On Double Stage Shrinkage Estimator For the Variance of Normal Distribution With Unknown Mean

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Abstract

This paper is concerned with preliminary test double stage shrinkage estimators to estimate the variance (σ^2) of normal distribution when a prior estimate (σ_0^2) of the actual value (σ^2) is a available when the mean is unknown , using specifying shrinkage weight factors $\psi(\cdot)$ in addition to pre-test region (R).

Expressions for the Bias, Mean squared error [MSE $(\cdot)$], Relative Efficiency [R.EFF $(\cdot)$], Expected sample size [$E(n/\sigma^2)$] and percentage of overall sample saved of proposed estimator were derived. Numerical results (using MathCAD program) and conclusions are drawn about selection of different constants including in the mentioned expressions. Comparisons between the suggested estimator with the classical estimator in the sense of Bias and Relative Efficiency are given. Furthermore, comparisons with the earlier existing works are drawn.

Keywords: Normal Distribution, Double Stage Shrinkage Estimator, Bias Ratio, Mean Squared Error , Relative Efficiency , Expected sample size. Percentage of overall sample saved and probability of avoiding the second sample .

Introduction

" The normal distribution is the most widely known and used of all distributions. Because the normal distribution approximates many natural phenomena so well, it has developed into a standard of reference for many probability problems. The question, Why is the normal distribution useful?, and the answer is many things actually are normally distributed, or very close to it. For example, height and intelligence are approximately normally distributed; measurement errors also often have a normal distribution , the normal distribution is easy to work with mathematically. In many practical cases, the methods developed using normal theory work quite well, even when the distribution is not normal . The normal distribution plays a vital role in many applied problems of biology, economics, engineering, financial risk management, genetics, hydrology, mechanics, medicine, number theory, statistics, physics, psychology, reliability, etc., and has been extensively studied, both from theoretical and applications point of view, by many researchers, since its inception.

There is a very strong connection between the size of a sample N and the extent to which a sampling distribution approaches the normal form.

Many sampling distributions based on large sample of size can be approximated by the normal distribution even though the population distribution itself is definitely not normal"; [1],[12] .

The probability density of the normal distribution is:

$$f(x|\mu,\sigma)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad , \quad -\infty < x < \infty \qquad \ldots(1)$$

Here, μ is the *mean* or *expectation* of the distribution (and also its median and mode). The parameter σ is its standard deviation with its variance then σ^2. If $\mu = 0$ and $\sigma = 1$, the distribution is called the standard normal distribution or the unit normal distribution denoted by $N(0,1)$ and a random variable with that distribution is a standard normal deviate.

Assume that $x_1, x_2, \ldots, x_n$ be a random sample of size (n) from a normal population with unknown mean (μ) and unknown variance (σ^2). In conventional notation, we write $X\sim N(\mu,\sigma^2)$.

In this work, we suggest the problem of estimating the variance (σ^2) when some prior information (σ_0^2) regarding the variance (σ^2) is available. More specifically, we assume that the prior information regarding is due the following reasons, [10]:

1. We believe that (σ_0^2) is close to the true value of σ^2, or

2. We fear that, (σ_0^2) may be near the true value of σ^2, i.e.; something bad happens if (σ^2) approximately equals to (σ_0^2) and we do not known about it.

In such a situation it is natural to start with the MVUE $(\hat{\sigma}^2)$ of σ^2 and modify it by moving it closure to (σ_0^2) using shrinkage weight factor [$\psi(\cdot)$], so that the resulting estimator though perhaps biased, has a smaller mean squared error [MSE] than that of $(\hat{\sigma}^2)$ in some interval around (σ_0^2). This method of constructing an estimator that incorporates the prior value leads to what is known as a shrinkage estimator

i.e.; $\psi_1(\hat{\sigma}^2)\hat{\sigma}^2+[1-\psi_1(\hat{\sigma}^2)]\sigma_0^2$; $0 \le \psi_1(\hat{\sigma}^2) \le 1$. …(2)

where $\psi_i(\hat{\sigma}^2)$, $0 \le \psi_i(\hat{\sigma}^2) \le 1$ is shrinkage weight factor specifying the belief in $\hat{\sigma}^2$ and $(1-\psi_i(\hat{\sigma}^2))$ specifying the belief in (σ_0^2) and $\psi_i(\hat{\sigma}^2)$ may be a function of $\hat{\sigma}^2$ or a constant.

Preliminary test double stage shrinkage estimator for the variance (σ^2) that utilize a prior estimate (σ_0^2) is represented as following steps:-

1. Select two positive integers (n_1) and (n_2).

2. Obtain a random sample of size (n_1) on x [first stage sample]. Compute sample variance $\hat{\sigma}_1^2(s_1^2)$ [MVUE], where $s_1^2 = \frac{\sum_{i=1}^{n_1}(x_i - \hat{\mu}_1)^2}{n_1 - 1}$ and $\hat{\mu}_1$ is the first sample mean.

3. Choose a suitable region (R) around σ_0^2.

Noted that, this work concerns with pre-test region.

4. If $\hat{\sigma}_1^2 \in R$, take the shrinkage estimator of the form defined in (1).

Here, we put forward $\psi_1(\cdot) = k = n_1\alpha^4$

However, if $\hat{\sigma}_1^2 \notin R$, obtain a second stage random sample of size n_2 on x and advise the estimator $\hat{\sigma}_p^2$ of σ^2 as polling estimator of two samples variance.

i.e.; $\hat{\sigma}_p^2 = (n_1\hat{\sigma}_1^2 + n_2\hat{\sigma}_2^2)/n$ …(3)

where $\hat{\sigma}_2^2$ is the variance of the second random sample and $n = n_1 + n_2$.

Thus, the suggested form of preliminary test double stage shrunken estimator (PDSE) has the following form:

$$\tilde{\sigma}_{DS}^2 = \begin{cases} k(\hat{\sigma}_1^2 - \hat{\sigma}_0^2) + \sigma_0^2 & , \quad \text{if } \hat{\sigma}_1^2 \in R, \\ \hat{\sigma}_p^2 & , \quad \text{if } \hat{\sigma}_1^2 \notin R, \end{cases} \quad \ldots(4)$$

where R is the pre-test region for testing the hypothesis $H_0 : \hat{\sigma}^2 = \sigma_0^2$ against $H_A : \hat{\sigma}^2 \ne \sigma_0^2$ with level of significance (α) using test statistic $T(\hat{\sigma}^2/\sigma_0^2) = \frac{(n_1-1)\hat{\sigma}^2}{\sigma_0^2}$.

$$\text{i.e.;} \quad R = \left[X^2_{1-\alpha/2,n-1}\frac{\sigma_0^2}{n-1}, X^2_{\alpha/2,n-1}\frac{\sigma_0^2}{n-1}\right] \quad \ldots(5)$$

$\hat{\sigma}_i^2$ (i=1,2) is the sample variance of stage (i), $X^2_{1-\alpha/2,n-1}$ and $X^2_{\alpha/2,n-1}$ are the lower and upper $100(\alpha/2)$ percentile point of chi-square distribution with degree of freedom (n – 1) and $n = n_1 + n_2$.

And for simplest we shall assume that R=[a,b].

The aim of this paper is to show that the proposed estimators defined in (4) is better than the classical estimators and some estimators introduced by some authors in the sense of mean squared error and relative efficiency.

Several authors have studied (PDSE) for estimating the mean and variance of Normal distribution and also for estimating the parameters of different distributions, for example, Katti [7],Thompson [10],Al-Joboori [3,4], Al-Joboori et al[5,6],Waikar, Schuurmann and Raghunathan [11] , Handa, Kambo and Al-Hemyari [8] and Maha [9].

Preliminary Test Double Stage Shrunken Estimator (PDSE)

In this section, recall (PDSE) defined in (3) for estimating the variance of normal distribution .

