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Contractive Mappings Having Mixed Finite Monotone Property in Generalized Metric Spaces

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Abstract

The concepts of the modified tuple coincidence points and the mixed finite monotone property is introduced in this paper. Also, the existence and uniqueness of modified tupled coincidence point is discusses without continuous condition for mappings having mixed finite monotone property in generalized metric spaces.

Keywords: generalized metric spaces, fixed point, coincidence point, monotone property.

1. Introduction

In 2000, Dhage [1] introduced$D$–metric space as a generalization of metric space and he proved many results in this space but in 2005, Mustafa and Sims [2] proved that the results presented by Dhage are invalid in topological structure and hence they introduced G–metric space and as a generalized of metric space. On other Bhashkar and Lakshmikantham in [3] introduced the concept of coupled fixed point and proved the existence of a coupled fixed theorem in partially ordered complete metric space. In 2009, Lakshmikantham and Ciric [4] defined mixed g–monotone property and coupled coincidence point in partially ordered metric spaces, also in 2011, Berinde and Borcut [5] introduced the concept of triple fixed point and proved some results a round and in 2012, Berinde and Borcut [6] defined the concept of triple Coincidence point and established some triple Coincidence point theorems in partially ordered metric space. In this paper, we will give a mixed finite monotone property and modified tupled coincidence point with study the existence of modified tupled coincidence point in partially ordered generalized metric space.

2.Background

In this section, we recall some definitions and properties introduced by Mustafa and Simis [2]

Definition 1

Let X be a nonempty set, $\mathcal{G}: X \times X \times X \to R_{+}$be a function satisfying:

1. $\mathcal{G}(x,y,z) = 0 \;\; if \;\; x = y = z.$
2. $0 < \mathcal{G}(x,x,y)$ for all $x, y \in X \; with \;\; x \neq y.$

3. $\mathcal{G}(x,x,y) \leq \mathcal{G}(x,y,z)$ for all $x,y,z \in X$ $with\ y \neq z$
4. $\mathcal{G}(x,y,z) = \mathcal{G}(x,z,y) = G(y,z,x) =$ ....
(symmetry in all hree variable)
5. $\mathcal{G}(x,y,z) \leq \mathcal{G}(x,a,a) + \mathcal{G}(a,y,x,z) for\ all\ x,y,z,a \in X$.

Then the function $\mathcal{G}$ is called generalized metric and the pair $(\mathrm{X},\mathcal{G})$ is called a generalized metric space or more specially $\mathcal{G}\_$ metric space.

Definition 2

Let $(\mathrm{X},\mathcal{G})$ be a $\mathcal{G}\_$ metric space, and let (x_n) be a sequence of points of X.We say that (x_n) is $\mathcal{G}$_convergent to x if $\lim_{n,m\to\infty} \mathcal{G}(x,x_n,x_m) = 0$, that is, for any $\varepsilon > 0$ there exist $N \in \mathbb{N}$ such that
$\mathcal{G}(x,x_n,x_m) < \varepsilon$, for all $n,m \geq N$.We call x the limit of sequence and write $x_n \to x$ or $\lim_{n,\to\infty} x_n = x$.

Definition 3

Let $(\mathrm{X},\mathcal{G})$ be a $\mathcal{G}\_$ metric space, A sequence (x_n) is called $\mathcal{G}$_cauchy sequence if for any $\varepsilon > 0$ there exist $N \in \mathbb{N}$ such that $\mathcal{G}(x_n,x_m,x_l) < \varepsilon$
for all $n,m,l \geq N$, that is, $\mathcal{G}(x_n,x_m,x_l) \to 0\ as\ n,m,l \to \infty^+$.

Proposition 4

Let $(\mathrm{X},\mathcal{G})$ be a $G\_$ metric space. A mapping is called $\mathcal{G}$_continuous at $x \in X$ if and if it is $\mathcal{G}$_sequentially ontinuous at x, that is ,whenever (x_n) is $\mathcal{G}$_convergent to x, $(f(x_n))\ is\ \mathcal{G}$_convergent to $f(x)$.

Proposition 5

A $\mathcal{G}\_$ metric space $(\mathrm{X},\mathcal{G})$ is called G_complete if every $\mathcal{G}$_cauchy sequence is $\mathcal{G}$_convergent in $(\mathrm{X},\mathcal{G})$.

