Ibn Al-Haitham Jour. for Pure & Appl. Sci. 32 (3) 2019

Ibn Al Haitham Journal for Pure and Applied Science

Journal homepage: http://jih.uobaghdad.edu.iq/index.php/j/index

Filter Bases and *j*-*ω*-Perfect Mappings

G. S. Ashaea **Y. Y. Yousif**

Department of Mathematics, College of Education for Pure Sciences, Ibn - Al-Haitham, University of Baghdad

ghidaasadoon@gmail.com yoyayousif@yahoo.com

Article history: Received 25 March 2019, Accepted 26 May 2019, Publish September 2019

Doi:10.30526/32.3.2289

Abstract

This paper consist some new generalizations of some definitions such: *j*-*ω*-closure converge to a point, *j*-*ω*-closure directed toward a set, almost *j*-*ω*-converges to a set, almost *j*-*ω*-cluster point, a set *j*-*ω*-H-closed relative, *j*-*ω*-closure continuous mappings, *j*-*ω*-weakly continuous mappings, *j*-*ω*-compact mappings, *j*-*ω*-rigid a set, almost *j*-*ω*-closed mappings and *j*-*ω*-perfect mappings. Also, we prove several results concerning it, where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

Keywords: Filter base, *j*-*ω*-closure converge, almost *j*-*ω*-converges, almost *j*-*ω*-cluster, *j*-*ω*-rigid a set, *j*-*ω*-perfect mappings.

Math Subject Classification 2010: 54C05, 54C08, 54C10.

1. Introduction

The notion "filter" first commence in Riesz [1]. and the setting of convergence in terms of filters sketched by Cartan in [2, 3]. And was sophisticatedly by Bourbaki in [4]. Whyburn in [5]. Introduces the notion directed toward a set and the generalization of this notion studied in Section 2. Dickman and Porter in [6]. Introduce the notion almost convergence, Porter and Thomas in [7]. introduce the notion of quasi-H-closed and the analogues of this notions are studied in Section 3. Levine in [8]. Introduce the notion θ-continuous functions, Andrew and Whittlesy in [9]. Introduce the notion weakly θ-continuous functions, in Dickman [6]. Introduce the notions θ-compact functions, θ-rigid a set, almost closed functions and the analogues of this notions are studied in Section 4. In [5]. The researcher introduces the notion of θ-perfect functions but the analogue of this notion studied in Section 5. The neighborhood denoted by nbd. The closure (resp. interior) of a subset K of a space G denoted by cl (K) (resp., int(K)). A point g in G is said to be condensation point of $K \subseteq G$ if every S in τ with $g \in S$, the set $K \cap S$ is uncountable [10]. In 1982 the *ω*-closed set was first exhibiting by Hdeib in [10]. and he know it a subset $K \subseteq G$ is called *ω*-closed if it incorporates each its condensation points and the *ω*-open set is the complement of the *ω*-closed set [12]. The *ω*-interior of the set $K \subseteq G$ defined as the union of all *ω*-open sets contain in K and is denoted by $\text{int}_\omega(K)$. A point $g \in G$ is said to θ-cluster points of $K \subseteq G$ if $\text{cl}(S) \cap K \neq \varphi$ for each open set S

of G containment g. The set of each θ-cluster points of K is called the θ-closure of K and is denoted by $cl\theta(K)$. A subset $K \subseteq G$ is said to be θ-closed [11]. if $K = cl\theta(K)$. The complement of θ-closed set said to be θ-open. A point $g \in G$ said to θ-ω-cluster points of $K \subseteq G$ if $_{\omega}cl\theta(S) \cap K \neq \varphi$ for each ω-open set S of G containment g. The set of each θ-ω-cluster points of K is called the θ-ω-closure of K and is denoted by $_{\omega}cl\theta(K)$. A subset $K \subseteq G$ is said to be θ-ω-closed [11]. if $K = {_{\omega}cl\theta(K)}$. The complement of θ-ω-closed set said to be θ-ω-open, δ-closed [12]. if $K = cl\delta(K) = \{g \in G: int(cl(S)) \cap K \neq \varphi, S \in \tau$ and $g \in S\}$. The complement of δ-closed said δ-open set, δ-ω-closed if $K = {_{\omega}cl\delta}(K) = \{g \in G: int_{\omega}(cl(S)) \cap K \neq \varphi, S \in \tau$ and $g \in S\}$. The complement of δ-ω-closed said δ-ω-open.

2. Filter

In this section we introduce definition of filter, filter base, nbd filter, finer ultrafilter and some other related concepts.

Definition 1 [4].

A nonempty family $\Im$ of nonempty subsets of G called filter if it satisfies the following conditions:

(a) If $M_1, M_2 \in \Im$, then $M_1 \cap M_2 \in \Im$.

(b) If $M \in \Im$ and $M \subseteq M^* \subseteq G$, then $M^* \in \Im$.

Definition 2 [4].

A nonempty family $\Im$ of nonempty subsets of G is called filter base if $M_1, M_2 \in \Im$ then $M_3 \subseteq M_1 \cap M_2$ for some $M_3 \in \Im$.

The filter generated by a filter base $\Im$ consists of all supersets of elements of $\Im$. An open filter base on a space G is a filter base with open members. The set $\aleph_g$ of all nbds of $g \in G$ is a filter on G, and any nbd base at g is a filter base for $\aleph_g$. This filter called the nbd filter at g.

Definition 3 [4].

Let $\Im$ and $\wp$ be filter bases on G. Then $\wp$ is called finer than $\Im$ (written as $\Im < \wp$) if for all $M \in \Im$, there is $\mathcal{G} \in \wp$ such that $\mathcal{G} \subseteq M$ and that $\Im$ meets $\mathcal{G}$ if $M \cap \mathcal{G} \neq \phi$ for all $M \in \Im$ and $\mathcal{G} \in \wp$. Notice, $\Im \to g$ iff $\aleph_g < \Im$.

Definition 4 [4].

A filter $\Im$ is called an ultrafilter if there is no strictly finer filter $\wp$ than $\Im$. The ultrafilter is the maximal filter.

Definition 5 [13].

A subset K of a space G called:

(a) α-ω-open if $K \subseteq int_{\omega}(cl(int_{\omega}(K)))$.

(b) *pre*-ω-open if $K \subseteq int_{\omega}(cl(K))$.

(c) b-ω-open if $K \subseteq cl(int_{\omega}(K)) \cup int_{\omega}(cl(K))$.

(d) β-ω-open if $K \subseteq cl(int_{\omega}(cl(K)))$.

The complement of an (resp. α-ω-open, *pre*-ω-open, b-ω-open, β-ω-open) called (resp. α-ω-closed, *pre*-ω-closed, b-ω-closed, β-ω-closed).

The j-ω-closure of $K \subseteq G$ is denoted by $cl\ j\text{-}\omega\text{-}(K)$ and defined by $cl\ j\text{-}\omega\text{-}(K) = \cap\{M \subseteq G$; G is j-ω-closed and $K \subseteq M\}$, where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$. Several characterizations of ω-closed sets were provided in [11],. [13-16]. Furthermore, we built some results about δ-ω-closed and δ-ω-open depending on the results in [17-19].

