Minimax Shrunken Technique for Estimate Burr X Distribution Shape Parameter

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Abstract:

The present paper concern with minimax shrinkage estimator technique in order to estimate Burr X distribution shape parameter, when prior information about the real shape obtainable as original estimate while known scale parameter.

Derivation for Bias Ratio, Mean squared error and the Relative Efficiency equations.

Numerical results and conclusions for the expressions mentioned above were displayed. Comparisons for proposed estimator with most recent works were made.

Keywords: Minimax Shrunken Technique, Burr X Distribution, MLE, MSE and Efficiency.

1. Introduction

Burr X distribution would be able to use effectively in modeling data and also modeling lifetime data. [2],[4],[8],[9],[10]and [11] .

Let X be a R.V. follows Burr X distribution with two parameters α and λ . The (C.D.F.) and (P.D.F) of X are represented respectively as below;

$$F(x;\alpha,\lambda) = (1 - e^{-(\lambda x)^2})^{\alpha} \qquad For\ \ x > 0, \alpha > 0\ , \lambda > 0 \qquad \dots (1)$$

$$f(x;\alpha,\lambda) = \begin{cases} 2\alpha\lambda^2 x e^{-(\lambda x)^2}(1 - e^{-(\lambda x)^2})^{\alpha-1} & for\ x > 0, \alpha > 0, \lambda > 0 \\ 0 & o.w \end{cases} \qquad \dots . (2)$$

Wherever, α & λ refer respectively to shape and scale parameters.

Pretest shrunken estimator of α determination as below:

$$\tilde{\alpha} = \begin{cases} \psi_1(\hat{\alpha})\hat{\alpha} + \left(1 - \psi_1(\hat{\alpha})\right)\alpha_0 & if\,\hat{\alpha} \in R \\ \psi_2(\hat{\alpha})\hat{\alpha} + \left(1 - \psi_2(\hat{\alpha})\right)\alpha_0 & if\,\hat{\alpha} \notin R \end{cases} \qquad \dots (3)$$

Anywhere , $\psi_i(\hat{\alpha}), 0 \le \psi_i(\hat{\alpha}) \le 1$, i = 1, 2 refer to weightiness shrunken identifying acceptance of $\hat{\alpha}$ while $(1 - \psi_i(\cdot))$ agreeing confidence of α_0, and well-known that $\psi_i(\hat{\alpha})$ may be constant or function depend on $\hat{\alpha}$ (ad hoc basis), and R refer to pretest region for testing H_o: $\alpha = \alpha_0$ opposite H_1: $\alpha \neq \alpha_0$ via statistics test T ($\hat{\alpha}/\alpha_0$).[1]and [12].

Minimax procedure of estimation is a non-classical approach for estimation presented by Abraham Wald (1945) from the model of game theory. The most significant basics in the minimax approach are the description of the prior distribution and the loss function used. Quadratic loss functions will be considered to find the Minimax estimators of Burr X distribution in present research; [3], [5], [6] and [7].

The goal of this paper, toward suggested shrunken estimator technique $\tilde{\alpha}$ defined by (3) through employment the Minimax estimator ($\hat{\alpha}_B = \hat{\alpha}_{MQL}$) instead of the classical estimator (MLE) $\hat{\alpha}$ to estimate α Burr X distribution with known λ.

Expressions for main statistical indicators of the proposed estimator $\tilde{\alpha}$ were derived and obtained. Discussions and numerical results were achieved and presented in appropriated table.

Comparisons were done between the suggested estimators with standard minimax estimator $\hat{\alpha}_B$ as well as with some of the last studies.

2. Minimax Estimator of α

In this section, we obtain the minimax estimators of the parameter α for the Burr X distribution depends on Lehmann s theorem in (1950).

Agreeing with this theorem, consequently obtain the minimax estimator of the parameter α as $\hat{\alpha}_B = \hat{\alpha}_{MQL} = (n-2)/\sum_{i=1}^{n} \ln\left(1 - e^{-(\lambda x_i)^2}\right)^{-1}$ of the Burr X distribution in case of quadratic loss function.

