Study the Shapes of Nuclei for Heavy Elements with Mass Number Equal to (226≤A≤252) Through Determination of Deformation Parameters for two Elements (U&Cf)

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Abstract
The current paper focuses on the studying the forms of (even-even) nuclei for the heavy elements with mass numbers in the range from (A=226 - 252) for ($^{226-240}_{92}U$ and $^{244-252}_{98}Cf$) isotopes. This work will consist of studying deformation parameters (β_2) which is deduced from the "Reduced Electric Transition Probability" $B(E2)\uparrow$ which is in its turn dependent on the first Excited State (2^+). The "Intrinsic Electric Quadrupole Moments" (non-spherical charge distribution) (Q_0) were also calculated. In addition to that the Roots Mean Square Radii (Isotope Shift) $< r^2 >^{1/2}$ are accounted for in order to compare them with the theoretical results.

The difference and variation in shapes of nuclei for the selected isotopes were detected using 3D-plots for them (with symmetric axes); a 2D-plot were also used for each isotope to discriminate between them by the values of semi-major is equal (a) axes and semi minor is equal (b) axes.

Keyword: Nuclear Shape, Isotopes, Electric Transition, Semi minor, Deformation.

1. Introduction

The state of the atomic of nucleus usually reflects the structure of the protons and neutrons shells from which it is shaped. In the case when the shells are completely filled, we attain a "magic" nucleus, which is spherical in form. Most nuclei have a tendency to be deformation on the basis that the shells are partially filled. Much of the shapes we encountered are either elongated (prolate) or flattened (oblate). These shapes can be modified between adjacent nucleus by capturing or give-away a proton or neutron. In some cases, it is sufficient re-configure the protons or neutrons within the same nucleus to change its shape. The same nucleus can therefore assume different shapes corresponding to states of different energy.

2.Theoretical

2.1. Nuclear Shape

In the stable state, the natural shape of nuclei is spherical. this configuration is the optimum shape to minimize the surface energy. Nevertheless, some small deformations can be

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observed, for instance, in area$150 < A < 190$. The shape irregularities can be presented using the ratio [1]:

$$\delta = \frac{\Delta R}{R} \quad (1)$$

R Is radius average of the nu

ΔR is the differences between semi- minor and semi- major axes.

$$\Delta R = (b - a) \quad (2)$$

But the sphere ΔR is equal zero.

2.2. Nuclear Surface Deformations

The collective motion can be clarified as vibrations and rotations of nuclear surface of the (collective model) that was initially suggested by Bohr and Mottelson [2].

The quadrupole deformation parameter β_2 is related to the spheroid axes [3]:

$$\beta_2 = \frac{4}{3}\sqrt{\frac{\pi}{5}}\frac{\Delta R}{R_{av}} = 1.06\frac{\Delta R}{R_{av}} \quad (3)$$

Where: The average radius$R_{av} = R_0 A^{1/3}$. ΔR: is the difference between both of the semi-major and minor axes. As long as the value of β_2 is larger, the nucleus becomes more deformed.

2.3. The Root Mean Square Charge Radius (Isotopes Shift)

The stated efficacy of nuclear structure, (e.g. shell closures and initiation of deformity), can be referred to by one key nuclear information which can be represented by the root mean square (rms) of nuclear charge radius $R = \langle r^2 \rangle^{1/2}$ with one of nuclear ground-state characteristics [4]. The root mean square (rms) of nuclear charge radius $R = \langle r^2 \rangle^{1/2}$, with one of nuclear ground-state properties, is considered the key nuclear materials information which refers to stated nuclear structure effectiveness, for instance: shell closures and a deformation starting. [4].This (rms) radius $\langle r^2 \rangle^{1/2}$ can be directly driven from scattered electrons distribution; for a uniformly charged spherical shape, the radius of square charge distribution is $\langle r^2 \rangle$ [5]:

$$\langle r^2 \rangle = \frac{3}{5}R^2 = \frac{3}{5}R_0^2 A^{2/3} > 100 \quad (4)$$

Where: A is mass number, R is the radius of the sphere, and $R = R_0 A^{1/3}$.

