Ibn Al Haitham Journal for Pure and Applied Science

Journal homepage: http://jih.uobaghdad.edu.iq/index.php/j/index

On Estimation of P(Y<X) in Case Inverse Kumaraswamy Distribution

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Article history: Received 2 April 2019, Accepted 30 June 2019, Publish January 2020.

Doi: 10.30526/33.1.2375

Abstract

The estimation of the stress -strength reliability of Invers Kumaraswamy distribution will be introduced in this paper based on the maximum likelihood, moment and shrinkage methods. The mean squared error has been used to compare among proposed estimators. Also a Monte Carlo simulation study is conducted to investigate the performance of the proposed methods in this paper.

Keyword: Invers Kumaraswamy distribution, Maximum likelihood estimator, Moment estimator, Shrinkage estimator, Stress - Strength reliability, Monte Carlo simulation.

1. Introduction

In the past few years, a majority application in survey sampling, medical research, biological sciences, engineering sciences, econometrics and life testing problems have been interested to apply inverse distribution [1-2]. Abedul Fattah et al. [3]. Presented the Invers Kumaraswamy Distribution (IKumD) based on Kumaraswamy distribution when it is presented in 1980 [4]. Which endorse in varied range of applications counting test scores, atmospheric temperature, height of individuals and many others [5 -7].

Abedul Fattah, depends on the r.v. T ($T = \frac{1-X}{X}$), as a function of a r.v. X when X follows Kumaraswamy distribution (~KumD(α, β)), where α *and* β are shape parameters.

Then, they explained how the (IKumD) affect long term reliability prediction, making optimistic predictions of rare events arising in the right tail of the distribution related to additional distributions.

The probability density function (PDF) of r.v. X which is distributed as IKumD is

$$f(x, \alpha, \beta) = \alpha\beta(1+x)^{-(\alpha+1)}(1-(1+x)^{-\alpha})^{\beta-1} \;, x > 0 \;;\; \alpha, \beta > 0 \quad (1)$$

Where, α *and* β are shape parameters.

And the cumulative distribution function (CDF) of X has the form below

$$F(x; \alpha, \beta) = (1-(1+x)^{-\alpha})^{\beta} \;, x > 0 \,, \alpha, \beta > 0 \quad (2)$$

On the other hand, for the reliability (R) in the stress- strength (S-S) model was attracted

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many statisticians for several years owing to their applicability in different and divers parts such as engineering, quality control, economics. In addition, in the previous thirty years, there have been many applications to medical problems and clinical trials [8-9].

The term stress-strength (S-S) refers to a component which has a random of strength X subject to a random stress Y to evaluate the reliability. The component fails if the stress applied to it exceeds the strength, while the component works whenever Y less than X ($Y< X$). Several researchers assuming various lifetime distributions for the stress - strength random variates.[11-14].

Accordingly, $R= P(Y < X)$ is a measure of component reliability in (S-S) model. Consequently, this study estimates the (S-S) reliability when the stress and strength follows the two parameters Invers Kumaraswamy Distribution (IKumD) via different estimation methods.

Now, if X and Y are two random variables follows the IKumD with parameters (α,β_1) and (α,β_2) as strength and stress respectively when the parameter α is known, then the (S-S) reliability is defined as below;

$$R= P(Y < X)$$
$$= \int_0^\infty \int_0^x f(x)g(y)\,dx\,dy$$
$$= \int_0^\infty F_y(x)f(x)\,dx$$
$$= \int_0^\infty (1-(1+x)^{-\alpha})^{\beta_2}\alpha\beta_1(1+x)^{-(\alpha+1)}(1-(1+x)^{-\alpha})^{\beta_1-1}\,dx$$
$$= \frac{\beta_1}{\beta_1+\beta_2} \qquad (3)$$

The rest of this paper organized as follows: Section 2 including some estimation methods of P(Y<X) . Section 3 offers simulation study. Section 4 demonstrates the effectiveness of the proposed method through results. Finally, a conclusion is provided in Section5.

