\{\emptyset, X, \{1,2\}, \{3\}, \{1,2,3\}\}$, $\mathcal{F}=\{X,\emptyset, \{1,2,4\},\{3,4\},\{4\}\}$
$PO(X) = \tau \cup \{\{1\},\{2\},\{1,3\},\{2,3\},\{1,3,4\},\{2,3,4\}\}$
$PC(X) = \mathcal{F} \cup \{\{2,3,4\},\{1,3,4\},\{2,4\},\{1,4\},\{2\},\{1\}\}$
$SPO(X) = PO(X) \cup \{\{1,4\},\{2,4\},\{3,4\},\{1,2,4\}\}$
now $\{1,4\} \in SPO(X)$, but $\{1\} \subseteq \{1,4\} \not\subset$ pre-cl $\{1\}=\{1\}$,thus $\{1,4\}$ is not semi-p-open set.

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