Soft Strongly Generalized Mapping With Respect to an Ideal in Soft Topological Space

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Abstract

In this work, we introduced and studied a new kind of soft mapping on soft topological spaces with an ideal, which we called soft strongly generalized mapping with respect an ideal I, we studied the concepts like SSIg-continuous, Contra-SSIg-continuous, SSIg-open, SSIg-closed and SSIg-irresolute mapping and the relations between these kinds of mappings and the composition of two mappings of the same type of two different types, with proofs or counter examples.

Key words: SSIg-continuous, contra-SSIg-continuous, SSIg-irresolute, SSIg-closed mapping, SSIg-open mapping.

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Introduction

The concept of soft sets was introduced, for the first time, by Molodtstov in (1999) , as a generalization of fuzzy sets, soft sets are used as a tool to ideal with uncertain objects. Recently, in (2013), the study of soft topological spaces was introduced by D. N. Georgiou, A. C. Megaritis, they used the concept of soft set to define a topology ,that leads to a new world in general topology, the study soft strongly generalized closed set with respect to an ideal in soft topological space, and is denoted by SSIg-closed set, introduced by Alyasaa J. and Narjis A. in [1]. We gave the definitions of SSIg-continuous mapping, SSIg-open, SSIg-closed, SSIg-irresolute, Contra-SSIg-continuous mapping. The composition of these mappings are also discussed.

1- Preliminary concepts and results

Definition (1.1)[2]: For $A \subseteq E$, the pair (F,*A*) is called a soft set over X, where *F* is a mapping given by $F:A \rightarrow P(X)$.

In other words, the soft set is a parametrized family of subsets of the set X. Every set $F(e),\ e \in E$, from this family may be considered as the set of e-elements of the soft set (*F,E*) , or as the set of e-approximate elements of the soft set. Clearly, a soft set is not a set.

Note(1.2): In what follows by *SS(X,E)* we denote the family of all soft sets over X.

Definition(1.3)[1]: For two soft sets *(F,A)* and *(G,B)* in *SS(X,E)* , we say that *(F, A)* is a soft subset of *(G, B)* if $A \subseteq B$ and $F(e) \subseteq G(e),\ \forall e \in A$ **.**

Definition (1.4) [1]: The union of two soft sets $(F,\ A)$ and (G, B) over the common universe X is the soft set (H, C), where $C = A \cup B$ and for all $e \in C$, $H(e) = F(e) \cup G(e)$.

Definition(1.5)[2]: The intersection of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \cap B$ and for all $e \in C$, $H(e) = F(e) \cap G(e)$.

Definition(1.6) [3]: Let (F,E) be a soft set over X and $x \in X$. We say that $x \tilde{\in} (F,E)$ whenever $x \in F(\alpha)$ for all $\alpha \in E$. Note that for $x \in X$, $x \tilde{\notin} (F,E)$ if $x \notin F(\alpha)$ for some $\alpha \in E$.

Definition(1.7)[3]: A soft set (F, A) over X is said to be a null soft set, denoted by ϕ_A, if for all $e \in A$, $F(e) = \phi$ (null set), where $\phi_A(e) = \phi \quad \forall e \in A$.

Definition(1.8) [3]: A soft set (F, A) over X is said to be an absolute soft set, denoted by X_A, if for all $e \in A$, $F(e) = X$. Clearly, we have $X_A{}^c = \phi_A$ and $\phi_A{}^c = X_A$.

Definition(1.9) [3]: Let τ be a collection of soft sets over X with the fixed set E of parameters , then $\tau \subseteq SS(X,E)$. We say that the family τ defines a soft topology on *X* if the following axioms are true :

1- X_A , $\phi_A \in \tau$,
2- If (G,*A*) , (H,*A*) $\in \tau$, then $(G,A) \tilde{\cap} (H,A) \in \tau$,
3- If $(G_i, A) \in \tau$ for every $i \in \Lambda$, then $\tilde{\bigcup}_{i \in \Lambda} (G_i, A) \in \tau$.

Then τ is called a soft topology on X and the triple (X,τ,E) is called soft topological spaces over X .

Definition(1.10) [4]: Let E be a set of parameters, A nonempty collections I of soft subsets over X is called a soft ideal on X if the following holds
(1) If $(F,A)\in I$ and $(G,B)\tilde{\subseteq}(F,A)$ implies $(G,B)\in I$ (heredity),
(2) If (F,A) and $(G,A)\in I$, then $(F,A)\tilde{\cup}(G,A)\in I$ (additivity).
If I is ideal on X and Y is subset of X, then $I_Y = \{Y_E \tilde{\cap} I_i : I_i \in I, i\in\Lambda\}$ is an ideal on Y.

Definition(1.11) [4] : Let (X,τ,E) be a soft topological space with an ideal I . A soft set $(F,E)\in SS(X,E)$ is called soft generalized closed set with respect to an ideal I (soft Ig-closed) if $cl(F,E)-(G,E)\in I$ whenever $(F,E)\tilde{\subseteq}(G,E)$ and $(G,E)\in\tau$. The relative complement $(F,E)^c$ is called soft generalized open set with respect to an ideal I (soft Ig-open).

Definition(1.12) [2]: Let (X,τ,E) be a soft topological space with an ideal I. A soft subset (A,E) of (X,τ,E) is said to be soft strongly generalized closed set with respect to an ideal I ,(briefly SSIg- closed), if $cl(int(A,E))-(B,E)\in I$ whenever $(A,E)\tilde{\subseteq}(B,E)$ and (B,E) is soft open set. the relative complement $(F,E)^c$ is soft strongly generalized open set with respect to an ideal I,(briefly SSIg- closed).

Proposition(1.13) [1]: Let (X,τ,E) be a soft topological space with an ideal I. Then every soft closed set is an SSIg-closed set.

Corollary(1.14) [1]: Let (X,τ,E) be a soft topological space with an ideal I. Then every soft open set is an SSIg-open set.

