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Approximate Solutions for Alcohol Consumption Model in Spain

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Abstract

In this paper, our aim is to solve analytically a nonlinear social epidemic model as an initial value problem (IVP) of ordinary differential equations. The mathematical social epidemic model under study is applied to alcohol consumption model in Spain. The economic cost of alcohol consumption in Spain is affected by the amount of alcohol consumed. This paper refers to the study of alcohol consumption using some analytical methods. Adomian decomposition and variation iteration methods for solving alcohol consumption model have used. Finally, a compression between the analytic solutions of the two used methods and the previous actual values from 1997 to 2007 years is obtained using the absolute and relative errors. The analysis results obtained have been discussed tabularly and graphically.

Keywords: Ordinary differential equations, Adomian decomposition method, Variation iteration method, social epidemiology.

1. Introduction

The type of epidemiological models had been used in the study of many social diseases. In recent years, several researchers were interested to study and analyze the social epidemics, ecstasy or heroin addiction [1, 2]. Smoking habit evaluation in Spain [3-5]. a cocaine abuse in Spain [6, 7]. Campaigns on reducing excess weight in Valencia [8-10].

Alcohol consumption habit is considered a disease that spread out rapidly by social pressure or social contact which influences the spread of social disease. Recently, the rate of alcohol consumption has increased more with the developing countries, so alcohol consumption represents a big problem that efforts not only on the human health, but also on the community economy. The public health care becomes more expensive when the number of people who are suffering from such diseases increased, so the community economy will be damaged. With regard to the health effects of chloride that impact on the healthy body, the chloride can damage some parts of the body, such as heart and liver, influence also on the other functions. An addition that, the cost of alcohol affects the economy [11, 12].

Adomian decomposition method (ADM) is a reliable method to solve many various kinds of problems so it is a trusty method which emerging in applied science, linear and nonlinear as well. The Adomian decomposition method was introduced and developed by George Adomian. This method has been used by many researchers as well as it has extensive applications of linear and nonlinear ordinary differential equations, partial differential equations and integral equations [13, 14]. The modified ADM and its applications on some equations have also been given; see examples in [15-18]. ADM was introduced and developed by George Adomian, 1976. This method took a large part of the researcher's work, where it was applied widely on the linear and nonlinear differential equations, as well as on the integral equations [19, 14]. Some researchers used the ADM to solve a system of ordinary differential equations [20]. and apply at epidemic model [21, 22]. As well as, ADM solved a system of integral–differential equations [23]. Recently, the accuracy of nonlinear singular initial value problems was discussed using a semi-analytic [24]. ADM is applied to solve fuzzy fractional order differential algebraic equations [25]. Modified Adomian Decomposition Method was applied on Integro-Differential Inequality [26].

The Variational Iteration method (VIM) was established by Ji-Huan in 1997 where it is considered as one of the reliable iterative methods that give approximate solutions for the of differential equations [27, 28]. This method is used widely in scientific and engineering applications, where it is used to solve linear, nonlinear and homogeneous and inhomogeneous equations. VIM is used for the analytical purpose because it is an effective and reliable way in the approaching quickly to the exact solution. The VIM method differs from the ADM, where VIM does not require specific treatments for nonlinear problems [14]. Many works to solve nonlinear problems using VIM [27]. With autonomous ordinary differential systems [29]. And to solve differential equations that have fractional order [30].

The application of real social epidemic alcohol consumption model is considered in this paper, in order to study the behavior of this social epidemic during sometimes under study. The feature of such model under study that the model has multiple parameters and multivariate. As well as, it is a nonlinear system of first-order ordinary differential equations for autonomous initial value problem that don't have mostly exact solution. Our interested is to solve analytically such these models. This study is organized as follows: in Section 2, the mathematical model of alcohol consumption is described; Section 3 derived the analytic methods ADM and VIM to solve the nonlinear system of alcohol consumption model in Spain. Following, in Section 4, the results of the presented methods is discussed. Finely, Section 5 is devoted to the conclusion of the study.