3. Filter Bases and *j*-ω-Closure Directed toward a Set

In this section we defined filter bases and *j*-ω-closure directed toward a set and the some theorems concerning of them.

Lemma 6 [15].

Let $\lambda : (G, \tau) \to (H, \sigma)$ be an injective mapping.

(a) If $\Im = \{M: M \subseteq G\}$ is a filter base in G, then $\lambda(\Im) = \{\lambda(M): M \in \Im\}$ is a filter base in H.

(b) If $\wp = \{\mathcal{G}: \mathcal{G} \subseteq \lambda(G)\}$ is a filter base in $\lambda(G)$, $\Im = \{\lambda^{-1}(\mathcal{G}): \mathcal{G} \in \wp\}$ is a filter base in G. For each $\phi \neq K \subseteq G$ and any filter base $\wp$ in $\lambda(K)$, then $\{K \cap \lambda^{-1}(\mathcal{G}): \mathcal{G} \in \wp\}$ is a filter base in K.

(c) If $\Im = \{M : M \subseteq G\}$ is a filter base in G, $\wp = \{\lambda(M) : M \in \Im\}$, $\mathcal{G}^*$ is finer than $\mathcal{G}$, and $\Im^* = \{\lambda^{-1}(\mathcal{G}^*) : \mathcal{G}^* \in \wp^*\}$, then the collection of sets $\Im^{**} = \{M \cap M^*$ for all $M \in \Im$ and $M^* \in \Im^*\}$ is finer than both of $\Im$ and $\Im^*$.

Definition 7 [4].

Let $\Im$ be a filter base on a space G. We say that $\Im$ converges to $g \in G$ (written as $\Im \to g$) iff each open set S about g contains some element $M \in \Im$. We say $\Im$ has g as a cluster point (or $\Im$ cluster at g) iff each open set S about g meets all element $M \in \Im$. Clear that if $\Im \to g$, then $\Im$ cluster at g.

Definition 8 [15].

Let $\Im$ be a filter base on a space G. We say that $\Im$ directed toward (shortly, *dir,- tow*) a set $K \subseteq G$, provided each filter base finer than $\Im$ has a cluster point in K. (Note: Any filter base can't be *dir,- tow* the empty set).

Now, we will generalizations Definitions 7 and 8 as follows.

Definition 9

Let $\Im$ be a filter base on a space G. We say that $\Im$ closure converges to $g \in G$ (written as $\Im \rightsquigarrow g$) iff all open set S about g, the cl(S) contains some element $M \in \Im$. We say $\Im$ has g as a closure cluster point (or $\Im$ closure cluster at g) iff all open set S about g the cl(S) meets all element $M \in \Im$.

Clear that if $\Im \rightsquigarrow g$, then $\Im$ closure cluster at g. cl $(\aleph_g)$ used to denote the filter base $\{\text{cl}(S): S \in \aleph_g\}$. Notice, $\Im \rightsquigarrow g$ if and only if cl $(\aleph_g) < \Im$. [10].

Definition 10

Let $\Im$ be a filter base on a space G. We say that $\Im$ closure directed toward (shortly, cl *dir,- tow)* a set $K \subseteq G$, provided each filter base finer than $\Im$ has a closure cluster point in K.

Theorem 11

Let $\Im$ be a filter base on a space G. $\Im \rightsquigarrow g \in G$ if and only if $\Im$ is cl *dir,- tow* g.

Proof: ($\Rightarrow$) Assume $\Im \rightsquigarrow g$, all open set S about g, cl(S) contains an element of $\Im$ and thus contains an element of every filter base $\Im^* < \Im$, therefore $\Im^*$ actually closure converges to g.

($\Leftarrow$) Assume $\Im$ is cl *dir,- tow* g, it must $\Im \rightsquigarrow g$. For if not, yond is an open set S in G about g such that cl(S) don't contains an element of $\Im$. Denote by $\Im^*$ the collection of sets $M^* = M \cap (G - \text{cl}(S))$ for $M \in \Im$, then the sets M^* are nonempty. And $\Im^*$ is a filter base and indeed $\Im^* < \Im$, because result in $M_1^* = M_1 \cap (G - \text{cl}(S))$ and $M_2^* = M_2 \cap (G - \text{cl}(S))$, so there is an $M_3 \subseteq M_1 \cap M_2$ and this perform to

$$M_3^* = M_3 \cap (G - \text{cl}(S)) \subseteq M_1 \cap M_2 \cap (G - \text{cl}(S))$$
$$= M_1 \cap (G - \text{cl}(S)) \cap M_2 \cap (G - \text{cl}(S)).$$

By construction, g is not a closure cluster point of $\Im^*$. This contradiction crops that, $\Im \rightsquigarrow g$.

Theorem 12

Let $\lambda : (G, \tau) \to (H, \sigma)$ be an injective mapping and given $L \subset H$. If for each filter base $\wp$ in $\lambda(G)$ cl *dir,- tow* a point $h \in L$, the inverse filter $\mathcal{M} = \{\lambda^{-1}(\mathcal{G}) : \mathcal{G} \in \wp\}$ is cl *dir,- tow* $\lambda^{-1}(h)$, then for any filter base $\Im$ in $\lambda(G)$ cl *dir,- tow* a set L, $\mathcal{E} = \{\lambda^{-1}(M) : M \in \Im\}$ is cl *dir,- tow* $K = \lambda^{-1}(L)$.

Proof: Suppose that the hypothesis is true and any $h \in L$ is a closure cluster point of a filter base finer than $\Im$ must be in $\lambda(G)$. Thus $L \cap \lambda(G) \neq \phi$, and $\Im$ is cl *dir,- tow* $L \cap \lambda(G)$. So we may assume $L \subseteq \lambda(G)$. Let $\mathcal{M}$ be a filter base finer than $\mathcal{E}$. Then $\wp = \{(\lambda(m): m \in \mathcal{M}\}$ finer than $\Im$ by Lemma (6, a). So $\wp$ has a closure cluster point l in L and a filter base $\wp^*$ finer than $\wp$ closure converges to l and so is cl *dir,- tow* l. By supposition $\mathcal{M}^* = \{\lambda^{-1}(\mathcal{G}^*): \mathcal{G}^* \in \wp^*\}$ is *cl dir,- tow* $\lambda^{-1}(l)$. In addition, by Lemma (6, c), $\mathcal{M}$ and $\mathcal{M}^*$ have a common filter base $\mathcal{M}^{**}$ finer than of them. So $\mathcal{M}^{**}$ has a closure cluster point g in $\lambda^{-1}(l)$. Since g is a closure cluster point of $\mathcal{M}$ and $g \in \lambda^{-1}(l) \subset K$, obtain result follows.

Theorem 13

Let $\lambda : G \to H$ be closed mapping and $\lambda^{-1}(h)$ compact for every $h \in H$ iff for every filter base $\Im$ in $\lambda(G)$ cl *dir,- tow* a set $L \subseteq H$, the collection $\mathcal{E} = \{\lambda^{-1}(M) : M \in \Im\}$ is cl *dir,- tow* $\lambda^{-1}(L)$.