2.4. Electric Quadrupole Moment

The electric multi-pole moments can fairly represent the spatial distribution and charge allocation in a nucleus in a straight forward application of classical electrostatic principles [6]. Constant quadruple moments can be measured experimentally for a number of nuclei. A symmetrical axis oval shape can be anticipated for these nuclei. This classic configuration has guided the way for the definition of intrinsic quadruple moment as in the Equation (5) [7]:

$$Q_0 = \int d^3\rho(r)(3z^2 - r^2) \quad (5)$$

Where: $\rho(r)$ is proton's density of radial charge and r radius of charge, Q_0 can be described as per Equation (6), given that it is calculated for a homogeneously charged ellipsoid, with a charge Ze and semi-axis (a) and (b) with later pointing along the (z) axis [8].

$$Q_0 = \frac{2}{5}Z(a^2 - b^2) \quad (6)$$

If the deviation from sphericity is not very large, the average radius: $R = 1/2\,(a+b)$ and $\Delta R = (b - a)$ from Equation (2) can be presented and with $\delta = \Delta R/R$, from Equation (1), the quadrupole moment is [8]:

$$Q_0 = \frac{4}{5}ZR^2\delta \quad (7)$$

The nucleus quadrupole distortion parameter values δ calculated from the Equation (8) [9]:

$$\delta = 0.75\, Q_0/(Z\langle r^2\rangle) \quad (8)$$

The semi-axes $(a)$and (b) are gained from the two following Equations (9 & 10) [10].

$$a = \sqrt{\langle r^2\rangle(1.66 - \frac{2\delta}{0.9})} \quad (9)$$

$$b = \sqrt{5\langle r^2\rangle - 2a^2} \quad (10)$$

2.5. Quadrupole Deformations

As a rule of thumb, nuclei with charge value Z or N deviating from the magic number are usually deformed. The most abundant type of deformation is in quadrupole. Accordingly, the nuclei shape may either be prolate or oblate where the quadrupole deformation has one symmetry axis (z) as shown in **Figure 1**. [11].

The Shape of nuclei deformation can be symmetrical, this can be explained by deformation factor β_2 which in turn associated to quadrupole moment Q_0 , this particular case which indicates a homogeneous distribution of charge [11,12]:

$$\beta_2 = \frac{\sqrt{5\pi}}{3}\frac{Q_0}{ZR_0^2} \quad (11)$$

Where: (Z) is the atomic number, $R_0 = 1.2 \times A^{1/3}$fm. And (β_2) is the deformation parameter and$(\beta_2 < 1)$.

2.6. The Reduced Electric quadrupole Transition probability $(E2)\uparrow$

In isotopes, the conversion between different nuclear states through radioactive electromagnetic transformation is the ideal way attains stable nuclear structure, and an opportunity to study different structure models [14]. The transmission of $B(E2)$ act as a decisive part in determining; the average lifetime of a nuclear state and nuclear deformation β_2. It is also responsible for the volume of essential electric quadrupole moment and the energy levels of low-lying nuclei. Highest moments and transmission forces of quadrupole indicate the participation of many nucleons in the combined effects [15]. In this case, the probability of reduced electric quadrupole transition B (E2) ↑ from the spin 0^+ ground state to the first excited spin 2^+ state is specified by [16]:

$$B(E2:0^+ \rightarrow 2^+) = \frac{5}{16\pi} e^2 Q_0^2 \quad (12)$$

Where: $B(E2)\uparrow$ is the reduced electric quadrupole transition probability in the unit of (e^2b^2) and Q_0 is the intrinsic quadrupole moment in unit of barn (b).

