On Shrinkage Estimation for R (s, k) in Case of Exponentiated Pareto Distribution

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Abstract

This paper concerns with deriving and estimating the reliability of the multicomponent system in stress-strength model $R_{(s,k)}$, when the stress and strength are identical independent distribution (iid), follows two parameters Exponentiated Pareto Distribution (EPD) with the unknown shape and known scale parameters. Shrinkage estimation method including Maximum likelihood estimator (MLE), has been considered. Comparisons among the proposed estimators were made depending on simulation based on mean squared error (MSE) criteria.

Keywords: Exponentiated Pareto Distribution(EPD), Reliability of multi-component Stress – Strength models $R_{(s,k)}$, Maximum likelihood estimator (MLE), Shrinkage estimator and mean squared error (MSE).

1. Introduction

The reliability of the multicomponent stress-strength model (s out of k) system, denoted by $R_{(s,k)}$refers to the system functioning when at minimums ($1 \le s \le k$) of components survive. In other words, this system works well if at least s out of k components resist the stress. Bhattacharyya and Johnson in (1974) was the first who studied and derived $R_{(s,k)}$ [1]. Noted that, when s=1 and s=k is respectively referring to parallel and series systems. The model mentioned used in many applications in physics and engineering and many authors had studied and estimated $R_{(s,k)}$for example: Afify in (2010), showed that the Exponentiated Pareto distribution denoted by EP (α, λ) used quite successfully in studying many lifetime data and the EP (α, λ) decreasing and upside-down bathtub shaped failure rates depending on shape parameter α[2]. Hassan& Basheikh in (2012), estimated $R_{(s,k)}$using Bayes and non-Bayes estimation methods when the strength and stress are non-identical and follows the Exponentiated Pareto distribution [3], Rao et al in (2016), estimated the reliability system in a multicomponent stress-strength when stress and strength follows Exponentiated Weibull distribution for different shape parameters [4], and in (2017) Abbas and Fatima, they estimated the reliability of the multicomponent system in stress–strength model for Exponentiated Weibull distribution, using; ML, MOM and the conclude results approved that the Shrinkage estimator using Shrinkage weight function was the best[5].

In this paper we estimate $R_{(s,k)}$ based on Exponentiated Pareto distribution EP(α, λ) with unknown shape parameter α and known scale parameter λ using several shrinkage estimation methods depends on (MLE) methods and make a comparison of the considered estimation methods through Monte Carlo simulation via mean squared error (MSE) criteria.

It is well known, the EP (α, λ) is a special case of Exponentiated Lomax distribution (ELD), when ($\epsilon = 1$) where the CDF of ELD has the form below [6].

$$F(X) = \left[1-(1+\epsilon x)^{-\lambda}\right]^{\alpha} \qquad x > 0; \alpha, \lambda and\ \epsilon > 0$$
(1)

Let X be a random variable follows two parameters Exponentiated Pareto distribution EPD (α, λ).

The probability density function (p. d. f.) of X will be.

$$f(x;\alpha,\lambda) = \begin{cases} \alpha\lambda(1+x)^{-(\lambda+1)}\left[1-(1+x)^{-\lambda}\right]^{\alpha-1}; \text{ For x} > 0;\ \alpha,\lambda > 0 \\ 0 \qquad\qquad\qquad\qquad otherwise \end{cases} \tag{2}$$

Here, α refers to shape parameter and λ refers to scale parameter [2].

Implies, the cumulative distribution function (CDF) of X as below:

$$F(x,\alpha,\lambda) = \left[1-(1+x)^{-\lambda}\right]^{\alpha} \qquad x > 0; \alpha, \lambda > 0 \tag{3}$$

Assume $X_1, X_2, \dots, X_k$ are strength random variable follows EP(α,λ) and subject to common stress random variable Y which is distributed as EP(β,λ) .