3. Main Results

In this section, the modification of tupled coincidence point is proposed as the follows:

Definition 6

Let $(X,\leq)$ be a partially ordered set. If $\mathcal{T}_n: X^n \to X$, $\mathcal{T}_1,\mathcal{T}_2, \ldots,\mathcal{T}_{n-1},\mathcal{U},\mathcal{V}: X \to X$ are there mappings. An element $(x_1,x_2,\ldots\ldots,x_n) \in X^n$ is called modified tupled coincidence point of $\mathcal{T}_1,\mathcal{T}_2,\ldots\ \mathcal{T}_n,\mathcal{U}$ and $\mathcal{V}$ if:

$$\mathcal{T}_1\left(\mathcal{T}_2\ \ldots\ \left(\mathcal{T}_n(x_1,x_2,\ldots\ldots,x_n)\right)\right) = \mathcal{U}\mathcal{V}(x_1)$$

$$\mathcal{T}_1\left(\mathcal{T}_2\ \ldots\ \left(\mathcal{T}_n(x_2,x_3,\ldots\ldots,x_n,x_1)\right)\right) = \mathcal{U}\mathcal{V}(x_2)$$

$$\vdots$$

$$\mathcal{T}_1\left(\mathcal{T}_2\ \ldots\ \left(\mathcal{T}_n(x_n,x_1,\ldots\ldots,x_{n-1})\right)\right) = \mathcal{U}\mathcal{V}(x_n)$$

Definition 7

Let $(X, \leq)$ be a partially ordered set. If $\mathcal{T}_n: X^n \to X$, $\mathcal{T}_1, \mathcal{T}_2, \ldots, \mathcal{T}_{n-1}, \mathcal{U}, \mathcal{V}: X \to X$ are there mappings we say that f have mixed finite monotone property if

$\mathcal{T}_1\left(\mathcal{T}_2 \ldots \left(\mathcal{T}_n(x_1, x_2, \ldots \ldots, x_n)\right)\right)$ is monotone finite increasing if n is odd, and

$\mathcal{T}_1\left(\mathcal{T}_2 \ldots \left(\mathcal{T}_n(x_1, x_2, \ldots \ldots, x_n)\right)\right)$ is monotone finite decreasing if n is even

That is, for each $x_1, x_2, \ldots \ldots, x_n \in X$

$$y_1, z_1 \in X, \quad \mathcal{UV}(y_1) \leq \mathcal{UV}(z_1) \Rightarrow \mathcal{T}_1\left(\mathcal{T}_2 \ldots \left(\mathcal{T}_n(y_1, x_2, x_3, \ldots, x_n)\right)\right) \leq \mathcal{T}_1\left(\mathcal{T}_2 \ldots \left(\mathcal{T}_n(z_1, x_2, x_3, \ldots, x_n)\right)\right)$$

$$y_2, z_2 \in X, \quad \mathcal{UV}(y_2) \leq \mathcal{UV}(z_2) \Rightarrow \mathcal{T}_1\left(\mathcal{T}_2 \ldots \left(\mathcal{T}_n(x_1, y_2, x_3, \ldots, x_n)\right)\right) \geq \mathcal{T}_1\left(\mathcal{T}_2 \ldots \left(\mathcal{T}_n(x_1, z_2, x_3, \ldots, x_n)\right)\right)$$

$$\vdots$$

$y_n, z_n \in X, \quad \mathcal{UV}(y_n) \leq \mathcal{UV}(z_n) \Rightarrow$

$$\mathcal{T}_1\left(\mathcal{T}_2 \ldots \left(\mathcal{T}_n(x_1, x_2, \ldots, y_n)\right)\right) \leq \mathcal{T}_1\left(\mathcal{T}_2 \ldots \left(\mathcal{T}_n(x_1, x_2, \ldots, z_n)\right)\right) \quad (if\ n\ is\ odd)$$

$$\mathcal{T}_1\left(\mathcal{T}_2 \ldots \left(\mathcal{T}_n(x_1, x_2, \ldots, y_n)\right)\right) \geq \mathcal{T}_1\left(\mathcal{T}_2 \ldots \left(\mathcal{T}_n(x_1, x_2, \ldots, z_n)\right)\right) \quad (if\ n\ is\ even)$$

Let

i. $\mathcal{A}$ is the set of all mappings $\mathcal{T}_n: X^n \to X$ and $\mathcal{T}_1, \mathcal{T}_2, \ldots, \mathcal{T}_{n-1}, \mathcal{U}$ and $\mathcal{V}: X \to X$ such that:
   1. $\mathcal{UV}(X)$ is complete subspace of X, containing $\mathcal{T}_1\left(\mathcal{T}_2 \ldots \left(\mathcal{T}_n(X^n)\right)\right)$
   2. $\mathcal{T}_1, \mathcal{T}_2, \ldots, \mathcal{T}_n, \mathcal{U}$ and $\mathcal{V}$ are continuous and commute mappings.
   3. $\mathcal{T}_1, \mathcal{T}_2, \ldots, \mathcal{T}_n$, have mixed finite monotone property.

ii. $\mathfrak{B}$ is the set of all mappings $\omega: [0, \infty) \to [0, \infty)$ increasing mapping Such that:
   1. $\omega(t) \leq t \quad \forall\, t > 0$
   2. $\omega(0) = 0$ *and* $\lim_{n \to 0} \omega^n(t) = 0$, where ω^n denotes the n the iterate of ω.