2. Estimation Methods of P($Y < X$)

2. 1. Maximum Likelihood Estimation (MLE)

Let $x_1, x_2, \ldots\ldots x_n$ be a random sample from IKumD (α,β_1) and $y_1, y_2, \ldots\ldots y_n$ be a random sample from IKumD (α,β_2), then the likelihood function is given by

$$l= L(x,y,\beta_1,\beta_2,\alpha)$$
$$= \prod_{i=1}^n f(x_i)\,.\prod_{j=1}^m g(y_j)$$
$$= \prod_{i=1}^n \alpha\beta_1(1+x_i)^{-(\alpha+1)}(1-(1+x_i)^{-\alpha})^{\beta_1-1}.\prod_{j=1}^m \alpha\beta_2(1+y_j)^{-(\alpha+1)}\left(1-\left(1+y_j\right)^{-\alpha}\right)^{\beta_2-1}$$
$$= \alpha^{n+m}\beta_1^n\beta_2^m \prod_{i=1}^n(1+x_i)^{-(\alpha+1)}\prod_{j=1}^m\left(1+y_j\right)^{-(\alpha+1)}.\prod_{i=1}^n(1-(1+x_i)^{-\alpha})^{\beta_1-1}.\prod_{j=1}^m\left(1-\left(1+y_j\right)^{-\alpha}\right)^{\beta_2-1} \qquad (4)$$

Take ln to both sides and the equation (4) will be

$$ln\,l = (n+m)\ln\alpha + n\ln\beta_1 + m\,ln\,\beta_2 - (\alpha+1)\sum_{i=1}^n ln(1+x_i) - (\alpha+1)\sum_{j=1}^m ln\left(1+y_j\right) + (\beta_1-1)\sum_{i=1}^n ln(1-(1+x_i)^{-\alpha}) + (\beta_2-1)\sum_{j=1}^m ln\left(1-\left(1+y_j\right)^{-\alpha}\right)$$

The partial derivative of ln l with respect to $\beta_1 and\ \beta_2$ and equate the result to zero ,we obtained the following

$$\frac{\partial \ln l}{\partial \beta_1} = \frac{n}{\beta_1} + \sum_{i=1}^{n} \ln(1-(1+x_i)^{-\alpha}) = 0$$
$$\frac{\partial \ln l}{\partial \beta_2} = \frac{m}{\beta_2} + \sum_{j=1}^{m} \ln\left(1-\left(1+y_j\right)^{-\alpha}\right) = 0$$
The ML's estimator for the unknown shape parameters β_i $(i = 1,2)$ is given by
$$\hat{\beta}_{1_{MLE}} = \frac{-n}{\sum_{i=1}^{n} \ln(1-(1+x_i)^{-\alpha})} \tag{5}$$
$$\hat{\beta}_{2_{MLE}} = \frac{-m}{\sum_{j=1}^{m} \ln\left(1-\left(1+y_j\right)^{-\alpha}\right)} \tag{6}$$

By substituting equation (5) and (6) in equation (3) we get
$$\hat{R}_{MLE} = \frac{\hat{\beta}_{1_{MLE}}}{\hat{\beta}_{1_{MLE}}+\hat{\beta}_{2_{MLE}}} \tag{7}$$

2. 2. Moments Method (MOM)

In this section, population moments for X and Y of IKumD were needed. Therefore, Moments method (MOM) was considered to estimate the parameter β for IKumD when the parameter α is known.
$$\mathrm{E}(X^r) = \beta \sum_{j=0}^{r} \binom{r}{j} -1^{r-j} B\left(1-\frac{j}{\alpha}, \beta\right) \quad r = 1, 2, \ldots$$
Hence, the population means of X and Y are given respectively as below
$$\mathrm{E}(X) = \beta_1 B\left(1-\frac{1}{\alpha}, \beta_1\right) - 1 \qquad , \alpha > 1$$
$$\mathrm{E}(Y) = \beta_2 B\left(1-\frac{1}{\alpha}, \beta_2\right) - 1 \qquad , \alpha > 1$$
where $B(.,.)$ refer to Beta distribution.
And when equality the sample mean with the corresponding population mean, we get
$$\bar{X} = \frac{\sum_{i=1}^{n} x_i}{n} = \beta_1 B\left(1-\frac{1}{\alpha}, \beta_1\right) - 1 \quad \text{and}$$
$$\bar{Y} = \frac{\sum_{j=1}^{m} y_j}{m} = \beta_2 B\left(1-\frac{1}{\alpha}, \beta_2\right) - 1$$
As a result , we obtained