2. Mathematical Model

Alcohol Ireland and the United Kingdom, this problem was discussed and the data were reported from the first year of 2002 until the end of 2014 [31]. In Spain, the effects of the different usage of alcohol between the female and male in Spanish university alumni was studied [32].The mathematical model of alcohol consumption was explained and described in the current study by Santonja et al., (2010). This model consists of three subpopulations of Spanish population of the age of 15-64 years old from 1997 to 2007 years that represented as the nonlinear system of three ordinary differential equations of the first order. This system is referred to analyze the changing in the social epidemic stages (non-drink alcohol people, non-risk-drink alcohol people and risk-drink alcohol people), see **Table 1.** The parameters of the model are described in **Table 2.** Alcohol consumption model for the Spanish public appears as the following nonlinear system of ordinary differential equations, [12]. consumption is a dangerous social habit that spread in many countries such as

$$a'(t) = \alpha + \beta * r - \mu * a - \xi * a * (m + r) - a * (\alpha - \mu * a - \sigma * m - \sigma * r) \quad (1)$$

$$m'(t) = \xi * a * (m + r) - \eta * m + \mu * a * m - \sigma * a * m - \alpha * m \quad (2)$$

$$r'(t) = \eta * m - \beta * r + \mu * a * r - \sigma * a * r - \alpha * r \quad (3)$$

The initial conditions of equations (1-3) in 1997 are $a(t = 0) = 0.362$, $m(t = 0) = 0.581$, $r(t = 0) = 0.057$, with the predicted parameters are given as: $\alpha = 0.01$, $\beta = 0.0014$, $\mu = 0.008$, $\xi = 0.0284534$, $\sigma = 0.009$, and $\eta = 0.000110247$,[12].

Table 1. Variables of alcohol consumption model, [12].

$\boldsymbol{a(t)}$	Non-drink alcohol people are subpopulations who never drink alcohol in their life.
$\boldsymbol{m(t)}$	Non-risk-drink alcohol people are subpopulations who drink a little liquid of alcohol alcohol that means the men who drink less than 50 cc and women who drink less than 30 cc of alcohol every day.
$\boldsymbol{r(t)}$	Risk-drink alcohol people are subpopulations who drink a lot of alcohol that means the men who drink more than 50 cc and the women who drink more than 30 cc of alcohol every day.

Table 2. Parameters of alcohol consumption model, [12].

$\boldsymbol{\alpha}$	Birth rate in Spain
$\boldsymbol{\beta}$	The rate at which a risk-drink alcohol people becomes a non-drink alcohol people
$\boldsymbol{\mu}$	Death rate in Spain
$\boldsymbol{\xi}$	Transmission rate because social pressure that leads to increasing the alcohol drinking
$\boldsymbol{\sigma}$	Growing death rate due to alcohol drinking
$\boldsymbol{\eta}$	A rate that transmits a non-risk-drink alcohol people moves to the risk-drink alcohol people

3. Analytic Methods

In this section some analytic methods are introduced such

3.1 Adomian Decomposition Method

We can solve the nonlinear system of equations (1-3) by using the Adomian decomposition method with the above initial condition and the given parameters. Let l be an operator that is given by $= \frac{d}{dt}$, the inverse of this operation is $l^{-1} = \int_0^t (.)dt$, then by applying l^{-1} for both sides of equations (1-3), we obtain:

$$a(t) - a(0) = l^{-1}\Big(\alpha + \beta * r(t) - \mu * a(t) - \xi * a(t) * \big(m(t) + r(t)\big) - a(t) * \big(\alpha - \mu * a(t) - \sigma * m(t) - \sigma * r(t)\big)\Big), \text{ where } a_0(t) = 0.362.$$

Similarity,

$m(t)-m(0)=l^{-1}\left(\xi*a(t)*\left(m(t)+r(t)\right)-\eta*m(t)+\mu*a(t)*m(t)-\sigma*a(t)*\right.$

$\left.m(t)-\alpha*m(t)\right)$, where $m_0(t)=0.581$, and

$r(t)-r(0)=l^{-1}\left(\eta*m(t)-\beta*r(t)+\mu*a(t)*r(t)-\sigma*a(t)*r(t)-\alpha*r(t)\right)$,

where $r_0(t)=0.057$.

The above equations equivalent the following equations (4-6):

$a_{k+1}(t)=l^{-1}\big(\alpha+\beta*r_k(t)-\mu*a_k(t)-\xi*A_k(t)-\xi*B_k(t)-\alpha*a_k(t)+\mu*$
$C_k(t)+\sigma*A_k(t)+\sigma*B_k(t)\big)$, forall$k\geq 0$. (4)

$m_{k+1}(t)=l^{-1}\big(\xi*\left(A_k(t)+B_k(t)\right)-\eta*m_k(t)+\mu*A_k(t)-\sigma*A_k(t)-\alpha*m_k(t)\big)$, for all $k\geq 0$. (5)

$r_{k+1}(t)=l^{-1}\big(\eta*m_k(t)-\beta*r_k(t)+\mu*B_k(t)-\sigma*B_k(t)-\alpha*r_k(t)\big)$,
for all $k\geq 0$. (6)

The general form of the non-linear borders $A_k(t)$, $B_k(t)$ and $C_k(t)$ have to be:

$A_k(t)=(\sum_{n=0}^{2}a_n(t))*(\sum_{n=0}^{2}m_n(t))$, for all $k=0,1,2$ (7)

$B_k(t)=(\sum_{n=0}^{2}a_n(t))*(\sum_{n=0}^{2}r_n(t))$, for all $k=0,1,2$ (8)

$C_k(t)=\left(\sum_{n=0}^{2}a_n(t)\right)^{,2}$ for all $k=0,1,2$ (9)

The non-linear borders of $A_0(t)$, $B_0(t)$ and $C_0(t)$ as the following:

$A_0(t)=a_{0(t)}*m_0(t), B_0(t)=a_0(t)*r_0(t),\ C_0(t)=\left(a_0(t)\right)^{.2}$

Substituting for all $a_0(t), r_0(t), A_0(t), B_0(t)$ and $C_0(t)$ by Eq.(4), to obtain

$a_1(t)=-0.00009881t$,

Substituting for all $m_0(t), A_0(t)$ and $B_0(t)$ by Eq.(5), to find

$m_1(t)=0.00048711t$,

Substituting for all $m_0(t), r_0(t)$ and $B_0(t)$ by Eq.(6), to get

$r_1(t)=-0.00060638t$,

Now we will find $a_2(t), m_2(t)$ and $r_2(t)$.

The non-linear borders of $A_1(t)$, $B_1(t)$ and $C_1(t)$ are given in the following formula:

$A_1(t)=a_0(t)*m_1(t)+m_0(t)+a_1(t)$,

$B_1(t)=a_0(t)*r_1(t)+r_0(t)*a_1(t)$,

$C_1(t) = 2 * a_0(t) * a_1(t),$

Substituting for all $a_1(t), r_1(t)$. $A_1(t), B_1(t)$ and $C_1(t)$ by Eq.(4), to obtain

$a_2(t) = 1.2117815 * 10^{-8} t^2$

Substituting also for all $m_1(t), A_1(t)$ and $B_1(t)$ by Eq.(5), to get

$m_2(t) = -0.00000403 t^2,$

Substituting for all $m_1(t), r_1(t) and\ B_1(t)$ by Eq.(6), to have

$r_2(t) = 0.00000359 t^2,$

At the same previous steps, the non-linear borders of $A_2(t)$, $B_2(t)$ and $C_2(t)$ are given as:

$A_2(t) = a_0(t) * m_2(t) + a_1(t) * m_1(t) + m_0(t) * a_2(t),$

$B_2(t) = a_0(t) * r_2(t) + a_1(t) * r_1(t) + r_0(t) * a_2(t),$

$C_2(t) = 2 * a_0(t) * a_2(t) + \left(a_1(t)\right)^2,$

Substituting for all $a_2(t), r_2(t). A_2(t), B_2(t)$ and $C_2(t)$ by Eq.(4), to be

$a_3(t) = 2.55439073 * 10^{-11} t^3,$

Substituting for all $m_2(t), A_2(t)$ and $B_2(t)$ by Eq.(5), to find

$m_3(t) = 1.27758808 * 10^{-8} t^3,$

Substituting for all $m_2(t), r_2(t)$ and $B_2(t)$ by Eq.(6), to get

$r_3(t) = -1.42663059 * 10^{-8} t^3.$

The Adomian decomposition method assumes that the unknown functions $a(t)$, $m(t)$ and $r(t)$ that can be written by series as follows:

$a(t) = \sum_{k=0}^{\infty} a_k(t), m(t) = \sum_{k=0}^{\infty} m_k(t), r(t) = \sum_{k=0}^{\infty} r_k(t),$

$$\sum_{k=0}^{\infty} a_k(t) = a_1(t) + a_2(t) + a_3(t) + \cdots$$

$$a(t) = 0.362 - 0.00009881 t + 1.21178155 * 10^{-8} t^2 + 2.55439073 * 10^{-11} t^3 + \quad (10)$$

$$\sum_{k=1}^{\infty} m_k(t) = m_1(t) + m_2(t) + m_3(t) + \cdots$$

$$m(t) = 0.581 + 0.00048711 t - 0.00000403 t^2 + 1.27758808 * 10^{-8} t^3 + \quad (11)$$

$$\sum_{k=1}^{\infty} r_k(t) = r_1(t) + r_2(t) + r_3(t) + \cdots$$

$$r(t) = 0.057 - 0.00060638t + 0.00000359t^2 - 1.42663059 * 10^{-8}t^3 + \qquad (12)$$

$a(t)$, $m(t)$ and $r(t)$ of ADM results are unsettled terms.