Proposition(1.15) [1]: Every soft g-closed set is a soft strongly generalized closed set with respect to an ideal I .

Corollary(1.16) [1]: Every soft g-open set is a soft strongly generalized open set with respect to an ideal I .

Theorem(1.17) [1]: Every soft Ig- closed set is a soft strongly generalized closed set with respect to a soft ideal I .

Definition(1.18) [1]: Let (A,E) be a soft set in a soft topological space (X,τ,E) with an ideal I, Then the interior of (A,E) is the union of all SSIg-open sets which are contained in (A,E). and denoted it by $int^*(A,E)$

Proposition(1.19) [1]: Let (A,E) be a soft open set in a soft topological space (X,τ,E) with an ideal I, then $int^*(A,E)=(A,E)$.

Proposition (1.20) [1]: Let (A,E) be any soft set in a soft topological space (X,τ,E) with an ideal I, then $int(A,E)\tilde{\subseteq}int^*(A,E)$.

Definition(1.22) [1]: For any soft subset (A,E) in a soft topological space (X,τ,E) with an ideal I, the SSIg-closure of (A,E), denoted by $cl^*(A,E)$, is defined by the intersection of all SSIg-closed sets containing (A,E).

Theorem (1.23) [1]: If (A,E) is SSIg-closed set in a soft topological space (X,τ,E) with an ideal I ,then $(A,E)= cl^*(A,E)$.

Proposition(1.24) [1]: Let (A,E) be any soft set of a soft topological space (X,τ,E) with an ideal I, then $cl^*(A,E)\tilde{\subseteq}cl(A,E)$.

2. Soft strongly generalized mapping with respect to an ideal in soft topological space.

Definition(2.1)[6]: Let SS(X,E) and SS(Y,B) be families of soft sets over X and Y respectively, $u : X \to Y$ and $p: E \to B$ be mappings. Then the mapping f_{pu}:SS(X,E) $\to$ SS(Y,B) is defined as :

1- If (F,E) $\in$ SS(X,E) , then the image of (F,E) under f_{pu} , written as f_{pu}(F,E) =(f_{pu}(F),p(E)) is a soft set in SS(Y,B) such that

$$f_{pu}(F)(b)=\begin{cases}\bigcup_{a\in p^{-1}(b)} u(F(a)) \ , & p^{-1}(b)\neq \phi. \\ \phi \ , & p^{-1}(b)=\phi.\end{cases} \quad \text{for all } b\in B.$$

2- If (G,B) $\in$ SS(Y,B) , then the inverse image of (G,B) under f_{pu} , written as f_{pu}^{-1} (G,B) =(f_{pu}^{-1} (G),p^{-1}(B)) is a soft set in SS(X,E) , such that

$$f_{pu}^{-1}(G)(a)=\begin{cases}u^{-1}(G(p(a))) \ , & p(a)\in B. \\ \phi \ , & \text{o.w.}\end{cases} \quad \text{for all } a\in E.$$

Definition(2.2): Let f_{pu}:SS(X,E) $\to$ SS(Y ,K) and g_{qs} : SS(Y,K) $\to$ SS(Z,H) be a soft mappings . Then $(g\circ f)_{(q\circ p)(s\circ u)}$: SS(X,E) $\to$ SS(Z,H), if (F,E) $\in$ SS(X,E), then the image of (F,E) under $(g\circ f)_{(q\circ p)(s\circ u)}$, written as $(g\circ f)_{(q\circ p)(s\circ u)}$ (F,E) = ($(g\circ f)_{(q\circ p)(s\circ u)}$ (F), $q\circ p$ (E)) is a soft set.

Remark(2.3): In Definition(3.2.3) $(g\circ f)_{(q\circ p)(s\circ u)} = g_{qs}\circ f_{pu}$.

Theorem (2.4)[5]: Let f_{pu}: SS(X, E) $\to$ SS(Y,K) , $u : X \to Y$, and $p : E \to K$ be mappings. Then for soft sets (F,) and (F_i, A_i) in the soft class SS(X, E) and (G,B) in SS(Y,K) ,we have the following properties :

1- $f_{pu}(\phi_E)=\phi_K$.
2- $f_{pu}(X_E)\tilde{\subseteq} Y_K$.
3- $f_{pu}((F,A)\tilde{\cup}(G,B))=f_{pu}(F,A)\tilde{\cup} f_{pu}(G,B)$, in general we get
   $f_{pu}(\tilde{\cup}_i(F_i,A_i))=\tilde{\cup}_i f_{pu}((F_i,A_i)) \quad \forall i\in\Lambda.$
4- $f_{pu}((F,A)\tilde{\cap}(G,B))\subseteq f_{pu}(F,A)\tilde{\cap} f_{pu}(G,B)$, in general we get
   $f_{pu}(\tilde{\cap}_i(F_i,A_i))\subseteq \tilde{\cap}_i f_{pu}((F_i,A_i)) \quad \forall i\in\Lambda.$
5- If $(F,A)\tilde{\subseteq}(G,B)$, then $f_{pu}(F,A)\tilde{\subseteq} f_{pu}(G,B)$.
6- $f_{pu}^{-1}(\phi_K)=\phi_E$.
7- $f_{pu}^{-1}(Y_K)=X_E$.
8- $f_{pu}^{-1}((F,A)\tilde{\cup}(G,B))=f_{pu}^{-1}(F,A)\tilde{\cup} f_{pu}^{-1}(G,B)$, in general we get
   $f_{pu}(\tilde{\cup}_i(F_i,A_i))=\tilde{\cup}_i f_{pu}((F_i,A_i)) \quad \forall i\in\Lambda.$
9- $f_{pu}^{-1}((F,A)\tilde{\cap}(G,B))=f_{pu}^{-1}(F,A)\tilde{\cap} f_{pu}^{-1}(G,B)$, in general we get
   $f_{pu}^{-1}(\tilde{\cap}_i(F_i,A_i))=\tilde{\cap}_i f_{pu}^{-1}((F_i,A_i)) \quad \forall i\in\Lambda.$

Note(2.5)[5]: If (X,τ,I) is a topological space with an ideal I , (Y,σ) is a topological space and f : (X,τ,I) →(Y,σ) is a function, then f(I) = {f(I$_i$) : I$_i$ ∈ I , $\forall i \in \Lambda$ } is an ideal of Y. So in this research we will depend I as an ideal over X and f(I) is an ideal over Y.