$$\hat{\beta}_{1_{MOM}} = \frac{1+\sum_{i=1}^{n} \frac{x_i}{n}}{B\left(1-\frac{1}{\alpha}, \beta_{1_0}\right)} \tag{8}$$
And,
$$\hat{\beta}_{2_{MOM}} = \frac{1+\sum_{j=1}^{m} \frac{y_j}{m}}{B\left(1-\frac{1}{\alpha}, \beta_{2_0}\right)} \tag{9}$$
Where β_{i_0} is an initial value for β_i $(i = 1,2)$.

Similarly, substitution the equations (8) and (9) in the equation (3) we get

$$\hat{R}_{MOM} = \frac{\hat{\beta}_{1_{MOM}}}{\hat{\beta}_{1_{MOM}}+\hat{\beta}_{2_{MOM}}} \tag{10}$$

2. 3. Shrinkage Estimation Method

In 1968, Thompson proposed the shrink usual estimator $\hat{\beta}$ (ex. MLE or Unbiased estimator) of the parameter β to prior information β_0 using shrinkage weight actor $\psi(\hat{\beta})$, such that $0 \leq \psi(\hat{\beta}) \leq 1$. Thompson say that "We are estimating β and we believe β_0 is closed to the true value of β and something bad happens if $\beta_0 \approx \beta$ and we do not use β_0". Thus, Thompson gave the form of shrinkage estimator of β say $\hat{\beta}_{Sh}$ as bellow:

$$\hat{\beta}_{Sh} = \psi(\hat{\beta})\hat{\beta}_{ub} + (1 - \psi(\hat{\beta}))\beta_0 \quad (11)$$

Where $\hat{\beta}_{ub}$ was an unbiased estimator and applied as usual estimator of β, $\psi(\hat{\beta})$ may be a function of sample size *(n,m)* , a constant, or found through minimizing mean square error of $\hat{\beta}_{sh}$ (ad hoc basis). On the other hand, β_0 is a very closed value of β which used as a prior information. [15, 16, 17, 18].

There is no doubt, if our assumption to take $\beta_{i_0} = [\beta_i - 0.001]$, as a prior information of β_i for i =1, 2 in this work. Noted that $\beta_{i_{MLE}}$ is biased estimator, since $E(\hat{\beta}_{i_{MLE}}) = \frac{\lambda}{\lambda-1}\beta_i \neq \beta_i$, λ refer to n or m.

Thus $\hat{\beta}_{i_{ub}} = \frac{\lambda-1}{\lambda}\hat{\beta}_{i_{MLE}}$ become unbiased estimator of β_i whereas $E(\hat{\beta}_{i_{ub}}) = \beta_i$.

Likewise,

$\text{var}(\hat{\beta}_{1_{ub}}) = \frac{(\beta_1)^2}{(n-2)}$ and $\text{var}(\hat{\beta}_{2_{ub}}) = \frac{(\beta_2)^2}{(m-2)}$

i.e.; $$\hat{\beta}_{1_{ub}} = \frac{n-1}{-\sum_{i=1}^{n}\ln(1-(1+x_i)^{-\alpha})} \quad (12)$$

And $$\hat{\beta}_{2_{ub}} = \frac{m-1}{-\sum_{j=1}^{m}\ln\left(1-(1+y_j)^{-\alpha}\right)} \quad (13)$$

2. 3. 1. Constant Shrinkage Weight Factor (Sh_1)

In this subsection, the constant shrinkage weight factor will be assumed as;

$\Psi\ (\hat{\beta}_1) = K_1 = 0.01$, and $\Psi(\hat{\beta}_2) = K_2 = 0.01$, then applying to the following shrinkage estimators;

$$\hat{\beta}_{1_{Sh1}} = K_1\hat{\beta}_{1_{ub}} + (1 - K_1)\beta_{1_0} \quad (14)$$

$$\hat{\beta}_{2_{Sh1}} = K_2\hat{\beta}_{2_{ub}} + (1 - K_2)\beta_{2_0} \quad (15)$$