3.2 Variation Iteration Method

The VIM gives a better approximate solution by constructing a correctional functional that uses an initial function. Where Lagrange multiplier considers the key of the correction functional which can be specified via variation theory [14]. The nonlinear system of alcohol model under study, [12]. can be solved by using the VIM with given initial condition and parameters. The correction functional of the system (1-3) becomes:

$$a_{k+1}(t) = a_k(t) + \int_0^t \lambda \Big(a'_k(t) - \big(\alpha + \beta * r_k(t) - \mu * a_k(t) - \xi * a_k(t) * (m_k(t) + r_k(t)) - a_k(t) * (\alpha - \mu * a_k(t) - \sigma * m_k(t) - \sigma * r_k(t))\big)\Big) dt, for\ all\ k \geq 0. \qquad (13)$$

$$m_{k+1}(t) = m_k(t) + \int_0^t \lambda \Big(m'_k(t) - \big(\xi * a_k(t) * (m_k(t) + r_k(t)) - \eta * m_k(t) + \mu * a_k(t) * m_k(t) - \sigma * a_k(t) * m_k(t) - \alpha * m_k(t)\big)\Big) dt, for\ all\ k \geq 0. \qquad (14)$$

$$r_{k+1}(t) = r_k(t) + \int_0^t \lambda \Big(r'_k(t) - \big(\eta * m_k(t) - \beta * r_k(t) + \mu * a_k(t) * r_k(t) - \sigma * a_k(t) * r_k(t) - \alpha * r_k(t)\big)\Big) dt, for\ all\ k \geq 0 \qquad (15)$$

The Lagrange multiplier is $\lambda = -1$ in equations (10-12). By substituting this value in equations (10), (11) and (12). The zero borders will be:

$a_0(t) = 0.362, m_0(t) = 0.581, r_0(t) = 0.057.$

In equations (13-15), if we substitute (k=0), we obtain the following $a_1(t), m_1(t)$ and $r_1(t)$.

$a_1(t) = 0.362 + 0.00011927t.$

$m_1(t) = 0.581 + 0.00048711t.$

$r_1(t) = 0.057 - 0.00060638t.$

By the same way, if we have (k=1), in equations (13-15) we get the following $a_2(t), m_2(t)$ and $r_2(t)$.

$a_2(t) = 0.362 + 0.00011927t - 0.00000147t^2 + 1.30183484 * 10^{-10}t^3,$

$m_2(t) = 0.581 + 0.00048711t - 0.00000212t^2 - 1.54291666 * 10^{-10}t^3,$

$r_2(t) = 0.057 - 0.00060638t + 0.00000359t^2 + 2.41081825 * 10^{-11}t^3,$

Continuing in the same manner when (k=2), we can a achieved the following $a_3(t), m_3(t)$ and $r_3(t)$.

$$a_3(t) = 0.362 + 0.00011928t - 0.00000147t^2 + 1.04339442 * 10^{-8}t^3 - 2.97475940 * 10^{-12}t^4 + 1.20789692 * 10^{-14}t^5 - 1.75446808 * 10^{-18}t^6 + 6.64675814 * 10^{-23}t^7 \qquad (16)$$

$$m_3(t) = 0.581 + 0.00048710t - 0.00000212t^2 + 3.66528836 * 10^{-9}t^3 + 3.38204598 * 10^{-12}t^4 - 1.31514568 \times 10^{-14}t^5 + 1.82643463 * 10^{-18}t^6 - 6.60192260 * 10^{-23}t^7 \qquad (17)$$

$$r_3(t) = 0.057 - 0.0006064t + 0.00000359t^2 - 1.40992325 * 10^{-8}t^3 - 4.07286579 * 10^{-13}t^4 + 1.07248763 * 10^{-15}t^5 - 7.19665484 * 10^{-20}t^6 - 4.48355311 * 10^{-25}t^7 \quad (18)$$

And so on, continue in order to get better approximations:
$a(t) = \lim_{k\to\infty} a_k(t), m(t) = \lim_{k\to\infty} m_k(t)$ and $r(t) = \lim_{k\to\infty} r_k(t)$.

