An Accurate MHD Flux Solutions of a Viscose Fluid and Generalized Burgers' Model fluxwithin an Annular Pipe Under Sinusoidal Pressure

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Abstract

The aim of this work presents the analytical studies of both the magnetohydrodynamic (MHD) flux and flow of the non-magnetohydro dynamic (MHD) for a fluid of generalized Burgers' (GB) withinan annular pipe submitted under Sinusoidal Pressure (SP)gradient. Closed beginning velocity's' solutions are taken by performing the finite Hankel transform (FHT) and Laplace transform (LT) of the successivefraction derivatives. Lastly, the figures were planned to exhibition the transformations effects of different fractional parameters (DFP) on the profile of velocity of both flows.

Keywords: Generalized Burgers', finite Hankel transform, Laplace transform, Sinusoidal Pressure gradient.

Preface

Lately, many attentions have been devoted to the project of nonNewtonian fluids. In general, the foremostobject is t
hat fluids (such as paints, the molten plastics, slurries, pulps, emulsions, the petroleum was drilled, blood and other identical entities), which do not follow the Newtonian assume that the stress tensor is immediately symmetricto the rate of turn of deformitytensor, and show characteristics of flow quite severalto those of Newtonian fluids, the models are usually distribution gas fluids of differential, average and integral types (Rajagopal, [1]; and Dunn and Rajagopal, [2]). Different studies were performed on a generalized Oldroyd-B(GO-B) fluid flux includes those from Zheng et al. [3], Khan et al. [4], and Sultan et al. [5]. Fetecau et al. [6], Kamranc et al [7] and Hyder Ali Muttaqi Shah [8] thought fulsome summary fluxes of (GO-B) fluid through two wall sides that perpendicular to a sheet. Zheng et al. [9], and Nazar et al. [10] talk overMaxwell fluid flux because of a plate, with fixed velocity. Mahmood et al. [11] investigated the unsteady flux of a non-Newtonian fluid between two infinite coaxial circular cylinders. Whereas Khan et al. [12], Khan et al. [13], with Khan and Shafie, [14] described the exact solutions for the flux of an MHD (GB) fluid.
In this paper, our target is to study the unsteady viscoelastic fluid flow with the model of fractional (GB) fluid within an annular pipe under (SP), and compare with flow under MHD (SP). The accurate solution for the distribution of velocity is performed by implying the (FHT) Garg et al. [15] and discrete (LT) of the sequential fractional derivatives.

Prevalent Equations

The constituent equations for an incompressible fractional(GB)fluid are agreed through

$$\mathbf{T} = -p\mathbf{I} + \mathbf{S}, (1 + \lambda_1^{\alpha} \widetilde{\mathrm{D}}_t^{\alpha} + \lambda_2^{\alpha} \widetilde{\mathrm{D}}_t^{2\alpha})\mathbf{S} = \mu(1 + \lambda_3^{\beta} \widetilde{\mathrm{D}}_t^{\beta})\mathbf{A_1} \qquad (1)$$

anywhere $\mathbf{T}$ fixed by Cauchy stress, $-p\,\mathbf{I}$ is undefined spherical stress, $\mathbf{S}$ means the additional stress tensor, $\mathbf{A}_1 = \mathbf{L} + \mathbf{L}^{\mathrm{T}}$ is the first tensor of Rivlin- Ericksen with the gradient of velocity anywhere $\mathbf{L} = \mathrm{grad}\,\mathbf{V}$, μ showed the efficient viscosity of fluid, λ_1 and λ_3 $(<\lambda_1)$ are the relaxation, and the obstruction times, respectively, λ_2 is the modern item parameter of (GB)fluid, α and β the (DFP) calculus like that $0 \le \alpha \le \beta \le 1$ and $\widetilde{\mathrm{D}}_t^{p}$ the upper convicted fractional derivative which described through

$$\widetilde{D}_t^{\alpha}\mathbf{S} = D_t^{\alpha}\mathbf{S} + (\mathbf{V}.\nabla)\mathbf{S} - \mathbf{L}\,\mathbf{S} - \mathbf{S}\mathbf{L}^{\mathrm{T}} \qquad (2)$$

$$\widetilde{D}_t^{\beta}\mathbf{A_1} = D_t^{\beta}\mathbf{A_1} + (\mathbf{V}.\nabla)\mathbf{A_1} - \mathbf{L}\mathbf{A_1} - \mathbf{A_1}\mathbf{L}^{\mathrm{T}} \qquad (3)$$

in whose D_t^{α} and D_t^{β} are the (DFP)of order α and β be contingent on the definition of Riemann- Liouville, identify as

$$D_t^{p}[F(t)] = \frac{1}{\Gamma(1-P)} \frac{d}{dt} \int_0^t \frac{F(\tau)}{(t-\tau)^p} d\tau \qquad ,0 \le P \le 1 \text{ and } D_t^{2p}\mathbf{S} = D_t^{p}(D_t^{p}\mathbf{S}) \qquad (4)$$