When substitution the equation (14) and (15) in equation (3), lead to the estimation of S-S reliability using shrinkage estimator $\hat{R}_{Sh1}$ as below

$$\hat{R}_{Sh1} = \frac{\hat{\beta}_{1_{Sh1}}}{\hat{\beta}_{1_{Sh1}} + \hat{\beta}_{2_{Sh1}}} \quad (16)$$

2. 3. 2. Shrinkage Weight Function (Sh_2)

In this subsection, we considered the shrinkage weight factor as a function of n and m, respectively in equation (11)

$\Psi\ (\hat{\beta}_1) = K_1 = e^{-n}$, and $\Psi\ (\hat{\beta}_2) = K_2 = e^{-m}$.

$$\hat{\beta}_{i_{Sh2}} = K_i\hat{\beta}_{i_{ub}} + (1 - K_i)\beta_{i_0} \text{ for i= 1, 2} \quad (17)$$

The corresponding S-S reliability estimation was used using equation (17) as follows;

$$\hat{R}_{Sh2} = \frac{\hat{\beta}_{1_{Sh2}}}{\hat{\beta}_{1_{Sh2}} + \hat{\beta}_{2_{Sh2}}} \quad (18)$$

2. 3. 3. Modified Thompson Type Shrinkage Weight Factor (Sh_{Th})

The shrinkage weight factor which was considered by Thompson in 1968 will be modified in this subsection as follows:

$$\varphi(\hat{\beta}_{i_{ub}}) = \frac{\left(\hat{\beta}_{i_{ub}} - \hat{\beta}_{i_0}\right)^2}{\left(\hat{\beta}_{i_{ub}} - \hat{\beta}_{i_0}\right)^2 + var(\hat{\beta}_{i_{ub}})} (0.001) \quad \text{for i=1, 2} \tag{19}$$

where, $var(\hat{\beta}_{i_{ub}})$ defined in section (2.1).

In consequence, the modified Thompson type shrinkage estimator will be

$$\hat{\beta}_{i_{Th}} = \varphi(\hat{\beta}_i)\hat{\beta}_{i_{ub}} + \left(1 - \varphi(\hat{\beta}_i)\right)\beta_{i_0} \quad \text{, for i = 1, 2.} \tag{20}$$

By substituting equation (20) in the equation (3), we get the modified Thompson type shrinkage estimation of the S-S reliability as below

$$\hat{R}_{Th} = \frac{\hat{\beta}_{1_{Th}}}{\hat{\beta}_{1_{Th}} + \hat{\beta}_{2_{Th}}} \tag{21}$$

3. Simulation Study

Simulation is the best criteria for investigate the performance comparison between the different estimators of reliability which is obtained in this section. Different samples were Mote Carlo used based on MSE criteria with size = (20, 30, 50 and 100), with 1000 trials. simulation was used for this purpose as a following steps;

Step1: Initialize random samples, which is follows the continuous uniform distribution defined on the interval (0,1) as $u_1, u_2, \ldots, u_n$

Step2: Generate a random sample follows the continuous uniform distribution over the interval (0,1) as $w_1, w_2, \ldots, w_m$

Step3: Transform the above uniform random samples to a random samples follows IKumD using the cumulative distribution function (CDF) as follow:

$$F(x) = (1 - (1 + x_i)^{-\alpha})^{\beta_1}$$

$$u_i = (1 - (1 + x_i)^{-\alpha})^{\beta_1}$$

$$x_i = \left[1 - (u_i)^{\frac{1}{\beta_1}}\right]^{-\frac{1}{\alpha}} - 1$$

As well as, by the same method, we get

$$y_j = \left[1 - (w_j)^{\frac{1}{\beta_2}}\right]^{-\frac{1}{\alpha}} - 1$$

Step4: Recall MLE estimator for reliability using equation (7).

Step5: Moment method was computed for reliability using equation (10).

Step6: Compute Shrinkage estimators of reliability using equation (16), (18) and (21).

Step7: Calculate the MSE based on (L=1000) trials as follows:

$$\text{MSE} = \frac{1}{L}\sum_{i=1}^{L}\left(\hat{R}_i - R\right)^2$$

4. Numerical Results

In this section, the simulation results of the proposed estimation methods are illustrated in **Table 1-8.** Below and distinguish the following results.

