Numerical Solutions of Fractional Integral and Fractional Integrodifferential Equations

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Abstract

In this paper, we introduce and discuss an algorithm for the numerical solution of some kinds of fractional integral and fractional integrodifferential equations. The algorithm for the numerical solution of these equations is based on iterative approach. The stability and convergence of the fractional order numerical method are described. Finally, some numerical examples are provided to show that the numerical method for solving the fractional integral and fractional integrodifferential equations is an effective solution method.

Introduction

Various fields of science and engineering deal with the dynamical systems, which can be described by fractional-order equations. This topic has received a great deal of attention in the last decade (1, 4, 5).

Numerical methods associated with integral order ordinary differential equations were treated extensively in the literature. On the other hand, theoretical studies of the numerical methods and the error estimate of fractional order differential equation are quite limited, because theoretical analysis of fractional–order numerical methods is very difficult(2).

In this paper, we find the general solution of fractional integral equations of the form:

- $\sum_{i=0}^{n} d_i w^{(v+i)} - bw = 0$, where v < 0 and n+v < 0 .
- $\sum_{i=0}^{n} d_i w^{(iv)} = 0$, where v < 0.

we offer fractional differintegrations calculated by Nishimoto, since this definition enables us to calculate some fractional differintegrations that are easier to calculate than the other definitions.

Fractional Integral Equations of Order n+v

It's general form is:

$$\sum_{i=0}^{n} d_i w^{(v+i)} - bw = 0 \text{, where } v < 0 \text{ and } n+v < 0 \text{ and } v = \frac{-m}{k},\ m,k \in z,\ k \neq 0 \qquad [1]$$

Now, to find a_i's that satisfies e^{az} solution $w = e^{az}$ then:

$$\sum_{i=0}^{n} d_i a^{i-\frac{m}{k}} - b = 0$$

$$\sum_{i=0}^{n} d_i a^i = a^{\frac{m}{k}} b$$

$$\left(\sum_{i=0}^{n} d_i a^i\right)^k = a^m b^k \qquad [2]$$

Equation [2] is an an algebraic equation of order max(m,nk) in the unknown a and by finding its roots a_i's we find the general solution of [1]:

$$w(z) = \sum_{i=0}^{\max(m,nk)} c_i e^{a_i z}$$

where $c's$ are arbitrary constants.

Example 1: Consider fractional integral equations of the form [1] where n =1, $v = -3/2$, d_1=1, d_0 =1 and $b = -2$, eps=0.000006

Program

$$\sum_{i=0}^{n} d_i w^{(v+i)} - bw = 0 \Rightarrow w^{(-3/2)} + w^{\left(\frac{-1}{2}\right)} + 2w = 0$$

Now, to find a_i's that satisfies $w = e^{az}$, then:

$$a^{\left(\frac{-3}{2}\right)} + a^{\left(\frac{-1}{2}\right)} + 2 = 0 \Rightarrow 4a^3 - \frac{1}{4}a^2 - a - 1 = 0$$