Then, the reliability system for a multicomponent in stress-strength model $R_{(s,k)}$ will be [1]:

$R_{(s,k)}$ =P(at least s of the $X_1, X_2, \dots, X_k$ exceed Y)

$$=\sum_{i=s}^{k}\binom{k}{i}\int_{o}^{\infty}\left(1-F_x(y)\right)^{i}\left(F_x(y)\right)^{k-i}f(y)dy \tag{4}$$

$$=\sum_{i=s}^{k}\binom{k}{i}\int_{o}^{\infty}(1-\left[1-(1+y)^{-\lambda}\right]^{\alpha})^{i}(\left[1-(1+y)^{-\lambda}\right]^{\alpha})^{k-i}\beta\lambda(1+y)^{-(\lambda+1)}\left[1-(1+y)^{-\lambda}\right]^{\beta-1}dy$$

$$R_{(s,k)} = \frac{\beta}{\alpha}\sum_{i=s}^{k}\frac{k!}{(k-i)!}\left[\prod_{j=0}^{i}\left(k+\frac{\beta}{\alpha}-j\right)\right]^{-1} \quad ; \text{k, i, j are integers} \tag{5}$$

Now we estimation methods of $R_{(s,k)}$ by the following:

2. Maximum Likelihood Estimator (MLE)

Suppose the strength random sample be of size n say $x_1, x_2, \dots, x_n$ follow EPD (α, λ) and $y_1, y_2, \dots \dots, y_m$ be the stress random sample of size m follow EPD (β, λ)

The Maximum Likelihood function for the observed sample is given as:

$$l = L(\alpha,\beta;x,y) = \prod_{i=1}^{n} f(x_i)\prod_{j=1}^{m} f(y_j) \tag{6}$$

From equation (2) and the equation (6) become

$$l = \alpha^n\beta^m\lambda^{(n+m)}\prod_{i=1}^{n}(1+x_i)^{-(\lambda+1)}\prod_{i=1}^{n}\left[1-(1+x_i)^{-\lambda}\right]^{\alpha-1}\prod_{j=1}^{m}(1+y_j)^{-(\lambda+1)}\prod_{j=1}^{m}\left[1-(1+y_j)^{-\lambda}\right]^{\beta-1}$$

Take Logarithm to both sides, we get:

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$\ln(l) = nln\alpha + mln\beta + (n+m)ln\lambda - (\lambda+1)\sum_{i=1}^{n} ln(1+x_i) + (\alpha-1)\sum_{i=1}^{n} \ln(1-(1+x_i)^{-\lambda}) - (\lambda+1)\sum_{j=1}^{m} ln(1+y_j) + (\beta-1)\sum_{j=1}^{m} ln(1-(1+y_j)^{-\lambda})$

(7)

$$\frac{d\mathrm{Ln}(l)}{d\alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n} ln(1-(1+x_i)^{-\lambda}) = 0$$

$$\frac{d\mathrm{Ln}(l)}{d\beta} = \frac{m}{\beta} + \sum_{j=1}^{m} ln(1-(1+y_j)^{-\lambda}) = 0$$

Thus, the Maximum Likelihood estimator for the unknown shape parameters α and β will be respectively as follows [2]:

$$\hat{\alpha}_{\mathrm{MLE}} = \frac{-\mathrm{n}}{\sum_{i=1}^{n} ln(1-(1+x_i)^{-\lambda})} \quad (8)$$

$$\hat{\beta}_{MLE} = \frac{-m}{\sum_{j=1}^{m} ln(1-(1+y_j)^{-\lambda})} \quad (9)$$

Note that, $\hat{\alpha}_{\mathrm{MLE}}$ and $\hat{\beta}_{MLE}$ are biased estimators for α and β respectively since E $(\hat{\alpha}_{\mathrm{MLE}}) = \frac{n\alpha}{n-1} \neq \alpha$, and E $(\hat{\beta}_{\mathrm{MLE}}) = \frac{m\beta}{m-1} \neq \beta$