The values of $B(E2)\uparrow$ are required as an experimental measures independent on nuclear model used. Nevertheless, if the model in hand believed to be dependent on the measured quantity is very useful and it represents the deformation parameter (β_2). If charge distribution is thought to be uniform reaching up to the distance $R(\theta,\varphi)$ and charge value at zero is below (β_2) can be associated with $B(E2)\uparrow$ through the Equation (13) [17]:

$$\beta_2 = (4\pi\,/\,3ZR_0^2)[B(E2)\uparrow/e^2]^{\frac{1}{2}} \quad (13)$$

$$R_0{}^2 = \left(1.2 \times A^{\frac{1}{3}}\mathrm{fm}\right)^2 = 0.0144A^{2/3}b \quad (14)$$

In accordance with the global system, the energy acknowledgement E (KeV) of the 2^+ state is whole that is required of creating a prediction for the corresponding $B(E2)\uparrow(e^2b^2)$ value [15]:

$$B(E2)\uparrow = 2.6 \times E^{-1}Z^2A^{-\frac{2}{3}} \quad (15)$$

3.Result

3.1. Deformity Parameters($\boldsymbol{\beta_2}$)

The deformations factor for the Uranium and Californium isotopes β_2 can be derived from $B(E2)\uparrow$ (Miniature Electric Transition Probability) of even-even nucleus by the application of Equation (13).

- Miniature Electric Transition Probability $B(E2)\uparrow: 0^+ \rightarrow 2^+$ from the ground 0^+ to the first excited of (2^+) states calculated by using Equation (15). The energy E(KeV) of the first excited states (2^+) was obtained from the references (18).
- Average Nuclear Radius R_0^2 calculated using Equation (14).
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3.2. Deformity Parameters: ($\boldsymbol{\delta}$)

Another methodology to calculate deformation parameters (δ) is available by utilizing the actual quadrupole moments Q_0 in Equation (8). To assess this method, the following variables need to available:

- $\langle r^2 \rangle$ which is calculated from Equation (4) for $A > 100$.
- Q_0 of nuclei were calculated from the Equation (11).

All these values were classify in Tables 1 and 2.

The major and minor axes (a and b) were calculated using Equations (9 and 10), respectively, The difference between them ΔR was calculated using Equations (1 - 3), respectively. The results are shown in **Tables 3. and 4**.

Table 1. Mass Number of Isotopes (A) , Neutron Number (N), Gamma Energy of the First Excited State $2^{+}(E_{\gamma})$, Nuclear Average Radius (R_0^2), Reduced Electric Transition Probability B(E2)↑ in unit of e^2b^2, Intrinsic Quadrupole Moment (Q_o) in unit of barn, and Deformation Parameters (β_2, δ) for $_{92}$U Isotopes.

(Z)	(A)	(N)	E_{γ}(KeV) [19]	Theoretical Value		Present Work				
				B(E2)↑ e^2b^2 For SSANM [32]	**β_2**	**R_0^2**	**B(E2)↑ e^2b^2**	**Q_o (b)**	**β_2**	**δ**
92	226	134	80.5 10	7.161	0.2280	0.5343	7.3680	8.6064	0.2313	0.2084
	228	136	59 10	7.966	0.2391	0.5374	9.9940	10.0235	0.2678	0.2413
	230	138	51.72 4	8.793	0.2498	0.5406	11.3346	10.6746	0.2836	0.2555
	232	140	47.522 7	9.580	0.2592	0.5437	12.2520	11.0982	0.2931	0.2641
	234	142	43.498 1	10.310	0.2674	0.5468	13.3230	11.5731	0.3039	0.2739
	236	144	45.242 3	10.880	0.2731	0.5499	12.7370	11.3157	0.2955	0.2663
	238	146	44.91 3	11.210	0.2756	0.5530	12.7591	11.3256	0.2941	0.2650
	240	148	45 1	11.490	0.2775	0.5561	12.6628	11.2827	0.2913	0.2625

Table 2. Mass Number of Isotopes (A) , Neutron Number (N), Gamma Energy of the First Excited State $2^{+}(E_{\gamma})$, Nuclear Average Radius (R_0^2), Reduced Electric Transition Probability B(E2)↑ in unit of e^2b^2, Intrinsic Quadrupole Moment (Q_o) in unit of barn, and Deformation Parameters (β_2, δ) for$_{98}$Cf Isotopes.