Proof: ($\Rightarrow$) Suppose that λ is closed mapping and $\lambda^{-1}(h)$ compact for every $h \in H$. Then by Theorem 11 and 12 it suffices to prove that if $\wp$ is a filter base in $\lambda(G)$ j-ω-closure converging to $h \in L$, then $\mathcal{M} = \{\lambda^{-1}(\mathcal{G}) : \mathcal{G} \in \wp\}$ is *cl-d-t* $\lambda^{-1}(h)$.In order to if not, yond is a filter base $\mathcal{M}^*$ finer than $\mathcal{M}$, no point of $\lambda^{-1}(h)$ is a j-ω-closure cluster point of $\mathcal{M}^*$. For all $g \in \lambda^{-1}(h)$, by supposition yond is an open set S_g about g and $\mathcal{M}_g^* \in \mathcal{M}^*$ with $\mathcal{M}_g^* \cap S_g = \phi$. Since $\lambda^{-1}(h)$ is compact, yond are a finite numbers of open sets S_{g_i} such that $\lambda^{-1}(h) \subseteq S = \cup S_{g_i}$, suppose $m^* \in \mathcal{M}^*$ such that $m^* \subseteq \cap m_{g_i}^*$ and let $T = H - \lambda(G - S)$ be the open set. Then $\lambda(m^*) \cap T = \phi$ because of $m^* \subset G - \text{cl}(S)$. So since $\lambda(m^*) \in \wp^*$, $\wp^*$ cannot have h as a closure cluster point.

($\Leftarrow$) Suppose that the hypothesis is true and λ is not closed. Let $K \subseteq G$ be a closed set and for some $h \in H - \lambda(K)$ is a closure cluster point of $\lambda(K)$. Suppose $\wp$ be a filter base of sets $\lambda(K) \cap T$ for every open sets $T \subseteq H$ such that $h \in T$, then $\wp$ is a filter base in $\lambda(G)$ and $\wp \rightsquigarrow h$. Let $\mathcal{M} = \{\lambda^{-1}(\mathcal{G}): \mathcal{G} \in \wp\}$ and $\mathcal{M}^* = \{K \cap m : m \in \mathcal{M}\}$. It apparent that $\mathcal{M}^* < \mathcal{M}$.

Nevertheless, $G - K$ is open and $\lambda^{-1}(h) \subseteq G - K$, $\mathcal{M}^*$ has no closure cluster point in $\lambda^{-1}(h)$. The contradiction crops that λ be a closed mapping. Finally, to prove $\lambda^{-1}(h)$ is compact, this is easy for $h \in H - \lambda(G)$. And for $h \in \lambda(G)$, $\{h\}$ is a filter base in $\lambda(G)$ cl *dir,- tow* h. By supposition, $\{\lambda^{-1}(h)\}$ cl *dir,- tow* $\lambda^{-1}(h)$. This means that every filter base in $\lambda^{-1}(h)$ has closure cluster point in $\lambda^{-1}(h)$, so that $\lambda^{-1}(h)$ is compact.

Corollary 14

Let $\lambda : G \to H$ be closed mapping and $\lambda^{-1}(h)$ compact for every $h \in H$ if and only if each filter base in $\lambda(G) \rightsquigarrow h \in H$ has *pre*-image filter base cl *dir,- tow* $\lambda^{-1}(h)$.

Corollary 15

Let $\lambda : G \rightarrow H$ be closed mapping and $\lambda^{-1}(h)$ compact for every $h \in Y$, for every compact set $W \subseteq H$, $\lambda^{-1}(W)$ is compact.

Proof. Let $W \subseteq H$ be a compact set and $\Im$ is a filter base in $\lambda^{-1}(W)$, $\wp = \{\lambda(M): M \in \Im\}$, is a filter base in W and in $\lambda(G)$ and is cl *dir,- tow* W. So $\Im^* = \{\lambda^{-1}(\mathcal{G}): \mathcal{G} \in \wp\}$ is cl *dir,- tow* $\lambda^{-1}(W)$, so that $\Im^* < \Im$ and $\Im^*$ has a closure cluster point in $\lambda^{-1}(W)$.

4. Filter Bases and Almost *j-ω*-Convergence

In this section, we defined filter bases, almost j-ω-closure, and the some theorems about them. We now introduce the definition of almost *j-ω*-closure, where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

Definition 16

Let $\Im$ be a filter base on a space G. We say $\Im$ almost *j-ω*-converges to a subset $K \subseteq G$ (written as $\Im_{j-\omega} \rightsquigarrow K$) if for each cover $\mathcal{K}$ of K by subsets open in G, there is a finite subfamily $\mathcal{L} \subseteq \mathcal{K}$ and $M \in \Im$ such that $M \subseteq \cup\{\text{cl}(L) : L \in \mathcal{L}\}$. We say $\Im$ almost *j-ω*-converges to $g \in G$ (written as $\Im_{j-\omega} \rightsquigarrow g$) if $\Im_{j-\omega} \rightsquigarrow \{g\}$. Now, cl $(\aleph_g) \rightsquigarrow g$, while, $_{j-\omega}$ cl $(\aleph_g)_{j-\omega} \rightsquigarrow$ g, where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

Also, we introduce the definitions of almost *j-ω*-cluster point, and quasi -*j-ω*-H-closed set where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

Definition 17

A point $g \in G$ is called an almost *j-ω*-cluster point of a filter base $\Im$ (written as $g \in$ (al- j-ω-c$_g$)$\Im$) if $\Im$ meets cl j-ω-($\aleph_g$), where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

For a set $K \subseteq G$, the almost *j-ω*-closure of K, denoted as (al- j-ω-cl (K)) is al j-ω-c$_g$ $\{K\}$ if $K \neq \phi$ i.e. $\{g \in G$: every *j-ω*-closed nbd of g meets $K\}$ and is ϕ if $K = \phi$; K is almost *j-ω*-closed if K = (al- j-ω-cl(K)). Correspondingly, the almost *j-ω*-interior of K, denoted as (al- j-ω-int$K$), is $\{g \in G$; cl j-ω- $(S) \subseteq K$ for some open set S containing $g\}$; K is almost *j-ω*-interior if K = (al-j-ω-int(K)), where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

Theorem 18

Let $\Im$ and $\wp$ be filter bases on a space G, $K \subseteq G$ and $g \in G$.

(a) If $\Im_{j-\omega} \rightsquigarrow k$, then cl$_{j-\omega}$ $(\aleph_k) < \Im$.

(b) If $\Im_{j-\omega} \rightsquigarrow g$, iff cl$_{j-\omega}$ $(\aleph_g) < \Im$.

(c) If $\Im < \wp$, then (al-$_{j-\omega}$-c$_g$ $\wp$) $\subseteq$ (al-$_{j-\omega}$ c$_g\Im$).

(d) If $\Im < \wp$ and $\Im_{j-\omega} \rightsquigarrow K$, then $\wp_{j-\omega} \rightsquigarrow K$.

(e) (al-$_{j-\omega}$ c$_g\Im$) = $\cap$ {cl$_{j-\omega}$ (M): $M \in \Im$}.

(f) If $\Im_{j-\omega} \rightsquigarrow g$ and $g \in K$, then $\Im_{j-\omega} \rightsquigarrow K$.