The expressions for Bias and Mean Squared Error [MSE(·)] of $\tilde{\sigma}^2_{DS}$ are respectively as follows:

$$\mathrm{Bias}(\tilde{\sigma}^2_{DS}\,|\hat{\sigma}^2,\mathrm{R})=\int_{\hat{\sigma}_2^2=0}^{\infty}\int_{\hat{\sigma}_1^2\in R}[k(\hat{\sigma}_1^2-\hat{\sigma}_0^2)+(\sigma_0^2-\sigma^2)]f(\hat{\sigma}_1^2)f(\hat{\sigma}_2^2)\mathrm{d}\hat{\sigma}_1^2\,\mathrm{d}\hat{\sigma}_2^2+$$
$$\int_{\hat{\sigma}_2^2=0}^{\infty}\int_{\hat{\sigma}_1^2\in\bar{R}}[\frac{1}{2}(\frac{n_1\hat{\sigma}_1^2+n_2\hat{\sigma}_2^2}{n_1+n_2}+\sigma_0^2)-\sigma^2]f(\hat{\sigma}_1^2)f(\hat{\sigma}_2^2)\mathrm{d}\hat{\sigma}_1^2\,\mathrm{d}\hat{\sigma}_2^2$$

Where ,

$$f(\hat{\sigma}_i^2\,|\sigma^2)=\begin{cases}\dfrac{\left[(n_i-1)/\sigma^2\right]^{\frac{n_i-1}{2}}\left[\hat{\sigma}_i^2\right]^{\frac{n_i-3}{2}}\exp\left[-\dfrac{(n_i-1)}{2\sigma^2}\hat{\sigma}_i^2\right]}{2^{\frac{n_i-1}{2}}\Gamma\left[\dfrac{(n_i-1)}{2}\right]}, & for\ \hat{\sigma}^2\geq 0\quad,\sigma^2\geq 0\quad,i=1,2\\ 0 & ,otherwise\end{cases}\qquad\ldots(6)$$

and by simple calculations, we get

$$\mathrm{Bias}(\tilde{\sigma}^2_{DS}\,|\hat{\sigma}^2,\mathrm{R})=\frac{\sigma^2}{n_1-1}\Big\{\ \mathrm{k}\{\mathrm{j}_1(\mathrm{a_i},\mathrm{b_i})-(\mathrm{n}_1-1)\lambda\mathrm{J}_0(\mathrm{a_i},\mathrm{b_i})\}$$
$$+(\mathrm{n}_1-1)(\lambda-1)\mathrm{J}_0(\mathrm{a_i},\mathrm{b_i})-\left\{\frac{1}{1+\mathrm{u}}[\mathrm{J}_1(\mathrm{a_i},\mathrm{b_i})-(\mathrm{n}_1-1)\mathrm{J}_0(\mathrm{a_i},\mathrm{b_i})\right\}\ \Big\}$$
…(7)

The Bias Ratio of $\tilde{\sigma}^2_{DS}$ = $\mathrm{Bias}(\tilde{\sigma}^2_{DS}\,|\hat{\sigma}^2,\mathrm{R})\ /\ (\dfrac{\sigma^2}{n_1-1})$ …(8)

And,

$$\mathrm{MSE}(\tilde{\sigma}^2_{DS}\,|\,\sigma^2,\mathrm{R})=\mathrm{E}[\tilde{\sigma}^2_{DS}-\sigma^2]^2$$
$$=\int_{\hat{\sigma}_2^2=0}^{\infty}\int_{\mathrm{R}}[\mathrm{k}(\hat{\sigma}_1^2-\hat{\sigma}_0^2)+(\sigma_0^2-\sigma^2)]^2\mathrm{f}(\mathrm{s}_1^2/\sigma^2)\mathrm{f}(\mathrm{s}_2^2/\sigma^2)\mathrm{d}\hat{\sigma}_1^2\mathrm{d}\hat{\sigma}_2^2+$$
$$\int_{\hat{\sigma}_2^2=0}^{\infty}\int_{\bar{\mathrm{R}}}\left[\mathrm{s}_\mathrm{p}^2-\sigma^2\right]^2\mathrm{f}(\mathrm{s}_1^2/\sigma^2)\mathrm{f}(\mathrm{s}_2^2/\sigma^2)\mathrm{d}\hat{\sigma}_1^2\mathrm{d}\hat{\sigma}_2^2$$

We conclude,

$$\mathrm{MSE}(\tilde{\sigma}^2_{DS}\,|\,\sigma^2,\mathrm{R})=\frac{\sigma^4}{(n_1-1)^2}\Big\{\ \mathrm{K}^2\{\mathrm{J}_2(\mathrm{a_i},\mathrm{b_i})-2(\mathrm{n}_1-1)\lambda\mathrm{J}_1(\mathrm{a_i},\mathrm{b_i})+$$
$$(\mathrm{n}_1-1)^2\lambda^2\mathrm{J}_0(\mathrm{a_i},\mathrm{b_i})\}+2\mathrm{k}\{(\mathrm{n}_1-1)(\lambda-1)[\mathrm{J}_1(\mathrm{a_i},\mathrm{b_i})-(\mathrm{n}_1-1)\lambda\mathrm{J}_0(\mathrm{a_i},\mathrm{b_i})]\}$$

$$+(n_1-1)^2(\lambda-1)^2J_0(a_i,b_i)+2\left(\frac{1}{1+u}\right)^2(n_1-1)+2\left(\frac{u}{1+u}\right)^2\cdot\frac{(n_1-1)^2}{(un_1-1)}$$

$$-\left\{\left(\frac{1}{1+u}\right)^2[J_2(a_i,b_i)-2(n_1-1)J_1(a_i,b_i)+(n_1-1)^2J_0(a_i,b_i)]\right.$$

$$\left.+2\left(\frac{u}{1+u}\right)^2\frac{(n_1-1)}{(un_1-1)}J_0(a_i,b_i)\right\}\ \}\qquad\ldots(9)$$

where $j_\ell(a_1,b_1)=\int_{a_1}^{b_1} y^\ell f(y)dy,\ \ell=0,1,2.$ …(10)

$$\lambda=\frac{\sigma_0^2}{\sigma^2},\ y_i=\frac{(n_i-1)s^2}{\sigma^2}\sim X^2_{n_i-1},\qquad\ldots(11)$$

$a_1=\lambda\ X^2_{1-\alpha/2,n_1-1},\ b_1=\lambda\ X^2_{\alpha/2,n_1-1},$ …(12)

also $u = n_2/n_1$, and $n = n_1 + n_2$,

The Expected sample size can be obtained as:

$E(n\,|\,\sigma^2,R) = n_1\,[1 + u(1 - j_0(a_1,b_1))]$ …(13)

The Efficiency of $\tilde{\sigma}^2_{DS}$ relative to $\hat{\sigma}^2(s^2)$ is given by :

$$R.Eff\,(\tilde{\sigma}^2_{DS}\,|\,\sigma^2,R)=\frac{MSE(\hat{\sigma}^2\,|\,\sigma^2)}{[MSE(\tilde{\sigma}^2_{DS}\,|\,\sigma^2,R)][E(n\,|\,\sigma^2,R)/n]}\qquad\ldots(14)$$

Where $[E\,(n/\sigma^2,R)/n]$ is the Expected sample size proportion.

The probability of avoiding the second sample computing by $p(\hat{\sigma}_1^2\in R)$.

Finally, the percentage of overall sample saved can be obtained by

$(n_2/n)\ p(\hat{\sigma}_1^2\in R)*100$ …(15)

See for example [3],[4], [9] ,[10] and [11].

Numerical Results and Discussion

The computations (using MathCAD program) of Relative Efficiency [R.Eff(·)], Bias ratio [B(·)], Expected sample size [E(n | μ,R)], Expected sample size proportion, Percentage of the overall sample saved and probability of avoiding the second sample were used for the estimator $\tilde{\sigma}^2_{DS}$. These computations were performed for n_1 = 5,7,9,11,13; u =,2,6,8,10; λ = 0.0(0.1)1,2; α = 0.01,0.05,0.1.

Some of these computations are given in annex tables (1)-(17) .

The observations mentioned in the tables lead to the following results:

i. R.Eff($\tilde{\sigma}^2_{DS}$) are adversely proportional with small value of α.

ii. R.Eff($\tilde{\sigma}^2_{DS}$) are maximum when $\sigma^2\approx\sigma_0^2$ and decreasing otherwise

iii. The Bias Ratio of $\tilde{\sigma}^2_{DS}[B(\tilde{\sigma}^2_{DS}) = \frac{n_1 - 1}{\sigma^2}Bias(\tilde{\sigma}_{DS})]$ are reasonably small when $\sigma^2 \approx \sigma^2_0$, otherwise $B(\tilde{\sigma}^2_{DS})$ will be maximum.

iv. $B(\tilde{\sigma}^2_{DS})$ are reasonably small with small value of α.

v. $R.Eff(\tilde{\sigma}^2_{DS})$ and $B(\tilde{\sigma}^2_{DS})$ are decreasing function with respect to first sample size n_1.

vi. The Expected values of sample size of $\tilde{\sigma}^2_{DS}$ is closer to n_1 specially when $\sigma^2 \approx \sigma^2_0$ and start faraway slowly with increasing of λ.

vii. Percentage of the overall sample saved $[\frac{n_2}{n} j_0(a_1,b_1)\times 100]$ is a decreasing function of λ, and has a maximum value when $\sigma^2 \approx \sigma^2_0$.

viii. $R.Eff(\tilde{\sigma}^2_{DS})$ is an increasing function with respect to u ($u = n_2/n_1$).

iv. The suggested estimator $\tilde{\sigma}^2_{DS}$ is more efficient than the estimators introduced by Al-Bermani [2] and AL-Joboori[4] .