(Z)	(A)	(N)	E_{γ}(KeV) [19]	Theoretical Value		Present Work				
				B(E2)↑ e^2b^2 For SSANM [32]	**β_2**	**R_0^2**	**B(E2)↑ e^2b^2**	**Q_o (b)**	**β_2**	**δ**
98	244	146	40 2	14.970	0.2941	0.5623	15.9872	12.6776	0.3039	0.2739
	248	150	41.53 6	15.510	0.2961	0.5684	15.2322	12.3746	0.2935	0.2645
	250	152	42.722	15.680	0.2962	0.5715	14.7281	12.1681	0.2870	0.2587
	252	154	45.72 5	15.840	0.2961	0.5745	13.6894	11.7312	0.2753	0.2481

Table 3. Mass number (A), Neutron Number (N), Root Mean Square Radii $< r^2 >^{1/2}$, Major and minor axes (a, b) and the difference between them (Δ R) by three methods for ($_{92}$U) Isotopes

(Z)	(A)	(N)	*Theoretical Value*	*Present Work*					
			$\langle r^2\rangle^{1/2}$ fm [18]	$\langle r^2\rangle^{1/2}fm$	$a\ (fm)$	$b\ (fm)$	ΔR_1 (fm)	ΔR_2 (fm)	ΔR_3 (fm)
92	226	134	-------	5.8017	2.6424	3.8787	1.3585	1.2363	1.5951
	228	136	-------	5.8188	2.5646	3.9924	1.5776	1.4278	1.8523
	230	138	-------	5.8357	2.5323	4.0440	1.6752	1.5117	1.9669
	232	140	-------	5.8526	2.5138	4.0774	1.7366	1.5636	2.0390
	234	142	5.8291	5.8694	2.4920	4.1142	1.8058	1.6222	2.1202
	236	144	5.8431	5.8860	2.5154	4.0958	1.7606	1.5804	2.0672
	238	146	5.8571	5.9026	2.5222	4.0975	1.7572	1.5753	2.0632
	240	148	-------	5.9191	2.5322	4.0953	1.7457	1.5631	2.0496

Table 4. (A) Mass number, (N) Neutron Number, $< r^2 >^{1/2}$ Root Mean Square Radii, (a, b) Major and minor axes and the difference between them (Δ R) by three methods for ($_{98}$Cf) Isotopes.

(Z)	(A)	(N)	Theoretical Value	Present Work					
			$\langle r^2\rangle^{1/2}$ fm [18]	$\langle r^2\rangle^{1/2}$ fm	a (fm)	b (fm)	Δ R_1 (fm)	Δ R_2 (fm)	Δ R_3 (fm)
98	244	146	----	5.9518	2.5094	4.1430	1.8313	1.6336	2.1501
	248	150	----	5.9841	2.5410	4.1240	1.7779	1.5830	2.0874
	250	152	----	6.0002	2.5596	4.1107	1.7435	1.5512	2.0471
	252	154	----	6.0161	2.5905	4.0816	1.6765	1.4911	1.9683

4. Discussion

The present study focuses on nuclei characterized by even-even form for the heavy elements with mass numbers equal to $(226 \leq A \leq 252)$, which were included in deformation parameters study $(\beta_2 \& \delta)$ derived from the $B(E2)\uparrow$ and Q_0 values.

It is found that the first excited state energy levels 2^+ of these nuclei (which show a collective behavior), begin to change smoothly when the mass number A increases, and the nucleons outside the core polarize either the whole or partial vibrations of the core to one direction and permanent deformation of the nucleus can be acquired.

Also from evident$\langle r^2\rangle^{1/2}$, it was noticed that the increase in $\langle r^2\rangle^{1/2}$ values is comparable with (A) increase. For evaluation reasons, it was noticed that the the calculated values of present work $\langle r^2\rangle^{1/2}$ nicely correlated with experimental values of $\langle r^2\rangle^{1/2}$ in Ref [18]. Also, the values of (ΔR) were calculated using three different methods, and it was found that these results were fairly close.