(g) If $\Im_{j-\omega} \rightsquigarrow K$ iff $\Im_{j-\omega} \rightsquigarrow K \cap$ (al-$_{j-\omega}$-c$_g\Im$).

(h) If $\Im_{j-\omega} \rightsquigarrow K$, then $K \cap$ (al-$_{j-\omega}$ c$_g\Im$) $\neq \phi$.

(i) If $S \subseteq G$ is open, then (al-$_{j-\omega}$-cl(S)) = cl(S).

(j) If $\Im$ is a open filter base, then (al-$_{j-\omega}$ cl$\Im$) = (al-$_{j-\omega}$ c$_g\Im$).

If S is an open ultrafilter on G. Then $S \rightsquigarrow g$ if and only if $S_{j-\omega} \rightsquigarrow g$, where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

Proof: The proof is easy, so it omitted.

Definition 19

The subset K of a space G is said to be quasi -j-ω-H-closed relative to G if every cover $\mathcal{K}$ of K by open subsets of G contains a finite subfamily $L \subseteq K$ such that $K \subseteq \cup\{\text{cl } j\text{-}\omega\text{-}(L) : L \in \mathcal{B}\}$. If G is Hausdorff, we say that K is j-ω-H-closed relative to G. If G is quasi- j-ω-H-closed relative to itself, then G is said to be quasi- j-ω-H-closed (resp. j-ω-H-closed), where $j \in\{\theta, \delta, \alpha, pre, b, \beta\}$.

Theorem 20

The following are equivalent for a subset $K \subseteq G$:

(a) K is quasi-j-ω-H-closed relative to G.

(b) For all filter base $\Im$ on K, $\Im_{\text{j-}\omega} \rightsquigarrow K$.

(c) For all filter base $\Im$ on K, (al $_{\text{-j-}\omega}$ $c_g\Im$) $\cap K \neq \phi$. Where $j \in\{\theta, \delta, \alpha, pre, , b, \beta\}$.

Proof: Clearly (a) $\Rightarrow$ (b), and by Theorem (18, h), (b) $\Rightarrow$ (c). To show (c) $\Rightarrow$ (a), let $\mathcal{K}$ be a cover of K by open subsets of G such that the j-ω-closed of the union of any finite subfamily of $\mathcal{K}$ is not cover K. Then $\Im = \{K - \text{cl } j\text{-}\omega\text{-}_g(\cup_k S_k)$: k is finite subfamily of $\mathcal{K}\}$ is a filter base on K and (al -j-ω-$c_g\Im$) $\cap$ $K = \phi$. This contradiction crop s that K is quasi- j-ω-H-closed relative to G, where $j \in\{\theta, \delta, \alpha, pre, b, \beta\}$.

By concepts of closure directed toward a set, almost j-ω-convergence characterized and related in the next result.

Theorem 21

Let $\Im$ be a filter base on a space G and $K \subseteq G$. Then:

(a) $\Im$ is cl-*dir,-tow* K iff for each cover $\mathcal{K}$ of K by open subsets of G, there is a finite subfamily $L \subseteq K$ and an $M \in \Im$ such that $M \subseteq \cup\{\text{cl } j\text{-}\omega\text{-}(L) : L\in \mathcal{B} \}$, where $j\in \{\theta, \delta, \alpha, pre, , b, \beta\}$.

(b) For every filter base $\wp$, $\Im < \wp$ implies (al- j-ω-$c_g\wp$) $\cap$ $K \neq \phi$ iff $\Im_{\text{j}} {}_{-\omega} \rightsquigarrow K$, where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

Proof: The proofs of the two facts are similar; so, we will only prove the fact (b):

($\Rightarrow$) Suppose for every filter base $\wp$, $\Im < \wp$ implies (al- j-ω-$c_g\wp$) $\cap$ $K \neq \phi$. If $\Im_{\text{j}} {}_{-\omega} \rightsquigarrow g$ for some $g \in K$, then by Theorem (3.3, f), $\Im_{\text{j-}\omega} \rightsquigarrow K$. So, assume that for each $g \in K$, $\Im$ does not $_{j\text{-}\omega} \rightsquigarrow g$. Let $\mathcal{K}$ be a cover of K by subsets open in G. For every $g \in K$, yond is an open set S_g containing g and $T_g \in$ K such that $S_g \subseteq T_g$ and $M-$ cl j-ω-$_g(S_g) \neq \phi$ for every $M \in \Im$. So, $\wp_g = \{M - \text{cl } j\text{-}\omega\text{-}_g(S_g) : M \in \Im\}$ is a filter base on G and $\Im < \wp_g$. Now, $g \notin$ (al- j-ω-$c_g\wp_g$).

Assume that $\cup\{\wp_g$: $g \in K\}$ forms a filter sub base with $\wp$ denoting the generated filter. Then $\Im < \wp$ and (al -j-ω-$c_g\wp$) $\cap K = \phi$. This contradiction implies yond is a finite subset $L \subseteq K$ and $M_g \in \Im$ for $g \in L$ such that, $\phi = \cap\{M_g - \text{cl } j\text{-}\omega\text{-}_g(S_g) : g \in L\}$.There is $M \in \Im$ such that $M \subset \cap\{M_g : g \in L\}$. It easily follows that $\phi = \cap\{M - \text{cl } j\text{-}\omega\text{-}_g(S_g) : g \in L$ and $M \subseteq \cup\{\text{cl } j\text{-}\omega\text{-}_g(T_g)$: $g \in L\}$. Thus $\Im_{\text{j-}\omega} \rightsquigarrow K$.

($\Leftarrow$) Suppose $\Im_{\text{j-}\omega} \rightsquigarrow K$ and $\wp$ is a filter base such that $\Im < \wp$. By Theorem (18, d), $\wp_{\text{j-}\omega} \rightsquigarrow K$, and Theorem (18, h), (al- j-ω-$c_g\wp$) $\cap$ $K \neq \phi$.

5. Filter Bases and *j-ω*-Rigidity

In the section, we defined filter bases, j-ω-rigidity, and the some theorems concerning of them.

Definition 22

A mapping $\lambda : G \to H$ is said to be *j-ω*-closure continuous (resp. *j-ω*-weakly continuous) if for every $g \in G$ and every nbd T of $\lambda(g)$, there exists a nbd S of g in G such that λ(cl *j-ω*-(S)) $\subseteq$ cl *j-ω*-(T) (resp. $\lambda(S) \subseteq$ cl *j-ω*-(T)).

Clearly, every continuous mapping is *j-ω*-closure continuous, where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

The notions of almost *j-ω*-convergence and almost *j-ω*-cluster can used to characterize *j-ω*-closure continuous.

Theorem 23

Let $\lambda : G \to H$ be a mapping. The following are equivalent:

(a) λ is *j-ω*-closure continuous.

(b) For all filter base $\Im$ on G, $\Im_{j\text{-}\omega} \rightsquigarrow g$ implies $\lambda(\Im) \to \lambda(g)$.