4. Results and Discussion

The analytical solutions for nonlinear alcohol consumption model in Spain are analyzed and discussed in this section then listed in **Table 3.** The predicted values of variables $a(t)$, $m(t)$ and $r(t)$ for alcohol consumption model, [12]. had been given. Since the exact solution is not available for the current model, the predicted values are used to compare between the current analytic solutions of ADM and VIM with the predicted values [12]. in the interval (0,10) from 1997 to 2007. For comparison purpose, the corresponding absolute error of $a(t)$, $m(t)$ and $r(t)$ for ADM and VIM methods are shown numerically in **Table 4.** where the absolute error in this study is the absolute value of the difference between the analytic solutions and the predicted values. Moreover, the relative error of $a(t)$, $m(t)$ and $r(t)$ for ADM and VIM methods is also listed in **Table 5.** which is defined as the absolute value of dividing the absolute error on the predicted values in this study. In **Table 4,5.** the absolute and relative errors of ADM for $a(t)$ have smaller values from 2003 to 2005 and 2007 than the VIM, while the absolute and relative errors of ADM for $m(t)$ are smaller than VIM from 1999 till 2001 and 2007. For $r(t)$, the absolute and relative errors of ADM in the interval (0,10) from 1997 to 2007 are oscillatory with VIM errors.

The figures describe the behavior of alcohol drinking habit from 1997 to 2007. **Figure 1a**. of $a(t)$ shows the ADM and VIM obtained results near to some predicted values in 2001 until 2005.While **Figure 1b.** Of $m(t)$ shows the predicted values around both ADM and VIM obtained results. Regarding to **Figure C.** of $r(t)$, both ADM and VIM curves obtained results converge to the predicted values in 1999 until 2005. For **Figure 1a.** That related to non-drink alcohol people $a(t)$, the ADM curve keeps on its level from 1997 to 2007. More other, there exists a variation between them such that the VIM curve is higher level than ADM curve. On the other hand, both ADM and VIM curves of non-risk-drink alcohol people $m(t)$ have higher that appears during the ten years from 1997 till 2007 in **Figure 1b. Figure 1c.** illustrates the decrease in the risk-drink alcohol people $r(t)$ through the ten years under study for both ADM and VIM curves. The results are calculated by Mathematica software, the figures are drawn by the Magic Plot program. Finally, the percentage of non-drink alcohol people $a(t)$ and the risk-drink alcohol people $r(t)$ are almost decrease, but increase with the non-risk-drink alcohol people $m(t)$.

Table 3. Analytic solutions and predicted values [12] .of alcohol consumption model (* means present result).

Subpop.	**Method**	**1997**	**1999**	**2001**	**2003**	**2005**	**2007**
$\boldsymbol{a(t)}$	**Santonja at. el.**	0.362	0.383	0.363	0.359	0.354	0.400

	in 2010						
	ADM*	0.362	0.36180243	0.36160497	0.36140759	0.36121033	0.36101316
	VIM*	0.362	0.36223274	0.36245419	0.36266487	0.36286526	0.36305586
$\boldsymbol{m}(\boldsymbol{t})$	**Santonja at al.in 2010**	0.581	0.578	0.581	0.588	0.591	0.566
	ADM*	0.581	0.58195819	0.58288472	0.58378022	0.58464529	0.58548056
	VIM*	0.581	0.58196578	0.58291479	0.58384724	0.58476327	0.58566309
$\boldsymbol{r}(\boldsymbol{t})$	**Santonja at el. in 2010**	0.057	0.039	0.056	0.053	0.055	0.034
	ADM*	0.057	0.05580150	0.05463109	0.05348808	0.05237178	0.051281517
	VIM*	0.057	0.05580148	0.05463101	0.05348789	0.05237147	0.05128105

Table 4. Absolute error for ADM and VIM solutions as relative the predicted values [12].

Subpop.	**Absolut error**	**1999**	**2001**	**2003**	**2005**	**2007**
$\boldsymbol{a}(\boldsymbol{t})$	**ADM**	0.02119757	0.00139503	0.00240759	0.00721033	0.03898684
	VIM	0.02076726	0.00054581	0.00366487	0.00886526	0.03694414
$\boldsymbol{m}(\boldsymbol{t})$	**ADM**	0.00395819	0.00188472	0.00421978	0.00635471	0.01948056
	VIM	0.00396578	0.00191479	0.00415276	0.00623673	0.01966309
$\boldsymbol{r}(\boldsymbol{t})$	**ADM**	0.01680150	0.00136891	0.00048808	0.00262822	0.017281517
	VIM	0.01680148	0.00136899	0.00048789	0.00262853	0.01728105

Table 5. Relative error for ADM and VIM solutions as relative the predicted values [12].