Definition (2.6)[5]: Let (X,τ$_X$,E,I) and (Y,τ$_Y$,E) be two soft topological spaces, f_{pu}: SS(X,E) → SS(Y,E) be a soft mapping. then f_{pu} is a said to be soft Ig-continuous if the inverse image under f_{pu} of every soft open set in SS(Y,E) is soft Ig-open set in SS(X,E).

Definition(2.7) : Let (X,τ$_X$,A) and (Y,τ$_Y$,B) be two soft topological spaces with an ideal I, f_{pu}: (X, τ$_X$,A) → (Y,τ$_Y$,B) be a soft mapping. If for each soft open set (G,E) over Y , f_{pu}^{-1} ((G,B)) is a SSIg-open set over X, then f_{pu} is said to be SSIg-continuous mapping.

Proposition (2.8): Every soft continuous mapping is SSIg-continuous .

Proof: Let (X,τ,E) soft topological space with an ideal I, (Y,ϑ,K) be a soft topological space and f_{pu} : (X,τ,E) ⟶(Y,ϑ,K) be a soft continuous mapping. Let (H,K) be a soft open set in (Y,ϑ,K), since f_{pu} is a soft continuous mapping. then $f_{pu}^{-1}(H,K)$ is soft open set, so $(f_{pu}^{-1}(H,K))^c$ is soft closed set. But we have every soft closed set is SSIg-closed from Proposition(1.13) , then $(f_{pu}^{-1}(H,K))^c$ is SSIg-closed, hence $f_{pu}^{-1}(H,K)$ is SSIg-open set, thus f_{pu} is a SSIg-continuous mapping. □

Remark (2.9): The converse of Proposition (2.8) need not to be true by the following example.

Example: Let X={a,b,c} , E = {e$_1$, e$_2$} , Y={h$_1$, h$_2$, h$_3$}, K = {k$_1$, k$_2$}, $I = \{\phi_E\}$ and τ = { ϕ_E , X_E }, ϑ = { ϕ_K , Y_K , (G,K)} be two soft topologies defined on X and Y respectively , where (G,E)={(k_1,{ h_1 }), (k_2 ,{ h_3 ,h_2 })}. Define $p: E \to K$ such that $p(e_1)=k_2$, $p(e_2)=k_1$ and $u: X \to Y$ such that $u(a)=h_3, u(b)=h_2, u(c)=h_1$. Then , f_{pu} : (X,τ,E) ⟶(Y,ϑ,K) is an SSIg-continuous mapping . But it is not soft continuous since (G,K) is soft open set in (Y,ϑ,K) but $f_{pu}^{-1}((G,K))=$ (H,E) which is not soft open set in (X,τ,E) . Therefore f_{pu} is not soft continuous . □

Definition(2.10) : Let (X,τ$_X$,A) and (Y,τ$_Y$,B) be two soft topological spaces with an ideal I, f_{pu}: (X,τ$_X$,A) → (Y,τ$_Y$,B) be a mapping. If for each soft open set (G,B) over Y , f_{pu}^{-1} ((G,B)) is a SSIg-closed set over X, then f_{pu} is said to be contra-SSIg-continuous mapping.

Proposition(2.11): Every soft contra-continuous mapping is contra-SSIg-continuous .
Proof: Let (X,τ,E) be a soft topological space with an ideal I, (Y,ϑ,K) be a soft topological space and f_{pu} : (X,τ,E) ⟶ (Y, ϑ ,K) be a soft contra-continuous mapping. Let (H,K) be a soft open set in (Y, ϑ ,K), since f_{pu} is a soft contra-continuous mapping. then $f_{pu}^{-1}(H,K)$ is soft closed set. But we have

every soft closed set is SSIg-closed from Proposition(1.13) , then $f_{pu}^{-1}(H,K)$ is SSIg-closed, thus f_{pu} is a contra-SSIg-continuous mapping. □

Remark(2.12): The converse of Proposition(2.11) need not be true by the following example.

Example : Let X={a,b,c}, E = {e1, e2}, Y={h1, h2, h3}, K = {k1, k2}, $I=\{\phi_E\}$, and τ = { ϕ_E , X_E }, ϑ = { ϕ_K , Y_K , (G,K)} be two soft topologies defined on X and Y respectively , where (G,K) $=\{(k_1,\{h_1\}),(k_2,\{h_3,h_2\})\}$. define f_{pu} same as Remark(2.9), then f_{pu} is SSIg-continuous. But it is not soft contra-continuous since (G,K) is soft open set in (Y, ϑ ,K) but $f_{pu}^{-1}((G,K))=\{(e_1,\{c\}),(e_2,\{a,b\})\}$ is not soft closed set in (X,τ,E) .

Remark(2.13): The concepts of contra-SSIg-continuous and SSIg-continuous are independent by the following examples.

Example : Let X={a,b,c} , E = {e1, e2} , Y={h1, h2, h3}, K = {k1, k2}, $I=\{\phi_E\}$ and τ = { ϕ_E , X_E ,(F,E) }, ϑ = { ϕ_K , Y_K , (G,K)} be two soft topologies defined on X and Y respectively , where (F,E)= $\{(e_1,\{a\}),(e_2,\{b,c\})\}$, (G,K)= $\{(k_1,\{h_1\}),(k_2,\{h_3,h_2\})\}$. Define $p:E\to K$ such that $p(e_1)=k_1$, $p(e_2)=k_2$ and $u:X\to Y$ such that $u(a)=h_1, u(b)=h_3, u(c)=h_2$.
Then , f_{pu} :(X,τ,E) →(Y, ϑ,K) is not contra-SSIg-continuous, since $f_{pu}^{-1}((G,K))=f_{pu}^{-1}(\{(k_1,\{h_1\}),(k_2,\{h_3,h_2\})\})=\{(e_1,\{a\}),(e_2,\{b,c\})\}$= (F, E) which is not SSIg-closed .