Conclusions

From the above discussions, it is obvious that by using guess point value one can improve the usual estimator by using shrinkage technique. It can be noted that if the guess point is very close to the actual value of the parameter, the proposed estimator perform is better than the usual estimator. If one has no confidence in the guessed value, then proposed preliminary test shrunken estimators can be suggested. We can safely use the proposed estimators for small sample size at the usual level of significance and moderate value of shrunken weight factor $\Psi(.)$. The difficulty of obtaining samples is because of the scarcity and high cost led researchers to use the double stage shrinkage estimators to reduce the sample size that we need and for achieving savings of the items in the sample and obtaining high efficiency estimators.

References

1. Ahsanullah,M. et al.,(2014)," Normal and Student’s t Distributions and Their Applications", Atlantis Press and the authors. DOI: 10.2991/978-94-6239-061-4_2.
2. Al-Bermani,M.H.,(2008), "comparison between Bayesian Shrinkage Estimators and Shrinkage Estimators for the variance of normal distribution by using simulation",ph.D. thesis, college of Administration and economics, university of Baghdad.
3. Al-Joboori,A.N.(2010) , “Pre-Test Single and Double Stage Shrunken Estimators for the Mean of Normal Distribution With Known Variance”, Baghdad J. for Science, 7 (4), 1432-1442.
4. Al-Joboori,A.N.,(2011), Single and Double Stage Shrunken Estimators for the Variance of Normal Distribution". Journal of Education, AL-Mustansiryia. University, 2 , 597-608.
5. Al-Joboori, A.N., et. al , (2014), Single and Double Stage Shrinkage Estimators for the Normal Mean with the Variance Cases , International Journal of Statistics ,.(38),2,.1127 - 1134.

6. Al-Joboori, A.N. and Ameen, M. M.(2016), On Double Stage Minimax-Shrinkage Estimator For Generalized Rayleigh Model, International Journal of Applied Mathematical Research, 5 (1), 39-47.
7. Katti, S.K., (1962), Use of Some a Prior Knowledge in the Estimation of Means from Double Samples, Biometrics,.18,.139-147.
8. Handa, B.K., Kambo, N.S. and Al-Hemyari; Z.A., (1988), "On Double Stage Shrunken Estimator of the Mean of Exponential Distribution", IAPQR Trans,.(13),1,.19-33.
9. Maha.A. M.(2011) , Double Stage Shrinkage Estimator of Two Parameters Generalized Rayleigh Distribution, Education college journal AL-Mustansiriya University (2), 566-573
10. Thompson, J.R., (1968), Some Shrinkage Techniques for Estimating the Mean, J. Amer. Statist. Assoc,.63.113-122.
11. Waikar, V.B., Schuurmann, F.J. and Raghunathar, T.E., (1984), On a Two-Stage Shrinkage Testimator of the Mean of a Normal Distribution, Commum. Statist-Theor. Meth., A,.13 (15),.1901-1913.
12. https://www3.nd.edu/~rwilliam/stats1/x21.pdf.

Table (1): Shows Bias ratio [B(·)] and R.E.ff w.r.t. α, n_1 and λ when u =2

			λ							
α	**n_1**	R.Eff(-) B(-)	0.25	0.50	0.75	1	1.25	1.50	1.75	2
0.01	5	R.Eff(-)	3.4225875	6.921051	17.5040876	34.055869	19.9017239	8.5142224	4.335779	2.5633057
		B(-)	-1.3026559	-1.5453646	-0.8971937	0.0153905	0.9868869	1.9653436	2.9382656	3.902073
	7	R.Eff(-)	2.5218328	4.5657778	12.7316908	28.0764855	14.2199712	5.5730906	2.7507472	1.6013574
		B(-)	-1.4307066	-2.20102	-1.3479203	0.0157813	1.4734652	2.9296484	4.3617489	5.76015
	9	R.Eff(-)	2.2344079	3.4244664	10.2563163	25.4964189	11.3593618	4.1642427	2.0076996	1.1539179
		B(-)	-1.3689823	-2.7706791	-1.7927305	0.0159786	1.9578157	3.8804139	5.7438821	7.5241258
	11	R.Eff(-)	2.1910011	2.7529803	8.6693885	24.0612878	9.5571863	3.326823	1.574551	0.8951149
		B(-)	-1.2082129	-3.2566881	-2.2313379	0.0160965	2.4399322	4.8161439	7.0770414	9.1721113
	13	R.Eff(-)	2.2811736	2.3156926	7.5393297	23.1480114	8.2890208	2.7683441	1.2902951	0.7267732
		B(-)	-1.0097075	-3.6626852	-2.6635711	0.0161745	2.9197667	5.7353787	8.3540981	10.684783
0.05	5	R.Eff(-)	3.6194909	6.1808134	14.5712829	27.1849786	17.6374379	7.933749	4.0761615	2.406342
		B(-)	-0.8929279	-1.208987	-0.7460543	0.0480284	0.9402995	1.8412993	2.7191497	3.5619491
	7	R.Eff(-)	2.9246212	4.2077426	10.8292699	22.9410048	12.8134551	5.1866676	2.5615348	1.4850268
		B(-)	-0.8955581	-1.6696883	-1.1277614	0.0490329	1.3835582	2.6961091	3.9248409	5.0444625
	9	R.Eff(-)	2.7826467	3.25824	8.8296177	21.0500992	10.3076319	3.8608721	1.8563702	1.0656958
		B(-)	-0.7871207	-2.0348087	-1.4992136	0.0495215	1.8205121	3.5130704	5.0225485	6.3026994
	11	R.Eff(-)	2.8635758	2.7085502	7.5250407	19.9818656	8.7026587	3.0727202	1.4509899	0.8312489
		B(-)	-0.6412647	-2.3148738	-1.8601538	0.0498112	2.2514014	4.2914965	6.008853	7.3316557
	13	R.Eff(-)	3.0579131	2.3588939	6.5853355	19.2959357	7.561666	2.5489063	1.1900405	0.6854149
		B(-)	-0.4968093	-2.5203237	-2.2105946	0.049997	2.6762825	5.0307925	6.88241	8.1350713
0.1	5	R.Eff(-)	3.809139	5.8752143	12.8310536	22.523826	15.5288307	7.3579544	3.8501684	2.2911515
		B(-)	-0.7040529	-1.0102327	-0.6359713	0.0714824	0.8853387	1.7015398	2.4774536	3.1963249
	7	R.Eff(-)	3.2170698	4.1163008	9.6938813	19.242412	11.4827459	4.8503737	2.4332978	1.4289644
		B(-)	-0.6720805	-1.3703967	-0.967212	0.0727881	1.2837426	2.4483068	3.4831665	4.3586444
	9	R.Eff(-)	3.1448576	3.2715374	7.9930075	17.7562484	9.3307197	3.630191	1.7788781	1.0457584
		B(-)	-0.5642185	-1.6389512	-1.2865157	0.0733923	1.6732909	3.1433387	4.3485916	5.2481266
	11	R.Eff(-)	3.2748593	2.7871275	6.8728943	16.9099419	7.9327245	2.9040967	1.4075725	0.8379534
		B(-)	-0.4403664	-1.8297872	-1.5937468	0.0737264	2.054479	3.7879236	5.0795818	5.886642
	13	R.Eff(-)	3.4937754	2.4844107	6.0604952	16.3637492	6.9291647	2.4226542	1.1723785	0.7136349
		B(-)	-0.3276427	-1.9555015	-1.8890279	0.0739318	2.4275048	4.3832542	5.6833953	6.3014258