From the above definition, we show the following modification

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Theorem 8

Let $(X,\mathcal{G},\leq)$be a partially ordered generalized metric space, $\mathcal{T}_n: X^n \to X$ and $\mathcal{T}_1, \mathcal{T}_2, \dots, \mathcal{T}_{n-1}, \mathcal{U}$ and $\mathcal{V} : X \to X$ are mappings lies in $\mathcal{A}$ and hold that following conditions, $\forall\ x_1, x_2, \dots, x_n, y_1, y_2, \dots, y_n \in X \text{ and } \mathcal{L} > 0$

$$\mathcal{G}\left(\mathcal{T}_1\left(\mathcal{T}_2 \ \dots\ \left(\mathcal{T}_n(x_1, x_2, \dots, x_n)\right)\right), \mathcal{T}_1\left(\mathcal{T}_2\ \dots\ \left(\mathcal{T}_n(y_1, y_2, \dots y_n)\right)\right), \mathcal{L}\right) \leq w\left\{\max\left\{\begin{matrix}\omega_1 \mathcal{G}(\mathcal{UV}(x_1), \mathcal{UV}(y_1), \mathcal{L}), \omega_2 \mathcal{G}(\mathcal{UV}(x_2), \mathcal{UV}(y_2), \mathcal{L}), \dots, \\ \omega_n \mathcal{G}(\mathcal{UV}(x_n), \mathcal{UV}(y_n), \mathcal{L})\end{matrix}\right\}\right\} \quad (1)$$

Where is w upper semicontinuous from R^+ into itself satisfying $w(x) < x$ for all $x > 0$ .If

$$\mathcal{UV}({x_0}^1) \leq \mathcal{T}_1\mathcal{T}_2\ \dots\ \mathcal{T}_n({x_0}^1, {x_0}^2, \dots \dots, {x_0}^n)$$

$$\mathcal{UV}({x_0}^2) \geq \mathcal{T}_1\mathcal{T}_2\ \dots\ \mathcal{T}_n({x_0}^2, {x_0}^3, \dots \dots, {x_0}^n, {x_0}^1)$$

$$\vdots \qquad (2)$$

$$\mathcal{UV}({x_0}^n) \leq \mathcal{T}_1\mathcal{T}_2\ \dots\ \mathcal{T}_n({x_0}^n, {x_0}^1, {x_0}^2, \dots \dots, {x_0}^{n-1}) \quad \text{if } n \text{ is odd}$$

$$\mathcal{UV}({x_0}^n) \geq \mathcal{T}_1\mathcal{T}_2\ \dots\ \mathcal{T}_n({x_0}^n, {x_0}^1, {x_0}^2, \dots \dots, {x_0}^{n-1}) \quad \text{if } n \text{ is even}$$

Then $\mathcal{T}_1, \mathcal{T}_2, \dots, \mathcal{T}_n, \mathcal{U}\ and \mathcal{V}$ have a modified tupled coincidence point.

Proof

Define
$$\mathcal{UV}({x_1}^1) = \mathcal{T}_1\mathcal{T}_2\ \dots\ \mathcal{T}_n({x_0}^1, {x_0}^2, \dots \dots, {x_0}^n)$$

$$\mathcal{UV}({x_1}^2) = \mathcal{T}_1\mathcal{T}_2\ \dots\ \mathcal{T}_n({x_0}^2, {x_0}^3, \dots \dots, {x_0}^n, {x_0}^1)$$

$$\vdots$$

$$\mathcal{UV}({x_1}^n) = \mathcal{T}_1\mathcal{T}_2\ \dots\ \mathcal{T}_n({x_0}^n, {x_0}^1, {x_0}^2, \dots \dots, {x_0}^{n-1})$$

$$\Rightarrow \mathcal{UV}({x_0}^1) \leq \mathcal{UV}({x_1}^1)$$

$$\mathcal{UV}({x_0}^2) \geq \mathcal{UV}({x_1}^2)$$

$$\vdots$$

$$\mathcal{UV}({x_0}^n) \leq \mathcal{UV}({x_1}^n) \qquad \text{if } n \text{ is odd}$$

$$\mathcal{UV}({x_0}^n) \geq \mathcal{UV}({x_1}^n) \qquad \text{if } n \text{ is even}$$