Four samples problem size 20,30,50,100 have been implemented 1000 trials based on two parameters values (β_i). The Mote Carlo simulation was coded using Matlab b 2010. All proposed estimation methods has reasonably MSE see **Table 2, 4, 6, 8.** Also showed that the shrinkage estimator based on shrinkage weight function ($\hat{R}_{Sh2}$) had minimum mean square error for the S-S reliability estimator of the Invers Kumaraswamy distribution. This implies that shrinkage estimator ($\hat{R}_{Sh2}$) of S-S reliability was the best than the others. While Modified Thompson type shrinkage estimator ($\hat{R}_{Th}$) had the second rank and followed by Sh_1, MOM and MLE, respectively. At most when n fixed and m change MSE decreases. The following **Table 1-8.** Will be presents the simulation results.

Table 1. Estimation value of R, when $\alpha = 5\ ,\ \beta_1 = 2\ and\ \beta_2 = 4$.

n	m	R	$\hat{R}_{MLE}$	$\hat{R}_{MOM}$	$\hat{R}_{Sh1}$	$\hat{R}_{Sh2}$	$\hat{R}_{Th}$
20	20	0.333333	0.344809	0.335369	0.333362123100458	0.333277759270399	0.333284750506442
	30	0.333333	0.328533	0.331329	0.333216093650363	0.333277759265131	0.333269872853556
	50	0.333333	0.333264	0.330096	0.333293257564652	0.333277759275897	0.333275729850324
	100	0.333333	0.336435	0.331783	0.333335012065777	0.333277759271973	0.333277077788890
30	20	0.333333	0.332185	0.334541	0.333231166393347	0.333277759228006	0.333277009208634
	30	0.333333	0.337846	0.331541	0.333295628413176	0.333277759253084	0.333279323627313
	50	0.333333	0.338030	0.334136	0.333331973551988	0.333277759253085	0.333282199655278
	100	0.333333	0.338454	0.333449	0.333341266226874	0.333277759253085	0.333280477716060
50	20	0.333333	0.320632	0.331299	0.333084778976879	0.333277759213775	0.333268023990180
	30	0.333333	0.336537	0.332339	0.333271373787237	0.333277759253084	0.333280641825118
	50	0.333333	0.338671	0.333546	0.333312103469809	0.333277759253085	0.333280220606030
	100	0.333333	0.340324	0.334073	0.333353617428845	0.333277759253085	0.333282003591840
100	20	0.333333	0.332727	0.334635	0.333206494736057	0.333277759233512	0.333279135957987
	30	0.333333	0.328593	0.332474	0.333195889013743	0.333277759253083	0.333274334748396
	50	0.333333	0.336768	0.334502	0.333298953792528	0.333277759253085	0.333282567198524
	100	0.333333	0.333861	0.333008	0.333281857836376	0.333277759253085	0.333278109789984

Table 2. MSE value of R= 0.333333 when $\alpha = 5\ ,\ \beta_1 = 2\ and\ \beta_2 = 4$.

n	m	$\hat{R}_{MLE}$	$\hat{R}_{MOM}$	$\hat{R}_{Sh1}$	$\hat{R}_{Sh2}$	$\hat{R}_{Th}$
20	20	0.005328	0.000396	0.000000606944029	0.000000003088476	0.000000006027609
	30	0.003583	0.000303	0.000000416241795	0.000000003088477	0.000000006254660
	50	0.003169	0.000278	0.000000373963720	0.000000003088476	0.000000005486328
	100	0.003641	0.000350	0.000000408208905	0.000000003088476	0.000000005655820
30	20	0.003890	0.000459	0.000000490653947	0.000000003088481	0.000000006363335
	30	0.002995	0.000336	0.000000335168930	0.000000003088478	0.000000005167804
	50	0.002506	0.000190	0.000000277353441	0.000000003088478	0.000000004467301
	100	0.002096	0.000130	0.000000245182027	0.000000003088478	0.000000004439858
50	20	0.002771	0.000237	0.000000396182755	0.000000003088483	0.000000006381154
	30	0.002947	0.000205	0.000000308374736	0.000000003088478	0.000000004792502
	50	0.002467	0.000206	0.000000263522730	0.000000003088478	0.000000004594002
	100	0.001636	0.000128	0.000000166214348	0.000000003088478	0.000000003705272
100	20	0.002889	0.000226	0.000000325225366	0.000000003088481	0.000000005030656
	30	0.001862	0.000144	0.000000248485615	0.000000003088478	0.000000005013088
	50	0.001453	0.000123	0.000000156553276	0.000000003088478	0.000000003699886
	100	0.007893	0.000819	0.00000084766426940	0.00000003088478396	0.00000003599008268