When $\Gamma(.)$ is the Gamma function.
The type diminished to the model of (GO-B) when $\lambda_2 = 0$ and if, addendum for that α = β = 1 the normal Oldroyd-B typeshall be earned.
So, we suppose that shear stress and the field of velocity of the format

$$\mathbf{V} = \omega\ (r\ ,t)e_{\ z}\ \ ,\mathbf{S} = \mathbf{S}\ \ (r\ ,t) \quad (5)$$

When $e_{\ z}$ meant vector unit along the z-direction .Equation (5) substituted into (1) and takeover an account for the first condition

$$S\ (r,0) = 0 \quad (6)$$

Obtain

$$(1+\lambda_1^{\alpha}\ D_{\ t}^{\ \alpha} + \lambda_2^{\alpha}\ D_{\ t}^{\ 2\alpha})S = \mu(1+\lambda_3^{\beta}\ D_{\ t}^{\ \beta})\partial_r \omega\ (r,t)$$
$$(1+\lambda_1^{\alpha}\ \mathrm{D}_{\ t}^{\ \alpha} + \lambda_2^{\alpha}\ \mathrm{D}_{\ t}^{\ 2\alpha})\mathrm{S}_{zz} - 2\mathrm{S}_{rz}(\lambda_1^{\alpha} + \lambda_2^{\alpha}\ \mathrm{D}_{\ t}^{\ \alpha})\partial_r\omega\ (r,t) = -2\mu\,\lambda_3^{\beta}(\partial_r\omega\ (r,t))^2 \quad (7)$$

When $\mathrm{S}_{rr} = \mathrm{S}_{\theta z} = \mathrm{S}_{r\theta} = \mathrm{S}_{\theta\theta} = 0$.

Thereafter the being gradient of pressure at z-direction, the motion equation provided next scalar equation:

$$\rho\ \frac{d\omega}{\mathrm{dt}} = \frac{-\partial \mathrm{p}}{\partial \mathrm{z}} + \frac{1}{r}\ *\frac{\partial}{\partial r}(r\ S_{\ rz})\ (8)$$

When ρ showed constant fluid density. The judge $\mathrm{S}_{\ rz}$ amidst Eqs. (7), and (8), we earn the next fractional differential equation

$$(1+\lambda_1^{\alpha}\ \mathrm{D}_{\ t}^{\ \alpha} + \lambda_2^{\alpha}\ \mathrm{D}_{\ t}^{\ 2\alpha})\frac{\partial\omega}{\partial \mathrm{t}} = \frac{-1}{\rho}(1+\lambda_1^{\alpha}\ \mathrm{D}_{\ t}^{\ \alpha} + \lambda_2^{\alpha}\ \mathrm{D}_{\ t}^{\ 2\alpha})\frac{\mathrm{dp}}{\mathrm{dz}} + v(1+\lambda_3^{\beta}\ \mathrm{D}_{\ t}^{\ \beta})(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r})\omega\ (9)$$

When $v = \frac{\mu}{\rho}$ indicated the kinematic viscosity.

First Problem of the Non-magnetohydrodynamic Flow

Regard that the fluxaffair of an incompressible (GB) fluid is firstly at rest in between two infinitely long coaxial cylinders of the radius R_0 and R_1 ($> R_0$). At time $t = 0^+$ fluid is generated because of(SP) gradient which acts on liquid in z-direction. Pointing to Eq. (9), the coinciding (DFP) equation that define such flux has the way

$$(1+\lambda_1^{\alpha}\ \mathrm{D}_{\ t}^{\ \alpha} + \lambda_2^{\alpha}\ \mathrm{D}_{\ t}^{\ 2\alpha})\frac{\partial\omega}{\partial \mathrm{t}} = -\mathrm{P}_0\cos\ (ut)\left(1+\lambda_{\ 1}^{\ \alpha}\frac{t^{-1-\alpha}}{\Gamma(-\alpha)} + \lambda_{\ 2}^{\ \alpha}\frac{t^{-1-2\alpha}}{\Gamma(-2\alpha)}\right) + v(1+\lambda_{\ 3}^{\ \beta}\ \mathrm{D}_{\ t}^{\ \beta})(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r})\omega\ (10)$$

When $\mathrm{P}_{\ 0}\cos\ (\mathrm{ut}) = \frac{1}{\rho}\frac{dp}{dz}$ showed the (SP) gradient

