The product of para – compact spaces

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Abstract

The product of m-paracompact and m-strongly paracompact are briefly discussed.

Introduction

In this paper we give a necessary and sufficient condition for the product of two m-paracompact (m-metacompact, m-strongly paracompact) spaces to be an m-paracompact (m--metacompact, m-strongly paracompact) space.

Also, we provide simple proofs for the results of Morita (1), katuta(2), Wenjen(3), and smith and Krajewski(4). Theorems 2.4 and 2.7 are the main results of this paper.

Definition 1.1:

A Collection $\{A_t : t \in T\}$ of subsets a space X is called order locally finite, if one can introduce a total order $\leq$ on T such that for each t in T, $\{A_s : s \leq t\}$ is locally finite at each point of A_t.

Definition 1.2:

Let F be a subset of a space (X,T), Then:

(I) F is called relatively m-paracompact if every open cover of cardinality $\leq m$ of F by members of T has an open locally finite refinement by members of T Covering.

(II) F is called relatively m-metacompact if every open cover of cardinality $\leq m$ by members T has an open point-finite refinement by members of T covering F.

(III) F is called relatively m-strongly paracompact if for every open cover U of cardinality $\leq$ m of F by members of T there is an open star-finite cover of X such that $\{v\cap F: v\in V\}$ refines U.

If a set F is relatively m-paracompact (relatively m-metacompact, relatively m-strongly paracompact) for each cardinal number m, then it is called relatively paracompact (relatively metacompact, relatively strongly paracompact).

Definition 1.3:

Let m be an infinite cardinal number and let S be a subspace of a space X. Then S is said to be p^m- embedded in X if every m-separable continuous pseudo-metric on S can be extended to an m-separable continuous pseado-metric on X.

We say S is p-embedded on X if every continuous pseudo-metric on S can be extended to X.

Lemma 1.1:

Let U be an open cover of X and let $V=\{v_i:i \text{ is in } N\}$ be a star-finite open cover of X. Let W be a star –finite open cover of X satisfying:

$\{v_i\cap w: w\in W\}$ is a refinement of U for each i in N. Then U has an open star-finite refinement.

Proof:

The proof is easy as $R=\bigcup_{i=1}^{\infty} R_i$ where $R_i=\{v_i\cap w: w\in W\}$ is a refinement of U by assumption , it is star- finite (because W is star-finite).■

Lemma1.2:

Let U be an open star-finite cover of X .Then $U=\{U_t: t\in T\}$ where $U_t=\{ U_{ti}:i\in N \}$ and the sets U_t^* are open and disjoint.

For the proof see (p.228 , lemmas 1 and 2 of (5)).

2. Some generalizations of perfect marppings:

Definition 2.1:

A continuous function f: X $\rightarrow$ Y is called m-paraperfect metaperfect) if for every open cover U of X of cardinality $\leq$ m there is an open cover V of Y and an open refinement W of U such that for each v in V, $f^{-1}(v)$ is contained in the union of a subfamily R of W, where R is locally finite (point – finite).

Definition 2.2:

A continuous function f: X $\rightarrow$ Y is called m-strongly paraperfect if for every open cover U of X of cardinality $\leq$ m there is an open cover V of Y such that for each countable subcollection V_1 of V there is an open star-finite cover W_1 of X such that $f^{-1}(V_1) \cap W_1$ is a refinement of U.

Note: We shall state and prove results for m-paracompact spaces except in cases where others need special consideration.

Theorem 2.1:

Any continous function from an m-paracompact space X onto any space Y is an m-paraparfect function.

The proof is obvious.

Theorem2.2:

Let f: X $\rightarrow$ Y be an m-paraperfect function .Then X is m-paracompact if Y is paracompact .

The proof of the above theorem in the paraperfect and metaperfect case is simple. The case for strong paraperfectness follows from lemmas 1.1 and 1.2.

Theorem2.3:

Let M be a closed m-paracompact subset of a space X and let F be a closed subset of the interior G of M. Then F is relatively m-paracompact if M is paracompact.

For the proof see (6).

Theorem2.4:

Let f: X $\rightarrow$ Y be a continuous and closed mapping of X onto Y. Then f is an m-paraperfect function if $f^{-1}(y)$ is relatively m-paracompact for each y in Y.