Table (2): shows Expected Sample Size of w.r.t. α, u, and λ when u=2

			λ							
u	**n_1**	**α**	**0.25**	**0.50**	**0.75**	**1**	**1.50**	**1.25**	**1.75**	**2**
2	5	0.01	9.4626764	6.1612363	5.2784085	5.1	5.0863241	5.1104802	5.1458085	5.1868903
		0.05	10.9604355	7.4027053	5.9395857	5.5	5.4507682	5.541906	5.6868973	5.8569183
		0.1	11.7142558	8.286365	6.5968836	6	5.9220739	6.0697617	6.3146586	6.6033556
	7	0.01	15.2774575	9.2324937	7.4605309	7.14	7.1388037	7.2101349	7.3118377	7.4375958
		0.05	17.2132402	11.2628078	8.4749133	7.7	7.6964458	7.9650779	8.349908	8.8028357
		0.1	18.0780715	12.5900999	9.4439234	8.4	8.3949103	8.8294659	9.4531645	10.1659641
	9	0.01	21.677695	12.6609409	9.6835974	9.18	9.2034457	9.3521743	9.5716994	9.8578205
		0.05	23.7805868	15.5644753	11.0974452	9.9	9.9834929	10.5230634	11.2806411	12.1806478
		0.1	24.6328946	17.3373257	12.4021872	10.8	10.9315681	11.7892456	12.9614272	14.2785969
	11	0.01	28.3740477	16.445059	11.9491813	11.22	11.2808331	11.5426699	11.9456985	12.4917937
		0.05	30.4266966	20.2635052	13.8075215	12.1000215	12.3123412	13.2217745	14.4951456	16.0118761
		0.1	31.1907751	22.4627683	15.4703011	13.2	13.5316881	0.4529807	0.5100832	0.5730448
	13	0.01	35.168353	20.576298	14.2587195	13.26	13.3715672	13.787624	14.4534252	15.380123
		0.05	37.030099	25.314775	16.604957	14.300013	14.6835055	16.0662498	18.00335	20.2990627
		0.1	37.6719203	27.9067068	18.6466128	15.6000225	16.1951122	18.3053181	21.0545513	24.0153376

Table (3): shows Expected Sample Size Proportion w.r.t. α, u, n_1 and λ

			λ							
u	**n_1**	**α**	**0.25**	**0.50**	**0.75**	**1**	**1.25**	**1.50**	**1.75**	**2**
2	5	0.01	0.6308451	0.4107491	0.3518939	0.34	0.3390883	0.3406987	0.3430539	0.3457927
		0.05	0.7306957	0.4935137	0.3959724	0.3666667	0.3633845	0.3694604	0.3791265	0.3904612
		0.1	0.7809504	0.5524243	0.4397922	0.4	0.3948049	0.4046508	0.4209772	0.4402237
	7	0.01	0.727498	0.4396426	0.3552634	0.34	0.339943	0.3433398	0.3481827	0.3541712
		0.05	0.8196781	0.5363242	0.4035673	0.3666667	0.3664974	0.3792894	0.3976147	0.4191827
		0.1	0.8608605	0.5995286	0.4497106	0.4	0.3997576	0.4204508	0.4501507	0.4840935
	9	0.01	0.8028776	0.4689237	0.3586518	0.34	0.3408684	0.3463768	0.3545074	0.3651045
		0.05	0.8807625	0.576462	0.4110165	0.3666667	0.369759	0.3897431	0.4178015	0.4511351
		0.1	0.9123294	0.6421232	0.4593403	0.4	0.4048729	0.4366387	0.4800529	0.5288369
	11	0.01	0.8598196	0.4983351	0.3620964	0.34	0.3418434	0.3497779	0.3619909	0.3785392
		0.05	0.9220211	0.6140456	0.4184097	0.3666673	0.3731012	0.4006598	0.4392468	0.4852084
		0.1	0.945175	0.6806899	0.468797	0.4	0.4100512	0.4529807	0.5100832	0.5730448
	13	0.01	0.9017526	0.5275974	0.3656082	0.34	0.3428607	0.3535288	0.3706006	0.3943621
		0.05	0.9494897	0.6490968	0.4257681	0.366667	0.3765001	0.4119551	0.4616244	0.5204888
		0.1	0.9659467	0.7155566	0.4781183	0.4000006	0.4152593	0.4693671	0.5398603	0.6157779

Table (4): shows Percentage of Overall Sample Saved w.r.t. α, u, n_1 and λ

			λ							
u	**n_1**	**α**	**0.25**	**0.50**	**0.75**	**1**	**1.25**	**1.50**	**1.75**	**2**
2	5	0.01	36.9154908	58.9250912	64.8106099	66	66.0911727	65.9301318	65.6946101	65.4207316
		0.05	26.93043	50.6486313	60.402762	63.3333333	63.6615452	63.0539602	62.0873511	60.9538781
		0.1	21.9049616	44.7575669	56.0207761	60	60.5195074	59.5349218	57.9022763	55.9776293
	7	0.01	27.2502024	56.035744	64.4736623	66	66.0056967	65.6660242	65.1817254	64.5828769
		0.05	18.0321895	46.3675819	59.6432699	63.3333333	63.350258	62.0710574	60.2385331	58.0817348
		0.1	13.9139454	40.0471431	55.0289363	60	60.0242367	57.9549245	54.9849309	51.5906472
	9	0.01	19.7122408	53.1076264	64.1348246	66	65.9131643	65.3623175	64.5492614	63.4895536
		0.05	11.9237526	42.3537951	58.898351	63.3333333	63.0241004	61.025691	58.2198476	54.8864896
		0.1	8.7670569	35.7876824	54.0659732	60	59.5127109	56.3361273	51.994714	47.1163077
	11	0.01	14.0180372	50.1664879	63.7903596	66	65.8156573	65.0222124	63.8009135	62.1460797
		0.05	7.797889	38.5954388	58.1590257	63.3332681	62.6898751	59.9340168	56.0753164	51.4791632
		0.1	5.4824996	31.9310052	53.1202997	60	58.9948844	54.7019278	48.9916837	42.6955182
	13	0.01	9.824736	47.2402616	63.4391808	66	65.7139302	64.6471179	62.9399354	60.5637872
		0.05	5.0510283	35.0903195	57.4231882	63.3332994	62.3499858	58.8044877	53.8375641	47.9511213
		0.1	3.4053325	28.4443414	52.1881723	59.9999423	58.4740712	53.0632869	46.0139709	38.4222114

Table (5): shows Bias ratio [B(·)] and R.E.ff w.r.t. α, n_1 and λ when u =6

			λ							
α	n_1	R.Eff(-) B(-)	0.25	0.50	0.75	1	1.25	1.50	1.75	2
0.01	5	R.Eff(-)	11.5006501	33.8896946	128.4109443	367.1232704	144.7705775	50.2690886	23.7539585	13.5001075
		B(-)	-1.5075366	-1.6724435	-0.9400311	6.5959247E-3	0.9894473	1.9725209	2.9485466	3.9153134
	7	R.Eff(-)	7.824044	19.8793331	85.9709005	293.141886	97.6234315	31.6149811	14.4828724	8.0291786
		B(-)	-1.6642392	-2.3842134	-1.4066272	6.7634277E-3	1.4801297	2.9441184	4.3834731	5.7900406
	9	R.Eff(-)	6.7130198	13.6628666	65.1865558	261.9441267	74.7424065	22.8642487	10.1646016	5.482838
		B(-)	-1.600479	-3.0082667	-1.8677671	6.8479789E-3	1.9690037	3.9040281	5.7813398	7.5781944
	11	R.Eff(-)	6.5571466	10.2569075	52.5095898	244.7941858	60.8410979	17.7495447	7.6624893	4.0189104
		B(-)	-1.418965	-3.5457157	-2.3232238	6.8984803E-3	2.4560206	4.8507276	7.1345046	9.2577111
	13	R.Eff(-)	6.9381658	8.1644261	43.8607296	233.9596269	51.3673362	14.3824044	6.0326161	3.0809526
		B(-)	-1.1906399	-3.9992212	-2.7728235	6.9319302E-3	2.9411151	5.782728	8.4356936	10.8086101
0.05	5	R.Eff(-)	11.9571477	26.3678738	96.7325926	279.9319496	120.1103596	41.7774658	19.2261266	10.5928915
		B(-)	-1.0751756	-1.3863891	-0.8374679	0.020581	0.9486396	1.8700222	2.7618359	3.6163367
	7	R.Eff(-)	9.2710531	16.0497764	64.8867595	226.6866491	80.7341533	25.5180318	11.1806818	5.9511695
		B(-)	-1.0793316	-1.9078691	-1.2501509	0.0210104	1.4074243	2.7515257	4.0054699	5.1488562
	9	R.Eff(-)	8.8439706	11.5218048	49.1298125	203.8020931	61.3853871	17.9695621	7.5716586	3.9374258
		B(-)	-0.9505521	-2.3241581	-1.6521752	0.0212187	1.8605786	3.5978076	5.1465369	6.4646175
	11	R.Eff(-)	9.371124	9.0678239	39.493593	191.1080902	49.5700178	13.6334173	5.5762673	2.8671892
		B(-)	-0.7761139	-2.6461336	-2.0434257	0.0213418	2.3081601	4.4076609	6.1798519	7.5543804
	13	R.Eff(-)	10.4950917	7.579985	32.9274671	183.0477624	41.5061007	10.8394335	4.3402899	2.2319648
		B(-)	-0.6025631	-2.8847259	-2.423926	0.0214203	2.7501513	5.1800754	7.102504	8.4182427
0.1	5	R.Eff(-)	12.7626413	23.7335622	78.9331919	215.928713	99.0740976	35.1681173	16.1459181	8.8558915
		B(-)	-0.8649671	-1.2001307	-0.7526722	0.030574	0.897929	1.7493915	2.5500459	3.2882648
	7	R.Eff(-)	10.5600862	15.0114678	53.6450635	176.6596949	66.9926153	21.3915861	5.0146261	5.0146261
		B(-)	-0.82466	-1.6168933	-1.1218638	0.0311074	1.3214546	3.6124198	4.5197047	4.5197047
	9	R.Eff(-)	10.5459862	11.1664975	41.008249	159.5533794	51.0635638	6.3899673	3.4077472	3.4077472
		B(-)	-0.692626	-1.929093	-1.4778917	0.0313403	1.7365953	3.2772928	4.5357948	5.4777418
	11	R.Eff(-)	11.4988003	33.2356514	33.2356514	150.0039513	41.2888982	11.4400968	4.782322	2.578555
		B(-)	9.0868879	-2.1522249	-1.8208237	0.0314575	2.1435705	3.9657723	5.3237704	6.179403
	13	R.Eff(-)	13.0387903	7.8389809	27.9187493	143.9156124	34.6026821	9.1401254	3.8043577	2.0999618
		B(-)	-0.403068	-2.3003672	-2.1508414	0.0315197	2.5424719	4.604823	5.9819398	6.6488272