The condition of initial and boundary relations are described as form

$$\omega\ \ (r\ \ ,0) = \frac{\partial}{\partial t}\omega\ \ (r,0) = \frac{\partial^2}{\partial t^2}\omega\ \ (r\ \ ,0) = 0 \qquad ,R_0 \le r\ \ \le R_1\ (11)$$

$$\omega\ (R_0,t) = \omega\ (R_1,t) = 0 \qquad ,t > 0\ (12)$$

For producing the accurate analytical solution of the previous problems (10)- (12), First, we perform(LT) rule Garg et al. [15] through respect to t, we got

$$s\,(1+\lambda_1^{\alpha}\ s^{\alpha} + \lambda_2^{\alpha}\ s^{2\alpha})\overline{\omega} = v(1+\lambda_3^{\beta}\ s^{\beta})(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r})\overline{\omega} - \frac{\mathrm{P}_0\mathrm{s}}{\mathrm{s}^2+u^2}(1+\lambda_1^{\alpha}\ s^{\alpha} + \lambda_2^{\alpha}\ s^{2\alpha})\ (13)$$

$$\overline{\omega}\ (r\ ,0) = 0$$
$$\overline{\omega}\ (R_0,s\ \ ) = \overline{\omega}\ (R_1,s\ \ ) = 0 \qquad ,\ \ t\ \ > 0 \quad (14)$$

When $\overline{\omega}\ (r\ ,s\ \ )$ denoted of function image of $\omega\ \ (r\ ,t\ \ )$ and s denoted the parameter transform. We imply the (FHT) Garg et al. [15], described as form

$$\overline{\omega}_{\ H} = \int_{R_0}^{R_1} r\ \ \overline{\omega}\ \ B_0(r\ k_i)dr \qquad ,\ i\ \ = 1\cdot 2\cdot 3\cdot\ldots\ \ (15)$$

While its inverse as

$$\overline{\varpi} = \frac{\pi^2}{2} \sum_{i=1}^{\infty} \frac{k_i^2 \overline{\varpi}_H \, B_0(rk_i) J_0^2(R_1 k_i)}{J_0^2(R_0 k_i) - J_0^2(R_1 k_i)} \quad (16)$$

Where k_i are the positive roots of equation $B_0(R_1 k_i) = 0$ and

$B_0(rk_i) = J_0(rk_i) Y_0(R_0 k_i) - Y_0(rk_i) J_0(R_0 k_i)$

When $J_0(.)$ while $Y_0(.)$ are the functions of Bessel of the first and second types of zero order.

Here using (FHT) to Eqs. (13)-(14) through respect to r, we take

$$\overline{\varpi}_H = -\mathrm{P}_0 \frac{\frac{\mathrm{s}}{\mathrm{s}^2 + u^2}(1 + \lambda_1^{\alpha} s^{\alpha} + \lambda_2^{\alpha} s^{2\alpha})}{\mathrm{s}(1 + \lambda_1^{\alpha} s^{\alpha} + \lambda_2^{\alpha} s^{2\alpha}) + \nu \, \mathrm{k}_\mathrm{i}^2 (1 + \lambda_3^{\beta} s^{\beta})} \quad (17)$$

Currently, lettering Eq. (17) In the form of a chain as

$$\overline{\varpi}_H = -\mathrm{P}_0 (1 + \lambda_1^{\alpha} s^{\alpha} + \lambda_2^{\alpha} s^{2\alpha}) \sum_{k=0}^{\infty} (-1)^k \sum_{a,b,c,d,e,f,j \geq 0}^{a+b+c+d+e+f+j=k} k! \frac{(u^2)^{h-k-n-1} (vk_i^2)^q (\lambda_1^{\alpha})^{m-k-n-1} (\lambda_2^{\alpha})^{n-l+r_*-q} (\lambda_3^{\beta})^z s^{\delta}}{a!b!c!d!e!f!j!h! \left(s^{\alpha+1} + \frac{\nu k_i^2}{\lambda_1^{\alpha}} \right)^{k+1}} \quad (18)$$

Where $\delta = 1 + 3k - 2l + q - 2h + \alpha(m + n - 2l + 2r_* - 2q) + \beta z$. Whileits discrete inverse (LT) Garg et al. (2007) will yield the form

$$\omega_H = -\mathrm{P}_0 \sum_{k=0}^{\infty} (-1)^k \sum_{a,b,c \geq 0}^{a+b+c=k} k! \frac{(u^2)^{h-k-n-1} (\nu k_i^2)^q (\lambda_1^{\alpha})^{m-k-n-1} (\lambda_2^{\alpha})^{n-l+r_*-q} (\lambda_3^{\beta})^z}{a!b!c!d!e!f!j!h!} t^{(\alpha+1)k+(\alpha+1-\delta)-1} \quad (19)$$

$$\left\{ E^{k}_{\alpha+1,\alpha+1-\delta}\left(-\frac{vk_i^2}{\lambda_1^{\alpha}} t^{\alpha+1} \right) + t^{\alpha} \lambda_1^{\alpha} E^{k}_{\alpha+1,1-\delta}\left(-\frac{vk_i^2}{\lambda_1^{\alpha}} t^{\alpha+1} \right) + t^{2\alpha} \lambda_2^{\alpha} E^{k}_{\alpha+1,1-\alpha-\delta}\left(-\frac{vk_i^2}{\lambda_1^{\alpha}} t^{\alpha+1} \right) \right\}$$