Use Fixed point at 0 < a < 1

Choose $a = \sqrt[3]{\frac{1}{16}a^2 + \frac{1}{4}a + \frac{1}{4}}$

$$f(a, c_1, c_2, c_3) = (c_1 + c_2 a + c_3 a^2)^{\frac{1}{3}}$$

$$D(a, c_1, c_2, c_3) = \frac{d}{d(a)} f(a, c_1, c_2, c_3)$$

$a = 0.5$ $c_1 = 0.25$ $c_2 = 0.25$ $c_3 = 0.063$

$|D(a, c_1, c_2, c_3)| \rightarrow .19493451588085773769$

.19493451588085773769 <1 then

$f(a, c_1, c_2, c_3) \rightarrow 0.73100443455321651638$

$f(0.73100443455321651638, c_1, c_2, c_3) \rightarrow 0.77536871880939399242$

$f(0.77536871880939399242, c_1, c_2, c_3) \rightarrow 0.78374322291556063677$

$f(0.78374322291556063677, c_1, c_2, c_3) \rightarrow 0.78531902901931962366$

$f(0.78531902901931962366, c_1, c_2, c_3) \rightarrow 0.78561536712613996803$

$f(0.78561536712613996803, c_1, c_2, c_3) \rightarrow 0.78567108872455685163$

$f(0.78567108872455685163, c_1, c_2, c_3) \rightarrow 0.78568156605114873639$

$f(0.78568156605114873639, c_1, c_2, c_3) \rightarrow 0.78568353609401988289$

$$f\,(0.785683536094019882 89, c_1, c_2, c_3) \to 0.78568390651924141194$$
$$f\,(0.78568390651924141194, c_1, c_2, c_3) \to 0.78568397616992137448$$
$$f\,(0.78568397616992137448, c_1, c_2, c_3) \to 0.78568398926626797715$$
$$f\,(0.78568398926626797715, c_1, c_2, c_3) \to 0.78568399172876071348$$
$$f\,(0.78568399172876071348, c_1, c_2, c_3) \to 0.78568399219178071858$$
$$f\,(0.78568399219178071858, c_1, c_2, c_3) \to 0.78568399227884189928$$
$$f\,(0.78568399227884189928, c_1, c_2, c_3) \to 0.78568399229521192455$$
$$f\,(0.78568399229521192455, c_1, c_2, c_3) \to 0.78568399229828996376$$
$$f\,(0.78568399229828996376, c_1, c_2, c_3) \to 0.78568399229886872434$$
$$f\,(0.78568399229886872434, c_1, c_2, c_3) \to 0.78568399229897754811$$
$$f\,(0.78568399229897754811, c_1, c_2, c_3) \to 0.78568399229899801013$$
$$f\,(0.78568399229899801013, c_1, c_2, c_3) \to 0.78568399229900185758$$

Stop condition:

|0.78568399229900185758 - 0.78568399229899801013|=3.886E-15 < eps.

Then

0.78568399229900185758 is approximate 1^{st} root

So, and in the same way, we find other roots for the equation and put it in:

$$w(z) = c_1 e^{a_1 z} + c_2 e^{a_2 z} + c_3 e^{a_3 z}$$

Where $c's$ are arbitrary costants and $a_i,\ i = 1,\ldots,3$ is the i-th approximate root of the equation.

Fractional Integral Equations of Order nv

It's general form is:

$$\sum_{i=0}^{n} d_i w^{(iv)} = 0,\ d_n \neq 0 \ \text{and} \ v = \frac{m}{k} < 0,\ m,k \in z,\ k \neq 0 \qquad [3]$$

Now, to find a_i's satisfies the solution $w = e^{az}$ then:

$$\sum_{i=0}^{n} d_i a^{vi} e^{az} = 0,\ \text{since } e^{az} \neq 0, \text{ then } \sum_{i=0}^{n} d_i a^{vi} = 0$$

Rewriting this equation in the form:

$$a^{v}\left(d_n a^n + d_{n-1} a^{n-1} + \ldots + d_1\right) = -d_0$$

$$a^{\frac{-m}{k}}\left(d_n a^n + d_{n-1} a^{n-1} + \ldots + d_1\right) = -d_0$$

$$\left(d_n a^n + d_{n-1} a^{n-1} + \ldots + d_1\right)^k = a^m\left(-d_0\right)^k \qquad [4]$$

Which is an algebraic equation of order max(m,nk) in the unknown a.

Finding its roots a_i's w solution of [3] in the form:

$$w(z) = \sum_{i=0}^{\max(m,nk)} c_i e^{a_i z}$$

Where $c's$ e find, are arbitrary constants.

Notice. We use numerical method to solve equations [2] and [4] since we can not obtain theoretical solution always because general roots don't exist to solve equation of order more than the one which equal 5.