Table 3. Estimation value of R, when $\alpha = 5$, $\beta_1 = 2$ and $\beta_2 = 1.5$.

n	m	R	$\hat{R}_{MLE}$	$\hat{R}_{MOM}$	$\hat{R}_{Sh1}$	$\hat{R}_{Sh2}$	$\hat{R}_{Th}$
20	20	0.571428	0.565418	0.570898	0.571436691408593	0.571469411085378	0.571468821676133
	30	0.571428	0.575999	0.571278	0.571580800458761	0.571469411136943	0.571477366116829
	50	0.571428	0.571258	0.570731	0.571538155015663	0.571469411121242	0.571470488585277
	100	0.571428	0.560984	0.570845	0.571405564640412	0.571469411087639	0.571458433287792
30	20	0.571428	0.571720	0.576066	0.571462591577377	0.571469411073979	0.571469931801052
	30	0.571428	0.568857	0.572765	0.571474641003307	0.571469411092053	0.571469814477133
	50	0.571428	0.572942	0.571064	0.571524903973974	0.571469411092054	0.571472757959768
	100	0.571428	0.572777	0.572013	0.571519136443476	0.571469411092053	0.571469148069890
50	20	0.571428	0.558760	0.570994	0.571297047177488	0.571469411053786	0.571458816445993
	30	0.571428	0.556923	0.569852	0.571313920162524	0.571469411092051	0.571459850543671
	50	0.571428	0.574199	0.570379	0.571506291371074	0.571469411092053	0.571472448384525
	100	0.571428	0.568220	0.570833	0.571457181783688	0.571469411092053	0.571466452241728
100	20	0.571428	0.560389	0.571697	0.571299432578466	0.571469411048078	0.571461064899951
	30	0.571428	0.570032	0.570274	0.571435249866571	0.571469411092051	0.571469693744469
	50	0.571428	0.568900	0.572045	0.571441022201234	0.571469411092053	0.571468347558121
	100	0.571428	0.573643	0.571769	0.571495477701192	0.571469411092053	0.571471319635579

Table 4. MSE value of $R= 0.571428$ when $\alpha = 5$, $\beta_1 = 2$ and $\beta_2 = 1.5$.

n	m	$\hat{R}_{MLE}$	$\hat{R}_{MOM}$	$\hat{R}_{Sh1}$	$\hat{R}_{Sh2}$	$\hat{R}_{Th}$
20	20	0.006315	0.000448	0.000000767437031	0.000000001667878	0.000000006170575
	30	0.004956	0.000347	0.000000725951140	0.000000001667882	0.000000006621191
	50	0.004430	0.000339	0.000000606951619	0.000000001667880	0.000000005436988
	100	0.002530	0.000264	0.000000249898428	0.000000001667878	0.000000002254632
30	20	0.005278	0.000384	0.000000662992946	0.000000001667877	0.000000005517575
	30	0.005006	0.000393	0.000000571214723	0.000000001667878	0.000000005112411
	50	0.003065	0.000248	0.000000400704653	0.000000001667878	0.000000004318326
	100	0.002887	0.000227	0.000000308254345	0.000000001667878	0.000000003266297
50	20	0.004811	0.000315	0.000000664165384	0.000000001667875	0.000000004828579
	30	0.003213	0.000206	0.000000349478890	0.000000001667878	0.000000002725463
	50	0.002675	0.000180	0.000000292883062	0.000000001667878	0.000000003460735
	100	0.002007	0.000138	0.000000211130641	0.000000001667878	0.000000002648511
100	20	0.005067	0.000278	0.000000631569442	0.000000001667875	0.000000004596756
	30	0.002638	0.000198	0.000000299913294	0.000000001667878	0.000000003341709
	50	0.001422	0.000133	0.000000148485486	0.000000001667878	0.000000002331929
	100	0.001176	0.000090	0.000000128239633	0.000000001667878	0.000000002525611