When $E^{m}_{\alpha,\beta}(z) = \sum_{j=0}^{\infty} \frac{(j+m)!z!}{j!\Gamma(\alpha j + \alpha m + \beta)}$ showed generalized Mittag- Leffler function Garg et al. [15] andto earn Eq. (19), the following property of inverse (LT) is used (20)

$$L^{-1}\left\{ \frac{m! s^{\lambda-\mu}}{(s^{\lambda} \mp c)} \right\} = t^{\lambda m + \mu - 1} E^{m}_{\lambda,\mu}(\pm c t^{\lambda}) \qquad , \left(\mathrm{Re}(s) > |c|^{\frac{1}{\lambda}} \right) (20)$$

eventually, the inverse (FHT) obtains the analytic solution of velocity classification

$$\omega(r,t) = -\frac{\mathrm{P}_0 \pi^2}{2} \sum_{i=1}^{\infty} \frac{k_i^2 B_0(rk_i) J(R_i k_i)}{J_0^2(R_0 k_i) - J_0^2(R_i k_i)} \left[\begin{array}{l} \sum_{k=0}^{\infty} (-1)^k \sum_{a,b,c,d,e,j,j \geq 0}^{a+b+c+f+e+d+j=k} k! \frac{(u^2)^{h-k+l-q-1} (\nu k_i^2)^q (\lambda_1^{\alpha})^{m-k-n-1} (\lambda_2^{\alpha})^{n-l+r_*-q} (\lambda_3^{\beta})^z}{a!b!c!h!e!j!f!d!} t^{(\alpha+1)k+(\alpha+1-\delta)-1} \\ \left\{ E^{k}_{\alpha+1,\alpha+1-\delta}\left(-\frac{vk_i^2}{\lambda_1^{\alpha}} t^{\alpha+1} \right) + t^{\alpha} \lambda_1^{\alpha} E^{k}_{\alpha+1,1-\delta}\left(-\frac{vk_i^2}{\lambda_1^{\alpha}} t^{\alpha+1} \right) + t^{2\alpha} \lambda_2^{\alpha} E^{k}_{\alpha+1,1-\alpha-\delta}\left(-\frac{vk_i^2}{\lambda_1^{\alpha}} t^{\alpha+1} \right) \right\} \end{array} \right] (21)$$

The Special Cases

Working the limits of Eq.(21) where $\alpha \neq 0$, $\lambda_2 \to 0$ (b=0) , we obtain the distribution of velocity for a (GO-B) fluid. So the field of velocity decreases to

$$\omega(r,t) = -\frac{\mathrm{P}_0 \pi^2}{2} \sum_{i=1}^{\infty} \frac{k_i^2 B_0(rk_i) J_0^2(R_1 k_i)}{J_0^2(R_0 k_i) - J_0^2(R_1 k_i)} \left[\sum_{k=0}^{\infty} (-1)^k \sum_{a+b+f+e+d+c=k}^{a,b,f,e,d,c \geq 0} k! \frac{(u^2)^{-1+n-k+f} (\nu k_i^2)^l (\lambda_1^{\alpha})^{m-k-n-1} (\lambda_3^{\beta})^h}{a!b!f!e!d!c!} t^{(\alpha+1)k+(\alpha+1-\delta)-1} \right.$$

$$\left. \left\{ E^{k}_{\alpha+1,\alpha+1-\delta}\left(-\frac{vk_i^2}{\lambda_1^{\alpha}} t^{\alpha+1} \right) + \frac{\lambda_1^{\alpha}}{t^{\alpha}} E^{k}_{\alpha+1,\alpha+1-\delta}\left(-\frac{vk_i^2}{\lambda_1^{\alpha}} t^{\alpha+1} \right) \right\} \right] \quad (22)$$

Where $\delta = 3k + 1 + \alpha(m + l + h - f - 2n) - l + h - 3f + \beta h$.

Second Problem of the Magnetohydrodynamic (MHD) Flow

Moreover, it believes that showing fluid is prevailed by imposing magnetic field $\mathbf{H} = [0, \mathrm{H}_0, 0]$ which work in positive z-direction. In the calculation of the low-magnetic Reynolds number, the magnetic power of the body is considered as $\sigma H_0^2 w$, when σ indicated electrical accessibility of fluid.Now, by adding magnetic field to Eq.(8) we get an Eq. (23):

$$\rho\frac{dw}{dt}=-\frac{\partial p}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}(r S_{rz})-\sigma H_0^2 w \quad (23)$$

the judge S_{rz} among Eqs. (7) and (23), we make the next fractional differential equation

$$(1+\lambda_1^{\alpha} D_t^{\alpha}+\lambda_2^{\alpha} D_t^{2\alpha})\frac{\partial\omega}{\partial t}=\frac{-1}{\rho}(1+\lambda_1^{\alpha} D_t^{\alpha}+\lambda_2^{\alpha} D_t^{2\alpha})\frac{dp}{dz}+ + v(1+\lambda_3^{\beta} D_t^{\beta})(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r})\omega-M(1+\lambda_1^{\alpha} D_t^{\alpha}+\lambda_2^{\alpha} D_t^{2\alpha})\omega \quad (24)$$

Anywhere $v=\frac{\mu}{\rho}$ denoted the kinetic viscosity while $M=\frac{\sigma H_0^2}{\rho}$ denoted the dimensionless magnetic number.