Proof:

Let U be an open cover of X of cardinality $\leq$ m. since $f^{-1}(y)$ is relatively m-paracompact, U has an open refinement Ry in X which is locally finite and covers $f^{-1}(y)$. Define $Oy=y-f(X-R^{*}y)$ for each y in Y. Since f is closed, Oy is open and $f^{-1}(Oy)$ is contained in $R^{*}y$. Now, $R=\cup$ {Ry:y in Y} is an open refinement of U which covers X and {Oy:y in Y} is an open cover of Y such that for each Oy, $f^{-1}(Oy)$ is contained in $R^{*}y$ where Ry is locally finite.■

Corollary 2.5:

Let $f:X \longrightarrow Y$ be a closed continuous function.Then X is m-para-compact if Y is paracompact and $f^{-1}(y)$ is relatively m-para-compact for each y in Y.

Remark:

In view of theorem 2.7 in (7), if a closed m-paracompact subset S is P^{m}-embedded in a normal space X, Then S is relatively m-paracompact.

Corollary 2.6:

Let $f: X \rightarrow Y$ be a continuous and closed function. Then:

(I) X is paracompact if Y is paracompact and $f^{-1}(y)$ is compact for all y in Y. (Hanai (8))

(II) A normal space X is paracompact if Y is paracompact and $f^{-1}(y)$ paracompact and p-embedded in X for all y in Y. (shapiro (9))

The next theorem is proved only for paracompact spaces.

Theorem 2.7:

Let $f: X \rightarrow Y$ be a continuous function, where X is a regular space. Let $\{v_s : s \in S\}$ and $\{h_s : s \in S\}$ be two coverings of Y satisfying:

a) $\{v_s : s \in S\}$ is closed covering of y and $\{h_s : s \in S\}$ is an open ordered locally finite covering of y such that v_s is contained hs for each s in S.
b) $f^{-1}(v_s)$ is relativety paracompact for each s in S. Then X is paracompact.

Proof:

Let U be an open cover of X Since $f^{1}(v_s)$ is relativety paracompact for each s in S.

i.e.: $\forall s \in S$, there is an open locally finite refinement say $W(v_s)$ in X of U which covers $f^{1}(v_s)$.

Now, we have $f^{1}(v_s) \subseteq W(v_s)$ and $f^{1}(Vs) \subset f^{1}(hs)$ $\forall s \in S$.Define $A=\{f^{-1}(hs) \cap W$: w is in $W(v_s)$ and s is in S}. Evidently, A is an ordered locally finite refinement of U which covers X. Hence X is paracompact by (2).■

3. Application of the results of seIn this section all spaces are assumedT_1.

Theorem 3.1:

Let X and Y be two spaces and let $P_x:X \times Y \rightarrow X$ be the projection map. Then P_x is an m-paraperfect function if for each x in X, there is a neighborhood O_x such that $P^{-1}{}_x(O_x)$ is relativity m-paracompact.

The proof follows easily.

The following theorem gives us as corollaries the results of Morita (1), Wenjen(3), and smith and krajewski(4).

Theorem 3.2:

Let X and Y be a paracompact spaces.Then $X \times Y$ is m-paracompact if and only if for each x in X, there is a neighberhood U_x of x such that $U_x \times Y$ is relativety m-paracompact.

The proof follows from theorem 2.2 and the observation that if $X \times Y$ is m-paracompact then it is relativly m-paracompact.

Theorem 3.3:

Let $\{v_s$:s is in S} be a closed covering of X and let $\{h_s$: s is in S} be an open ordered locally finite covering X such that $v_s \subset h_s$ for each s in S and $p_x{}^{-1}(v_s)$ is relatively paracompact in XxY for each s in S. Then $X \times Y$ is paracompact if and only if Y is paracompact.

The proof follows from theorem 2.1.

Finally, we state an analogue of ponomarev's.

Theorem 3.4:

Let X be a space, then X is m-paracompact if and only if there exists an m-paraperfect function from X onto and a paracompact space Y.

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جداء الفضاءات فوق التراص

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الخلاصة

الهدف من هذا البحث ما يأتي:

1. دراسة العلاقة بين جداء الفضاءات م- فوق التراص والفضاءات م- فوق التـراص القوية والفضاءات م- فوق التامـــــة القوية.

2. إثبات أن كل دالة مستمرة من الفضاء م- فوق التراص إلى فضاء أخر تكون دالة من النمط م- فوق التامة.

3. تعريف الفضاء م- فوق التراص نسبيا كتعميم للفضاءات م- فوق التراص ومـن ثـم الحصول على نتائج تتعلق بالفضاءات فوق التراص.

4. تعميم نظرية موريتا (1963).