On the other hand, since $f_{pu}^{-1}((G,K))$ = (F, E) which is soft open set and so it is SSIg-open set by Corollary(1.15). Therefore f_{pu} is an SSIg-continuous. □

Example: Let X={a,b,c} , E = {e1, e2}, Y={h1, h2, h3}, K = {k1, k2}, $I=\{\phi_E\}$ and τ = { ϕ_E , X_E , (F, E) }, ϑ= { ϕ_K , Y_K , (G, K)} be two soft topologies defined on X and Y respectively , where (F, E)= $\{(e_1,\{b,c\}),(e_2,\{a\})\}$, (G, K)= $\{(k_1,\{h_1\}),(k_2,\{h_3,h_2\})\}$. Define f_{pu} same as Remark(2.13), then it is an contra-SSIg-continuous. But it is not SSIg-continuous since (G,K) is soft open set in (Y, ϑ,K) but $f_{pu}^{-1}((G,K))=f_{pu}^{-1}(\{(k_1,\{h_1\}),(k_2,\{h_3,h_2\})\})$ ={(e_1 ,{a}), $(e_2,\{b,c\})\}$= (F, E), then $cl(int(F,E)^c)-(F,E)^c\notin I$. Hence , $(F,E)^c$ is not SSIg-closed , therefore $f_{pu}^{-1}((G,K))$ is not SSIg-open set.

Proposition(2.14) : Let (X,τ,E) be a soft topological space with an ideal I, (Y, ϑ,K) be a soft topological space and f_{pu}:(X,τ,E) →(Y, ϑ,K) is a soft closed mapping. If (G, E) is a soft closed set in (X,τ,E) , then $f_{pu}(G,E)$ is SS f_{pu} (I)g-closed in (Y, ϑ,K) .

Proof : Suppose that (G, E) is a closed SSIg-closed in (X,τ,E). Let (H, K) be a soft open set in (Y, ϑ,K) such that $f_{pu}(G,E)\tilde{\subseteq}$ (H, K), then $f_{pu}(G,E)$ is soft closed set in (Y, ϑ,K), then by Proposition(1.13) we have get that $f_{pu}(G,E)$ is SS $f_{pu}(I)$ g-closed. □

Corollary(2.15): Let (X,τ,E) be a soft topological space with an ideal I , (Y, ϑ ,K) be a discrete soft topological space and f_{pu}:(X,τ,E) →(Y, ϑ,K) is a soft mapping . If (G,E) is a SSIg-closed in (X,τ,E) , then $f_{pu}(G,E)$ is SS f_{pu} (I)g-closed in (Y, ϑ,K) .

Proof : It is clear . □

Proposition (2.16): Every soft g-continuous mapping is SSIg-continuous .

Proof: Let (X,τ,E) soft topological space with an ideal I, (Y, ϑ ,K) be a soft topological space and f_{pu}: (X,τ,E)→(Y, ϑ,K) be a soft g-continuous mapping. Let (H,K) be a soft open set in (Y, ϑ,K), since f_{pu} is an soft g-continuous mapping. then $f_{pu}^{-1}(H,K)$ is soft g-open set. But we have every soft g-open set is SSIg-open from Corollary(1.16), then $f_{pu}^{-1}(H,K)$ is SSIg-open, thus f_{pu} is an SSIg-continuous mapping. □

Remark(2.17): The converse of Proposition (2.16) need not be true by the following example.

Example: Let X={a,b,c} , E = {e1,e2} , Y={h1, h2, h3}, K = {k1, k2}, $I=\{\phi_E\}$ and τ = { ϕ_E , X_E ,(F,E) }, ϑ= { ϕ_K , Y_K , (G,K)} be two soft topologies defined on X and Y respectively , where (F,E)={(e_1,{ a,c }), (e_2 ,{ a,b })} and (G,K)={(k_1,{ h_1,h_2 }), (k_2 ,{ h_3 })}. Define $p:E\rightarrow K$ such that $p(e_1)=k_1$, $p(e_2)=k_2$ and $u:X\rightarrow Y$ such that $u(a)=h_1, u(b)=h_3, u(c)=h_2$. Then , f_{pu}: (X,τ,E) →(Y, ϑ,K) is a soft mapping and it is a SSIg-continuous . But it is not soft g-continuous since (G,K) is soft open set in (Y, ϑ,K) since (V,E)$^c \tilde{\subseteq}$ (F,E) but cl(V,E)^c=X_E $\tilde{\not\subset}$ (F,E). Hence $f_{pu}^{-1}((G,K))$ is not soft g-open set, thus f_{pu} is not soft g-continuous. □

Proposition(2.18) : Let f_{pu} : (X,τ,E) →(Y, ϑ,K) be an SSIg-continuous mapping and g_{qs} : (Y, ϑ,K)→(Z, η,H) is a soft continuous mapping. Then $g_{qs}\circ f_{pu}$:(X,τ,E)→ (Z, η,H) is SSIg-continuous mapping .