Also, we define,
$$\mathcal{UV}({x_2}^1) = \mathcal{T}_1\mathcal{T}_2\ \dots\ \mathcal{T}_n({x_1}^1, {x_1}^2, \dots \dots, {x_1}^n)$$

$$\mathcal{UV}({x_2}^2) = \mathcal{T}_1\mathcal{T}_2\ \dots\ \mathcal{T}_n({x_1}^2, {x_1}^3, \dots \dots, {x_1}^n, {x_1}^1)$$

$$\vdots$$

$$\mathcal{UV}({x_2}^n) = \mathcal{T}_1\mathcal{T}_2 \ \dots \ \mathcal{T}_n({x_1}^n, {x_1}^1, \dots \dots, {x_1}^{n-1})$$

Since f has mixed finite monotone property

$$\mathcal{UV}({x_0}^1) \le \mathcal{UV}({x_1}^1) \le \mathcal{UV}({x_2}^1)$$

$$\mathcal{UV}({x_0}^2) \ge \mathcal{UV}({x_1}^2) \ge \mathcal{UV}({x_2}^2)$$

$$\vdots$$

$$\mathcal{UV}({x_0}^n) \le \mathcal{UV}({x_1}^n) \le \mathcal{UV}({x_2}^n) \quad \text{if } n \text{ is odd}$$

$$\mathcal{UV}({x_0}^n) \ge \mathcal{UV}({x_1}^n) \ge \mathcal{UV}({x_2}^n) \quad \text{if } n \text{ is even}$$

Continue process until we get to

$$\mathcal{UV}({x_0}^1) \le \mathcal{UV}({x_1}^1) \le \dots\dots \le \mathcal{UV}({x_k}^1)$$

$$\mathcal{UV}({x_0}^2) \ge \mathcal{UV}({x_1}^2) \ge \dots\dots \ge \mathcal{UV}({x_k}^2)$$

$$\vdots$$

$$\mathcal{UV}({x_0}^n) \le \mathcal{UV}({x_1}^n) \le \dots\dots \le \mathcal{UV}({x_k}^n) \quad \text{if } n \text{ is odd}$$

$$\mathcal{UV}({x_0}^n) \ge \mathcal{UV}({x_1}^n) \ge \dots\dots \ge \mathcal{UV}({x_k}^n) \quad \text{if } n \text{ is even}$$

In general:

$$\mathcal{UV}({x_k}^1) = \mathcal{T}_1\mathcal{T}_2 \ \dots \ \mathcal{T}_n({x_{k-1}}^1, {x_{k-1}}^2, \dots \dots, {x_{k-1}}^n)$$

$$\mathcal{UV}({x_k}^2) = \mathcal{T}_1\mathcal{T}_2 \ \dots \ \mathcal{T}_n({x_{k-1}}^2, {x_{k-1}}^3, \dots \dots, {x_{k-1}}^n, {x_{k-1}}^1)$$

$$\vdots$$

$$\mathcal{UV}({x_k}^n) = \mathcal{T}_1\mathcal{T}_2 \ \dots \ \mathcal{T}_n({x_{k-1}}^n, {x_{k-1}}^1, {x_{k-1}}^2, \dots \dots, {x_{k-1}}^{n-1})$$

And

$$\mathcal{UV}({x_0}^1) \le \mathcal{UV}({x_1}^1) \le \dots\dots \le \mathcal{UV}({x_{k-1}}^1) \le \mathcal{UV}({x_k}^1) \le \dots\dots\dots$$

$$\mathcal{UV}({x_0}^2) \ge \mathcal{UV}({x_1}^2) \ge \dots\dots \ge \mathcal{UV}({x_{k-1}}^2) \ge \mathcal{UV}({x_k}^2) \ge \dots\dots\dots$$

$$\vdots$$

$$\mathcal{UV}({x_0}^n) \le \mathcal{UV}({x_1}^n) \le \dots\dots \le \mathcal{UV}({x_{k-1}}^n) \le \mathcal{UV}({x_k}^n) \le \dots\dots \quad \text{if } n \text{ is odd}$$

$$\mathcal{UV}({x_0}^n) \ge \mathcal{UV}({x_1}^n) \ge \dots\dots \ge \mathcal{UV}({x_{k-1}}^n) \ge \mathcal{UV}({x_k}^n) \ge \dots\dots \quad \text{if } n \text{ is even}$$

Now, we have

$< \mathcal{UV}({x_k}^1) >, < \mathcal{UV}({x_k}^2) >, \dots\dots and \ < \mathcal{UV}({x_k}^n) >$ are sequence in $gT(X)$

$\mathcal{UV}({x_k}^1) \to r^1, \mathcal{UV}({x_k}^2) \to r^2, \dots\dots\dots, \mathcal{UV}({x_k}^n) \to r^n$

Since $r^1, r^2, \ldots \ldots, r^n \in \mathcal{UV}(X)$, then there exists $x^1, x^2, \ldots \ldots, x^n \in X$.