It is known, the MLE of α in Burr X distribution when λ equal to one has the following form ; [8]and[9]

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$$\hat{\alpha}_{mle} = -\frac{n}{\sum_{i=1}^{n} \ln\left(1-e^{-x_i{}^2}\right)} \qquad \dots (4)$$

Reminder, when xi ~ Burr X (α,1), then $-\alpha \sum_{i=1}^{n} \ln\left(1-e^{-x_i{}^2}\right)$ follow Gamma population including n shape and 1 scale; G(n,1). [4], [8] and [9].

i.e.;

$$E(\hat{\alpha}_{mle}) = \frac{n}{n-1}\alpha \qquad and \qquad var(\hat{\alpha}_{mle}) = \frac{n^2}{(n-1)^2(n-2)}\alpha^2.$$

By using (4), then

$$\mathrm{Bias}(\hat{\alpha}_B) = E(\hat{\alpha}_B) - \alpha$$
$$= -\frac{\alpha}{(n-1)}$$

$$Var(\hat{\alpha}_B) = \left(\frac{n-1}{n}\right)^2 . \mathrm{Var}(\hat{\alpha}_{mle})$$
$$= \frac{(n-2)}{(n-1)^2}\alpha^2$$

$$\mathrm{MSE}(\hat{\alpha}_B) = \mathrm{Var}(\hat{\alpha}_B) + \mathrm{Bias}(\hat{\alpha}_B)^2 = \frac{\alpha^2}{n-1}$$

3. Minimax Shrinkage Estimator $\tilde{\alpha}$

This section involves the suggestion of special case of pretest shrunken estimator $\tilde{\alpha}$ which is defined in (3) using Minimax estimator $\hat{\alpha}_B$ for estimate α of Burr X distribution when λ = 1,

$$\tilde{\alpha}\,(\hat{\alpha}_B, R_i) = \begin{cases} \alpha_0 & , if\, \hat{\alpha}_B \in R_i \\ k(\hat{\alpha}_B - \alpha_0) + \alpha_0 & , if\, \hat{\alpha}_B \notin R_i \end{cases} \qquad \dots (5)$$

When $\psi 1(\hat{\alpha}_B) = 0$ and $\psi 2(\hat{\alpha}_B) = k = \frac{(n-2)\alpha_0}{\hat{\alpha}_B . C}$; $c = \chi^2{}_{(1-\frac{\Delta}{2}, 2n)} \times 100$ such that (0 < k < 1) , and R refer to pretest region for testing H_o: $\alpha = \alpha_0$ vs. H_1: $\alpha \neq \alpha_0$ through level of significance (Δ) via statistic test function $\mathrm{TT} = \frac{2(n-2)\alpha}{\hat{\alpha}_B}$

i.e.; $R = \left[a < \frac{2(n-2)\alpha_0}{\hat{\alpha}_B} < b\right]$ …(6)

Where $a = X^2{}_{(1-\Delta, 2n)}$ and $b = X^2{}_{(\Delta, 2n)}$

The derivation of Bias equation for $\tilde{\alpha}$ is

$$Bias(\tilde{\alpha}|\alpha; R) = E(\tilde{\alpha} - \alpha)$$
$$= \int_R (\alpha_0 - \alpha) f(\hat{\alpha}) d\,\hat{\alpha} + \int_{\bar{R}} [k(\hat{\alpha} - \alpha_0) + (\alpha_0 - \alpha)] f(\hat{\alpha}) d\hat{\alpha}$$

Wherever, $\bar{R}$ refers to supplemented region of R , and

$$f(\hat{\alpha}_B) = \frac{\left(\frac{(n-2)\alpha}{\hat{\alpha}_B}\right)^{n+1} . e^{-\frac{(n-2)\alpha}{\hat{\alpha}_B}}}{\Gamma(n).(n-2)\alpha}, \hat{\alpha}_B > 0 \qquad \dots (7)$$

We conclude,

$$Bias(\tilde{\alpha}|\alpha; R) = \alpha\left\{\frac{n-2}{c}\zeta - \frac{n}{c}\zeta^2 + (\zeta - 1) - \frac{n-2}{c}\zeta J_0(a^*, b^*) + \frac{1}{c}\zeta^2 J_1'(a^*, b^*)\right\}$$

Anywhere, $J_l(a^*, b^*) = \int_{a^*}^{b^*} \frac{1}{y^l} \frac{y^{n-1}e^{-y}}{\Gamma(n)} dy \quad ; l = 0,1,2, \dots$ …(8)

Also

$$\zeta = \frac{\alpha_0}{\alpha}, a^* = (2\zeta)^{-1} . a, b^* = (2\zeta)^{-1} . b \text{ and } y = \left({(n-2)}/{\hat{\alpha}_B}\right)\alpha \qquad \dots (9)$$