In the same way as calculating the flux of the first problem we find Eq. (24), the according fractional partial differential equation that term such fluxhas the shape

$$(1+\lambda_1^{\alpha} D_t^{\alpha}+\lambda_2^{\alpha} D_t^{2\alpha})\frac{\partial\omega}{\partial t}=-P_0\cos(ut)\left(1+\lambda_1^{\alpha}\frac{t^{-\alpha-1}}{\Gamma(-\alpha)}+\lambda_2^{\alpha}\frac{t^{-2\alpha-1}}{\Gamma(-2\alpha)}\right)+v(1+\lambda_3^{\beta} D_t^{\beta})(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r})\omega-M(1+\lambda_1^{\alpha} D_t^{\alpha}+\lambda_2^{\alpha} D_T^{2\alpha})\omega \quad (25)$$

Where $P_0\cos(ut)=\frac{1}{\rho}\frac{dp}{dz}$ indicated the continual pressure gradient

To earn the accurate analytical solution of the previous problems (25)- (12), First, we perform (LT) rule Garg et al. [15] through respect to t, we obtain

$$s(1+\lambda_1^{\alpha}s^{\alpha}+\lambda_2^{\alpha}s^{2\alpha})\overline{\omega}=-P_0\frac{s}{s^2+u^2}(1+\lambda_1^{\alpha}s^{\alpha}+\lambda_2^{\alpha}s^{2\alpha})+v(1+\lambda_3^{\beta}s^{\beta})(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r})\overline{\omega}-M(1+\lambda_1^{\alpha}S^{\alpha}+\lambda_2^{\alpha}S^{2\alpha})\overline{\omega} \quad (26)$$

Now using (FHT) to Eqs. (26) -(14) through respect to r, we obtain

$$\overline{\omega}_H=-P_0\frac{\frac{s}{s^2+u^2}(1+\lambda_1^{\alpha}s^{\alpha}+\lambda_2^{\alpha}s^{2\alpha})}{(s+M)(1+\lambda_1^{\alpha}s^{\alpha}+\lambda_2^{\alpha}s^{2\alpha})+v\,k_i^2(1+\lambda_3^{\beta}s^{\beta})} \quad (27)$$

Now, inscription Eq. (27) in sequence form as

$$\overline{\omega}_H=-P_0(1+\lambda_1^{\alpha}s^{\alpha}+\lambda_2^{\alpha}s^{2\alpha})\sum_{k=0}^{\infty}(-1)^k\sum_{l,q,z,n,h,f,j,t_1,w_0,w_1,w_2w_3\geq 0}^{a+b+c+d+e+g+i+o+c_1+c_2+c_3=k}k!\frac{(u^2)^{*1}(vk_i^2)^{*2}(M)^{*3}(\lambda_1^{\alpha})^{*4}(\lambda_2^{\alpha})^{*5}(\lambda_3^{\beta})^{*6}s^{\delta}}{a!b!c!d!e!g!i!o!c_0!c_1!c_2!c_3!w_3!\left(s^{\alpha+1}+M+\frac{v\,k_i^2}{\lambda_1^{\alpha}}\right)^{k+1}} \quad (28)$$

Where $*1=-1-k+f-w_1+w_3$, $*2=w_1$, $*3=z-f+t_1-w_2$, $*4=-1-k+l-q+n-h-t_1+w_0$, $*5=q-z+h-f+j-t_1+w_0-w_1$, $*6=w_2$, $\delta=1+3k+\alpha(l+q+n+h)-(1+2\alpha)z-(1+2\alpha)f-(1+2\alpha)t_1+2\alpha(j+w_0)+\beta w_2-2w_3$.

While its discrete inverse (LT) Garg et al [15] will yield the form

$$\omega_H=-P_0\sum_{k=0}^{\infty}(-1)^k\sum_{j,q,z,\ldots\geq 0}^{a+b+c+\ldots=k}k!\frac{(u^2)^{*1}(vk_i^2)^{*2}(M)^{*3}(\lambda_1^{\alpha})^{*4}(\lambda_2^{\alpha})^{*5}(\lambda_3^{\beta})^{*6}}{a!b!c!d!e!f!j!h!}t^{(\alpha+1)k+(\alpha+1-\delta)-1} \quad (29)$$
$$\left\{E^{k}_{\alpha+1,\alpha+1-\delta}\left(-(\frac{M}{\lambda_1^{\alpha}}+\frac{vk_i^2}{\lambda_1^{\alpha}})t^{\alpha+1}\right)+t^{\alpha}\lambda_1^{\alpha}E^{k}_{\alpha+1,1-\delta}\left(-(\frac{M}{\lambda_1^{\alpha}}+\frac{vk_i^2}{\lambda_1^{\alpha}})t^{\alpha+1}\right)+t^{2\alpha}\lambda_2^{\alpha}E^{k}_{\alpha+1,1-\alpha-\delta}\left(-(\frac{M}{\lambda_1^{\alpha}}+\frac{vk_i^2}{\lambda_1^{\alpha}}t^{\alpha+1}\right)\right\}$$