Such that, $r^1 = \mathcal{UV}(x^1), r^2 = \mathcal{UV}(x^2), \ldots \ldots., r^n = \mathcal{UV}(x^n)$

Considering the hypotheses (i) and (ii) give in the theorem we get

$$\mathcal{UV}({x_k}^1) \leq \mathcal{UV}(x^1) = r^1$$

$$\mathcal{UV}({x_k}^2) \geq \mathcal{UV}(x^2) = r^2$$

$$\vdots$$

$$\mathcal{UV}({x_k}^n) \leq \mathcal{UV}(x^n) = r^n \ \ (if\ n\ is\ odd)$$

$$\mathcal{UV}({x_k}^n) \geq \mathcal{UV}(x^n) = r^n \ \ (if\ n\ is\ even)$$

Since $\mathcal{U}$ and $\mathcal{V}$ are continuous mapping, then we have:

$$\mathcal{UV}(\mathcal{UV}({x_k}^1)) \to \mathcal{UV}(r^1)$$

$$\mathcal{UV}(\mathcal{UV}({x_k}^2)) \to \mathcal{UV}(r^2)$$

$$\vdots$$

$$\mathcal{UV}(\mathcal{UV}({x_k}^n)) \to \mathcal{UV}(r^n)$$

And hence, $\mathcal{UV}(\mathcal{UV}({x_k}^1)) \leq \mathcal{UV}(r^1)$

$$\mathcal{UV}(\mathcal{UV}({x_k}^2)) \geq \mathcal{UV}(r^2)$$

$$\vdots$$

$$\mathcal{UV}\big(\mathcal{UV}({x_k}^n)\big) \leq \mathcal{UV}(r^n) \ \ if\ n\ is\ odd$$

$$\mathcal{UV}\big(\mathcal{UV}({x_k}^n)\big) \geq \mathcal{UV}(r^n) \ \ if\ n\ is\ even$$

Choose $\mathcal{L}$ satisfy:

$$\mathcal{G}(\mathcal{UV}(r^1), \mathcal{T}_1\mathcal{T}_2 \ \ldots\ \mathcal{T}_n(r^1, r^2, \ldots \ldots, r^n), \mathcal{L}) \leq \mathcal{G}\left(\mathcal{T}_1\mathcal{T}_2 \ \ldots\ \mathcal{T}_n(r^1, r^2, \ldots \ldots, r^n), \mathcal{L}, \mathcal{UV}\big(\mathcal{UV}({x_{k+1}}^1)\big)\right)$$

$$= \mathcal{G}\big(\mathcal{T}_1\mathcal{T}_2 \ \ldots\ \mathcal{T}_n(r^1, r^2, \ldots \ldots, r^n), \mathcal{UV}\big(\mathcal{UV}({x_{k+1}}^1)\big), \mathcal{L}\big)$$

$$= \mathcal{G}\big(\mathcal{T}_1\mathcal{T}_2 \ \ldots\ \mathcal{T}_n(r^1, r^2, \ldots \ldots, r^n), \mathcal{T}_1\mathcal{T}_2 \ \ldots\ \mathcal{T}_n\big(\mathcal{UV}({x_k}^1), \mathcal{UV}({x_k}^2), \ldots \mathcal{UV}({x_k}^n)\big), \mathcal{L}\big)$$

$$\leq \mathcal{w}\left\{\max\left\{\begin{matrix}\omega_1\mathcal{G}(\mathcal{V}(r^1), \mathcal{UV}(\mathcal{UV}({x_k}^1)), \mathcal{L}), \omega_2\mathcal{G}(\mathcal{UV}(r^2), \mathcal{UV}(\mathcal{UV}({x_k}^2)), \mathcal{L}), \ldots, \\ \omega_n\mathcal{G}(\mathcal{UV}(r^n), \mathcal{UV}(\mathcal{UV}({x_k}^n)), \mathcal{L})\end{matrix}\right\}\right\}$$

$$\leq \mathcal{w}\left\{\max\left\{\begin{matrix}\mathcal{G}(\mathcal{UV}(r^1), (\mathcal{UV}({x_k}^1)), \mathcal{L}), \mathcal{G}(\mathcal{UV}(r^2), \mathcal{UV}(\mathcal{UV}({x_k}^2)), \mathcal{L}), \ldots, \\ \mathcal{G}(\mathcal{UV}(r^n), \mathcal{UV}(\mathcal{UV}({x_k}^n)), \mathcal{L})\end{matrix}\right\}\right\}$$