Proof : Let f_{pu} : (X,τ,E) →(Y, ϑ,K) is a SSIg-continuous mapping and g_{pu} : (Y, ϑ,K) →(Z, η,H) is a soft continuous mapping. to prove that $(g\circ f)_{pu}$: (X,τ,E)→(Z, η,H) is SSIg-continuous mapping . Let (M,H) be a soft open set in (Z, η,H) . Since g_{qs} is a soft continuous mapping . Then $g_{qs}^{-1}(M,H)$ is soft open set in (Y, ϑ ,K) and since f_{pu} is SSIg-continuous mapping and $g_{qs}^{-1}(M,H)$ is soft open set in (Y, ϑ,K), So $f_{pu}^{-1}(g_{qs}^{-1}(M,H))$ is SSIg-open set in (X,τ,E) . Then $(g_{qs}\circ f_{pu})^{-1}(M,H)=f_{pu}^{-1}(g_{qs}^{-1}(M,H))$. Hence, $g_{qs}\circ f_{pu}$ is SSIg-continuous mapping . □

Proposition(2.19) : Let f_{pu}:(X,τ,E) → (Y, ϑ,K) is a SSIg-continuous mapping and g_{qs} : (Y, ϑ,K) → (Z, η,H) is a soft contra-continuous mapping . Then $g_{qs} \circ f_{pu}$: (X,τ,E) → (Z, η,H) is contra-SSIg-continuous mapping .

Proof : Let f_{pu}:(X,τ,E) → (Y, ϑ,K) is a SSIg-continuous mapping and g_{pu} : (Y, ϑ,K) → (Z, η,H) is a soft contra-continuous mapping . to prove that $g_{qs} \circ f_{pu}$: (X,τ,E) → (Z, η,H) is contra-SSIg-continuous mapping . Let (M,H) be a soft open set in (Z, η,H) . Since g_{qs} is a soft contra-continuous mapping . Then $g_{qs}^{-1}(M,H)$ is soft closed set in (Y, ϑ,K) and since f_{pu} is SSIg-continuous mapping and $g_{qs}^{-1}(M,H)$ is soft closed set in (Y, ϑ,K), therefore $f_{pu}^{-1}(g_{qs}^{-1}(M,H))$ is SSIg-closed set in (X,τ,E). Hence $(g_{qs} \circ f_{pu})^{-1}(M,H) = f_{pu}^{-1}(g_{qs}^{-1}(M,H))$. Thus, $g_{qs} \circ f_{pu}$ is contra -SSIg-continuous mapping . □

Theorem(2.20): Let f_{pu}: (X,τ,E) → (Y, ϑ,K) be a mapping from a soft space (X,τ,E) with an ideal I to soft space (Y, ϑ,K). If f_{pu} is SSIg-continuous mapping then for each soft singleton (P,E) in X and each soft open set (O,K) in Y and $f_{pu}(P,E) \tilde{\subseteq} (O,K)$, there exists a SSIg-open set (U,E) in X such that $(P,E) \tilde{\subseteq} (U,E)$ and $f_{pu}(U,E) \tilde{\subseteq} (O,K)$.

Proof : Suppose that f_{pu} is SSIg-continuous mapping . Let (P,E) be a soft singleton in X and (O,K) be a soft open set in Y such that $f_{pu}(P,E) \tilde{\subseteq} (O,K)$. Then $(P,E) \tilde{\subseteq} f_{pu}^{-1}(O,K)$,but f_{pu} is SSIg-continuous mapping and (O,K) be a soft open set in Y . By definition of SSIg-continuous mapping we get that $f_{pu}^{-1}(O,K)$ is SSIg-open set in X . Put (U,E)= $f_{pu}^{-1}(O,K)$. Therefore , $(P,E) \tilde{\subseteq} (U,E)$ and $(O,K) \tilde{\subseteq} f_{pu}(U,E)$.□

Remark(2.21): The converse of Proposition (2.20) need not be true by the following example.

Example : Let X={a,b}, E = {e1, e2}, Y={d, c}, K = {k1, k2}, $I = \{\phi_E\}$ and τ = {ϕ_E, X_E , (F,E)}, ϑ = {ϕ_K, Y_K, (G,K)} be two soft topologies defined on X and Y respectively , where (F,E)={(e_1,{b}), (e_2,ϕ)} , (G,K)={(k_1,{d}), (k_2,Y)} define
$p: E \to K$ such that $p(e_1) = k_1,\ p(e_2) = k_2$ and $u: X \to Y$ such that $u(a) = d$,$u(b) = c$. Then , f_{pu} : (X,τ,E) → (Y, ϑ,K) is a soft mapping .
Since (G,K) is SS $f_{pu}(I)$ g-open and $f_{pu}^{-1}((G,K)) = f_{pu}^{-1}$ ({(k_1 ,{d}), (k_2 ,Y)}) = {(e_1,{a}), (e_2,X)} which is not SSIg-open set. So it is not SSIg-continuous mapping.

On the other hand a ∈X, $f_{pu}(a,E) \tilde{\subseteq} (G,K) \in \tau_Y$ and a$\tilde{\in}$ {(e_1,{a}), (e_2,{a})} where {(e_1,{a}), (e_2,{a})} is SSIg-open set over X and $(a,E) \tilde{\subseteq}$ {(e_1,{a}), (e_2,{a})} , f_{pu} {(e_1,{a}), (e_2,{a})} $\tilde{\subseteq} f_{pu}(G,K)$. Also b ∈X and $f_{pu}(b,E) \tilde{\subseteq} Y_K$, while X_E is SSIg-open set and $(b,E) \tilde{\subseteq} X_E$, $f_{pu}(X_E) \tilde{\subseteq} Y_K$.□

Proposition(2.22): Let f_{pu}: (X,τ,E) → (Y, ϑ,K) be a mapping from a soft space (X,τ,E) with an ideal *I* to soft space (Y, ϑ,K).Then the following statements are equivalent :

1- f_{pu} is SSIg-continuous mapping .

2- the inverse image under f_{pu} for any soft closed set over Y is SSIg-closed set over X .

proof : (1)⇒(2) suppose that f_{pu} is SSIg-continuous mapping .

To prove that the inverse image under f_{pu} for any SSIg-closed set over Y is SSIg-closed set over X . Let (F,E) be a soft closed set over Y .We have to show that f_{pu}^{-1}(F,E) is SSIg-closed over X . Since (F,E) is a soft closed set in Y ,then $(F,E)^c$ is a soft open set over Y . Because f_{pu} is SSIg-continuous mapping .Then $f_{pu}^{-1}(F,E)^c$ is SSIg-open over X . Hence, f_{pu}^{-1}(F,E) is SSIg-closed over X .