Subpop.	**Relative error**	**1999**	**2001**	**2003**	**2005**	**2007**
$\boldsymbol{a}(\boldsymbol{t})$	**ADM**	0.05534613	0.00384306	0.00670638	0.02036816	0.0974671
	VIM	0.05422261	0.00150361	0.01020855	0.02504311	0.09236035
$\boldsymbol{m}(\boldsymbol{t})$	**ADM**	0.00684808	0.00324392	0.00717649	0.01075247	0.03441795
	VIM	0.00686121	0.00329567	0.00706252	0.01055284	0.03474044
$\boldsymbol{r}(\boldsymbol{t})$	**ADM**	0.43080769	0.02444482	0.00920906	0.04778582	0.50827991
	VIM	0.43080718	0.02444625	0.00920547	0.04779145	0.50826617

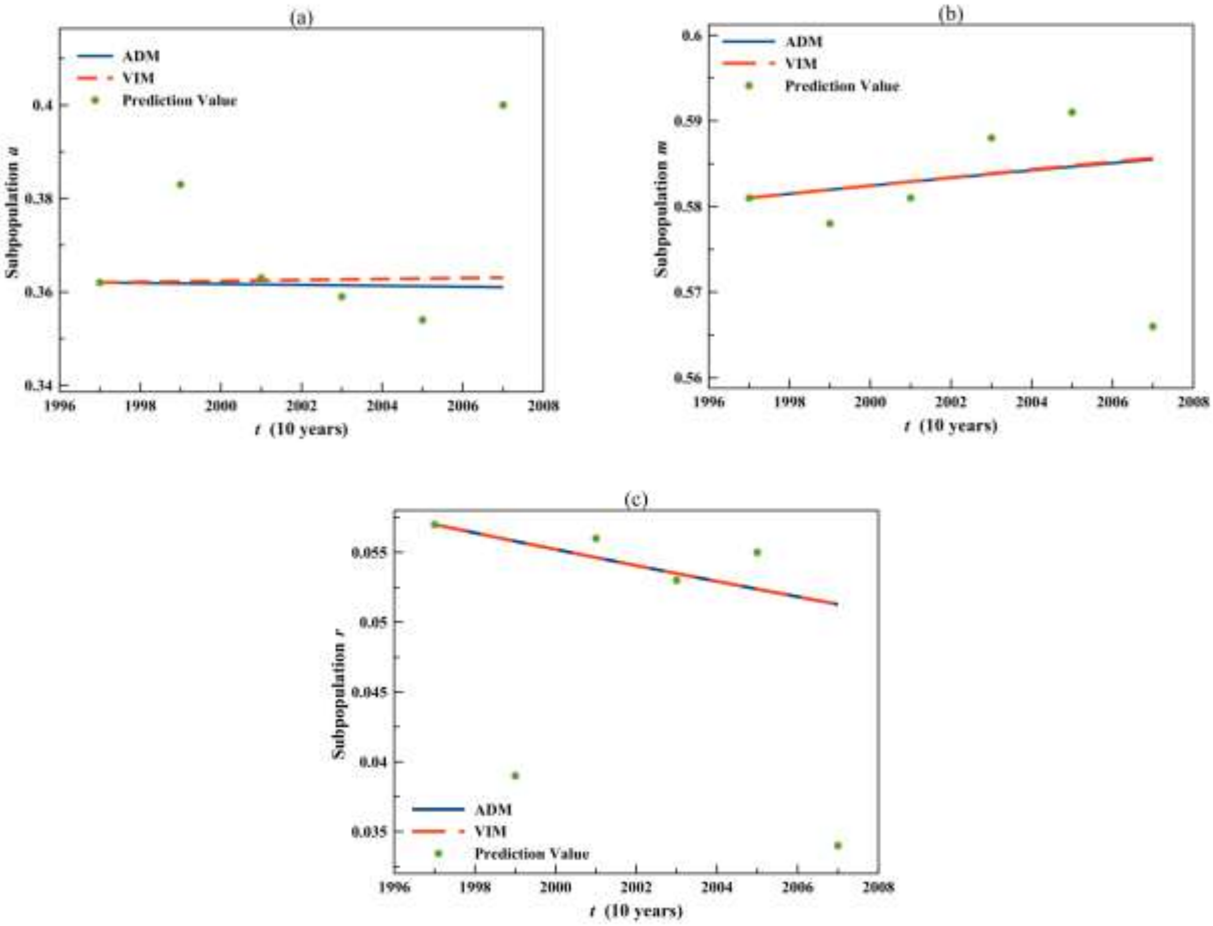

Figure 1. Variation of analytic solutions for ADM and VIM around predicted values [12].1 of (a) $a(t)$, (b) $m(t)$ and (c) $r(t)$ from 1997 to 2007 years.