But, $\mathcal{UV}(\mathcal{UV}({x_k}^1)) \to \mathcal{UV}(r^1), \mathcal{UV}(\mathcal{UV}({x_k}^2)) \to \mathcal{UV}(r^2)$,……….and

$$\mathcal{UV}(\mathcal{UV}({x_k}^n)) \rightarrow \mathcal{UV}(r^n)$$

Which is implies, by definition $\mathcal{G}$-convergent in $\mathcal{G}$-metric space,

$$\mathcal{G}(\mathcal{UV}(r^1), \mathcal{T}_1\mathcal{T}_2 \ldots \mathcal{T}_n(r^1, r^2, \ldots\ldots, r^n), \mathcal{L}) = 0 \implies \mathcal{T}_1\mathcal{T}_2 \ldots \mathcal{T}_n(r^1, r^2, \ldots\ldots, r^n) = \mathcal{UV}(r^1)$$

Also, choose $\mathcal{L}^0$ satisfy:

$$\mathcal{G}(\mathcal{UV}(r^2), \mathcal{T}_1\mathcal{T}_2 \ldots \mathcal{T}_n(r^2, r^3, \ldots, r^1), \mathcal{L}^0) \leq \mathcal{G}\left(\mathcal{T}_1\mathcal{T}_2 \ldots \mathcal{T}_n(r^1, r^2, \ldots, r^n), \mathcal{L}^0, \mathcal{UV}(\mathcal{UV}({x_{k+1}}^2))\right)$$

$$= \mathcal{G}(\mathcal{T}_1\mathcal{T}_2 \ldots \mathcal{T}_n(r^2, r^3, \ldots, r^1), \mathcal{UV}(\mathcal{UV}({x_{k+1}}^2)), \mathcal{L}^0)$$

$$= \mathcal{G}(\mathcal{T}_1\mathcal{T}_2 \ldots \mathcal{T}_n(r^2, r^3, \ldots, r^1), \mathcal{T}_1\mathcal{T}_2 \ldots \mathcal{T}_n(\mathcal{UV}({x_k}^2), \mathcal{UV}({x_k}^3), \ldots, \mathcal{UV}({x_k}^1), \mathcal{L}^0))$$

$$\leq \mathcal{w}\left\{\max\left\{\begin{matrix}\omega_1\mathcal{G}(\mathcal{UV}(r^2), \mathcal{UV}(\mathcal{UV}({x_k}^2), \mathcal{L}^0)), \omega_2\mathcal{G}(\mathcal{UV}(r^3), \mathcal{UV}(\mathcal{UV}({x_k}^3), \mathcal{L}^0)), \ldots, \\ \omega_n\mathcal{G}(\mathcal{UV}(r^1), \mathcal{UV}(\mathcal{UV}({x_k}^1)), \mathcal{L}^0)\end{matrix}\right\}\right\}$$

$$\leq \mathcal{w}\left\{\max\left\{\begin{matrix}\mathcal{G}(\mathcal{UV}(r^2), \mathcal{UV}(\mathcal{UV}({x_k}^2), \mathcal{L}^0)), \mathcal{G}(\mathcal{UV}(r^3), \mathcal{UV}(\mathcal{UV}({x_k}^3)), \mathcal{L}^0), \ldots, \\ \mathcal{G}(\mathcal{UV}(r^1), \mathcal{UV}(\mathcal{UV}({x_k}^1)), \mathcal{L}^0)\end{matrix}\right\}\right\}$$

But, $\mathcal{UV}(\mathcal{UV}({x_k}^1)) \rightarrow \mathcal{UV}(r^1), \mathcal{UV}(\mathcal{UV}({x_k}^2)) \rightarrow \mathcal{UV}(r^2)$,……….. and

$$\mathcal{UV}(\mathcal{UV}({x_k}^n)) \rightarrow \mathcal{UV}(r^n)$$

Which is implies, by definition $\mathcal{G}$ -convergent in $\mathcal{G}$-metric space,

$$\mathcal{G}(\mathcal{UV}(r^2), \mathcal{T}_1\mathcal{T}_2 \ldots \mathcal{T}_n(r^1, r^2, \ldots\ldots, r^1), \mathcal{L}) = 0 \implies \mathcal{T}_1\mathcal{T}_2 \ldots \mathcal{T}_n(r^2, r^3, \ldots\ldots, r^1) = \mathcal{UV}(r^2)$$