the following property of inverse (LT) is used (20). finally, the inverse (FHT) gets the analytic solution of velocity classification

$$\omega(r,t)=-\frac{P_0\pi^2}{2}\sum_{i=1}^{\infty}\frac{k_i^2B_0(rk_i)J(R_ik_i)}{J_0^2(R_0k_i)-J_0^2(R_ik_i)}\left[\begin{array}{l}\sum_{k=0}^{\infty}(-1)^k\sum_{l,q,z,\ldots\geq 0}^{a+b+c\ldots=k}k!\frac{(u^2)^{*1}(vk_i^2)^{*2}(M)^{*3}(\lambda_1^{\alpha})^{*4}(\lambda_2^{\alpha})^{*5}(\lambda_3^{\beta})^{*6}}{a!b!c!d!e!f!j!h!}t^{(\alpha+1)k+(\alpha+1-\delta)-1}\\ \left\{E^{k}_{\alpha+1,\alpha+1-\delta}\left(-(\frac{M}{\lambda_1^{\alpha}}+\frac{vk_i^2}{\lambda_1^{\alpha}})t^{\alpha+1}\right)+t^{\alpha}\lambda_1^{\alpha}E^{k}_{\alpha+1,1-\delta}\left(-(\frac{M}{\lambda_1^{\alpha}}+\frac{vk_i^2}{\lambda_1^{\alpha}})t^{\alpha+1}\right)+t^{2\alpha}\lambda_2^{\alpha}E^{k}_{\alpha+1,1-\alpha-\delta}\left(-(\frac{M}{\lambda_1^{\alpha}}+\frac{vk_i^2}{\lambda_1^{\alpha}})t^{\alpha+1}\right)\right\}\end{array}\right] \quad (30)$$

The Special cases

Working the limitsfor Eq.(30) where $\alpha \neq 0$, $\lambda_2 \to 0$ (b=0) while $M \to 0$ (c=d=0) , we obtain the distribution of velocity for (GO-B) fluid. So the field of velocity decreases to

$$\omega(r,t) = \frac{P_0 \pi^2}{2} \sum_{i=1}^{\infty} \frac{k_i^2 B_0(rk_i) J_0^2(R_1 k_i)}{J_0^2(R_0 k_i) - J_0^2(R_1 k_i)} \left[\sum_{k=0}^{\infty} (-1)^k \sum_{\substack{a+b+c+e+d+f=k \\ a,b,d,c,f,e \geq 0}} k! \frac{(u^2)^{-1-k+z} (\nu k_i^2)^{i-j} (\lambda_1^{\alpha})^{l+n+w-i-y-k-1} (\lambda_3^{\beta})^{j-z+y}}{a!b!c!d!e!f!h_0!h_1!} t^{(\alpha+1)k+(\alpha+1-\delta)-1} \right.$$
$$\left. M^{q-i+w-y} \left\{ E^{k}_{\alpha+1,\alpha+1-\delta}\left(-\frac{\nu k_i^2}{\lambda_1^{\alpha}} t^{\alpha+1} \right) + \frac{\lambda_1^{\alpha}}{t^{\alpha}} E^{k}_{\alpha+1,\alpha+1-\delta}\left(-\frac{\nu k_i^2}{\lambda_1^{\alpha}} t^{\alpha+1} \right) \right\} \right] \quad (31)$$

Where $\delta = 1 + 3k + \alpha(l - q + n - i + w - y) - q - z - w + \beta(j - z + y)$.

Results Discussion

In the present study, we have been discussed MHD flux of (GB) fluid that passed an annular pipe. The accurate solution for the field of velocity u is gotten by performing the (LT) and (FHT). Furthermore, figures were plotted to show the behavior of diverse parameters included the velocity expressions u .

A comparison between the effect of magnetic parameter (M≠0) (Panel (a)) and the effect of non-magnetic parameter (M=0) (Panel (b)) were also done graphically in figures (1-6).

figures (1) and (2) the velocity is increased with the increasing of the α with both cases (M=0 & M≠0), while it increased with β (M≠0) more than with β (M=0).

figures (3), (4) and (5) showed the relaxation parameter effect λ_1 on the fields of velocity. Velocity is decreased for the incensement of λ_1 for (M≠0), and it did not affected with the increase of λ_1 for (M=0). Velocity is increased with the incensement of λ_2 when (M=0), and it oscillated with the increase of λ_2 for (M≠0).The velocity is decreased with the incensement of λ_3 when (M=0), and decreased more with the incensement of λ_3 for (M≠0).

figure (6) has shown the effect of the magnetic parameter M inshort as well as in long time. It is detected that the velocity profile is increased with the increase of t = 0.5 – 1.2 for (M≠0) more than for (M=0).