Hence, $\hat{\alpha}_{\mathrm{ub}} = \frac{n-1}{n}\hat{\alpha}_{\mathrm{MLE}}$ and $\hat{\beta}_{\mathrm{ub}} = \frac{m-1}{m}\hat{\beta}_{\mathrm{MLE}}$ are respectively unbiased estimators for α and β.Therefore,

$$\mathrm{E}\,(\hat{\alpha}_{\mathrm{ub}}) = \alpha \text{ and } \mathrm{Var}\,(\hat{\alpha}_{\mathrm{ub}}) = \frac{\alpha^2}{n-2} \quad (10)$$

$$\mathrm{E}\,(\hat{\beta}_{\mathrm{ub}}) = \beta \text{ and } \mathrm{Var}\,(\hat{\beta}_{\mathrm{ub}}) = \frac{\beta^2}{m-2} \quad (11)$$

By substitute Equations (8) and (9) in equation (5), we obtain the maximum likelihood estimator for $R_{(s,k)}$ as below:

$$\hat{R}_{(s,k)MLE} = \frac{\hat{\beta}_{MLE}}{\hat{\alpha}_{MLE}} \sum_{i=s}^{k} \frac{k!}{(k-i)!} [\prod_{j=0}^{i}(k + \frac{\hat{\beta}_{MLE}}{\hat{\alpha}_{MLE}} - j)]^{-1}; \text{ k, i, j are integers} \quad (12)$$

3. Shrinkage Estimation Method (Sh)

As Thompson suggested in (1968), the shrinkage estimator of α denoted by $(\hat{\alpha}_{\mathrm{Sh}})$ is defined as below:

$$\hat{\alpha}_{\mathrm{Sh}} = \emptyset(\hat{\alpha})\hat{\alpha} + (1-\emptyset(\hat{\alpha}))\hat{\alpha}_0 \quad (13)$$

He shrinks a usual estimator $\hat{\alpha}$ to prior information α_0 using shrinkage weight factor $\emptyset(\hat{\alpha})$ and he believed α_0 is closed to α [5], [7].

We apply the unbiased estimator $\hat{\alpha}_{ub}$ as a usual estimator and $\alpha_0 \approx \alpha$ as a prior information of α in this paper.

Thus, the shrinkage estimator of the shape parameter α of EP (α,λ) will be as follows:

$$\hat{\alpha}_{sh} = \emptyset_1(\hat{\alpha})\hat{\alpha}_{ub} + (1-\emptyset_1(\hat{\alpha}))\alpha_0 \quad (14)$$

And the same way, the shrinkage estimator for the shape parameter β of EP(β,λ) will be as the following:

$$\hat{\beta}_{sh} = \emptyset_2(\hat{\beta})\hat{\beta}_{ub} + (1-\emptyset_2(\hat{\beta}))\beta_0 \quad (15)$$

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3.1. The Shrinkage Weight Function (Sh1):

In this subsection, the shrinkage weight factor will be considered as a function of sizes n and m respectively and taking the forms below:

$\emptyset_1(\hat{\alpha}) = |\sin n/n|$, and $\emptyset_2(\hat{\beta}) = |\sin m/m|$

where, n and m refer to the sample size of X and Y.

Therefore, the shrinkage estimator using shrinkage weight function of α and β which is defined in equations (14) and (15), will be.