Table (6): shows Expected Sample Size of w.r.t. α, u, and λ

			λ							
u	**n_1**	**α**	**0.25**	**0.50**	**0.75**	**1**	**1.50**	**1.25**	**1.75**	**2**
6	5	0.01	18.3880292	8.4837089	5.8352256	5.3	5.2589723	5.3314407	5.4374254	5.5606708
		0.05	22.8813065	12.2081159	7.8187571	6.5	6.3523047	6.6257179	7.060692	7.5707549
		0.1	25.1427673	14.8590949	9.7906508	8	7.7662217	8.2092852	8.9439757	9.8100668
	7	0.01	31.8323725	13.6974812	8.3815927	7.42	7.4164111	7.6304048	7.935513	8.3127875
		0.05	37.6397206	19.7884234	11.42474	9.1	9.0893375	9.8952338	11.0497241	12.4085071
		0.1	40.2342144	23.7702998	14.3317701	11.2	11.1847309	12.4883976	14.3594935	16.4978923
	9	0.01	47.033085	19.9828226	11.0507921	9.54	9.610337	10.0565228	10.7150982	11.5734615
		0.05	53.3417604	28.693426	15.2923357	11.7	11.9504787	13.5691903	15.8419234	18.5419434
		0.1	55.8986839	34.0119772	19.2065617	14.4	14.7947042	17.3677369	20.8842816	24.8357908
	11	0.01	63.1221431	27.335177	13.847544	11.66	11.8424993	12.6280097	13.8370956	15.4753811
		0.05	69.2800899	38.7905156	19.4225645	14.3000646	14.9370236	17.6653234	21.4854368	26.0356284
		0.1	71.5723254	45.3883048	24.4109033	17.6	18.5950644	22.8450915	28.4982332	34.731437
	13	0.01	79.5050589	35.7288939	16.7761585	13.78	14.1147016	15.362872	17.3602755	20.140369
		0.05	85.0902969	49.9443262	23.8148698	16.9000397	18.0505166	22.1987494	28.01005	34.8971881
		0.1	87.015761	57.7201205	29.9398384	20.8000675	22.5853367	28.9159543	37.163654	46.0460127

Table (7): shows Expected Sample Size Proportion w.r.t. α, u, n_1 and λ

			λ							
u	**n_1**	**α**	**0.25**	**0.50**	**0.75**	**1**	**1.25**	**1.50**	**1.75**	**2**
6	5	0.01	0.5253723	0.2423917	0.1667207	0.1514286	0.1502564	0.1523269	0.155355	0.1588763
		0.05	0.6537516	0.3488033	0.2233931	0.1857143	0.1814944	0.1893062	0.2017341	0.2163073
		0.1	0.7183648	0.4245456	0.2797329	0.2285714	0.221892	0.234551	0.2555422	0.2802876
	7	0.01	0.6496403	0.2795404	0.1710529	0.1514286	0.1513553	0.1557225	0.1619492	0.1696487
		0.05	0.7681576	0.4038454	0.233158	0.1857143	0.1854967	0.2019435	0.2255046	0.2532348
		0.1	0.8211064	0.4851082	0.2924851	0.2285714	0.2282598	0.2548653	0.2930509	0.3366917
	9	0.01	0.7465569	0.3171877	0.1754094	0.1514286	0.152545	0.1596273	0.1700809	0.1837057
		0.05	0.8466946	0.4554512	0.2427355	0.1857143	0.1896901	0.215384	0.2514591	0.2943166
		0.1	0.8872807	0.5398727	0.3048661	0.2285714	0.2348366	0.2756784	0.3314965	0.3942189
	11	0.01	0.8197681	0.3550023	0.1798382	0.1514286	0.1537987	0.1640001	0.1797025	0.200979
		0.05	0.8997414	0.5037729	0.2522411	0.1857151	0.1939873	0.2294198	0.2790316	0.338125
		0.1	0.9295107	0.5894585	0.3170247	0.2285714	0.2414943	0.2966895	0.3701069	0.4510576
	13	0.01	0.873682	0.3926252	0.1843534	0.1514286	0.1551066	0.1688228	0.1907723	0.2213227
		0.05	0.9350582	0.5488387	0.2617019	0.1857147	0.1983573	0.2439423	0.3078027	0.3834856
		0.1	0.9562172	0.634287	0.3290092	0.2285722	0.2481905	0.3177577	0.4083918	0.5060001

Table (8): shows Percentage of Overall Sample Saved w.r.t. α, u, n_1 and λ

			λ							
u	**n_1**	**α**	**0.25**	**0.50**	**0.75**	**1**	**1.25**	**1.50**	**1.75**	**2**
6	5	0.01	47.4627739	75.7608316	83.327927	84.8571429	84.9743648	84.7673123	84.4644987	84.1123692
		0.05	34.6248385	65.1196688	77.660694	81.4285714	81.8505581	81.0693774	79.8265943	78.3692718
		0.1	28.1635221	57.5454432	72.0267121	77.1428571	77.8107952	76.5448994	74.4457838	71.9712376
	7	0.01	35.0359746	72.0459566	82.8947087	84.8571429	84.8644672	84.4277453	83.8050755	83.0351275
		0.05	23.1842436	59.6154625	76.6842041	81.4285714	81.4503317	79.8056453	77.4495426	74.6765161
		0.1	17.8893584	51.489184	70.7514895	77.1428571	77.1740186	74.5134743	70.6949112	66.3308321
	9	0.01	25.3443096	68.281234	82.4590602	84.8571429	84.7454969	84.0372654	82.9919076	81.6294261
		0.05	15.330539	54.4548794	75.7264513	81.4285714	81.0309862	78.4616028	74.8540898	70.5683438
		0.1	11.2719303	46.0127345	69.5133941	77.1428571	76.5163426	72.4321637	66.8503466	60.5781099
	11	0.01	18.0231907	64.4997701	82.0161766	84.8571429	84.6201308	83.5999874	82.029746	79.9021024
		0.05	10.0258572	49.622707	74.7758902	81.4284876	80.601268	77.0580216	72.0968354	66.1874956
		0.1	7.0489281	41.0541495	68.2975282	77.1428571	75.8505657	70.3310501	62.9893076	54.8942377
	13	0.01	12.6318034	60.7374792	81.564661	84.8571429	84.4893389	83.1177231	80.9227741	77.8677263
		0.05	6.4941792	45.1161251	73.8298134	81.4285277	80.1642675	75.6057699	69.2197253	61.6514417
		0.1	4.3782846	36.5712961	67.0990787	77.1427829	75.1809487	68.224226	59.1608198	49.3999861