5. Conclusion

The current study, the convergence of the results for the analytic methods which are Adomian decomposition and variation iteration methods are examined in the nonlinear case. These methods have been known as a powerful device for solving a system of ordinary, partial differential equations or Integral equations and so on. In our work, they are used for solving a system of non-linear ordinary differential equations. The behavior of unhealthy social habit which is alcohol consumption in Spain are analyzed, based on the epidemiological model through ten years under study. The ADM and VIM methods help to analyze the effects of the unhealthy social habit of alcohol consumption. The obtained results are shown that there is increasing in alcohol consumption with non- risk-drink consumers and declining the risk-drink consumers during the ten years under study. For the number of the non-drink consumers has a small increase with the variation iteration method and maintains its level with respect to the Adomian decomposition method. The most predicted values [12]. around the ADM and VIM curves. Other analytical methods can solve such system under study like homotopy perturbation method, homotopy analysis method and Semi Analytical Iterative Method Temimi and Ansari.

References

1. Sánchez, E.; Llorens, N.; Barrio, G.; Sánchez, A.; Suelves, J.M. *Modeling of Drinking Behavior in Spain. Modelling for Medicine, Business and Engineering*.**2009**, 213-221, I.S.B.N.: 978-84-692-9972-2.
2. White, E.; Comiskey, C. Heroin epidemics, treatment and ODE modelling. *Mathematical biosciences*. **2007**, *208*, *1*, 312-324, https://doi.org/10.1016/j.mbs.2006.10.008.
3. Guerrero, F.; Santonja, F.J.; Villanueva, R.J. Analysing the Spanish smoke-free legislation of 2006: a new method to quantify its impact using a dynamic model. *International Journal of Drug Policy*.**2011**, *22*, *4*, 247-251, doi: 10.1016/j.drugpo.2011.05.003.
4. Guerrero, F.; Santonja, F.; Villanueva, R. Solving a model for the evolution of smoking habit in Spain with homotopy analysis method. Nonlinear Analysis. *Real World Applications*.**2013**, *14*, *1*, 549-558, https://doi.org/10.1016/j.nonrwa.2012.07.015.
5. Haq, F.; Shah, K.; Rahman, G.ur.; Shahzad, M. Numerical solution of fractional order smoking model via laplace Adomian decomposition method. *Alexandria Engineering Journal*.**2018**, *57*, 1061-1069, https://doi.org/10.1016/j.aej.2017.02.015.
6. Mohammed, M.A.; Noor, N.; Ibrahim, A.; Siri, Z. A non-conventional hybrid numerical approach with multi-dimensional random sampling for cocaine abuse in Spain. *International Journal of Biomathematics*.**2018**, *11*, *08*, 1850110, https://doi.org/10.1142/S1793524518501103.
7. Sánchez, E.; Villanueva, R.J.; Santonja, F.J.; Rubio, M. Predicting cocaine consumption in Spain. A mathematical modelling approach. Drugs. Education, *Prevention and Policy*.**2011**, *18*, *2*, 108-115, https://doi.org/10.3109/09687630903443299.
8. Mohammed, M.; Ibrahim, A.; Siri, Z.; Noor, N. Mean Monte Carlo finite difference method for random sampling of a nonlinear epidemic system. *Sociological Methods & Research*. **2019**, *48*, *1*, 34-61, https://journals.sagepub.com/doi/abs/10.1177/0049124116672683.
9. Mohammed, M.A.; Noor, N.F.M.; Siri, Z.; Ibrahim, A.I.N.; Numerical solution for weight reduction model due to health campaigns in Spain, in *AIP Conference Proceedings*.**2015**, AIP Publishing, 1-6, https://doi.org/10.1063/1.4932414.
10. Santonja, F.J.; Morales, A.; Villanueva, R.J. Analysing the effect of public health campaigns on reducing excess weight. A modelling approach for the Spanish Autonomous Region of the Community of Valencia. *Evaluation and Program Planning*.**2012**, *35*, *1*, 34-39, https://doi.org/10.1016/j.evalprogplan.2011.06.004.
11. Fierro, I.; Gonzalez, L.J.C.; Sequi, G.M.; Alvarez, F.J. Alcohol and drug use by Spanish drivers. Comparison of two cross-sectional road-side surveys *(2008–9/2013). International Journal of Drug Policy*.**2015**, *26*, *8*, 794-797, https://doi.org/10.1016/j.drugpo.2015.04.021.
12. Santonja, F.J.; Sánchez, E.; Rubio, M.; Morera, J.L. Alcohol consumption in Spain and its economic cost. a mathematical modeling approach. *Mathematical and Computer Modelling*.**2010**, *52*, 7-8, 999-1003, https://doi.org/10.1016/j.mcm.2010.02.029.
13. Adomian, G.; Rach, R. On the solution of algebraic equations by the decomposition method. *Journal of mathematical analysis and applications*.**1985**, *105*, *1*, 141-166, https://doi.org/10.1016/0022-247X(85)90102-7.
14. Wazwaz, A.M. *Partial differential equations and solitary waves theory;* 1st ed; Publishing Springer Science & Business Media.**2010**, ISSN 1867-8440.