Table 5. Estimation value of R, when $\alpha = 5$, $\beta_1 = 1.5 and\ \beta_2 = 2$.

n	m	R	$\hat{R}_{MLE}$	$\hat{R}_{MOM}$	$\hat{R}_{Sh1}$	$\hat{R}_{Sh2}$	$\hat{R}_{Th}$
20	20	0.428571	0.426216	0.427667	0.428499176135113	0.428530588901450	0.428530671931656
	30	0.428571	0.436086	0.427873	0.428643265773266	0.428530588952929	0.428538657119244
	50	0.428571	0.431015	0.427382	0.428600534764464	0.428530588937226	0.428531934876214
	100	0.428571	0.419576	0.427517	0.428467710362867	0.428530588903618	0.428520515627275
30	20	0.428571	0.431923	0.432813	0.428524999814827	0.428530588889961	0.428532133408948
	30	0.428571	0.428914	0.429389	0.428537009362929	0.428530588907947	0.428531008897794
	50	0.428571	0.431907	0.427614	0.428587166971384	0.428530588907949	0.428534074306687
	100	0.428571	0.431603	0.428634	0.428581318295701	0.428530588907948	0.428530659761990
50	20	0.428571	0.427726	0.430724	0.428473495982272	0.428530588885358	0.428530584436506
	30	0.428571	0.433572	0.430752	0.428540048829947	0.428530588907947	0.428532667398454
	50	0.428571	0.434124	0.428382	0.428577386378101	0.428530588907947	0.428534347244274
	100	0.428571	0.422278	0.425756	0.428471308507441	0.428530588907947	0.428524467817398
100	20	0.428571	0.421956	0.427712	0.428403361576469	0.428530588863387	0.428527175934630
	30	0.428571	0.427808	0.4280613	0.428488126663042	0.428530588907947	0.428531400079066
	50	0.428571	0.426419	0.427354	0.428486850288367	0.428530588907947	0.428529425614141
	100	0.428571	0.434333	0.429112	0.428585964386336	0.428530588907947	0.428535299823111

Table 6. MSE value of R= 0.428571 when $\alpha = 5$, $\beta_1 = 1.5\ and\ \beta_2 = 2$.

n	m	$\hat{R}_{MLE}$	$\hat{R}_{MOM}$	$\hat{R}_{Sh1}$	$\hat{R}_{Sh2}$	$\hat{R}_{Th}$
20	20	0.006255	0.000450	0.000000772703228	0.000000001667879	0.000000006187307
	30	0.005167	0.000345	0.000000709137244	0.000000001667874	0.000000005218974
	50	0.004552	0.000333	0.000000596670408	0.000000001667876	0.000000005099792
	100	0.002444	0.000258	0.000000260225013	0.000000001667878	0.000000003858325
30	20	0.005241	0.000390	0.000000663812611	0.000000001667880	0.000000005462927
	30	0.005007	0.000390	0.000000570321279	0.000000001667878	0.000000004989129
	50	0.003191	0.000243	0.000000392020094	0.000000001667878	0.000000003690361
	100	0.002892	0.000218	0.000000300246386	0.000000001667878	0.000000003180100
50	20	0.004315	0.000359	0.000000516976424	0.000000001667880	0.000000004621862
	30	0.002932	0.000261	0.000000352796511	0.000000001667878	0.000000003644106
	50	0.002819	0.000221	0.000000307535973	0.000000001667878	0.000000003157659
	100	0.001554	0.000158	0.000000180345696	0.000000001667878	0.000000003242982
100	20	0.003516	0.000309	0.000000443835525	0.000000001667882	0.000000004264359
	30	0.002307	0.000212	0.000000250681322	0.000000001667878	0.000000002930059
	50	0.001744	0.000164	0.000000201750387	0.000000001667878	0.000000002876528
	100	0.001426	0.000084	0.000000153365133	0.000000001667878	0.000000002279107

Table 7. Estimation value of R, when $\alpha = 5$, $\beta_1 = 3.2$ and $\beta_2 = 1.4$.