4.1. Uranium isotopes$^{226-240}_{92}U$

In Table 1 [19], stated that the minimum value of $(\beta_2 = 0.2313)$, corresponding to the energy of the first excited state $(E_\gamma = 80.5 \text{ MeV})$ for (U-226), and the highest is $(\beta_2 = 0.3039)$ corresponding to the of the first excited state energy [20] $(E_\gamma = 43.498 \text{ KeV})$ for (U-234). Other values are ranging between these values as shown in the **Figure 1**. It is also noted that these values of (β_2) are considered large values, which means large deformed shapes as shown in **Figures (3. & 5.)**, corresponds small values of the first excited state energies (the gaps between shells are low spaces), also the number of nucleons in the sub-shell outside closed shell are filled with many nucleons and the collective motion of these nucleons will be rotational motion. On the other hand, and (Q_0) are significant.

4.2.Californium isotopes$^{244-252}_{98}Cf$

Table 2. shows that there are four isotopes chosen form this element, the highest value of the deformation parameter belongs to $(^{244}_{98}Cf)$and equals to $(\beta_2 = 0.3039)$, corresponding to the first excited state energy $(E_\gamma = 40$ KeV).And the lowest value of the deformation parameter belongs to $(^{252}_{98}Cf)$ equals to $(\beta_2 = 0.2753)$ corresponds to the energy of the first excited state $(E_\gamma = 45.72$ KeV). **Figure 2.** shows this behavior. Also we note that the number of nucleons $(Z = 98, 244 \leq N \leq 252)$ that fill the shells outside closed shell are large, the energy values of the first excited state of the selected Californium isotopes are considered very small if compared with the energy of the closed shell (the distances between the ground

state and the first excited state are small so that the transition of the nucleons between these two states is very easy). Therefore, the values of the reduced electrical transition will be large. In addition to that, the values of intrinsic electric Quadrupole moments are large too. All these factors work to make the deformation parameters large, which in turn make the nuclei of these isotopes permanently deformed, and elliptical shapes as shown in **Figure 4.** In addition, the collective motion of nucleons in the external orbits is rotational motion.

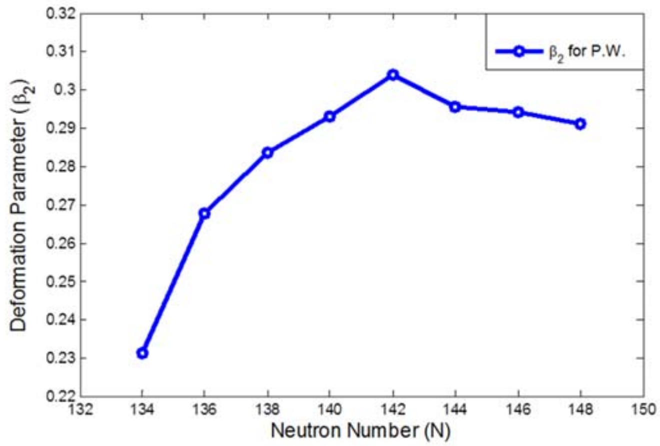

Figure 1. Deformation Parameter (β_2) value of the ($_{92}$U) Isotopes as function of neutron Number.

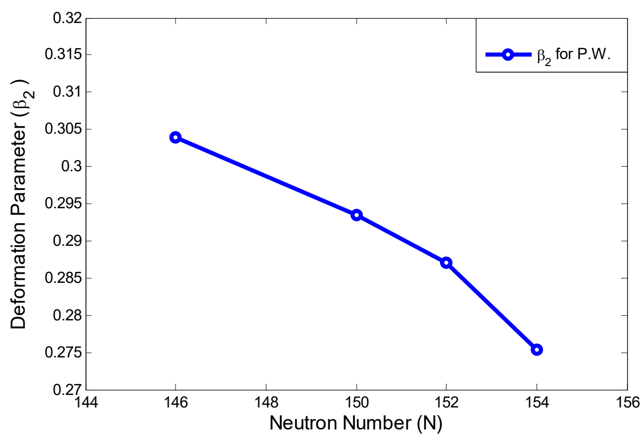

Figure 2. Deformation Parameter (β_2) value of the ($_{98}$Cf) Isotopes as function of neutron Number.