(2)⇒(1) Suppose that the inverse image under f_{pu} any soft closed set over Y is SSIg-closed set over X and to prove that f_{pu} is SSIg-continuous mapping .

Let (F,E) be a soft open set over Y .We have to show that f_{pu}^{-1}(F,E) is SSIg-open set in X . Since (F,E) is a soft open set over Y , then $(F,E)^c$ is a SSIg-closed set over Y . Then $f_{pu}^{-1}(F,E)^c$ is SSIg-closed set over X and $f_{pu}^{-1}(F,E)^c = (f_{pu}^{-1}(F,E))^c$.

Hence, f_{pu}^{-1}(F,E) is SSIg-open set over X . Therefore, f_{pu}^{-1} is SSIg-continuous mapping . □

Proposition(2.23) : Let f_{pu}: (X,τ,E) → (Y, ϑ,K) be an SSIg-continuous mapping from a soft topological space (X,τ,E) with an ideal *I* to soft topological space (Y, ϑ,K). If (A,E) is any soft set in over X , then $f_{pu}(cl^*(A,E)) \tilde{\subseteq} cl(f_{pu}(A,E))$.

Proof : Let (A,E) be any soft set over X . Then $f_{pu}(A,E)$ is a soft set over Y and $cl(f_{pu}(A,E))$ is soft closed set over Y. But f_{pu} is a SSIg-continuous mapping . Then $f_{pu}^{-1}(cl^*(f_{pu}(A,E)))$ is a SSIg-closed set over X . Then $cl^*(f_{pu}^{-1}(cl(f_{pu}(A,E))) = f_{pu}^{-1}(cl(f_{pu}(A,E)))$ by Proposition(1.26).

Then $(A,E) \tilde{\subseteq} f_{pu}^{-1}((f_{pu}(A,E)) \tilde{\subseteq} f_{pu}^{-1}(cl(f_{pu}(A,E)))$ *and* $(A,E) \tilde{\subseteq} f_{pu}^{-1}(cl(f_{pu}(A,E)))$

Therefore $cl^*(A,E) \tilde{\subseteq} cl^*(f_{pu}^{-1}(cl(f_{pu}(A,E))) = f_{pu}^{-1}(cl(f_{pu}(A,E))$.

Thus, $f_{pu}(cl^*(A,E)) \tilde{\subseteq} cl(f_{pu}(A,E))$.□

Remark (2.24): The equality of Proposition(2.23) need not to be true by the following example

Example: Let X={a,b,c} , E = {e1, e2} , Y={h1, h2, h3}, K = {k1, k2}, $I = \{\phi_E\}$ and τ = { ϕ_E , X_E ,(F,E)}, ϑ = { ϕ_K , Y_K , (G,K)} be two soft topologies defined on X and Y respectively, (F, E)={(e_1 ,{ b }), (e_2 ,{ a,c })}, (G,K)={(k_1 ,{ h_1 }), (k_2 ,{ h_2,h_3 })} , define $p: E \to K$ such that $p(e_1) = k_1$, $p(e_2) = k_2$ and $u: X \to Y$ such that $u(a) = h_1, u(b) = h_3, u(c) = h_2$. Then , f_{pu} : (X,τ,E) → (Y, ϑ,K) is a soft mapping . Since $f_{pu}^{-1}((G,K)) = f_{pu}^{-1}$ ({(k_1 ,{ h_3 }), (k_2 ,{ h_1,h_2 })}) ={(e_1 ,{ b }), (e_2 ,{ a,c })} =(F,E) which is soft open set, so it is soft continuous mapping . Now, let (A, E)={(e_1 , X), (e_2 ,{ a,b })} be a soft set in X .

Then $cl^*(A,E)=$ (A, E). We have $f_{pu}(A,E)=\{(k_1,Y),(k_2,\{h_1,h_3\})\}$. Then $cl(f_{pu}(A,E))=Y_K$, Hence, $f_{pu}(cl^*(A,E))\tilde{\subseteq}Y_K=cl(f_{pu}(A,E))$, which mean that $cl(f_{pu}(A,E))\ \tilde{\not\subseteq}(f_{pu}cl^*(A,E))$. □

Remark(2.25) : The converse of Proposition(2.23) need not to be true by the following example.

Example: Let X={a,b} , E = {e1, e2} , Y={h1, h2}, K = {k1, k2}, $I=\{\phi_E\}$ and τ = {ϕ_E, X_E, (F,E)}, ϑ= {ϕ_K, Y_K, (G,K)} be two soft topologies defined on X and Y respectively , where (F, E)={(e_1,ϕ), (e_2,{b})} , (G,K)={(k_1,Y), (k_2,{h_1})} , define $p:E\rightarrow K$ such that $p(e_1)=k_1$, $p(e_2)=k_2$ and $u:X\rightarrow Y$ such that $u(a)=h_1, u(b)=h_2$. Then , f_{pu}: (X,τ,E) →(Y, ϑ,K) is a soft mapping . Since $f^{-1}_{pu}((G,K))=f^{-1}_{pu}(\{(k_1,Y),(k_2,\{h_1\})\})=\{(e_1,X),(e_2,\{a\})\}$ which is not SSIg-open set. So it is not SSIg-continuous mapping .
On the other hand for each soft set (A, E) in SS(X, E) , then $f_{pu}(cl^*(A,E))\tilde{\subseteq}cl(f_{pu}(A,E))$. □

Proposition(2.26) : Let f_{pu}: (X,τ,E) →(Y, ϑ,K) be a SSIg-continuous mapping from a soft space (X,τ,E) with an ideal I to soft space (Y, ϑ,K). If (A,E) is any soft set in X ,then f^{-1}_{pu} *(int(A,E)* $\tilde{\subseteq}$ $int^*(f^{-1}_{pu}(A,E))$ **.**

Proof : Let (A,K) is any soft set in (Y, ϑ,K) .then *int(A,K)* is a soft open set in (Y, ϑ,K). But f_{pu} is SSIg-continuous, so f^{-1}_{pu} *(int(A,K))* is SSIg-open by Proposition(1.19) we have f^{-1}_{pu} *(int(A,K))*= $int^* f^{-1}_{pu}$ *(int(A,K))*. But *int(A,K)*$\tilde{\subseteq}$*(A,K)*, then f^{-1}_{pu} *(int(A,K))*$\tilde{\subseteq}$ f^{-1}_{pu} *(A,K)*. Hence, f^{-1}_{pu} *(int(A,K))*$\tilde{\subseteq}$ $int^* f^{-1}_{pu}$ *(A,K)*. □

Remark(2.27): The converse of Proposition (2.26) need not be true by the following example.