n	m	R	$\hat{R}_{MLE}$	$\hat{R}_{MOM}$	$\hat{R}_{Sh1}$	$\hat{R}_{Sh2}$	$\hat{R}_{Th}$
20	20	0.695652	0.698574	0.697915	0.695815269349488	0.695737277093191	0.695741078313743
	30	0.695652	0.691805	0.693986	0.695752268068552	0.695737277099320	0.695735422932987
	50	0.695652	0.690957	0.697203	0.695755093068527	0.695737277086360	0.695734322497301
	100	0.695652	0.693813	0.695859	0.695787596389732	0.695737277088173	0.695734619568542
30	20	0.695652	0.695140	0.697120	0.695740494751221	0.695737277062622	0.695737475508695
	30	0.695652	0.690159	0.697153	0.695693017315603	0.695737277076990	0.695732707125729
	50	0.695652	0.696888	0.695996	0.695795180678449	0.695737277076991	0.695738795459388
	100	0.695652	0.691288	0.695656	0.695741969015759	0.695737277076990	0.695733471720934
50	20	0.695652	0.677644	0.694686	0.695543285102023	0.695737277037347	0.695725927625687
	30	0.695652	0.694141	0.696227	0.695734633716281	0.695737277076989	0.695739339129245
	50	0.695652	0.694247	0.694868	0.695731670078937	0.695737277076988	0.695736576753283
	100	0.695652	0.699558	0.696755	0.695799211433752	0.695737277076988	0.695740317699564
100	20	0.695652	0.682233	0.695121	0.695584190595241	0.695737277045690	0.695730583868550
	30	0.695652	0.694780	0.695037	0.695720177095366	0.695737277076990	0.695737213007891
	50	0.695652	0.692752	0.696609	0.695708826932018	0.695737277076988	0.695736010915983
	100	0.695652	0.693149	0.695976	0.695717387439208	0.695737277076988	0.695735769739067

Table 8. MSE value of $R = 0.695652$ when $\alpha = 5$, $\beta_1 = 3.2$ and $\beta_2 = 1.4$.

n	m	$\hat{R}_{MLE}$	$\hat{R}_{MOM}$	$\hat{R}_{Sh1}$	$\hat{R}_{Sh2}$	$\hat{R}_{Th}$
20	20	0.004723	0.000359	0.000000576797968	0.000000007242551	0.000000010986471
	30	0.003827	0.000242	0.000000427720564	0.000000007242552	0.000000009327424
	50	0.003017	0.000267	0.000000378836873	0.000000007242550	0.000000009035571
	100	0.002962	0.000220	0.000000343245951	0.000000007242550	0.000000008697600
30	20	0.003800	0.000218	0.000000499424739	0.000000007242546	0.000000010446698
	30	0.002959	0.000200	0.000000360304775	0.000000007242549	0.000000008652092
	50	0.002627	0.000190	0.000000323750433	0.000000007242549	0.000000009427161
	100	0.002320	0.000148	0.000000241786558	0.000000007242549	0.000000008204097
50	20	0.003917	0.000205	0.000000468561104	0.000000007242542	0.000000008183235
	30	0.002502	0.000151	0.000000278982601	0.000000007242549	0.000000009102433
	50	0.001971	0.000124	0.000000217789936	0.000000007242549	0.000000008333216
	100	0.001224	0.000086	0.000000160735972	0.000000007242549	0.000000008692135
100	20	0.002748	0.000235	0.000000304691320	0.000000007242543	0.000000007794913
	30	0.001921	0.000115	0.000000220162503	0.000000007242549	0.000000008479394
	50	0.001183	0.000116	0.000000129626103	0.000000007242549	0.000000007717318
	100	0.008696	0.000760	0.000000908710753	0.00000072425485	0.000000075162844

5. Conclusion

Stress -strength reliability for two parameters based on Invers Kumaraswamy distribution was introduced in this paper by using different estimation methods namely; Maximum likelihood Estimation, Moments method, Constant shrinkage weight factor (Sh1), Shrinkage weight function (Sh2), and Modified Thompson type shrinkage weight factor. After that, Monte Carlo Simulation was exhibited to analyses(compare) and investigate these methods. Based on the results, the performance of shrinkage weight function ($\hat{R}_{Sh2}$) was appropriate behavior and it is efficient estimator than the others in the sense of MSE.

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