As a special case, consider the fractional integral equations of order 2v.

$$w^{(2v)} + bw^{(v)} + \lambda w = 0, \quad v < 0, \quad v = \frac{-m}{k}, \quad m, k \in z, \quad k \neq 0$$

Example 2: Consider fractional integral equations of the form [3] where n =4, $v = -1/2$, $d_0 = -1$, $d_1 = d_2 = d_3 = d_4 = 1$ and eps=0.00005.

Program

$$\sum_{i=0}^{n} d_i w^{(vi)} = 0 \Rightarrow -w^{\left(0.\frac{-1}{2}\right)} + w^{\left(1.\frac{-1}{2}\right)} + w^{\left(2.\frac{-1}{2}\right)} + w^{\left(3.\frac{-1}{2}\right)} + w^{\left(4.\frac{-1}{2}\right)} = 0$$

Now, to find a_i's that satisfies $w = e^{az}$, then:

$$-1 + a^{\left(\frac{-1}{2}\right)} + a^{(-1)} + a^{\left(3.\frac{-1}{2}\right)} + a^{(-2)} = 0 \Rightarrow a^{12} - 3a^9 - 3a^6 + a^3 + 1 = 0$$

Use a fixed point at 1 < a < 2

Choose $a = \left(3a^9 + 3a^6 - a^3 - 1\right)^{\frac{1}{12}}$

$$f(a, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}) = (c_1 + c_2 a + c_3 a^2 + c_4 a^3 + c_5 a^4 + c_6 a^5 + c_7 a^6 + c_8 a^7 + c_9 a^8 + c_{10} a^9)^{\frac{1}{12}}$$

$$D(a, c_1, c_2, c_3 c_4, c_5, c_6, c_7, c_8, c_9, c_{10}) = \frac{d}{d(a)} f(a, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10})$$

$a = 1.5 \quad c_1 = -1 \quad c_2 = 0 \quad c_3 = 0 \quad c_4 = -1 \quad c_5 = 0 \quad c_6 = 0 \quad c_7 = 3 \quad c_8 = 0 \quad c_9 = 0 \quad c_{10} = 3$

$|D(a, c_1, c_2, c_3 c_4, c_5, c_6, c_7, c_8, c_9, c_{10})| \to 0.68025578889276714799$

0.68025578889276714799 <1 then

$f(a, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}) \to 1.5253332893390171162$

$f\ (1.5253332893390171162, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}) \to 1.5425681077739530993$

$f\ (1.5425681077739530993, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}) \to 1.5542943084644395413$

$f\ (1.5542943084644395413, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}) \to 1.5622724402627148754$

$f\ (1.5622724402627148754, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}) \to 1.5677002760803632163$

$f\ (1.5677002760803632163, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}) \to 1.5713928861397996957$

$f\ (1.5713928861397996957, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}) \to 1.5739049148507729346$

$f\ (1.5739049148507729346, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}) \to 1.5756137644497782316$

$f\ (1.5756137644497782316, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}) \to 1.5767762147279333541$

$f\ (1.5767762147279333541, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}) \to 1.5775669638893329789$

$f\ (1.5775669638893329789, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}) \to 1.5781048604366752736$

$f\ (1.5781048604366752736, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}) \to 1.5784707548380016783$

$f\ (1.5784707548380016783, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}) \to 1.5787196467007653374$