Comparison displays that velocity sketch with the effect of magnetic field is greater when compared with velocity sketch without the effect of magnetic field. The result is demonstrated in long time.

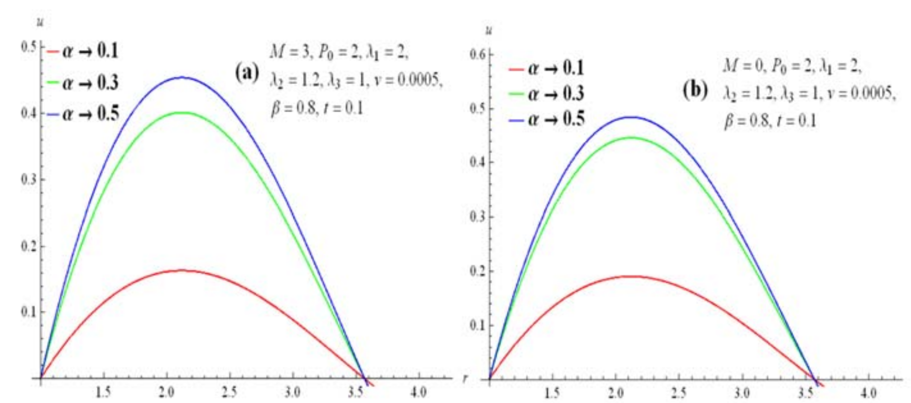

Figure (1): Shows velocity for various values of α while remaining another parameters constant (a) $M = 3$, and (b) $M = 0$

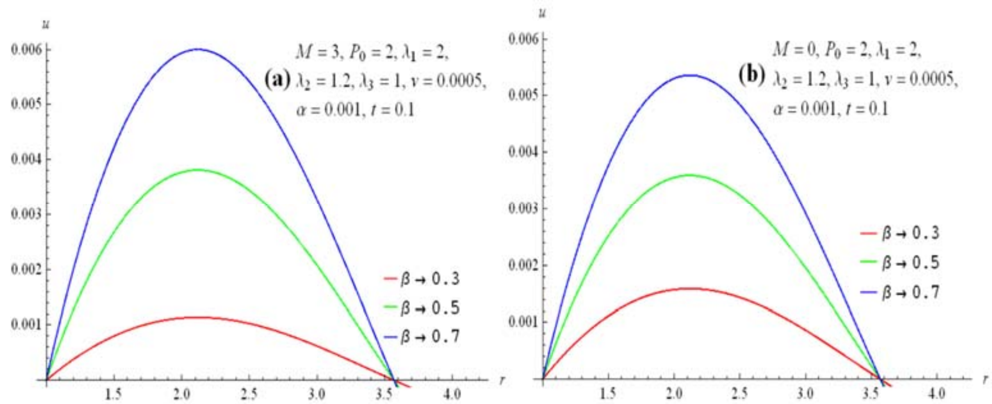

Figure (2): Shows velocity for various values of β while remaining another parameters constant (a) $\mathrm{M}=3$ and (b) $\mathrm{M}=0$

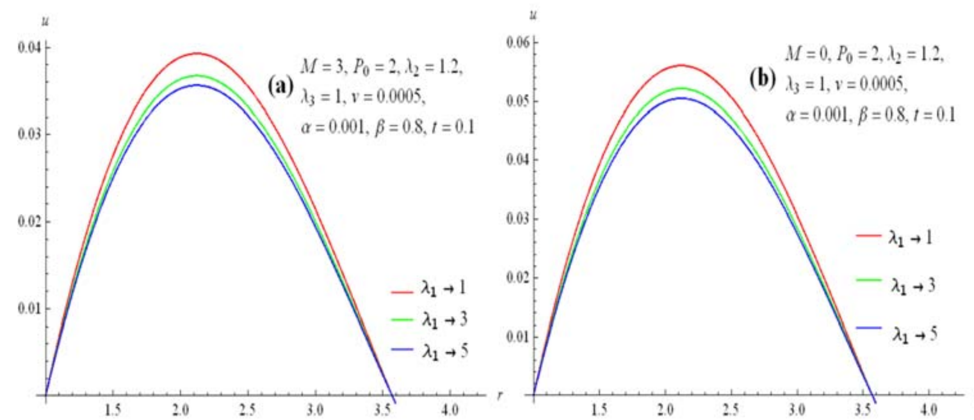

Figure (3) Shows velocity for various values of λ_1 while remaining another parameters constant (a) $\mathrm{M}=3$ and (b) $\mathrm{M}=0$