For all filter base $\Im$ on G, λ (al- *j-ω*-c$\Im$) $\subseteq$ (al- *j-ω*-c $\lambda(\Im)$. For all open $S \subseteq H$, $\lambda^{-1}(S) \subseteq$ (al- *j-ω*-intλ^{-1}(al- *j-ω*-cl(S))). Where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

Proof: The proof of the equivalence of (a), (b) and (d) is straightforward.

(a) $\Rightarrow$ (c) Suppose $\Im$ is a filter base on G, $g \in$ (al- *j-ω*-c$\Im$), $M \in \Im$ and T is a nbd of $\lambda(g)$, yond is a nbd S of g such that λ (cl *j-ω*- (S)) $\subseteq$ cl *j-ω*-(T). Since cl *j-ω*-(S) $\cap M \neq \phi$, then cl *j-ω*-(T) $\cap \lambda(M) \neq \phi$. So, $\lambda(g) \in$ (al- *j-ω*-c $\lambda(\Im)$). This shows that λ(al- *j-ω*-c$\Im$) $\subseteq$ (al- *j-ω*-c $\lambda(\Im)$).

(c) $\Rightarrow$ (a) Let $\mathcal{S}$ be an ultrafilter containing λ(cl *j-ω*-($\aleph_g$)). Now, $\lambda^{-1}(\mathcal{S})$ is a filter base since $\lambda(G) \in \mathcal{S}$ and $\lambda^{-1}(\mathcal{S})$ meets cl *j-ω*-($\aleph_g$). So, $\lambda^{-1}(\mathcal{S}) \cup$ cl *j-ω*-($\aleph_g$) is contained in some ultrafilter $\mathcal{T}$. Now $\lambda\lambda^{-1}(\mathcal{S})$ is an ultrafilter base that generates $\mathcal{S}$. Since $\lambda\lambda^{-1}(\mathcal{S}) < \lambda(\mathcal{T})$, then $\lambda(\mathcal{T})$ also generates $\mathcal{S}$; hence (al- *j-ω*-c$\lambda(\mathcal{T})$) = (al- *j-ω*-c $\mathcal{S}$). Since $g \in$ (al- *j-ω*-c($\mathcal{T}$)), then $\lambda(g) \in \lambda$(al- *j-ω*-c $\mathcal{T}$) $\subseteq$ (al- *j-ω*-c $\lambda(\mathcal{T})$) = (al- *j-ω*-c $\mathcal{S}$). So, $\mathcal{S}$ meets cl *j-ω*- ($\aleph\lambda_{(g)}$) and cl *j-ω*- ($\aleph\lambda_{(g)}$) $\subseteq \cap\{\mathcal{S} : \mathcal{S}$ ultrafilter, $\mathcal{S} \supseteq \lambda$(cl *j-ω*-($\aleph_g$))$\}$, (denote this intersection by $\wp$). Nevertheless, $\wp$ is the filter generated by (cl *j-ω*-($\aleph_g$)) (see [4]. Proposition I.6.6), so $\text{cl}_{j\text{-}\omega}$ ($\aleph\lambda_{(g)}$) $< \lambda$(cl $_{j\text{-}\omega}$ ($\aleph_g$)). Hence λ is *j-ω*-closure continuous, where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

Corollary 24

If $\lambda: G \to H$ is *j-ω*-closure continuous and $K \subseteq G$, then λ(al- *j-ω*-cl(K)) $\subseteq$ (al- *j-ω*-cl($\lambda(K)$), where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

Here are some similarly proven facts about *j-ω*-weakly continuous mapping.

Theorem 25

Let $\lambda : G \to H$ be a mapping. The following are equivalent:

(a) λ is *j-ω*-weakly continuous.

(b) For all filter base $\Im$ on G, $\Im \to g$ implies $\lambda(\Im)_{j\text{-}\omega} \rightsquigarrow \lambda(g)$.

(c) For all filter base $\Im$ on G, λ(al- *j-ω*-c$\Im$) $\subseteq$ (al- *j-ω*-c $\lambda(\Im)$).

(d) For all open $S \subseteq H$, $\lambda^{-1}(S) \subseteq$ int λ^{-1}(cl *j-ω*-(S)). Where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

Theorem 26

If $\lambda : G \to H$ is *j*-ω-weakly continuous mapping, then

(a) For all $K \subseteq G$, λ(cl *j*-ω-(K)) $\subseteq$ (al- *j*-ω-cl $\lambda(K)$).

(b) For all $L \subseteq H$, λ(cl *j*-ω-(int(cl *j*-ω-$\lambda^{-1}(L)$))) $\subseteq$ cl *j*-ω-(L).

(c) For all open $S \subseteq H$, λ (cl *j*-ω-(S)) $\subseteq$ cl *j*-ω-$\lambda(S)$. Where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

Now, We introduce the definitions of *j*-ω-compact, *j*-ω-rigid set, almost *j*-ω-closed, and *j*-ω-urysohn space as follows.

Definition 27

A mapping $\lambda : G \to H$ is said to be *j*-ω- compact if for every subset C quasi- *j*-ω-H-closed relative to H, $\lambda^{-1}(C)$ is quasi- *j*-ω-H-closed relative to G, where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

Definition 28

A subset K of a space G is said to be *j*-ω-rigid provided whenever $\Im$ is a filter base on G and $K \cap$ (al- *j*-ω-c$_g\Im$) = ϕ, there is an open S containing K and $M \in \Im$ such that cl *j*-ω-(S) $\cap$ $M = \phi$, where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

Definition 29

A mapping $\lambda : G \to H$ is said to be almost *j*-ω-closed if for any set $K \subseteq G$, λ(al- *j*-ω-cl(K)) = (al- *j*-ω-cl $\lambda(K)$), where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

Definition 30

A space G is said to be *j*-ω-Urysohn if every pair of distinct points are contained in disjoint *j*-ω-closed nbds, where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

Before characterizing *j*-ω-rigidity, we can show that a *j*-ω-closure continuous, *j*-ω-compact mapping into a *j*-ω-Urysohn space with a certain property (the "*j*-ω-closure" and "quasi- *j*-ω-H-closed relative" analogue of property α in [15].) is almost *j*-ω-closed.

Theorem 31

Suppose $\lambda : G \to H$ is a *j*-ω-closure continuous mapping and *j*-ω-compact and H is *j*-ω-Urysohn with this property: For each $L \subseteq H$ and $h \in$ (al- *j*-ω-cl(L), there is a subset C quasi-*j*-ω-H-closed relative to H such that $h \in$ (al- *j*-ω-cl($C \cap L$)). Then λ is almost *j*-ω-closed, where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

Proof: Let $K \subseteq H$. By corollary (24), λ (al- *j*-ω-cl (K)) $\subseteq$ (al- *j*-ω-clλ (K)). Suppose $h \in$ (al- *j*-ω-clλ (K)).Yond is a subset C quasi- *j*-ω-H-closed relative to H such that $h \in$(al- *j*-ω-cl($C \cap \lambda(K)$)). Then $\Im$ = {cl *j*-ω-(S) $\cap$ $C \cap \lambda(K)$: $S \in \aleph_h$}, is a filter base on H such that $\Im$ $_{\text{j-}\omega} \rightsquigarrow h$. Now, $\wp$ = {$K \cap \lambda^{-1}(M)$: $M \in \Im$} is a filter base on $K \cap \lambda^{-1}(C)$. Since $\lambda^{-1}(C)$ is quasi- *j*-ω-H-closed relative to H, then there is $g \in$ (al -*j*-ω-c$_g \wp$) $\cap$ $\lambda^{-1}(C)$. By theorem 23, $\lambda(g) \in$ (al- *j*-ω-c$_h\lambda(\wp)$) $\subseteq$ (al$_{\text{- j-}\omega}$ c$_h\Im$). Since $\Im$ $_{\text{j-}\omega} \rightsquigarrow$ h and H is *j*-ω-Urysohn, (al- *j*-ω-c$_h\Im$) = {h}. So, $h \in$ λ(al- *j*-ω-cl (K)), where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

Theorem 32

Let K be a subset of a space G. The following are equivalent:

(a) K is *j*-ω-rigid in G.