$\hat{\alpha}_{sh1} = |\sin n/n|\hat{\alpha}_{ub} + (1 - |\sin n/n|)\alpha_0$
(16)

$\hat{\beta}_{sh1} = |\sin m/m|\hat{\beta}_{ub} + (1 - |\sin m/m|)\beta_0$
(17)

Then, the estimation of$R_{(s,k)}$ which is defined in Equation (5) using shrinkage weight function will be:-

$$\hat{R}_{(s,k)_{sh1}} = \frac{\hat{\beta}_{sh1}}{\hat{\alpha}_{sh1}} \sum_{i=s}^{k} \frac{k!}{(k-i)!} \left[\prod_{j=0}^{i} (k + \frac{\hat{\beta}_{sh1}}{\hat{\alpha}_{sh1}} - j) \right]^{-1} \quad (18)$$

3.2. Constant Shrinkage Weight Function (Sh2)

We suggest in this subsection constant shrinkage weight factor $\emptyset_1(\hat{\alpha}) = 0.1$, and$\emptyset_2(\hat{\beta}) = 0.1$. Therefore, the shrinkage estimator using specific constant weight factor will be as follows:

$$\hat{\alpha}_{sh2} = (0.1)\, \hat{\alpha}_{ub} + (0.9)\alpha_0 \quad (19)$$

$$\hat{\beta}_{sh2} = (0.1)\hat{\beta}_{ub} + (0.9)\beta_0 \quad (20)$$

Substitute equation (19) and (20) in equation (5) to obtain the shrinkage estimation of $R_{(s,k)}$ using the above constant shrinkage weight factor as below:

$$\hat{R}_{(s,k)_{sh2}} = \frac{\hat{\beta}_{sh2}}{\hat{\alpha}_{sh2}} \sum_{i=s}^{k} \frac{k!}{(k-i)!} \left[\prod_{j=0}^{i} (k + \frac{\hat{\beta}_{sh2}}{\hat{\alpha}_{sh2}} - j) \right]^{-1}; \text{ k, i, j are integers} \quad (21)$$

3.3. Modified Thompson Type Shrinkage Weight Function (Th)

In this subsection, we modify the shrinkage weight factor of Thompson type estimator as below, [7]

$$\gamma(\hat{\alpha}) = \frac{(\hat{\alpha}_{ub} - \alpha_0)^2}{(\hat{\alpha}_{ub} - \alpha_0)^2 + Var(\hat{\alpha}_{ub})} * 0.001 \quad (22)$$

$$\gamma(\hat{\beta}) = \frac{(\hat{\beta}_{ub} - \beta_0)^2}{(\hat{\beta}_{ub} - \beta_0)^2 + Var(\hat{\beta}_{ub})} * 0.001 \quad (23)$$

where Var $(\hat{\alpha}_{ub}) = \frac{\alpha^2}{n-2}$ and Var $(\hat{\beta}_{ub}) = \frac{\beta^2}{m-2}$

Therefore, the shrinkage estimator ofα and β using modified shrinkage weight factor are respectively as below:

$$\hat{\alpha}_{Th} = \gamma(\hat{\alpha})\hat{\alpha}_{ub} + (1 - \gamma(\hat{\alpha}))\alpha_0 \quad (24)$$

$$\hat{\beta}_{Th} = \gamma(\hat{\beta})\hat{\beta}_{ub} + \left(1 - \gamma(\hat{\beta})\right)\beta_0 \quad (25)$$

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Substitute equation (24) and (25) in equation (5), then the shrinkage estimation of $R_{(s,k)}$ based on modified Thompson type shrinkage weight factor will be :

$$\hat{R}_{(s,k)_{Th}} = \frac{\hat{\beta}_{Th}}{\hat{\alpha}_{Th}} \sum_{i=s}^{k} \frac{k!}{(k-i)!} \left[\prod_{j=0}^{i} \left(k + \frac{\hat{\beta}_{Th}}{\hat{\alpha}_{Th}} - j\right) \right]^{-1}; \text{ k, i, j are integers} \qquad (26)$$