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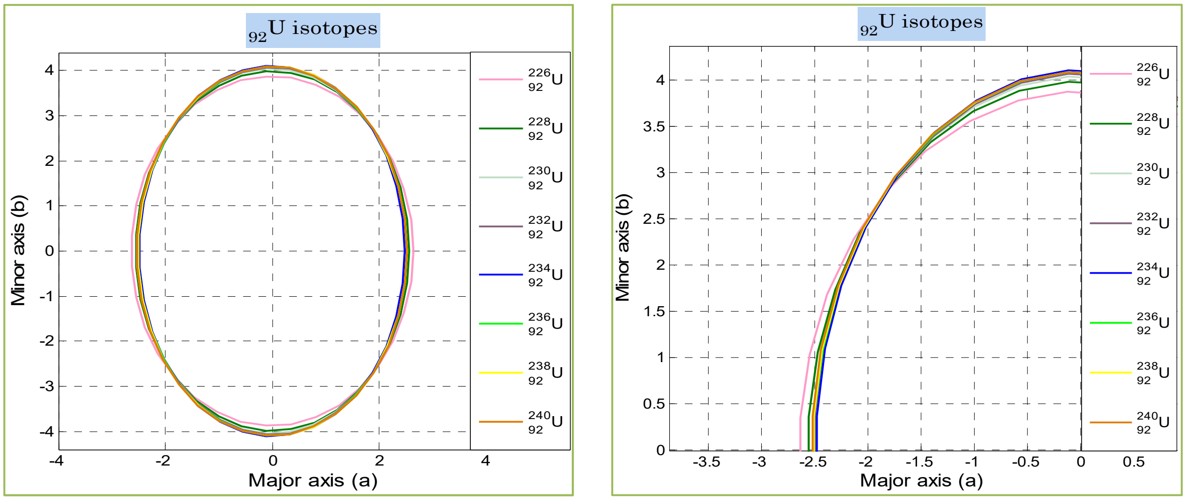

Figure 3. Shapes of axially symmetric quadrupole deformation for $_{92}U$isotope from major (a) and minor (b) axes.

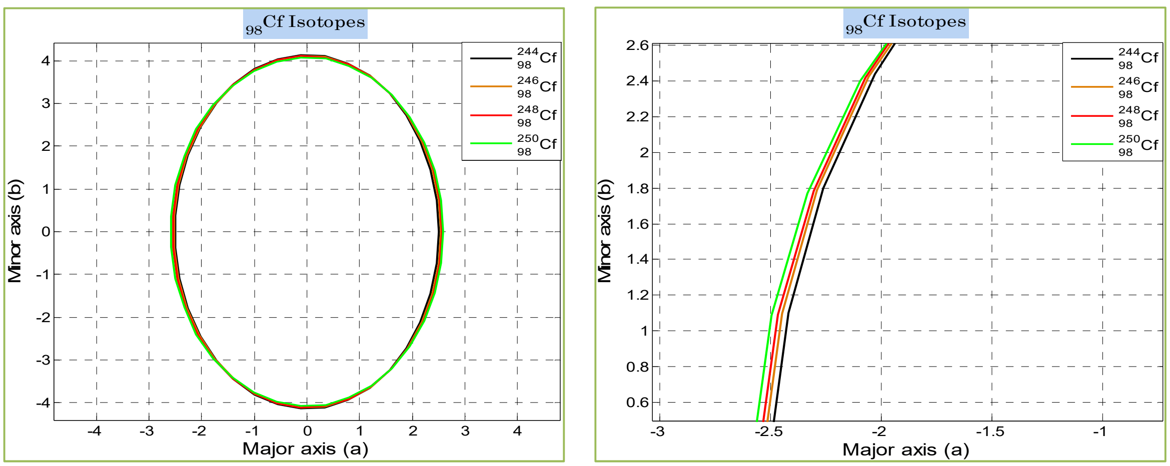

Figure 4. Shapes of axially symmetric quadrupole deformation for $_{98}Cf$ isotope from major (a) and minor (b) axes.