Bias Ratio [B ($\tilde{\alpha}$)] well-defined as:

$$B(\tilde{\alpha}) = \frac{Bias(\tilde{\alpha}|\alpha; R)}{\alpha} \qquad \dots (10)$$

Derivation of MSE ($\tilde{\alpha}$) will be as below:-

$$MSE(\tilde{\alpha}|\alpha; R) = E(\tilde{\alpha} - \alpha)^2$$

$$= \int_R (\alpha_0 - \alpha)^2 f(\hat{\alpha}) d\,\hat{\alpha} + \int_{\bar{R}} [k(\hat{\alpha} - \alpha_0) + (\alpha_0 - \alpha)]^2 f(\hat{\alpha}) d\hat{\alpha}$$

$$= \int_R (\alpha_0 - \alpha)^2 f(\hat{\alpha}) d\,\hat{\alpha} + \int_0^\infty \left[\left(\frac{(n-2)\alpha_0}{c} - \frac{(n-2)\alpha_0{}^2}{\hat{\alpha}c}\right) + (\alpha_0 - \alpha)\right]^2 f(\hat{\alpha}) d\hat{\alpha}$$
$$- \int_R \left[\left(\frac{(n-2)\alpha_0}{c} - \frac{(n-2)\alpha_0{}^2}{\hat{\alpha}c}\right) + (\alpha_0 - \alpha)\right]^2 f(\hat{\alpha}) d\hat{\alpha}$$

Some by simplification, the above equation became:

$$MSE(\tilde{\alpha}|\alpha; R) = \alpha^2\left\{\frac{(n-2)^2}{c^2}\zeta^2 - 2\frac{n(n-2)}{c^2}\zeta^3 + \frac{n(n+1)}{c^2}\zeta^4 + 2\frac{(n-2)}{c}\zeta(\zeta - 1)\right.$$
$$- 2\frac{n}{c}\zeta^2(\zeta - 1) + (\zeta - 1)^2 - \frac{(n-2)^2}{c^2}\zeta^2 J_0(a^*, b^*) + 2\frac{(n-2)}{c^2}\zeta^3 J_1'(a^*, b^*)$$
$$\left. - \frac{1}{c}\zeta^4 J_2'(a^*, b^*) - 2\frac{(n-2)}{c}\zeta(\zeta - 1) J_o(a^*, b^*) + 2\frac{1}{c}\zeta^2(\zeta - 1) J_1'(a^*, b^*)\right\}$$

…(11)

The Relative Efficiency of $\tilde{\alpha}$ to $\hat{\alpha}_B$ is :

$$R.Eff(\tilde{\alpha}|\alpha; R_i) = \frac{MSE(\hat{\alpha}_B)}{MSE(\tilde{\alpha}|\alpha; R)} \qquad \dots (12)$$

4. Discussion and Numerical Results

The calculations of the equations (10),(11) and (12), were performed for n = 4,6,8,10,12, $\zeta = \alpha_0/\alpha = 0.25(0.25)2$ and $\Delta = 0.01, 0.05, 0.1$, for the estimator $\tilde{\alpha}$.
These calculations are prearranged in attached table and consequence the following:

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i. R.Eff($\tilde{\alpha}$) has opposite relational with small value of Δ especially when $\zeta = 1$
ii. When ζ =1, the R.Eff ($\tilde{\alpha}$) takes height value for each n , Δ, and lessening otherwise ($\zeta \neq 1$).
iii. B($\tilde{\alpha}$) are almost small when $\zeta = 1$ for all n, Δ , and increases else.
iv. Effective interval of $\tilde{\alpha}$. [Value of ζ that makes R.Eff($\tilde{\alpha}$) larger one] is [0.5, 1.5].
v. R.Eff($\tilde{\alpha}$.) is reducing function with n, for each Δ and ζ.
vi. The suggested estimator $\tilde{\alpha}$ dominate $\hat{\alpha}$ especially at $\zeta = 1$.
vii. The suggested estimator $\tilde{\alpha}$ dominate the estimators presented by [5],[7] in the sense of minimum mean squared error.