4. Simulation Study

In this section, numerical results were studied to compare the performance of the suggested estimators for $R_{(s,k)}$, using different sample size n and m = (15, 25, 50 and 100), based on 1000 replication via MSE criteria. For this purpose, Mote Carlo simulation was employed by generating the random sample from continuous uniform distribution defined on the interval (0,1) as $u_1, u_2, \dots, u_n$; $v_1, v_2, \dots, v_m$. Transform uniform random samples to follow EPD (α, λ) using (c.d.f) as below, [8]:

$$F(x) = (1-(1+x)^{-\lambda})^{\alpha} \rightarrow \qquad U_i = (1-(1+x_i)^{-\lambda})^{\alpha} \rightarrow$$

$$x_i = \left[1-(U_i)^{(\frac{1}{\alpha})}\right]^{-(\frac{1}{\lambda})} - 1 \text{ ; i=1,2,\dots,n.}$$

And by the same way, calculate V_j to obtain the y_j:

$$y_j = \left[1-(V_j)^{(\frac{1}{\beta})}\right]^{-(\frac{1}{\lambda})} - 1 \text{ ; j=1,2,\dots,m.}$$

Compute the real value of $R_{(s,k)}$ in equation (5) and the value of estimation methods of all suggested methods $\hat{R}_{(s,k)_{MLE}}$, $\hat{R}_{(s,k)_{sh1}}$, $\hat{R}_{(s,k)_{sh2}}$ and $\hat{R}_{(s,k)_{Th}}$ in Equations (12), (18), (21) and (26) respectively.

Based on (L=1000) replication, we calculate the MSE for all proposed estimation methods of $R_{(s,k)}$ as follows:

$$\text{MSE} = \frac{1}{L}\sum_{i=1}^{L}(\hat{R}_{(s,k)_i} - R_{(s,k)})^2$$

where $\hat{R}_{(s,k)}$ refers to the proposed estimators of real value of $R_{(s,k)}$.

All the results are put on the tables listed below, noted that the odd label **Tables (1-7).** contain the value of real reliability $R_{(s,k)}$ and the value of reliability estimator $\hat{R}_{(s,k)}$ for different methods when (s,k)=(2,3),(2,4) and (α,β)=(2.5,4) and (4,2.5) and $(\alpha_0,\beta_0) = (\alpha + 0.001,\ \beta + 0.001)$.

and the even label tables (2-8) contain the value of MSE for the reliability estimator $\hat{R}_{(s,k)}$ for different methods when (s,k)=(2,3),(2,4) and (α,β)=(2.5,4) and (4,2.5) and (α_0,β_0)=$(\alpha + 0.001, \beta + 0.001)$.

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Table 1. Shows $\hat{R}_{(s,k)}$ for (s,k)=(2,3), α=2.5,β=4 & λ=2 when $R_{(s,k)} = 0.36232$,.

(n,m)	$\hat{R}_{(s,k)MLE}$	$\hat{R}_{(s,k)sh1}$	$\hat{R}_{(s,k)sh2}$	$\hat{R}_{(s,k)Th}$
(15,15)	0.37128	0.36262	0.36297	0.36237
(15,25)	0.36447	0.36214	0.36193	0.36236
(15,50)	0.37216	0.36244	0.36245	0.36236
(15,100)	0.37221	0.36231	0.36224	0.36236
(25,15)	0.36163	0.36240	0.36253	0.36236
(25,25)	0.36432	0.36236	0.36235	0.36236
(25,50)	0.37004	0.36238	0.36272	0.36236
(25,100)	0.36997	0.36237	0.36253	0.36236
(50,15)	0.35514	0.36228	0.36217	0.36236
(50,25)	0.35773	0.36234	0.36203	0.36236
(50,50)	0.36239	0.36236	0.36228	0.36236
(50,100)	0.36249	0.36235	0.36213	0.36236
(100,15)	0.35464	0.36229	0.36224	0.36236
(100,25)	0.35810	0.36235	0.36228	0.36236
(100,50)	0.35886	0.36235	0.36208	0.36236
(100,100)	0.36441	0.36237	0.36252	0.36236

Table 2. Shows MSE of $\hat{R}_{(s,k)}$ for (s,k)=(2,3), α=2.5,β=4 & λ=2 when $R_{(s,k)} = 0.36232$.