$$f\,(1.5787196467007653374, c_1,c_2,c_3,c_4,c_5,c_6,c_7,c_8,c_9,c_{10}) \to 1.5788889495273038146$$
$$f\,(1.5788889495273038146, c_1,c_2,c_3,c_4,c_5,c_6,c_7,c_8,c_9,c_{10}) \to 1.5790041135285882146$$
$$f\,(1.5790041135285882146, c_1,c_2,c_3,c_4,c_5,c_6,c_7,c_8,c_9,c_{10}) \to 1.5790824508319440980$$
$$f\,(1.5790824508319440980, c_1,c_2,c_3,c_4,c_5,c_6,c_7,c_8,c_9,c_{10}) \to 1.5791357376804271389$$
$$f\,(1.5791357376804271389, c_1,c_2,c_3,c_4,c_5,c_6,c_7,c_8,c_9,c_{10}) \to 1.5791719846033924980$$
$$f\,(1.5791719846033924980, c_1,c_2,c_3,c_4,c_5,c_6,c_7,c_8,c_9,c_{10}) \to 1.5791966405705758888$$
$$f\,(1.5791966405705758888, c_1,c_2,c_3,c_4,c_5,c_6,c_7,c_8,c_9,c_{10}) \to 1.5792134121050226560$$
$$f\,(1.5792134121050226560, c_1,c_2,c_3,c_4,c_5,c_6,c_7,c_8,c_9,c_{10}) \to 1.5792248204712984177$$
$$f\,(1.5792248204712984177, c_1,c_2,c_3,c_4,c_5,c_6,c_7,c_8,c_9,c_{10}) \to 1.5792325806915453767$$
$$f\,(1.5792325806915453767, c_1,c_2,c_3,c_4,c_5,c_6,c_7,c_8,c_9,c_{10}) \to 1.5792378593625278867$$
$$f\,(1.5792378593625278867, c_1,c_2,c_3,c_4,c_5,c_6,c_7,c_8,c_9,c_{10}) \to 1.5792414500293495853$$
$$f\,(1.5792414500293495853, c_1,c_2,c_3,c_4,c_5,c_6,c_7,c_8,c_9,c_{10}) \to 1.5792438924789067742$$
$$f\,(1.5792438924789067742, c_1,c_2,c_3,c_4,c_5,c_6,c_7,c_8,c_9,c_{10}) \to 1.5792455538860883029$$
$$f\,(1.5792455538860883029, c_1,c_2,c_3,c_4,c_5,c_6,c_7,c_8,c_9,c_{10}) \to 1.5792466840112748059$$

Stop condition:

|1.5792466840112748059- 1.5792455538860883029| = 1.13E-6 < eps.

Then

1.5792466840112748059 is approximate 1st root

As in the same way, we find the remaining roots for the equation and put them in:

$$w(z) = c_1 e^{a_1 z} + c_2 e^{a_2 z} + c_3 e^{a_3 z} + c_4 e^{a_4 z} + c_5 e^{a_5 z} + c_6 e^{a_6 z} + c_7 e^{a_7 z} + c_8 e^{a_8 z} + c_9 e^{a_9 z} + c_{10} e^{a_{10} z} + c_{11} e^{a_{11} z} + c_{12} e^{a_{12} z}$$

Where $c's$ are arbitrary costants and $a_i,\ i = 1,...,12$ is the i-th approximate root of the equation.

Example 3: Consider fractional integral equations of the form $w^{(2v)} + bw^{(v)} + \lambda w = 0,$ where $v = -3/2$, b= -1, $\lambda = -1$ and eps = 0.00001

Program

$$w^{(2v)} + bw^{(v)} + \lambda w = 0 \Rightarrow w^{\left(2.\frac{-3}{2}\right)} - w^{\left(\frac{-3}{2}\right)} - w = 0$$

Now, to find a_i's that satisfies $w = e^{az}$, then:

$$a^{-3} - a^{\left(\frac{-3}{2}\right)} - 1 = 0 \Rightarrow -a^6 + 3a^3 - 1 = 0$$

Use a fixed point at 0 < a < 1

Choose $a = \sqrt[3]{\dfrac{a^6+1}{3}}$

$$f(a,c_1,c_2,c_3,c_4,c_5,c_6,c_7) = (c_1 + c_2 a + c_3 a^2 + c_4 a^3 + c_5 a^4 + c_6 a^5 + c_7 a^6)^{\frac{1}{3}}$$

$$D(a,c_1,c_2,c_3,c_4,c_{5,}c_6,c_7) = \frac{d}{d(a)} f(a,c_1,c_2,c_3,c_4,c_5,c_6,c_7)$$