Example: Let X={a,b} , E = {e1, e2} , Y={d, c}, K = {k1, k2}, $I=\{\phi_E\}$ and τ = {ϕ_E, X_E, (F, E)}, ϑ= {ϕ_K, Y_K, (G, K)} be two soft topologies defined on X and Y respectively , where (F, E)={(e_1,{b}), (e_2,ϕ)} , (G,K)={(k_1,{d}), (k_2,Y)} , define $p:E\rightarrow K$ such that $p(e_1)=k_1$, $p(e_2)=k_2$ and $u:X\rightarrow Y$ such that $u(a)=d, u(b)=c$. Then, f_{pu} : (X,τ,E) →(Y, ϑ,K) is a soft mapping . Since $f^{-1}_{pu}((G,K))=f^{-1}_{pu}(\{(k_1,\{d\}),(k_2,Y)\})=\{(e_1,\{a\}),(e_2,X)\}$ which is not SSIg-open set. So it is not SSIg-continuous mapping .
On the other hand for each soft set (A, K) in SS(Y, K) , then get
f^{-1}_{pu} *(int(A,K))* $\tilde{\subseteq}$ $int^* f^{-1}_{pu}$ *(A,K)*. □

Remark(2.28): The equality in Proposition(2.26) need not be true in general by the following example.

Example : Let X={a,b} , E = {e1, e2} , Y={d, c}, K = {k1, k2}, $I=\{\phi_E\}$ and τ = {ϕ_E, X_E , (F, E)}, ϑ= {ϕ_K, Y_K, (G, K)} be two soft topologies defined on X and Y respectively , where (F, E)={(e_1,{a}), (e_2,X)} , (G,K)={(k_1,{d}), (k_2,Y)} , define $p: E \to K$ such that $p(e_1)=k_1$, $p(e_2)=k_2$ and $u: X \to Y$ such that $u(a) = d$, $u(b) = c$. Then, f_{pu}:(X,τ,E) →(Y, ϑ,K) is a soft mapping . It is clear that f_{pu} is an SSIg-continuous mapping .

On the other hand let (A, K) be soft set in SS(X, E) such that (A, K) ={(k_1,{d}), (k_2, ϕ)}, then int(A, E) $=\phi_K$ so $f_{pu}^{-1}(int(A,K))=\phi_E$ and $f_{pu}^{-1}(A,K)$ ={(e_1,{a}), (e_2,ϕ)}, so $\text{int}^* f_{pu}^{-1}(A,K)=f_{pu}^{-1}(A,K)$={($e_1$,{a}), ($e_2$,$\phi$)}, therefore int^* $f_{pu}^{-1}(A,K) \tilde{\not\subseteq} f_{pu}^{-1}(int(A,K))$. □

Proposition(2.29): Let f_{pu} : (X,τ,E) →(Y, ϑ,K) be a SSIg-continuous mapping from a soft topological space (X,τ,E) with an ideal I to soft topological space (Y, ϑ,K). If (A,E) is any soft set in over X , then $\text{int}(f_{pu}(A,E)) \tilde{\subseteq} f_{pu}(\text{int}^*(A,E))$.

Proof : Let (A,E) be any soft set over X . Then $f_{pu}(A,E)$ is a soft set over Y and $int(f_{pu}(A,E))$ is soft open set over Y. But f_{pu} is a SSIg-continuous mapping . Therefore $f_{pu}^{-1}(int(f_{pu}(A,E)))$ is a SSIg-open set over X. Then $int^*(f_{pu}^{-1}(int(f_{pu}(A,E)))) = f_{pu}^{-1}(int(f_{pu}(A,E)))$ by Proposition(1.19).
$\text{int}^*(f_{pu}^{-1}(\text{int}(f_{pu}(A,E))) \tilde{\subseteq} \text{int}^*(f_{pu}^{-1}(f_{pu}(A,E))) \tilde{\subseteq} \text{int}^*(A,E)$, therefore $f_{pu}^{-1}(\text{int}(f_{pu}(A,E)))$ $\tilde{\subseteq} \text{int}^*(A,E)$. Thus $\text{int}(f_{pu}(A,E)) \tilde{\subseteq} f_{pu}(\text{int}^*(A,E))$.□

Remark(2.30): The converse of Proposition(2.29) need not be true in general by the following example.

Example: Let X={a,b}, E = {e1, e2}, Y={d, c}, K = {k1, k2}, $I=\{\phi_E\}$ and τ = {ϕ_E, X_E , (F, E)}, ϑ= {ϕ_K, Y_K, (G, K)} be two soft topologies defined on X and Y respectively, where (F, E)={(e_1,{b}), (e_2,ϕ)}, (G,K)={(k_1,{d}), (k_2,Y)} , define $p: E \to K$ such that $p(e_1) = k_1$, $p(e_2) = k_2$ and $u: X \to Y$ such that $u(a) = d$, $u(b) = c$. Then, f_{pu} :(X,τ,E) → (Y, ϑ ,K) is a soft mapping. Since $f_{pu}^{-1}((G,K))=f_{pu}^{-1}$ ({(k_1 ,{d}), (k_2 ,Y)}) ={(e_1 ,{a}), (e_2,X)} which is not SSIg-open set. So it is not SSIg-continuous mapping .