Table (9): shows Bias ratio [B(·)] and R.E.ff w.r.t. α, n_1 and λ when u =8

			λ							
α	**n_1**	R.Eff(-) B(-)	0.25	0.50	0.75	1	1.25	1.50	1.75	2
0.01	5	R.Eff(-)	15.7992081	51.243284	221.9193302	750.5055682	255.3246925	83.6829607	38.7486865	21.7570903
		B(-)	-1.5416833	-1.6936233	-0.9471706	5.1301631E-3	0.989874	1.9737171	2.9502601	3.9175201
	7	R.Eff(-)	10.5963984	29.171536	145.3474248	596.7733005	169.5630469	51.9979693	23.2798893	12.68689
		B(-)	-1.7031613	-2.4147456	-1.4164116	5.260443E-3	1.4812404	2.9465301	4.3870937	5.7950224
	9	R.Eff(-)	9.045174	19.616913	108.3130188	532.1530326	128.2543243	37.1936914	16.0918181	8.4817049
		B(-)	-1.6390618	-3.0478647	-1.8802732	5.3262048E-3	1.9708684	3.9079638	5.7875827	7.5872058
	11	R.Eff(-)	8.8373127	14.486276	86.0098203	496.6870436	103.3795428	28.5774649	11.945746	6.0866627
		B(-)	-1.4540904	-3.5938869	-2.3385382	5.3654835E-3	2.458702	4.8564916	7.1440818	9.2719778
	13	R.Eff(-)	9.3905281	11.3847566	70.9699266	474.3030401	86.5662921	22.9288293	9.2631688	4.5730995
		B(-)	-1.2207954	-4.0553105	-2.7910323	5.3914999E-3	2.9446732	5.7906195	8.4492929	10.8292479
0.05	5	R.Eff(-)	16.3784606	38.1751751	158.3777524	548.2465274	202.6710004	65.8214265	29.3966741	15.8692206
		B(-)	-1.1055502	-1.4159561	-0.8527035	0.0160065	0.9500296	1.8748094	2.7689502	3.6254012
	7	R.Eff(-)	12.6272074	22.6975465	103.5790628	441.2897603	133.5242593	39.3949258	16.6710742	8.6530577
		B(-)	-1.1099605	-1.9475659	-1.2705491	0.01634	1.411402	2.7607618	4.0189081	5.1662551
	9	R.Eff(-)	12.074578	16.0400529	76.9340319	395.5866532	99.9286385	27.2668572	11.0550078	5.5915848
		B(-)	-0.9777906	-2.372383	-1.6776688	0.0165016	1.8672564	3.6119305	5.1672016	6.4916038
	11	R.Eff(-)	12.8963957	12.4842686	60.9012672	370.3070947	79.6540237	20.38264	8.0009247	3.9977397
		B(-)	-0.7985888	-2.7013436	-2.0739711	0.0165969	2.3176199	4.4270216	6.2083517	7.5915012
	13	R.Eff(-)	14.6253932	10.3513612	50.1327912	354.283013	65.9664258	15.9974695	6.1379857	3.0679499
		B(-)	-0.6201888	-2.9454596	-2.4594812	0.0166575	2.7624628	5.2049559	7.1391863	8.465438
0.1	5	R.Eff(-)	17.5479115	33.8859019	125.6270148	408.0886038	161.466766	53.2351603	23.6664404	12.711558
		B(-)	-0.8917862	-1.2317803	-0.7721223	0.0237559	0.9000274	1.7573668	2.5621447	3.3035882
	7	R.Eff(-)	14.5043071	21.0108329	83.2637693	331.625787	106.8234348	31.7118932	13.4014449	7.0154928
		B(-)	-0.8500899	-1.6579761	-1.1476391	0.0241606	1.3277399	2.5537544	3.633962	4.5465481
	9	R.Eff(-)	14.5867575	15.4296581	62.481054	298.5410309	80.0743853	21.9405279	8.9865301	4.682014
		B(-)	-0.7140273	-1.97745	-1.5097877	0.0243316	1.747146	3.2996185	4.5669953	5.5160111
	11	R.Eff(-)	16.1093432	12.4465036	49.9081068	280.1330665	63.8832813	16.4581892	6.6336712	3.4968988
		B(-)	-0.5579119	-2.2059645	-1.8586699	0.0244127	2.1584191	3.9954137	5.3644685	6.2281965
	13	R.Eff(-)	18.5877299	10.6714167	41.4304458	268.4201894	52.94306	13.0032137	5.2203146	2.8206194
		B(-)	-0.4156389	-2.3578448	-2.194477	0.024451	2.561633	4.6417512	6.0316972	6.7067275

Table (10): shows Expected Sample Size w.r.t. α, u, and λ when u=8

			λ							
u	**n_1**	**α**	**0.25**	**0.50**	**0.75**	**1**	**1.50**	**1.25**	**1.75**	**2**
		0.01	22.8507055	9.6449453	6.1136341	5.4	5.3452964	5.4419209	5.5832339	5.7475611
	5	0.05	28.841742	14.6108212	8.7583428	7	6.8030729	7.1676239	7.7475893	8.4276732
		0.1	31.857023	18.1454599	11.3875344	9	8.6882956	9.2790469	10.2586342	11.4134224
		0.01	40.1098299	15.929975	8.8421237	7.56	7.5552147	7.8405397	8.2473507	8.7503834
	7	0.05	47.8529608	24.0512312	12.8996533	9.8	9.7857833	10.8603118	12.3996322	14.2113428
		0.1	51.3122859	29.3603998	16.7756935	12.6	12.5796412	14.3178634	16.812658	19.6638564
		0.01	40.1098299	15.929975	8.8421237	7.56	7.5552147	7.8405397	8.2473507	8.7503834
8	9	0.05	68.1223472	35.2579013	17.3897809	12.6	12.9339716	15.0922537	18.1225646	21.7225912
		0.1	71.5315786	42.349303	22.608749	16.2	16.7262722	20.1569825	24.8457089	30.1143877
		0.01	59.7107799	23.6437634	11.7343894	9.72	9.8137826	10.4086971	11.2867976	12.4312821
	11	0.05	88.7067866	48.0540208	22.230086	15.4000861	16.2493648	19.8870978	24.9805823	31.0475046
		0.1	91.7631005	56.8510731	28.8812044	19.8	21.1267526	26.7934553	34.3309775	42.641916
		0.01	101.6734119	43.3051919	18.034878	14.04	14.4862688	16.150496	18.8137007	22.520492
	13	0.05	109.1203959	62.2591015	27.4198264	18.200053	19.7340221	25.2649992	33.0133999	42.1962508
		0.1	111.6876814	72.6268274	35.5864512	23.4000901	25.7804489	34.2212724	45.2182054	57.0613502

Table (11): shows Expected Sample Size Proportion w.r.t. α, u, n_1 and λ

			λ							
u	**n_1**	**α**	**0.25**	**0.50**	**0.75**	**1**	**1.25**	**1.50**	**1.75**	**2**
		0.01	0.5077935	0.2143321	0.1358585	0.12	0.1187844	0.1209316	0.1240719	0.1277236
	5	0.05	0.6409276	0.3246849	0.1946298	0.1555556	0.1511794	0.1592805	0.1721687	0.1872816
		0.1	0.7079338	0.4032324	0.2530563	0.2	0.1930732	0.206201	0.2279696	0.2536316
		0.01	0.636664	0.2528567	0.1403512	0.12	0.119924	0.124453	0.1309103	0.138895
	7	0.05	0.7595708	0.3817656	0.2047564	0.1555556	0.1553299	0.1723859	0.1968196	0.2255769
		0.1	0.8144807	0.4660381	0.2662808	0.2	0.1996768	0.2272677	0.2668676	0.3121247
		0.01	0.636664	0.2528567	0.1403512	0.12	0.119924	0.124453	0.1309103	0.138895
8	9	0.05	0.8410166	0.4352827	0.2146887	0.1555556	0.1596787	0.1863241	0.2237354	0.2681801
		0.1	0.8831059	0.5228309	0.2791204	0.2	0.2064972	0.2488516	0.3067371	0.3717826
		0.01	0.7371701	0.2918983	0.144869	0.12	0.1211578	0.1285024	0.1393432	0.1534726
	11	0.05	0.8960281	0.4853941	0.2245463	0.1555564	0.164135	0.2008798	0.2523291	0.3136112
		0.1	0.9269	0.5742533	0.2917293	0.2	0.2134015	0.270641	0.3467776	0.4307264
		0.01	0.8690035	0.3701298	0.1541443	0.12	0.1238143	0.1380384	0.1608009	0.1924828
	13	0.05	0.932653	0.5321291	0.2343575	0.155556	0.1686669	0.2159402	0.2821658	0.3606517
		0.1	0.9545956	0.6207421	0.3041577	0.2000008	0.2203457	0.2924895	0.3864804	0.4877038