(b) For all filter base $\Im$ on G, if $K \cap$ (al- j-ω-c$_g\Im$) = ϕ, then for some $M \in \Im$, $K \cap$ (al- j-ω-cl(M) = ϕ.

(c) For all cover $\mathcal{K}$ of K by open subsets of G, there is a finite subfamily $\mathcal{B} \subseteq \mathcal{K}$ such that $K \subseteq$ int cl j-ω-($\cup \mathcal{B}$). Where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

Proof: The proof that (a) $\Rightarrow$ (b) is straightforward. (b) $\Rightarrow$ (c) Let $\mathcal{K}$ be a cover of K by open subsets of G and $\Im = \{\cap_{S\in\mathcal{B}}(G -$ cl j-ω-$(S))$: $\mathcal{B}$ is a finite subset of $\mathcal{K}\}$. If $\Im$ is not a filter base, then for some finite subfamily $\mathcal{B} \subseteq \mathcal{K}$, $G \subseteq \cup\{$cl j-ω-(S) : $S \in \mathcal{B}\}$; thus, $K \subseteq G \subseteq$ int cl j-ω-($\cup \mathcal{B}$) which completes the proof in the case that $\Im$ is not a filter base. So, suppose $\Im$ is a filter base. Then $K \cap$ (al- j-ω-c$\Im$) = ϕ and there is an $M \in \Im$ such that $K \cap$ (al- j-ω-cl (M)) = ϕ. For each $x \in K$, yond is open T_g of g such that cl j-ω-(T_g) $\cap M = \phi$. Let $T = \cup\{T_g : g \in K\}$. Now, $T \cap M = \phi$. Since $M \in \Im$, then for some finite subfamily $\mathcal{B} \subseteq \mathcal{K}$, $M = \cap\{G -$ cl j-ω-(S) : $S \in \mathcal{B}\}$. It follows that $T \subseteq$ cl j-ω- ($\cup\mathcal{B}$) and hence, $K \subseteq$ int cl j-ω-($\cup \mathcal{B}$), where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

(c) $\Rightarrow$ (a) Let $\Im$ be a filter base on G such that $K \cap$ (al -j-ω-c$\Im$) = ϕ. For all $g \in K$ yond is open T_g of g and $M_g \in \Im$ such that cl j-ω-(T_g) $\cap M_g = \phi$. Now $\{T_g$: $g \in K\}$ is a cover of K by open subsets of G; so, there is finite subset $L \subseteq K$ such that $K \subseteq$ int cl j-ω- ($\cup\{T_g$: $g \in T\}$). Let S = int cl j-ω-($\cup\{T_g$: $g \in L\}$). Yond is $M \in \Im$ such that $M \subseteq \cap\ \{M_g$: $g \in L\}$. Since cl j-ω-(S) = $\cup\{$cl j-ω-(T_g): $g \in L\}$, then cl j-ω-(S) $\cap M = \phi$. So K is j-ω-rigid in G, where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

6. Filter Bases and *j-ω*-Perfect Mappings

In the section, we defined filter bases, j-ω-perfect mappings, and the some theorems about them.

In Corollary 14, we show that a mapping $\lambda : G \to H$ is perfect (i.e. closed and $\lambda^{-1}(y)$ compact for each $h \in H$) iff for all filter base $\Im$ on $\lambda(G)$, $\Im \rightsquigarrow h \in H$, implies $\lambda^{-1}(\Im)$ is (*cl-dir-tow)* $\lambda^{-1}(y)$ and in Corollary 15, proved that a perfect mapping is compact (i.e. inverse image of compact sets are compact). In view Theorem 21, we say that a mapping $\lambda : G \to H$ is j-ω-perfect if for every filter base $\Im$ on $\lambda(G)$, $\Im_{\text{j-}\omega} \rightsquigarrow h \in H$ implies $\lambda^{-1}(\Im)_{\text{j-}\omega} \rightsquigarrow \lambda^{-1}(h)$, where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

Theorem 33

Let $\lambda : G \to H$ be a mapping. The following are equivalent:

(a) λ is j-ω-perfect.

(b) For all filter base $\Im$ on G, (al- j-ω-(c $\lambda(\Im)$) $\subseteq \lambda$(al- j-ω-(c$\Im$).

(c) For all filter base $\Im$ on $\lambda(G)$, $\Im_{\text{j-}\omega} \rightsquigarrow L \subseteq H$, implies $\lambda^{-1}(\Im)_{\text{j-}\omega} \rightsquigarrow \lambda^{-1}(L)$. Where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

Proof: (a) $\Rightarrow$ (b) Assume $\Im$ is a filter base on G and $h \in$ (al- j-ω-c $\lambda(\Im)$). For if not. Assume that $\lambda^{-1}(h) \cap$ (al- j-ω-(c$\Im$) = ϕ. For each $g \in \lambda^{-1}(h)$, yond is open S_g of g and $M_g \in \Im$ such that cl j-ω-(S_g) $\cap M_g = \phi$. Since λ^{-1}(cl j-ω-($\aleph_h$))$_{\text{j-}\omega} \rightsquigarrow \lambda^{-1}(y)$ and $\{S_g : g \in \lambda^{-1}(h)\}$ is an open cover of $\lambda^{-1}(y)$, yond is a $V \in \aleph_h$ and a finite subset $B \subseteq \lambda^{-1}(y)$ such that λ^{-1}(cl j-ω-(T)) $\subseteq$ $\cup\{$cl j-ω-(T_g): $g \in L$ $\}$. Yond is an $M \in \Im$ such that $M \subseteq \cap\ \{M_g$: $g \in L\}$. Thus, $M \cap \lambda^{-1}$(cl j-

ω-(T)) = ϕ implying cl j-ω-(T) $\cap$ λ(M) = ϕ, a contradiction as $h \in$ (al- j-ω-c $\lambda(\Im)$). This shows that $h \in \lambda$(al- j-ω-c$\Im$), Where $j \in \{pre, , b, \alpha, \beta\}$.