$a = 0.5 \quad c_1 = 0.333 \quad c_2 = 0 \quad c_3 = 0 \quad c_4 = 0 \quad c_5 = 0 \quad c_6 = 0 \quad c_7 = 0.333$

$\left|D(a,c_1,c_2,c_3,c_4,c_{5,}c_6,c_7)\right| \to 4.2889469781380487238 \cdot 10^{-2}$

$4.2889469781380487238 \cdot 10^{-2} < 1$ then

$f(a,c_1,c_2,c_3,c_4,c_5,c_6,c_7) \to 0.59674159596366847153$

$f(0.59674159596366847153,\ c_1,c_2,c_3,c_4,c_5,c_6,c_7) \to 0.70364454056999046660$

$f(0.70364454056999046660, c_1,c_2,c_3,c_4,c_5,c_6,c_7) \to 0.72034885326896709841$

$f(0.72034885326896709841, c_1,c_2,c_3,c_4,c_5,c_6,c_7) \to 0.72425624597214515841$

$f(0.72425624597214515841, c_1,c_2,c_3,c_4,c_5,c_6,c_7) \to 0.72523130986763758800$

$f(0.72523130986763758800, c_1,c_2,c_3,c_4,c_5,c_6,c_7) \to 0.72547834796395703979$

$f(0.72547834796395703979, c_1,c_2,c_3,c_4,c_5,c_6,c_7) \to 0.72554117386534124342$

$f(0.72554117386534124342, c_1,c_2,c_3,c_4,c_5,c_6,c_7) \to 0.72555716687026665442$

$f(0.72555716687026665442, c_1,c_2,c_3,c_4,c_5,c_6,c_7) \to 0.72556123905387340202$

$f(0.72556123905387340202, c_1,c_2,c_3,c_4,c_5,c_6,c_7) \to 0.72556227598903008752$

$f(0.72556227598903008752, c_1,c_2,c_3,c_4,c_5,c_6,c_7) \to 0.72556254003692763325$

$f(0.72556254003692763325, c_1,c_2,c_3,c_4,c_5,c_6,c_7) \to 0.72556260727504995250$

$f(0.72556260727504995250, c_1,c_2,c_3,c_4,c_5,c_6,c_7) \to 0.72556262439682887969$

$f(0.72556262439682887969, c_1,c_2,c_3,c_4,c_5,c_6,c_7) \to 0.72556262875678689238$

$f(0.72556262875678689238, c_1,c_2,c_3,c_4,c_5,c_6,c_7) \to 0.72556262986702404804$

$f(0.72556262986702404804, c_1,c_2,c_3,c_4,c_5,c_6,c_7) \to 0.72556263014973928500$

$f(0.72556263014973928500, c_1,c_2,c_3,c_4,c_5,c_6,c_7) \to 0.72556263022173102582$

Stop condition:

|0.72556263022173102582-0.72556263014973928500|= 7.199E-11 < eps

Then

0.72556263022173102582 is approximate 1st root

So, and in the same way, we find the remaining roots for the equation and put them in:

$$w(z) = c_1 e^{a_1 z} + c_2 e^{a_2 z} + c_3 e^{a_3 z} + c_4 e^{a_4 z} + c_5 e^{a_5 z} + c_6 e^{a_6 z}$$

Where $c's$ are arbitrary costants and $a_i,\ i = 1,\dots,6$ is the i-th approximate root of the equation.

4. Fractional Integro-Differential Equations

We study these of the form:

$$\sum_{i=0}^{n} d_i w^{(v+i)} - bw = 0,\quad v = \frac{-m}{k},\ m,k \in z,\ k \neq 0 \text{ and } n-1 < |v| < n \qquad [5]$$

To solve such kind, suppose $w = e^{az}$, then:

$$\sum_{i=0}^{n} d_i a^{v+i} = b$$

and as in section 2,we get the solution.

Notice. Special kinds of fractional differintegral equations, of variable coefficient are solved theoretically by Nishimoto (3).