Table (12): shows Percentage of Overall Sample Saved w.r.t. α, u, n_1 and λ

			λ							
u	**n_1**	**α**	**0.25**	**0.50**	**0.75**	**1**	**1.25**	**1.50**	**1.75**	**2**
		0.01	49.2206544	78.5667883	86.4141465	88	88.1215635	87.9068424	87.5928135	87.2276421
	5	0.05	35.9072399	67.5315084	80.537016	84.4444444	84.8820603	84.0719469	82.7831348	81.2718374
		0.1	29.2066155	59.6767559	74.6943681	80	80.6926765	79.3798957	77.2030351	74.636839
		0.01	36.3336033	74.7143254	85.9648831	88	88.0075957	87.5546989	86.9089672	86.1105026
	7	0.05	24.0429193	61.8234426	79.5243598	84.4444444	84.4670106	82.7614099	80.3180442	77.442313
		0.1	18.5519272	53.3961908	73.3719151	80	80.0323156	77.2732327	73.3132412	68.7875296
		0.01	36.3336033	74.7143254	85.9648831	88	88.0075957	87.5546989	86.9089672	86.1105026
8	9	0.05	15.8983367	56.4717268	78.5311347	84.4444444	84.0321339	81.3675881	77.6264635	73.1819862
		0.1	11.6894092	47.7169099	72.0879642	80	79.3502812	75.1148365	69.3262854	62.8217436
		0.01	26.2829877	70.8101686	85.5130995	88	87.884219	87.1497567	86.0656819	84.6527382
	11	0.05	10.3971853	51.460585	77.5453677	84.4443575	83.5865002	79.9120224	74.7670885	68.6388843
		0.1	7.3099995	42.5746736	70.8270663	80	78.6598459	72.9359038	65.3222449	56.9273576
		0.01	13.0996479	62.9870155	84.5855744	88	87.6185737	86.1961572	83.9199139	80.7517162
	13	0.05	6.7347044	46.7870927	76.5642509	84.4443991	83.1333144	78.4059836	71.7834189	63.9348284
		0.1	4.5404433	37.9257886	69.5842298	79.999923	77.9654283	70.7510492	61.3519612	51.2296152

Table (13): shows Bias ratio [B(·)] and R.E.ff w.r.t. α, n_1 and λ when u =10

			λ							
α	**n_1**	R.Eff(-) B(-)	0.25	0.50	0.75	1	1.25	1.50	1.75	2
0.01	5	R.Eff(-)	20.1608911	70.0901591	337.8975179	1.3283519E+3	397.6598069	124.8418315	56.9138232	31.6400086
		B(-)	-1.5634131	-1.7071014	-0.951714	4.1974058E-3	0.9901455	1.9744783	2.9513505	3.9189244
	7	R.Eff(-)	13.3973709	39.0274664	217.5781913	1.0535918E+3	261.0147471	76.7975902	33.7481223	18.1241341
		B(-)	-1.7279299	-2.4341752	-1.4226381	4.3039982E-3	1.4819473	2.9480647	4.3893978	5.7981926
	9	R.Eff(-)	11.3994496	25.8338403	159.9465795	938.3261324	195.5715793	54.4183679	23.0147832	11.8933723
		B(-)	-1.6636145	-3.0730633	-1.8882317	4.3578032E-3	1.972055	3.9104683	5.7915555	7.5929403
	11	R.Eff(-)	11.1412572	18.8554033	125.5839342	875.1259032	156.4276203	41.440367	16.8569689	8.382714
		B(-)	-1.4764429	-3.6245414	-2.3482836	4.38994E-3	2.4604083	4.8601595	7.1501764	9.2810565
	13	R.Eff(-)	11.8744639	14.6867545	102.6244565	835.2613431	130.1326415	32.965176	12.9025777	6.1950229
		B(-)	-1.2399852	-4.0910038	-2.8026196	4.411226E-3	2.9469375	5.7956414	8.4579469	10.8423811
0.05	5	R.Eff(-)	20.8613574	50.5030161	229.6000232	929.6749514	302.3386783	93.3190785	40.7379243	21.6438516
		B(-)	-1.1248795	-1.4347714	-0.8623989	0.0130954	0.9509142	1.8778557	2.7734775	3.6311696
	7	R.Eff(-)	16.029387	29.5492404	147.3846837	745.5066524	196.1790652	54.9541643	22.6409719	11.5270072
		B(-)	-1.1294517	-1.9728274	-1.2835298	0.0133679	1.4139333	2.7666393	4.0274596	5.1773272
	9	R.Eff(-)	15.3578356	20.6607021	107.9248367	667.0983423	145.0392399	37.5186889	14.7696354	7.3170393
		B(-)	-0.9951243	-2.4030716	-1.6938919	0.0134998	1.8715058	3.6209178	5.1803519	6.508777
	11	R.Eff(-)	16.497595	15.9609391	84.4687135	623.805499	114.4561662	27.7215424	10.5487783	5.1616786
		B(-)	-0.812891	-2.7364772	-2.093409	0.0135774	2.3236398	4.4393421	6.226488	7.6151236
	13	R.Eff(-)	18.8799178	13.1624716	68.8833075	596.3929771	93.9822353	21.5408041	8.0061526	3.9211192
		B(-)	-0.6314051	-2.9841083	-2.4821073	0.0136267	2.7702973	5.2207889	7.1625296	8.4954713
0.1	5	R.Eff(-)	22.4114451	44.3832194	178.0034482	670.6915364	233.5318898	72.9627158	31.6895138	16.7617537
		B(-)	-0.9088528	-1.251921	-0.7844997	0.0194171	0.9013627	1.7624419	2.5698439	3.3133394
	7	R.Eff(-)	18.521388	27.157685	115.868162	542.716628	152.0394491	42.7926667	17.6322489	9.0783545
		B(-)	-0.8662726	-1.6841197	-1.1640416	0.0197399	1.3317397	2.5633405	3.6476707	4.5636303
	9	R.Eff(-)	18.7244342	19.7741578	85.7923384	487.5814411	112.5621374	29.2462296	11.6709842	5.9814853
		B(-)	-0.7276463	-2.0082226	-1.5300851	0.0198716	1.7538601	3.3138258	4.5868502	5.5403642
	11	R.Eff(-)	20.8738002	15.8588915	67.8167002	456.9695181	88.9104098	21.7230208	8.5311762	4.4272972
		B(-)	-0.5685978	-2.2401624	-1.8827538	0.0199297	2.1678682	4.0142765	5.3903673	6.2592469
	13	R.Eff(-)	24.4004725	13.5423265	55.8224208	437.5157543	73.0752811	17.0242182	6.6628313	3.5477961
		B(-)	-0.4236386	-2.3944215	-2.2222451	0.0199527	2.5738265	4.6652509	6.063361	6.7435731

Table (14): shows Expected Sample Size of w.r.t. α, u, and λ

u	n_1	α	λ 0.25	0.50	0.75	1	1.50	1.25	1.75	2
10	5	0.01	27.3133819	10.8061816	6.3920426	5.5	5.4316205	5.5524012	5.7290424	5.9344513
		0.05	34.8021775	17.0135265	9.6979285	7.5	7.2538411	7.7095298	8.4344867	9.2845914
		0.1	38.5712788	21.4318248	12.984418	10	9.6103695	10.3488087	11.5732928	13.016778
	7	0.01	48.3872874	18.1624687	9.3026546	7.7	7.6940184	8.0506746	8.5591883	9.1879792
		0.05	58.0662011	28.314039	14.3745667	10.5	10.4822291	11.8253897	13.7495402	16.0141785
		0.1	62.3903573	34.9504997	19.2196169	14	13.9745515	16.1473293	19.2658226	22.8298205
	9	0.01	72.3884749	27.3047043	12.4179868	9.9	10.0172283	10.7608713	11.8584971	13.2891026
		0.05	82.902934	41.8223766	19.4872261	13.5	13.9174644	16.6153171	20.4032057	24.903239
		0.1	87.1644732	50.6866287	26.0109362	18	18.6578403	22.9462281	28.8071361	35.3929846
	11	0.01	97.8702386	38.225295	15.7459067	12.1	12.4041654	13.7133495	15.7284927	18.4589686
		0.05	108.1334832	57.3175261	25.0376075	16.5001076	17.561706	22.1088723	28.4757279	36.0593807
		0.1	111.9538756	68.3138414	33.3515054	22	23.6584407	30.7418191	40.1637219	50.5523949
	13	0.01	123.8417649	50.8814899	19.2935975	14.3	14.857836	16.93812	20.2671259	24.900615
		0.05	133.1504949	74.5738769	31.024783	19.5000662	21.4175277	28.3312491	38.0167499	49.4953135
		0.1	136.3596017	87.5335342	41.2330639	26.0001126	28.9755611	39.5265905	53.2727567	68.0766878