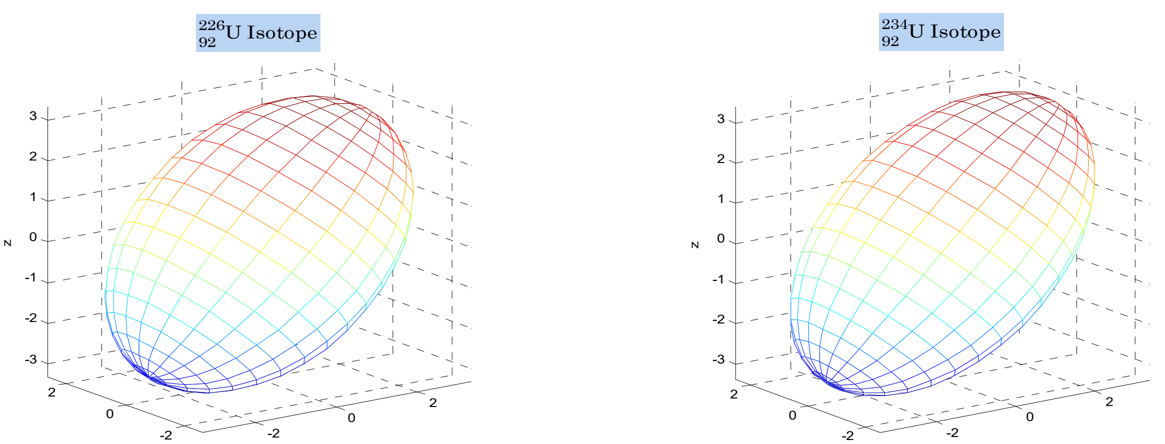

Figure 5. Axially-symmetric quadrupole deformations, the prolate shape of Nucleus with $(x = y < z)$ (where z is the minor axis (b) (symmetric axis) and (x, y) are major axes (a) for the U isotopes.

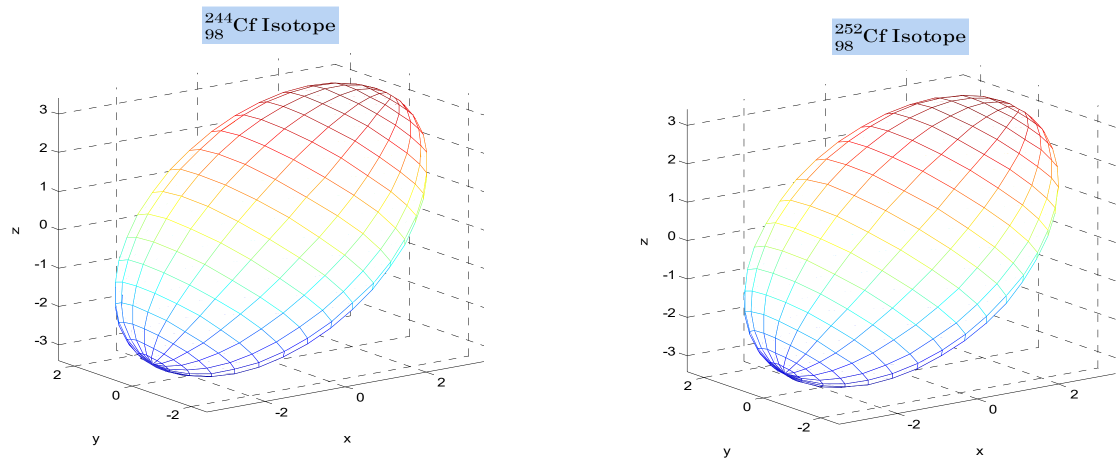

Figure 6. Axially-symmetric quadrupole deformations, the prolate shape of Nucleus with $(x = y < z)$ (where z is the minor axis (b) (symmetric axis) and (x, y) are major axes (a) for Cf isotopes.

5. Conclusions

From the results and the Figures of 2D and 3D shapes of the nuclei we conclude the following:

- It is found that the energy of the first excited state (2^{+}) of these nuclei begin to change smoothly when the mass number A increases, and the nucleons outside the core polarize either the whole or partial vibrations of the core to one direction and permanent deformation of the nucleus can be acquired and the nuclei are stable and non-spherical shape.
- Far from magic numbers, the motion of the nucleons out of closed shell will be rotational motion and the nucleus will be more distorted especially in the region $(242 \leqslant A \leqslant 252)$.
- The prolate shape is the dominant form of deformed nuclei.

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