(b) $\Rightarrow$ (c) Assume $\Im$ is a filter base on $\lambda(G)$ and $\Im$ $_{j\text{-}\omega} \rightsquigarrow L \subseteq H$. Let $\wp$ be a filter base on G such that $\lambda^{-1}(\Im) < \wp$. Then $\Im < \lambda(\wp)$ and (al$_{\text{-} j\text{-}\omega}$ c $\lambda(\wp)$) $\cap$ $L \neq \phi$. Therefore λ(al- j-ω-c$\wp$) $\cap$ L $\neq \phi$ and (al- j-ω-c$\wp$) $\cap$ $\lambda^{-1}(L) \neq \phi$. By Theorem (3.6, b), $\lambda^{-1}(\Im)$ $_{j\text{-}\omega} \rightsquigarrow \lambda^{-1}(L)$, where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

(c) $\Rightarrow$ (a) Clearly.

Corollary 34

If $\lambda : G \rightarrow H$ is j-ω-perfect mapping, then:

(a) For all $K \subseteq G$, (al- j-ω-cl $\lambda(K)$) $\subseteq \lambda$(al- j-ω-cl (K)).

(b) For all almost j-ω-closed $K \subseteq G$, $\lambda(K)$ is almost j-ω-closed.

(c) λ is j-ω-compact. Where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

Proof: (a) is an immediate consequence of Theorem 33, and (b) follows easily from (a). To prove (c) Let C be quasi- j-ω-H-closed relative to H , and $\wp$ be a filter base on $\lambda^{-1}(C)$, then $\lambda(\wp)$ is a filter base on C. By Theorem 20, (al- j-ω-c$\lambda(\wp)$) $\cap$ $C \neq \phi$ and by Theorem (33, b), (al- j-ω-c$\wp$) $\cap$ $\lambda^{-1}(C) \neq \phi$. By Theorem 20, $\lambda^{-1}(C)$ is quasi- j-ω-H-closed relative to G, where j$\in \{\theta, \delta, \alpha, pre, b, \beta\}$.

Theorem 35

An j-ω-closure continuous mapping $\lambda : G \rightarrow H$ is j-ω-perfect if and only if

(a) λ is almost j-ω-closed, and

(b) $\lambda^{-1}(y)$ j-ω-rigid for each $h \in H$, where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

Proof: ($\Rightarrow$) If λ is j-ω-closure continuous and j-ω-perfect mapping, then by Corollaries 34 and 24, λ is almost j-ω-closed. To show $\lambda^{-1}(h)$, for $h \in H$, is j-ω-rigid, Let $\Im$ be a filter base on G such that $\lambda^{-1}(h)$ $\cap$ (al -j-ω-c$\Im$) = ϕ. So, $h \notin \lambda$(al- j-ω-c$\Im$) and by Theorem (33, b), $h \notin$ (al$_{j\text{-}\omega}$ c $\lambda(\Im)$). Yond is open S of h and $M \in \Im$ such that cl j-ω-(S) $\cap$ $\lambda(M)$ = ϕ. So, λ^{-1}(cl j-ω-(S)) $\cap$ M = ϕ. Since λ is j-ω-closure continuous, then for any $g \in \lambda^{-1}(h)$, yond is open T of g such that cl j-ω-(T) $\subseteq \lambda^{-1}$(cl j-ω-(S)). So, $\lambda^{-1}(h)$ $\cap$ cl j-ω-(M) = ϕ, where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

($\Leftarrow$) Assume that j-ω-closure continuous mapping λ satisfies (a) and (b). Let $\Im$ be a filter base on $\lambda(G)$ such that $\Im$ $_{j\text{-}\omega} \rightsquigarrow h$. Let $\wp$ be a filter base on G such that $\lambda^{-1}(\Im) < \wp$. So, $\Im < \lambda(\wp)$ implying that $h \in$ (al- j-ω-c $\lambda(\wp)$). Therefore, for each $\mathcal{G} \in \wp$, $h \in$ (al- j-ω-cl$\lambda(\mathcal{G})$) $\subseteq \lambda$(al- j-ω-cl $\mathcal{G}$). Hence, $\lambda^{-1}(h)$ $\cap$ (al- j-ω-cl $\mathcal{G}$) $\neq \phi$ for each $\mathcal{G} \in \wp$. By (b), $\lambda^{-1}(h)$ $\cap$ (al- j-ω-c$\wp$) $\neq \phi$. By Theorem 33, λ is j-ω-perfect mapping, where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

Actually, in the proof of the converse of Theorem 35, we have shown that property (a) of Theorem 35 can reduced to this statement: For each $K \subseteq G$, al j-ω-cl $\lambda(K) \subseteq \lambda$ (al j-ω-cl (K); in fact, we have shown the next corollary (the mapping is not necessarily j-ω-closure continuous).

Corollary 36

Let $\lambda : G \rightarrow H$ be a mapping if

(a) For all $K \subseteq G$, (al- j-ω-cl $\lambda(K)$) $\subseteq \lambda$(al- j-ω-cl (K)

(b) $\lambda^{-1}(h)$ *j*-ω-rigid for each $h \in H$, then λ is *j*-ω-perfect, where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

Corollary 37

Let $\lambda : G \to H$ be a mapping.

(a) λ is almost *j*-ω closed

(b) $\lambda^{-1}(h)$ *j*-ω rigid for each $h \in H$, then λ^{-1} preserves *j*-ω rigidity, where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

Proof. Let $C \subseteq H$ be *j*-ω rigid and $\Im$ be a filter base on G such that al *j*-ω $c_g\Im \cap \lambda^{-1}(C) = \phi$. By Corollary 36 and Theorem 33, (al- *j*-ω c$\lambda(\Im)) \cap C = \phi$. So, there is $M \in \Im$ such that (al- *j*-ω cl $\lambda(M)) \cap C = \phi$. Nevertheless (al- *j*-$\omega$ clλ $(M)) = \lambda$(al- *j*-ω cl (M)). So, (al- *j*-ω cl $(M)) \cap \lambda^{-1}(C) = \phi$. So, by Theorem 32, $\lambda^{-1}(C)$ is *j*-ω rigid, where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

Theorem 38

Suppose $\lambda : G \to H$ has *j*-ω rigid point-inverses. Then:

(a) λ is *j*-ω closure continuous iff for each $h \in H$ and open set T containing h, there is an open set S containing $\lambda^{-1}(h)$ such that λ(cl *j*-ω $(S)) \subseteq$ cl *j*-$\omega(T)$, where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

(b) If for each $h \in G$ and open set S containing $\lambda^{-1}(h)$, there is an open set T of h such that λ^{-1}(cl *j*-ω $(T)) \subseteq$ cl *j*-ω (S), then for each $K \subseteq G$, (al- *j*-ω cl$(\lambda(K)) \subseteq \lambda$(al- *j*-$\omega$ cl(K)), where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

Proof. (a) ($\Rightarrow$) Is obvious.

($\Leftarrow$) Is straightforward using Theorem (32, c) (b) Let $\phi \neq K \subseteq G$ and $h \notin \lambda$(al- *j*-ω cl (K)). Then $\lambda^{-1}(h) \cap$ (al- *j*-ω cl $(K)) = \phi$. Now, $\Im = \{K\}$ is a filter base and (al -*j*-ω c$\Im) \cap \lambda^{-1}(h) = \phi$. So, yond is open set S continuing $\lambda^{-1}(h)$ such that cl *j*-ω $(S) \cap K = \phi$, yond is open T of h such that λ^{-1}(cl *j*-$\omega(T)) \subseteq$ cl *j*-$\omega(S)$. Therefore, cl *j*-ω $(T) \cap \lambda$ $(K) = \phi$. Hence $h \notin$ (al- *j*-ω cl $\lambda(K)$), where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

The next result related to Theorem (38, b); the proof is straightforward.