(n,m)	$\hat{R}_{(s,k)MLE}$	$\hat{R}_{(s,k)sh1}$	$\hat{R}_{(s,k)sh2}$	$\hat{R}_{(s,k)Th}$	Best
(15,15)	0.01029	0.23E-4	0.12E-3	0.76E-8	Th
(15,25)	0.00843	0.12E-4	0.96E-4	0.60E-8	Th
(15,50)	0.00714	0.13E-4	0.81E-4	0.56E-8	Th
(15,100)	0.00605	0.11E-4	0.66E-4	0.46E-8	Th
(25,15)	0.00752	0.10E-4	0.88E-4	0.59E-8	Th
(25,25)	0.00619	0.19E-6	0.68E-4	0.47E-8	Th
(25,50)	0.00489	0.15E-6	0.53E-4	0.44E-8	Th
(25,100)	0.00406	0.12E-6	0.45E-4	0.38E-8	Th
(50,15)	0.00666	0.12E-4	0.77E-4	0.53E-8	Th
(50,25)	0.00465	0.14E-6	0.50E-4	0.39E-8	Th
(50,50)	0.00326	0.97E-7	0.34E-4	0.34E-8	Th
(50,100)	0.00215	0.61E-7	0.22E-4	0.27E-8	Th
(100,15)	0.00622	0.12E-4	0.72E-4	0.52E-8	Th
(100,25)	0.00423	0.13E-6	0.45E-4	0.39E-8	Th
(100,50)	0.00244	0.70E-7	0.26E-4	0.30E-8	Th
(100,100)	0.00177	0.49E-7	0.18E-4	0.27E-8	Th

Note: 0.95E-5=0.0000095

Table 3. Shows $\hat{R}_{(s,k)}$for (s,k)=(2,4) α=2.5,β=4 & λ=2when $R_{(s,k)}$ =0.46584.

(n,m)	$\hat{R}_{(s,k)MLE}$	$\hat{R}_{(s,k)sh1}$	$\hat{R}_{(s,k)sh2}$	$\hat{R}_{(s,k)Th}$
(15,15)	0.46058	0.46565	0.46535	0.46588
(15,25)	0.47149	0.46584	0.46604	0.46588
(15,50)	0.47133	0.46592	0.46576	0.46588
(15,100)	0.47616	0.46589	0.46605	0.46588
(25,15)	0.45646	0.46576	0.46541	0.46588
(25,25)	0.46517	0.46588	0.46589	0.46588
(25,50)	0.46751	0.46588	0.46577	0.46588
(25,100)	0.47039	0.46588	0.46589	0.46588
(50,15)	0.46224	0.46605	0.46635	0.46588
(50,25)	0.46428	0.46589	0.46609	0.46589
(50,50)	0.46608	0.46589	0.46593	0.46588
(50,100)	0.46628	0.46588	0.46576	0.46588
(100,15)	0.45569	0.46576	0.46576	0.46588
(100,25)	0.46121	0.46588	0.46596	0.46588
(100,50)	0.46567	0.46589	0.46603	0.46588
(100,100)	0.46616	0.46588	0.46592	0.46588

Table 4. Shows MSE of$\hat{R}_{(s,k)}$for(s,k)=(2,4), α=2.5,β=4 & λ=2 when$R_{(s,k)} = 0.46584$.