15. Abbasbandy, S. Improving Newton–Raphson method for nonlinear equations by modified Adomian decomposition method. *Applied Mathematics and Computation*. **2003**, *145*, 2-3, 887-893, doi:10.1016/S0096-3003(03)00282-0.
16. Abbasbandy, S. Numerical solutions of the integral equations: Homotopy perturbation method and Adomian's decomposition method. *Applied Mathematics and Computation*. **2006**, *173, 1*, 493-500, https://doi.org/10.1016/j.amc.2005.04.077.
17. Chun, C. Construction of Newton-like iteration methods for solving nonlinear equations. *Numerische Mathematik*.**2006**, *104*, *3*, 297-315.
18. Noor, M.A.; Noor, K.I. Improved iterative methods for solving nonlinear equations. *Applied mathematics and computation*. **2007**, *184, 2,* 270-275, https://doi.org/10.1016/S0096-3003(97)10060-1.
19. Adomian, G. Nonlinear stochastic differential equations. *Journal of Mathematical Analysis and Applications*. **1976**, *55*, *2*, 441-452, https://doi.org/10.1515/IJNSNS.2006.7.1.27.
20. Biazar, J.; Babolian, E.; Islam, R. Solution of the system of ordinary differential equations by Adomian decomposition method. *Applied Mathematics and Computation*. **2004**, *147, 3,* 713-719, https://doi.org/10.1016/S0096-3003(02)00806-8.
21. Biazar, J. Solution of the epidemic model by Adomian decomposition method. *Applied Mathematics and Computation*.**2006**, *173*, *2*, 1101-1106, https://doi.org/10.1016/j.amc.2005.04.036.
22. Makinde, O.D. Adomian decomposition approach to a SIR epidemic model with constant vaccination strategy. *Applied Mathematics and Computation*.**2007**, *184*, *2,* 842-848, https://doi.org/10.1016/j.amc.2006.06.074.
23. Biazar, J. Solution of systems of integral–differential equations by Adomian decomposition method. *Applied Mathematics and Computation*.**2005**, *168*, *2*, 1232-1238, https://doi.org/10.1016/j.amc.2004.10.015
24. Tawfiq, L.N.; Hussein, R.W. Efficient Semi-Analytic Technique for Solving Nonlinear Singular Initial Value Problems. *Ibn AL-Haitham Journal For Pure and Applied Science*.**2014**, *27*, *3*, 513-522,
25. Ateea, A.k. Approximate Solution for Fuzzy Differential Algebraic Equations of Fractional Order Using Adomian Decomposition Method. *Ibn AL-Haitham Journal For Pure and Applied Science*.**2017**, *30*, *2*, 202-213.
26. Razzaq, E.A.L.A.; Yassein, S.M. Analytic Solutions For Integro-Differential Inequalities Using Modified Adomian Decomposition Method. *Ibn ALHaitham Journal For Pure and Applied Science*.**2017**, *30*, *1*, 177-191.
27. He, J.H. Variational iteration method–a kind of non-linear analytical technique. some examples. *International journal of non-linear mechanics*.**1999**, *34*, *4,* 699-708, https://doi.org/10.1016/S0020-7462(98)00048-1.
28. He, J. A new approach to nonlinear partial differential equations. *Communications in Nonlinear Science and Numerical Simulation*. **1997**, *2*, *4,* 230-235, https://doi.org/10.1016/S1007-5704(97)90007-1.
29. He, J.H. Variational iteration method for autonomous ordinary differential systems. *Applied Mathematics and Computation*. **2000**, *114*, 2-3, 115-123, https://doi.org/10.1016/S0096-3003(99)00104-6.
30. Odibat, Z.; Momani, S. Application of variational iteration method to nonlinear differential equations of fractional order. *International Journal of Nonlinear Sciences and Numerical Simulation*.**2006**, *7*, *1*, 27-34, https://doi.org/10.1515/IJNSNS.2006.7.1.27.

31. Davoren, M.P.; Demant, J.; Shiely, F.; Perry, I.J. Alcohol consumption among university students in Ireland and the United Kingdom from 2002 to 2014: a systematic review. *BMC Public Health.***2016**, *16*, *1*, 173-185, https://doi.org/10.1186/s12889-016-2843-1.
32. Gómez, P.; Moure-Rodríguez, L.; López-Caneda, E.; Rial, A.; Cadaveira, F.; Caamaño-Isorna, F. Patterns of alcohol consumption in Spanish university alumni: nine years of follow-up. *Frontiers in psychology*.**2017**, *8*, 756. https://doi.org/10.3389/fpsyg.2017.00756.