Table (1): B($\tilde{\alpha}$) and R.Eff ($\tilde{\alpha}$) w.r.t. Δ, n_1 and ζ

			ζ							
Δ	n	R.Eff(-) B(-)	0.25	0.50	0.75	1	1.25	1.50	1.75	2
0.01	4	R.Eff(-)	0.5926898	1.3334412	5.3336397	1.8766999E+6	5.3411594	1.336686	0.594938	0.3352064
		B(-)	-0.7499385	-0.4999798	-0.2499928	-2.507439E-5	0.2498139	0.4993667	0.7485095	0.9971855
	6	R.Eff(-)	0.3557834	0.8002317	3.20053	8.5776432E+5	3.2066372	0.8031576	0.3578644	0.201871
		B(-)	-0.7497598	-0.4999276	-0.2499792	-2.524196E-5	0.2497366	0.499006	0.7475579	0.9953259
	8	R.Eff(-)	0.2542997	0.5718069	2.286422	4.9532531E+5	2.2919673	0.5746331	0.2563705	0.1448072
		B(-)	-0.7495111	-0.4998344	-0.2499612	-2.532473E-5	0.2496521	0.4985883	0.7464494	0.9932013
	10	R.Eff(-)	0.1979233	0.4449856	1.778652	3.234304E+5	1.7839172	0.4477738	0.2000606	0.1131537
		B(-)	-0.749256	-0.4996955	-0.2499383	-2.537393E-5	0.2495605	0.4981157	0.7452037	0.9908772
	12	R.Eff(-)	0.1620404	0.3643486	1.4555841	2.280651E+5	1.4606951	0.3671231	0.1642812	0.0930405
		B(-)	-0.7490176	-0.4995104	-0.2499104	-2.540652E-5	0.2494616	0.4975907	0.7438406	0.9884106
0.05	4	R.Eff(-)	0.5927787	1.3336565	5.3342047	5.9151045E+5	5.3539023	1.3401523	0.5965971	0.336158
		B(-)	-0.7498823	-0.4999394	-0.2499795	-9.939819E-5	0.2495134	0.4987173	0.7474654	0.9957716
	6	R.Eff(-)	0.3558701	0.8005953	3.201717	2.6800421E+5	3.2167515	0.8060428	0.3592242	0.2026157
		B(-)	-0.7496684	-0.499814	-0.2499328	-9.980886E-5	0.2493392	0.4981078	0.7461391	0.9934961
	8	R.Eff(-)	0.2543477	0.5722693	2.2879665	1.5388055E+5	2.3009895	0.5772602	0.2575699	0.1454268
		B(-)	-0.7494403	-0.4996323	-0.2498766	-1.000067E-4	0.2491563	0.4974474	0.7447081	0.991088
	10	R.Eff(-)	0.1979429	0.4455051	1.78046	1.0007843E+5	1.7923586	0.4502451	0.201143	0.1136772
		B(-)	-0.749219	-0.499404	-0.249811	-1.001240E-4	0.2489647	0.4967414	0.7431971	0.9886024
	12	R.Eff(-)	0.1620468	0.3648894	1.4576081	7.0362889E+4	1.4687862	0.3694813	0.1652655	0.0934839
		B(-)	-0.7490028	-0.4991402	-0.2497362	-1.002015E-4	0.2487646	0.495995	0.7416269	0.9860785
0.1	4	R.Eff(-)	0.592807	1.3337917	5.334351	3.7785035E+5	5.3636228	1.3423159	0.5974921	0.3366172
		B(-)	-0.7498643	-0.4999141	-0.249976	-1.770097E-4	0.2492857	0.4983142	0.7469051	0.9950928
	6	R.Eff(-)	0.3558868	0.8008029	3.2025271	1.7103776E+5	3.2241357	0.8077279	0.3598975	0.2029403
		B(-)	-0.7496508	-0.4997492	-0.249901	-1.775499E-4	0.2490514	0.4975869	0.7454414	0.9927035

	8	R.Eff(-)	0.2543533	0.5724995	2.2890869	9.8133771E+4	2.3073582	0.5787131	0.2581224	0.1456745
		B(-)	-0.7494321	-0.4995319	-0.2498152	-1.778086E-4	0.2488095	0.4968218	0.7439125	0.9902491
	10	R.Eff(-)	0.1979443	0.445731	1.7817676	6.3788371E+4	1.798152	0.4515479	0.2016105	0.1138709
		B(-)	-0.7492164	-0.4992775	-0.2497191	-1.779616E-4	0.2485602	0.4960242	0.7423382	0.987767
	12	R.Eff(-)	0.1620471	0.3650962	1.4590432	4.4829877E+4	1.474204	0.3706716	0.1656663	0.0936367
		B(-)	-0.7490021	-0.4989989	-0.2496131	-1.780627E-4	0.2483035	0.4951987	0.7407343	0.9852809

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