Continue these processes

Choose $\mathcal{L}^*$ satisfy: $\mathcal{G}(\mathcal{UV}(r^n), \mathcal{T}_1\mathcal{T}_2 \ldots \mathcal{T}_n(r^n, r^1, \ldots\ldots, r^{n-1}), \mathcal{L}^*)$

$$\leq \mathcal{G}\left(\mathcal{T}_1\mathcal{T}_2 \ldots \mathcal{T}_n(r^n, r^1, \ldots\ldots, r^{n-1}), \mathcal{L}^*, \mathcal{UV}(\mathcal{UV}({x_{k+1}}^n))\right)$$

$$= \mathcal{G}(\mathcal{T}_1\mathcal{T}_2 \ldots \mathcal{T}_n(r^n, r^1, \ldots\ldots, r^{n-1}), \mathcal{UV}(\mathcal{UV}({x_{k+1}}^n)), \mathcal{L}^*)$$

$$= \mathcal{G}(\mathcal{T}_1\mathcal{T}_2 \ldots \mathcal{T}_n(r^n, r^1, \ldots\ldots, r^{n-1}), \mathcal{T}_1\mathcal{T}_2 \ldots \mathcal{T}_n(\mathcal{UV}({x_k}^n), \mathcal{UV}({x_k}^1), \ldots\ldots, \mathcal{UV}({x_k}^{n-1})), \mathcal{L}^*)$$

$$\leq \mathcal{w} \max\left\{\begin{matrix}\omega_1\mathcal{G}(\mathcal{UV}(r^n), \mathcal{UV}(\mathcal{UV}({x_k}^n)), \mathcal{L}^*), \omega_2\mathcal{G}(\mathcal{UV}(r^1), \mathcal{UV}(\mathcal{UV}({x_k}^1)), \mathcal{L}^*) \\ , \ldots\ldots, \omega_n\mathcal{G}(\mathcal{UV}(r^{n-1}), \mathcal{UV}(\mathcal{UV}({x_k}^{n-1})), \mathcal{L}^*)\end{matrix}\right\}$$

$$\leq \mathcal{w} \max\left\{\begin{matrix}\mathcal{G}(\mathcal{UV}(r^n), \mathcal{UV}(\mathcal{UV}({x_k}^n)), \mathcal{L}^*), \mathcal{G}(\mathcal{UV}(r^1), \mathcal{UV}(\mathcal{UV}({x_k}^1)), \mathcal{L}^*) \\ , \ldots\ldots, \mathcal{G}(\mathcal{UV}(r^{n-1}), \mathcal{UV}(\mathcal{UV}({x_k}^{n-1})), \mathcal{L}^*)\end{matrix}\right\}$$

But , $\mathcal{UV}(\mathcal{UV}({x_k}^1))$ is $\mathcal{G}$–convergent to $\mathcal{UV}(r^1)$

$$\mathcal{UV}\left(\mathcal{UV}({x_k}^2)\right) \text{ is } \mathcal{G}\text{–convergent to } \mathcal{UV}(r^2)$$

$$\vdots$$

$$\mathcal{UV}\left(\mathcal{UV}({x_k}^{n-1})\right) \text{ is } \mathcal{G}\text{–convergent to } \mathcal{UV}(r^{n-1})$$

$$\mathcal{UV}\left(\mathcal{UV}({x_k}^{n})\right) \text{ is } \mathcal{G}\text{–convergent to } \mathcal{UV}(r^{n})$$

Which is implies, by definition of $\mathcal{G}$–convergent in G–metric space,

$$\mathcal{G}(\mathcal{UV}(r^n), f(r^n, r^1, \dots \dots, r^{n-1}), \mathcal{L}^*) = 0$$

$$\Rightarrow \mathcal{T}_1\mathcal{T}_2 \dots \mathcal{T}_n(r^n, r^1, \dots \dots, r^{n-1}) = \mathcal{UV}(r^n)$$

So $(r^n, r^1, \dots \dots, r^{n-1})$ is a modified tupled coincidence point of $\mathcal{T}_1, \mathcal{T}_2, \dots, \mathcal{T}_n, \mathcal{U}$ and $\mathcal{V}$.