Theorem 39

Let $\lambda : G \to H$. The following are equivalent:

(a) For all *j*-ω-closed $K \subseteq G$, $\lambda(K)$ is *j*-ω-closed, where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

(b) For all $L \subseteq H$ and *j*-ω open S containing $\lambda^{-1}(L)$, there is *j*-ω-open T containing L such that $\lambda^{-1}(T) \subseteq S$, where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

Theorem 40

If $\lambda : G \to H$ is *j*-ω closure continuous and H is *j*-ω Urysohn, then λ is *j*-ω perfect if and only if for all filter base $\Im$ on G, if $\lambda(\Im)_{j\text{-}\omega} \rightsquigarrow h \in H$, then (al- *j*-$\omega$ $_{cg}\Im) \neq \phi$, where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

Proof. ($\Rightarrow$) Assume that λ is *j*-ω perfect and $\lambda(\Im)_{j\text{-}\omega} \rightsquigarrow h$. Therefore, $\lambda^{-1}(\Im)_{j\text{-}\omega} \rightsquigarrow \lambda^{-1}(h)$. Since $\lambda^{-1}\lambda(\Im) < \Im$, then by Theorem (18, d), $\Im_{j\text{-}\omega} \rightsquigarrow \lambda^{-1}(h)$, by Theorem (18, h), (al - *j*-ω c $\Im) \neq \phi$.

($\Leftarrow$) Assume that for each filter base $\Im$ on G, if $\lambda(\Im)_{j\text{-}\omega} \rightsquigarrow h \in G$, then (al- *j*-$\omega$ $c_g\Im) \neq \phi$. Suppose $\wp$ is a filter base on $\lambda(G)$ such that $\wp_{j\text{-}\omega} \rightsquigarrow h \in H$, and assume $\pounds$ is a filter base on G such that $\lambda^{-1}(\wp) < \pounds$. Then $\wp = \lambda\lambda^{-1}(G) < \lambda(\pounds)$. So, $\lambda(\pounds)_{j\text{-}\omega} \rightsquigarrow h$. Therefore, (al- *j*-$\omega$-$c_g$

$\mathcal{L}$) $\neq \phi$. Let $i \in H - \{h\}$. Because of H j-ω-Urysohn, yond are open sets S_i of i and S_h of h such that cl j-ω-(S_i) ∩ cl j-ω-(S_h) = ϕ. Yond is $H \in \mathcal{L}$ such that $\lambda(H) \subseteq$ cl j-ω- (S_h). For every $g \in \lambda^{-1}(i)$, there is open T_i of i such that λ (cl j-ω- (T_i)) $\subseteq$ cl j-ω- (S_i). So, cl j-ω- (T_g) ∩ H = ϕ. It follows that $\lambda^{-1}(i)$ ∩ (al- j-ω-c$_g$ $\mathcal{L}$) = ϕ for each $i \in H - \{h\}$. So, (al- j-ω-c$_g$ $\mathcal{L}$) ∩ $\lambda^{-1}(h) \neq \phi$ and λ is j-ω-perfect, where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

Corollary 41

If $\lambda : G \rightarrow H$ be a mapping is j-ω-closure continuous, G is quasi- j-ω-H-closed, and H is j-ω-Urysohn, then λ is j-ω-perfect, where $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

Proof. Since G is quasi- j-ω-H-closed, then all filter base on G has non void almost j-ω-cluster; now, the corollary follows directly from Theorem 35, Where $j \in \{\theta, \delta, \alpha, pre, , b, \beta\}$.

7. Conclusions

The starting point for the application of abstract topological structures in j-ω-perfect mapping is presented in this paper. We use filter base to introduce a new notion namely filter base and j-ω-perfect mapping. Finally, certain theorems and generalization concerning these concepts of studied; $j \in \{\theta, \delta, \alpha, pre, b, \beta\}$.

References

1. Riesz, F. Stetigkeitsbegriff and und abstrakte Mengenlehre. *Atti ael IV Congresso Internazionale dei Matematici*.**1909**, *2*, 18-24.
2. Cartan, H. *Theorie des filters*. Acad. Paris. 1st edition,**1937**.
3. Cartan, H. *Filters Ultrafilters*. Acad. Paris. 2nd edition,**1937**a.
4. Bourbaki, N. *General Topology*. Part I. Addison-Wesly. Reding. Mass,**1975**.
5. Whyburn, G.T. Directed families of sets and closedness of functions. *Proc. Nat. Acad. Sci.***1965**, *54*, 688–692.
6. Dickman, R.F.; Porter, J.R. θ-perfect and θ- absolutelyclosed functions. *Illinois Jour. Math*.**1977**, *21*, 42-60.
7. Porter, J.R.; Thomas, J.D. On H-closed and minimal Hausdorff spaces. *Trans. Amer. Math. Soc*.**1969**, *138*, 159–170.
8. Levine, N.A. Decomposition of Continuity in Topological Spaces. *American Mathematical Monthly*.**1961**, *38*, 413-418.
9. Andrew, D.R; Whittlesy, E.K. Closure Continuity. *American Mathematical Monthly*. **1966**, *73*, 758-759.
10. Willard, S. *General Topology*. Addison Wesley Publishing Company, Inc. 1st edition,**1970.**
11. Hdeib, H.Z. ω-closed mapping. *Revista Colombian a de Mathematics*.**1982**, *XVI*, 65-78.
12. Yousif, Y.Y. ω-Perfect Mappings. *Ibn Al– Haitham Journal for Pure and Applied Science*.**2007**, *20*, *2*, 141-149.
13. Mashhour, A.S.; Abd El-Monsef, M.E.; El-Deeb, S.N. On pre continuous and weak pre continuous mappings. *Proc. Math. Phys. Soc*.**1982**, *53*, 47-53.
14. Abd El-Monsef, M.E.; El-Deeb, S.N.; Mohmoud, R.A. β-open sets and β-continuous mappings. *Bull. Fac. Sc. Assuit Univ*.**1983**, *12*, 77-90.
15. Andrijevic, D. On b-open sets. *Mat. Vesnik*.**1996**, *48*, 59-64.
16. Velicko, N.N. H-closed topological spaces. *American Mathematical Society Translations*.**1968**, *78*, *2*, 103–118.

17. Yousif, Y.Y. Filter Bases and Supra Perfect Functions. *Science International*.**2019**, *31*, *1*, 23-27.
18. Yousif, Y.Y.; Hussain, L.A. Fibrewise IJ-Perfect Bitopological Spaces. *Conf. Series: Journal of Physics*.**2018**, *1003*, *012063*, 1-12, doi:10.1088/1742-6596/1003/1/012063.
19. Yousif, Y.Y.; Jassim, N.S. IJ-ω-Perfect Functions between Bitopological Spaces. *Ibn Al–Haitham Journal for Pure and Applied Science*.**2010**, *23*, *2*, 135-140.