(n,m)	$\hat{R}_{(s,k)MLE}$	$\hat{R}_{(s,k)sh1}$	$\hat{R}_{(s,k)sh2}$	$\hat{R}_{(s,k)Th}$	Best
(15,15)	0.01047	0.00002	0.00012	0.69E-8	Th
(15,25)	0.00880	0.00001	0.00009	0.65E-8	Th
(15,50)	0.00711	0.00001	0.00008	0.56E-8	Th
(15,100)	0.00658	0.00001	0.00007	0.53E-8	Th
(25,15)	0.00904	0.00001	0.00010	0.63E-8	Th
(25,25)	0.00656	0.21E-6	0.00007	0.51E-8	Th
(25,50)	0.00516	0.16E-6	0.00006	0.45E-8	Th
(25,100)	0.00448	0.14E-6	0.00005	0.42E-8	Th
(50,15)	0.00713	0.00001	0.00008	0.58E-8	Th
(50,25)	0.00469	0.14E-6	0.00005	0.43E-8	Th
(50,50)	0.00322	0.79E-7	0.00003	0.35E-8	Th
(50,100)	0.00245	0.70E-7	0.00003	0.30E-8	Th
(100,15)	0.006889	0.00001	0.00008	0.56E-8	Th
(100,25)	0.00407	0.12E-6	0.00004	0.39E-8	Th
(100,50)	0.00251	0.73E-8	0.00003	0.33E-8	Th
(100,100)	0.00173	0.47E-7	0.00002	0.28E-8	Th

Table 5. Shows $\hat{R}_{(s,k)}$ for (s,k)=(2,3), α=4,β=2.5 & λ=2 when $R_{(s,k)} = 0.63054$.

(n,m)	$\hat{R}_{(s,k)MLE}$	$\hat{R}_{(s,k)sh1}$	$\hat{R}_{(s,k)sh2}$	$\hat{R}_{(s,k)Th}$
(15,15)	0.62297	0.63042	0.63028	0.63050
(15,25)	0.62694	0.63041	0.630291	0.63050
(15,50)	0.63207	0.63043	0.63036	0.63050
(15,100)	0.63831	0.63059	0.63078	0.63050
(25,15)	0.61813	0.63038	0.63009	0.63050
(25,25)	0.62791	0.63052	0.63056	0.63050
(25,50)	0.63026	0.63050	0.63042	0.63050
(25,100)	0.63394	0.63051	0.63063	0.63050
(50,15)	0.62026	0.63055	0.63051	0.63050
(50,25)	0.62475	0.63050	0.63050	0.63050
(50,50)	0.62789	0.63049	0.63042	0.63050
(50,100)	0.62923	0.63049	0.63036	0.63050
(100,15)	0.61780	0.63046	0.63038	0.63050
(100,25)	0.62514	0.63051	0.63061	0.63050
(100,50)	0.62882	0.63051	0.63059	0.63050
(100,100)	0.62890	0.63049	0.63042	0.63050

Table 6. Shows MSE of $\hat{R}_{(s,k)}$ for (s,k)=(2,3), α=4,β=2.5 & λ=2 when $R_{(s,k)} = 0.63054$

(n,m)	$\hat{R}_{(s,k)MLE}$	$\hat{R}_{(s,k)sh1}$	$\hat{R}_{(s,k)sh2}$	$\hat{R}_{(s,k)Th}$	Best
(15,15)	0.00938	0.00002	0.00010	0.63E-8	Th
(15,25)	0.00664	0.00001	0.00008	0.51E-8	Th
(15,50)	0.00564	0.95E-5	0.00006	0.46E-8	Th
(15,100)	0.00489	0.94E-5	0.00006	0.39E-8	Th
(25,15)	0.00746	0.99E-5	0.00008	0.52E-8	Th
(25,25)	0.00531	0.16E-6	0.00006	0.41E-8	Th
(25,50)	0.00362	0.11E-6	0.00004	0.33E-8	Th
(25,100)	0.00326	0.98E-7	0.00003	0.30E-8	Th
(50,15)	0.00616	0.98E-5	0.00007	0.43E-8	Th
(50,25)	0.00390	0.11E-6	0.00004	0.32E-8	Th
(50,50)	0.00257	0.75E-7	0.00003	0.27E-8	Th
(50,100)	0.00193	0.56E-7	0.00002	0.25E-8	Th
(100,15)	0.00554	0.97E-4	0.00006	0.40E-8	Th
(100,25)	0.00370	0.11E-6	0.00004	0.32E-8	Th
(100,50)	0.00194	0.54E-7	0.00002	0.24E-8	Th
(100,100)	0.00131	0.36E-7	0.00001	0.21E-8	Th

Table 7. Shows $\hat{R}_{(s,k)}$ for, (s,k)=(2,4), α=4,β=2.5 & λ=2 when $R_{(s,k)} = 0.71575$.