From theorem(8),we can get the following corollaries

Corollary 9

Let $(X, \mathcal{G}, \leq)$be a partially ordered generalized metric space .Under the same assumptions of theorem(8) but

$$\mathcal{G}\left(\mathcal{T}_1\left(\mathcal{T}_2 \quad \dots \quad \left(\mathcal{T}_n(x_1, x_2, \dots, x_n)\right)\right), \mathcal{T}_1\left(\mathcal{T}_2 \dots \left(\mathcal{T}_n(y_1, y_2, \dots y_n)\right)\right), \mathcal{L}\right) \leq w\left\{\frac{1}{n}\left\{\begin{matrix}\omega_{1\mathcal{G}}(\mathcal{UV}(x_1), \mathcal{UV}(y_1), \mathcal{L}) + \omega_2\mathcal{G}(\mathcal{UV}(x_2), \mathcal{UV}(y_2), \mathcal{L}) + \cdots \\ +\omega_n\mathcal{G}(\mathcal{UV}(x_n), \mathcal{UV}(y_n), \mathcal{L})\end{matrix}\right\}\right\}$$

Then $\mathcal{T}_1, \mathcal{T}_2, \dots, \mathcal{T}_n, \mathcal{U}$ and$\mathcal{V}$ have a modified tupled coincidence point.

Corollary 10

Let $(X, G, \leq)$be a partially ordered generalized metric space

$$\mathcal{G}\left(\mathcal{T}_1\left(\mathcal{T}_2 \quad \dots \quad \left(\mathcal{T}_n(x_1, x_2, \dots, x_n)\right)\right), \mathcal{T}_1\left(\mathcal{T}_2 \dots \left(\mathcal{T}_n(y_1, y_2, \dots y_n)\right)\right), \mathcal{L}\right) \leq w\left\{\frac{1}{n}\left\{\begin{matrix}k_1\mathcal{G}(\mathcal{UV}(x_1), \mathcal{UV}(y_1), \mathcal{L}) + k_2\mathcal{G}(\mathcal{UV}(x_2), \mathcal{UV}(y_2), \mathcal{L}) + \cdots \\ +k_n\mathcal{G}(\mathcal{UV}(x_n), \mathcal{UV}(y_n), \mathcal{L})\end{matrix}\right\}\right\}$$

,such that $k_i \in [0,1]$ $for\ all\ i = 1,2, \dots \dots, n$.Then $\mathcal{T}_1, \mathcal{T}_2, \dots, \mathcal{T}_n, \mathcal{U}$ and$\mathcal{V}$ have a modified tupled coincidence point.

Corollary 11

Let $(X, \mathcal{G}, \leq)$be a partially ordered generalized metric space .Under the same assumptions of theorem(8) but

$$\mathcal{G}\left(\mathcal{T}_1\left(\mathcal{T}_2 \quad \dots \quad \left(\mathcal{T}_n(x_1, x_2, \dots, x_n)\right)\right), \mathcal{T}_1\left(\mathcal{T}_2 \dots \left(\mathcal{T}_n(y_1, y_2, \dots y_n)\right)\right), \mathcal{L}\right) \leq w(\max(k_1\mathcal{G}(\mathcal{UV}(x_1), \mathcal{UV}(y_1), \mathcal{L}), k_2\mathcal{G}(\mathcal{UV}(x_2), \mathcal{UV}(y_2), \mathcal{L}), \dots, k_n\mathcal{G}(\mathcal{UV}(x_n), \mathcal{UV}(y_n), \mathcal{L})\ )$$

such that $k_i \in (0,1]$ $for\ all\ i = 1,2, \dots \dots, n$.Then $\mathcal{T}_1, \mathcal{T}_2, \dots, \mathcal{T}_n, \mathcal{U}\ and \mathcal{V}$ have a modified tupled tupled coincidence point.

Corollary 12

Let $(X, \mathcal{G}, \leq)$be a partially ordered generalized metric space .Under the same assumptions of theorem(8) but

$$\mathcal{G}\left(\mathcal{T}_1\left(\mathcal{T}_2 \quad \dots \quad \left(\mathcal{T}_n(x_1, x_2, \dots, x_n)\right)\right), \mathcal{T}_1\left(\mathcal{T}_2 \dots \left(\mathcal{T}_n(y_1, y_2, \dots y_n)\right)\right), \mathcal{L}\right) \leq \frac{\omega_1 \mathcal{G}(\mathcal{UV}(x_1), \mathcal{UV}(y_1)\mathcal{L}) + \omega_2 \mathcal{G}(\mathcal{UV}(x_2), \mathcal{UV}(y_2), \mathcal{L}) + \dots + \omega_n(\mathcal{UV}(x_n), \mathcal{UV}(y_n), \mathcal{L})}{n}.$$

Then $\mathcal{T}_1, \mathcal{T}_2, \dots, \mathcal{T}_n, \mathcal{U}\ and \mathcal{V}$ have a modified tupled coincidence point .

4.Conclusion

The new concepts of modified tupled coincidence points and mixed finite monotone property are introduced. Also, we established some modified tupled coincidence theorems in partially ordered generalized metric space.

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