On the other hand for each soft set (A, E) in SS(X, E) , then $\text{int}(f_{pu}(A,E)) \tilde{\subseteq} f_{pu}(\text{int}^*(A,E))$. □

Remark(2.31): The equality in Proposition(2.29) need not be true in general by the following example.

Example: Let X={a,b}, E = {e1, e2}, Y={d, c}, K = {k1, k2}, $I=\{\phi_E\}$ and τ = {ϕ_E, X_E , (F, E)}, ϑ= {ϕ_K, Y_K, (G, K)} be two soft topologies defined on X and Y respectively , where (F, E)={(e_1,{a}), (e_2,X)}, (G,K)={(k_1,{d}), (k_2,Y)}, define $p: E \to K$ such that $p(e_1) = k_1$, $p(e_2) = k_2$ and $u: X \to Y$ such that $u(a) =$

$d, u(b) = c$. Then, f_{pu}: (X,τ,E)→(Y, ϑ,K) is a soft mapping . It is clear that is an SSIg-continuous mapping .

On the other hand let (A, E) be soft set in SS(X, E) such that (A, E) = {(e_1 ,{a}), (e_2 ,{b})}, then $f_{pu}(int^*(A,E))$ = {(k_1 ,{d}), (k_2 ,{c})} and $f_{pu}(A,E)$ ={(k_1,{d}), (k_2,{c})}, therefore $f_{pu}(\text{int}^*(A,E)) \tilde{\subseteq} \text{int}(f_{pu}(A,E))$. □

Definition(2.32) : Let (X,τ_X,A) and (Y,τ_Y,B) be two soft topological spaces with an ideal I. Let f_{pu}: (X, τ_X,A) → (Y, τ_Y ,B) be a mapping. If f_{pu}^{-1} ((G,B)) is a SSIg-open set over X for each SSIg-open set (G,B) over Y, then f_{pu} is said to be SSIg-irresolute mapping.

Proposition(2.33) : Every SSIg-irresolute mapping is an SSIg-continuous mapping.
Proof: Let (X,τ_X,A) and (Y,τ_Y,B) be two soft topological spaces with an ideal I. Let f_{pu}: (X, τ_X,A) → (Y, τ_Y ,B) be an SSIg- irresolute mapping. To show that f_{pu} is SSIg-continuous. Let (G,B) be a soft open set over Y. Then by Corollary(1.15), (G,B) is an SSIg-open set over Y. Since f_{pu} is an SSIg- irresolute. Then f_{pu}^{-1} ((G,B)) is a SSIg-open set over X. Therefore, f_{pu} is an SSIg-continuous mapping. □

Remark(2.34) : The converse of Proposition(2.33) need not be true in general by the following example.
Example: Let X={a,b} , E = {e_1, e_2}, Y={d, c}, K = {k_1, k_2}, $I = \{\phi_E\}$ and τ = {ϕ_E , X_E , (F, E)}, ϑ = {ϕ_K , Y_K} be two soft topologies defined on X and Y respectively , where (F, E)={(e_1 ,{b}),(e_2 , ϕ)}. Define f_{pu} the same as in Remark(2.31). It is clear that f_{pu} is an SSIg-continuous mapping. But f_{pu} is not SSIg- irresolute mapping since (G,K)= {(k_1 ,{d}), (k_2 ,Y)} is an SSIg-open set over Y, but $f_{pu}^{-1}((G,K))=f_{pu}^{-1}$({(k_1,{d}), (k_2 ,Y)}) = {(e_1 ,{a}), (e_2 ,X)} which is not SSIg-open set over X. □
Remark(2.35) : The notions SSIg-irresolute mapping and soft continuous mapping are independent. The observation follows by Example of Remark(2.34), such that f_{pu} is SSIg-continuous mapping but f_{pu} is not SSIg- irresolute mapping. Also in the following example shows that f_{pu} is SSIg- irresolute mapping but f_{pu} is not SSIg-continuous mapping.

Example: Let X={a,b} , E = {e_1, e_2} , Y={d, c}, K = {k_1, k_2}, $I = \{\phi_E\}$ and τ_X = {ϕ_E , X_E }, ϑ = {ϕ_K , Y_K , (G,K)} be two soft topologies defined on X and Y respectively , where (G,K)= {(k_1,{d}), (k_2 ,Y)} Let f_{pu} define in Remark(2.34) . Since $f_{pu}^{-1}((G,K))=f_{pu}^{-1}$({(k_1,{d}), (k_2 ,Y)}) = {(e_1,{a}), (e_2,X)} which is not SSIg-open set over X. Therefore f_{pu} is not SSIg-continuous mapping. But f_{pu} is an SSIg-irresolute mapping , since the inverse image under f_{pu} for every SSIg-open set, (SSIg-closed set),over Y is an SSIg-open set, (SSIg-closed set) . □

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التطبيقات من النمط SSIg بالنسبة لفضاء تبولوجي ناعم مع مثالي

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الخلاصة

في هذا العمل، قدمت الدراسة نوعا جديدا من التطبيقات الناعمة في الفضاءات التبولوجية الناعمة مع مثالي، والتي قد سُميت " التطبيقات من النمط SSIg في فضاء تبولوجي ناعم مع مثالي I، حيث درست مفاهيم مثل مستمرة، عكس-مستمرة ، المفتوح ، المغلقة و متردد بالنسبة للتطبيقات الناعمة من النمط SSIg والعلاقات بين هذه الأنواع من التطبيقات وتركيب اثنين من هذه التطبيقات من النوع نفسه او من نوعين مختلفين، مع البراهين أو أمثلة مضادة.

الكلمات المفتاحية: الاستمرارية من النمط SSIg ، عكس الاستمرارية من النمط SSIg، الارتداد من النمط SSIg، التطبيق المغلق من النمط SSIg، التطبيق المفتوح من النمط SSIg.