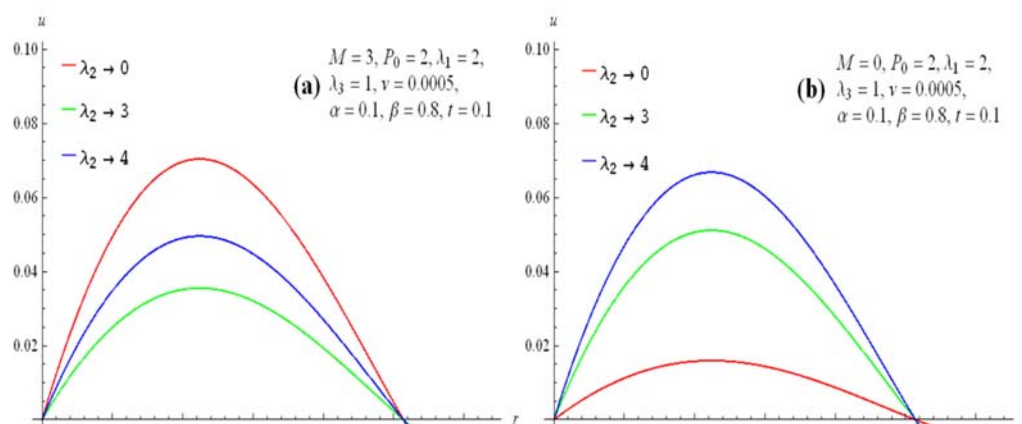

Figure (4): Shows velocity for various values of λ_2 while remaining another parameters constant (a) $\mathrm{M}=3$ and (b) $\mathrm{M}=0$

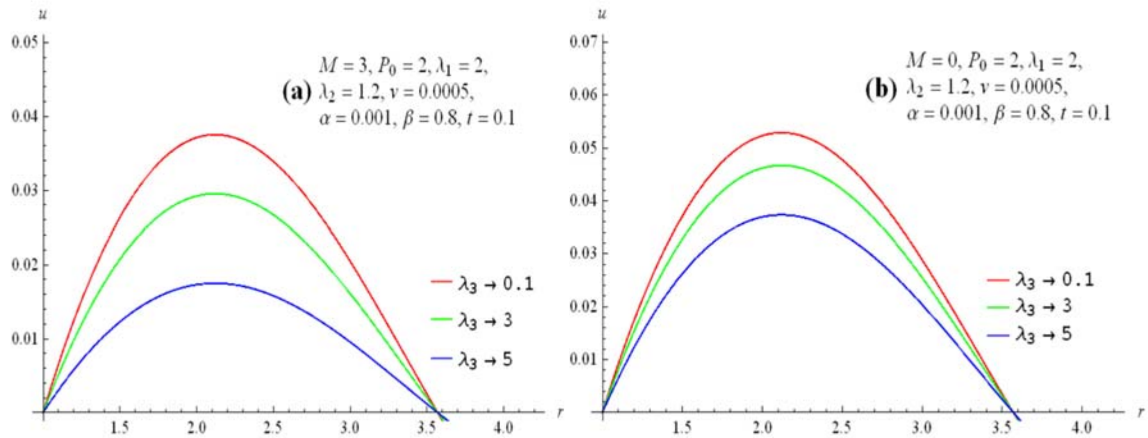

Figure (5): Shows velocity for various values of λ_3 while remaining another parameters constant

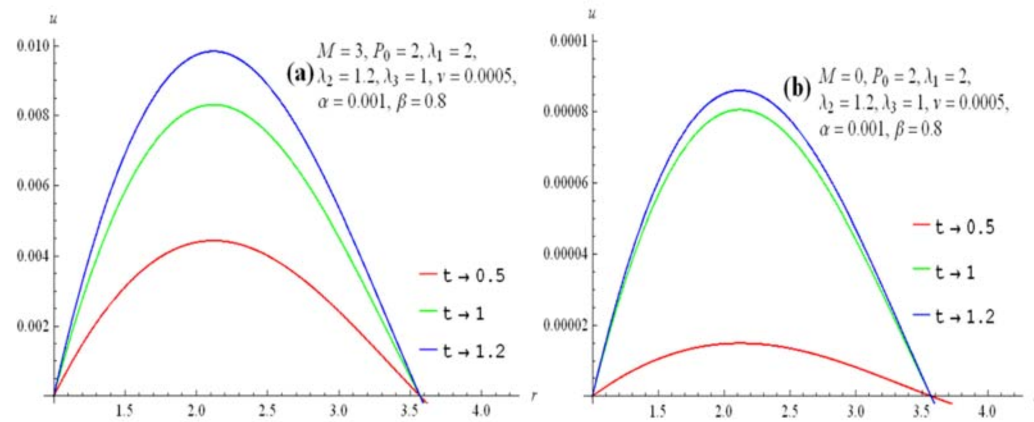

Figure (6): Shows velocity for various values of M while remaining another parameters constant (a) $t = 0.1$ and (b) $t = 0.5$

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