Table (15): shows Expected Sample Size Proportion w.r.t. α, u, n_1 and λ

u	n_1	α	λ 0.25	0.50	0.75	1	1.25	1.50	1.75	2
10	5	0.01	0.4966069	0.196476	0.116219	0.1	0.0987567	0.1009527	0.1041644	0.1078991
		0.05	0.6327669	0.3093368	0.176326	0.1363636	0.131888	0.1401733	0.1533543	0.1688108
		0.1	0.701296	0.3896695	0.2360803	0.1818182	0.174734	0.1881602	0.2104235	0.2366687
	7	0.01	0.6284063	0.2358762	0.1208137	0.1	0.0999223	0.1045542	0.1111583	0.1193244
		0.05	0.7541065	0.3677148	0.1866827	0.1363636	0.1361328	0.1535765	0.1785655	0.2079763
		0.1	0.8102644	0.4539026	0.2496054	0.1818182	0.1814877	0.2097056	0.2502055	0.2964912
	9	0.01	0.7311967	0.2758051	0.1254342	0.1	0.1011841	0.1086957	0.1197828	0.1342334
		0.05	0.8374034	0.4224482	0.1968407	0.1363636	0.1405804	0.1678315	0.206093	0.2515479
		0.1	0.8804492	0.5119861	0.2627367	0.1818182	0.188463	0.2317801	0.2909812	0.3575049
	11	0.01	0.8088449	0.3159115	0.1301315	0.1	0.1025138	0.1133335	0.1299875	0.1525535
		0.05	0.8936652	0.4736986	0.2069224	0.1363645	0.1451381	0.182718	0.2353366	0.2980114
		0.1	0.9252386	0.5645772	0.2756323	0.1818182	0.1955243	0.2540646	0.3319316	0.4177884
	13	0.01	0.8660263	0.3558146	0.1349203	0.1	0.103901	0.1184484	0.1417282	0.1741302
		0.05	0.9311223	0.5214956	0.2169565	0.1363641	0.1497729	0.1981206	0.2658514	0.3461211
		0.1	0.9535636	0.6121226	0.2883431	0.181819	0.2026263	0.2764097	0.3725368	0.4760608

Table (16) :shows Percentage of Overall Sample Saved w.r.t. α, u, n_1 and λ

u	n_1	α	λ 0.25	0.50	0.75	1	1.25	1.50	1.75	2
10	5	0.01	50.3393056	80.3523972	88.3781044	90	90.1243263	89.9047252	89.5835592	89.2100885
		0.05	36.7233136	69.0663154	82.3674027	86.3636364	86.811198	85.982673	84.6645697	83.1189246
		0.1	29.8704022	61.0330458	76.3919674	81.8181818	82.5266009	81.1839842	78.9576495	76.3331308
	7	0.01	37.159367	76.4123782	87.9186304	90	90.0077683	89.5445784	88.884171	88.0675594
		0.05	24.5893493	63.2285208	81.3317316	86.3636364	86.3867154	84.642351	82.1434543	79.2023656
		0.1	18.9735619	54.6097406	75.0394586	81.8181818	81.8512318	79.0294425	74.9794512	70.3508825
	9	0.01	26.8803284	72.4194906	87.456579	90	89.8815876	89.130433	88.0217202	86.5766641
		0.05	16.2596626	57.7551751	80.3159332	86.3636364	85.9419551	83.2168514	79.3907013	74.8452131
		0.1	11.9550776	48.8013851	73.7263271	81.8181818	81.1536967	76.8219918	70.9018828	64.2495105
	11	0.01	19.1155053	68.4088471	86.986854	90	89.7486236	88.6666533	87.0012457	84.7446541
		0.05	10.633485	52.6301438	79.3077624	86.3635474	85.4861934	81.7282047	76.4663405	70.1988589
		0.1	7.4761358	43.5422798	72.4367724	81.8181818	80.4475696	74.5935379	66.8068414	58.2211612
	13	0.01	13.3973672	64.4185385	86.5079738	90	89.6099049	88.1551608	85.8271847	82.5869825
		0.05	6.8877658	47.8504357	78.3043475	86.36359	85.0227079	80.1879377	73.4148602	65.3878927
		0.1	4.6436352	38.7877383	71.1656895	81.8181031	79.7373699	72.3590276	62.746324	52.3939246

Table (17): shows Probability of a Voiding Second Sample w.r.t. α, n_1 and λ

		ζ							
n_1	α	**0.25**	**0.50**	**0.75**	**1**	**1.25**	**1.50**	**1.75**	**2**
5	0.01	0.5537324	0.8838764	0.9721591	0.99	0.9913676	0.988952	0.9854192	0.981311
	0.05	0.4039564	0.7597295	0.9060414	0.95	0.9549232	0.9458094	0.9313103	0.9143082
	0.1	0.3285744	0.6713635	0.8403116	0.9	0.9077926	0.8930238	0.8685341	0.8396644
7	0.01	0.408753	0.8405362	0.9671049	0.99	0.9900855	0.9849904	0.9777259	0.9687432
	0.05	0.2704828	0.6955137	0.894649	0.95	0.9502539	0.9310659	0.903578	0.871226
	0.1	0.2087092	0.6007071	0.825434	0.9	0.9003636	0.8693239	0.824774	0.7738597
9	0.01	0.2956836	0.7966144	0.9620224	0.99	0.9886975	0.9804348	0.9682389	0.9523433
	0.05	0.1788563	0.6353069	0.8834753	0.95	0.9453615	0.9153854	0.8732977	0.8232973
	0.1	0.1315059	0.5368152	0.8109896	0.9	0.8926907	0.8450419	0.7799207	0.7067446
11	0.01	0.2102706	0.7524973	0.9568554	0.99	0.9872349	0.9753332	0.9570137	0.9321912
	0.05	0.1169683	0.5789316	0.8723854	0.949999	0.9403481	0.8990103	0.8411297	0.7721874
	0.1	0.0822375	0.4789651	0.7968045	0.9	0.8849233	0.8205289	0.7348753	0.6404328
13	0.01	0.147371	0.7086039	0.9515877	0.99	0.985709	0.9697068	0.944099	0.9084568
	0.05	0.0757654	0.5263548	0.8613478	0.9499995	0.9352498	0.8820673	0.8075635	0.7192668
	0.1	0.05108	0.4266651	0.7828226	0.8999991	0.8771111	0.7959493	0.6902096	0.5763332

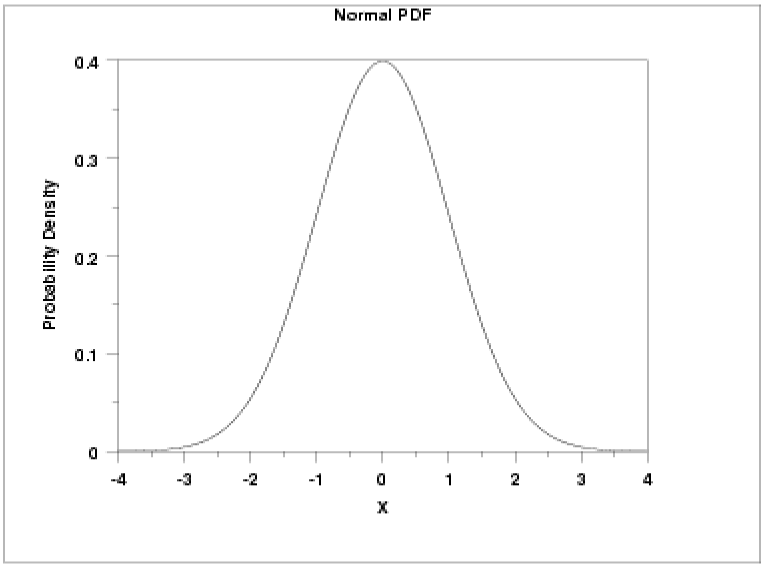

Figure (1): Graph for PDF of Normal Distribution