5. Existence and Uniqueness

To show the existence of a fixed point of fractional integral and fractional integrodifferential equations we give the following theorem.

Theorem. Let g(x) be a function in interval I= [a,b] for all $x \in I$ such that continuous, differentiable and there exists a constant λ, $0 \le \lambda < 1$ such that $|g'(x)| \le \lambda \quad \forall x \in I$ .Then g has exactly a fixed point $\alpha \in I$ and if $x_0 \in I$, then the sequence defined from $x_{n+1} = g(x_n)$, $n = 0,1,...$ .Converges to α .

Proof. To show the existence of a fixed point α.

suppose $g(a) \ne a,\ g(b) \ne b,$ then a <g(a) <b, a<g(b) <b

Let $h(a) = a - g(a) < 0, \quad h(b) > 0$

Then, there exist $\alpha \in (a,b)$ such that $h(\alpha) = 0,$ then $\alpha = g(x)$ by mean value theorem.

To show $x_{n+1} = g(x_n),$ let $x_0 \in I$ converges to α

Let $e_n = \alpha - x_n$ (error)

$e_n = g(\alpha) - g(x_{n+1})$ by mean value theorem between $\alpha,\ x_{n-1}$

$g(\alpha) - g(x_{n-1}) = g'(c)(\alpha - x_{n-1})$

Then $e_n = g'(c)e_{n-1} \Rightarrow |e_n| = |g'(c)||e_{n-1}|$

$|e_n| \le \lambda |e_{n-1}| \le \lambda \cdot \lambda |e_{n-2}|$

Then the sequence $\{e_n\} \to 0,$ then $\{x_n\} \to \alpha$.

To show uniqueness of α

let θ be another fixed point of g and θ $\in I$.

let θ = x_0, x_1= g(θ)= θ

$e_0 = \alpha - \theta,\ e_1 = \alpha - x_1 = \alpha - \theta$

Then $|e_0| = |e_1|$ but $|e_1| \le \lambda |e_0|,$ then $|e_1| < |e_0|$ which is a contradiction, then $e_0 = e_1 = 0,\ \alpha = \theta$.

6. Rate of convergence of iterative method

Give any iterative method its order which is said to be equals p if $|e_{n+1}| = c|e_n|^p$ for some number c depending on f where $e_n = \alpha - x_n,$ that is $|e_{n+1}| \alpha |e_n|^p$ if, $p = 1$ is linear

as p increases the method converges faster.

Since $x = g(x),\ |g'(x)| = \lambda,\ 0 \le \lambda < 1$

$|e_{n+1}| = \lambda |e_n|$

Then general iterative method is linear.

Finally, the method converge if $0 \le \lambda < 1$ and c must be less then 1.

7. Stability

The issue of stability is very important when implementing the method on a computer in finite-precision arithmetic because we must take into account the effects introduced by rounding errors. It is known (6) that the classical iterative method is a reasonable and practically useful compromise in the sense that its stability properties allow for a save application to mildly stiff equations without undue propagation of rounding errors, whereas the implementation does not require extremely time consuming elements. From the results of (7), we can see that these properties remain unchanged when we look at the fractional version of the algorithm instead of the classical one, and therefore it is also clear that the behaviour does not depend on the order of the differential operators involved.

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الحلول العددية للمعادلات التكاملية الكسرية والتفاضلية التكاملية الكسرية

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الخلاصة

في هذا البحث قدمنا وناقشنا خوارزمية للحل العددي لبعض انواع المعادلات التكاملية الكسرية والتفاضلية التكاملية الكسرية. وان خوارزمية الحل العددي لتلك المعادلات قائمة على اساس التقارب التكراري. كما ناقشنا الاستقرارية والتقارب للطريقة العددية ذي الرتبةالكسورية.

اخيرا قدمنا بعض الامثلة العددية التي تثبت ان الطرائق العددية لحل المعادلات التكاملية الكسرية والتفاضلية التكاملية الكسرية هي طرائق ذي حل مؤثر فعال مقبول .