(n,m)	$\hat{R}_{(s,k)MLE}$	$\hat{R}_{(s,k)sh1}$	$\hat{R}_{(s,k)sh2}$	$\hat{R}_{(s,k)Th}$
(15,15)	0.71125	0.71581	0.71589	0.71572
(15,25)	0.71194	0.71574	0.71567	0.71572
(15,50)	0.71423	0.71557	0.71534	0.71571
(15,100)	0.71529	0.71562	0.71539	0.71571
(25,15)	0.70901	0.71582	0.71589	0.71572
(25,25)	0.71098	0.71572	0.71567	0.71572
(25,50)	0.71496	0.71572	0.71569	0.71572
(25,100)	0.71776	0.71573	0.71583	0.71572
(50,15)	0.70333	0.71558	0.71542	0.71572
(50,25)	0.71236	0.71573	0.71592	0.71572
(50,50)	0.71332	0.71572	0.71567	0.71572
(50,100)	0.71743	0.71573	0.71593	0.71572
(100,15)	0.70434	0.71572	0.71566	0.71572
(100,25)	0.70971	0.71572	0.71575	0.71572
(100,50)	0.71124	0.71571	0.71554	0.71571
(100,100)	0.71516	0.71572	0.71577	0.71572

Table 9. Shows MSE of $\hat{R}_{(s,k)}$ for (s,k)=(2,4), α=4,β=2.5 & λ=2 when $R_{(s,k)} = 0.63054$.

(n,m)	$\hat{R}_{(s,k)MLE}$	$\hat{R}_{(s,k)sh1}$	$\hat{R}_{(s,k)sh2}$	$\hat{R}_{(s,k)Th}$	Best
(15,15)	0.00631	0.00001	0.00007	0.39E-8	Th
(15,25)	0.00549	0.00001	0.00006	0.39E-8	Th
(15,50)	0.00382	0.00001	0.00004	0.30E-8	Th
(15,100)	0.00376	0.68E-5	0.00004	0.31E-8	Th
(25,15)	0.00528	0.71E-5	0.00006	0.35E-8	Th
(25,25)	0.00409	0.12E-6	0.00004	0.30E-8	Th
(25,50)	0.00277	0.86E-7	0.00003	0.25E-8	Th
(25,100)	0.00239	0.73E-7	0.00003	0.23E-8	Th
(50,15)	0.00509	0.90E-5	0.00006	0.39E-8	Th
(50,25)	0.00306	0.90E-7	0.00003	0.24E-8	Th
(50,50)	0.00185	0.52E-7	0.00002	0.20E-8	Th
(50,100)	0.00140	0.40E-7	0.00001	0.17E-8	Th
(100,15)	0.00425	0.75E-5	0.00004	0.30E-8	Th
(100,25)	0.00251	0.71E-7	0.00003	0.22E-8	Th
(100,50)	0.00147	0.41E-7	0.00001	0.18E-8	Th
(100,100)	0.00105	0.28E-7	0.00001	0.15E-8	Th

5. Discussion Numerical Simulation Results

From the tables above, for all n=(15,25,50,100) and m=(15,25,50,100) we conclude that, the shrinkage estimator using Modified Thompson type shrinkage weight factor to estimate the reliability $R_{(s,k)}$ is the bestsince the (MSE)of $R_{Th(s,k)}$ was less than in the other methods.

6. Conclusion

Form the numerical results, we conclude that the shrinkage estimation method performance good behavior especially when using modified Thompson type shrinkage weight factor.

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