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CONVERGENCES VIA İ-PRE -g -OPEN SET

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Abstract

The main aim of this paper is to use the notion İ-*pre -g- openness*, which was introduced in [1]. To offered new classes of separation axioms in ideal spaces. So, we offered new type of notions of convergence in ideal spaces via the İ-pre -g- open set. Relations among several types of separation axioms that offered were explained.

Keyword: Ideal, separation axioms, İ-pre -g -open set, İ-pre -g -closed set, İ-pre -g- Convergent, İ-pre -g -open function, İ-pre -g -cotinuous function.

1. Introduction

In 1933, Kuratowski [2]. Presented the concept of ideals on non-empty sets. A collection İ $\subset$ P(X) is namely an ideals on a nonempty set X when the following two conditions are met; (i) B $\in$ İ whenever B $\subset$ A and A $\in$ İ, and (ii) A $\cup$ B $\in$ İ whenever A and B are belong to İ. Vaidyanathaswamy [3]. Had offered for initial the idea of ideal spaces by introduced the set operator $(\ )^*$: P(X) $\longrightarrow$ P(X), namely local function. So he founded new generalize of the topological spaces, namely ideal space and symbolizes by (X, T , İ), [4, 5].

The concept of "pre-open set" was introduced by Mashhour, Abd El- Monsef and El-Deeb, a set A in (X, T) is a pre-open when A $\subseteq$ cl(int(A)) [6]. From that time many researchers have submitted many studies in this field [7-9]. Latterly, Ahmed and Esmaeel [1]. had submitted the concept of İ-pre -g -closed set (simply, İpg -closed) A set A in (X, T , İ) is İpg -closed, if the condition A-U $\in$ İ and U is pre-open set, implies to cl(A) - U $\in$ İ. So, the set A in X is namely İ-pre -g -open set (simply, İpg -open), if X - A is İpg -closed. The collection of all İpg -closed set (respectively, İpg -open set) in (X, T , İ) simply İpg -C(X) (respectively, İpg -O(X)). For a space (X, T , İ), İpg -O(X) is finer than T [1]. The main target of this article is to introduce new kinds of separation axioms in ideal spaces by using the notion İpg -open set.

2. İ-Pre-g- separation axioms

This portion is to submit new classes of separation axioms by using the notion of İpg -openness. Properties of these sorts are studied and the relations between it are discussed.also.

Definition 2.1: A space (Ҳ, Ṭ, İ) is namely İ-pre-g-Ṭ$_0$-space (frugally,İpg -Ṭ$_0$-space) if for each elements $r_1 \neq r_2$, there exist an İpg-open set containing only one of them.
When (Ҳ, Ṭ) is Ṭ$_0$-space, it will lead to that (Ҳ, Ṭ, İ) is İpg -Ṭ$_0$-space for any ideal İ on Ҳ.
Remark 2.2: For a space(Ҳ, Ṭ, İ), the below sentences are rewards;

i. (Ҳ, Ṭ, İ) Is an İpg -Ṭ$_0$-space.
ii. For each element $r_1 \neq r_2$, there is an İpg-closed set containing only one of them.

Definition 2.3: A space (Ҳ, Ṭ, İ) is namely İ-pre-g-Ṭ$_1$-space (frugally,İpg -Ṭ$_1$-space) if for each elements $r_1 \neq r_2$, there are İpg-open sets Ų$_1$ and Ų$_2$, satisfies ($r_1 \in$ Ų$_1$-Ų$_2$) and ($r_2 \in$ Ų$_2$-Ų$_1$).

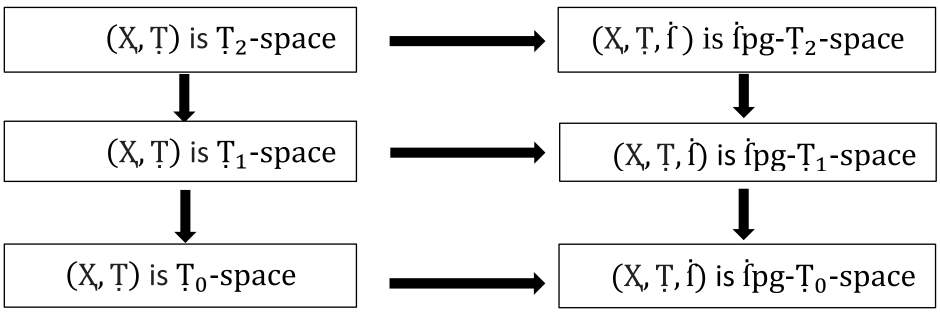
When (Ҳ, Ṭ) is Ṭ$_1$-space, it will lead to that (Ҳ, Ṭ, İ) is İpg -Ṭ$_1$-space, for any ideal İ on Ҳ.
Remark 2.4: If Ҳ is İpg- Ṭ$_1$-space, implies that İpg - Ṭ$_0$-space.
The inverse meaning implied in Remark 2.4, does not valid, in general.
Example 2.5: A space (Ҳ, Ṭ, İ) is a İpg - Ṭ$_0$-space where Ҳ $= \{r_1, r_2, r_3\}$, Ṭ $= \{$Ҳ$, \emptyset, \{r_1\}\}$ and İ $= \{\phi, \{r_2\}\}$. The space (Ҳ, Ṭ, İ) is not İpg-Ṭ$_1$-space, since for the elements $r_1 \neq r_3$, there is no İpg-open set Ų containing r_3 which does not contain r_1.
Remark 2.6: For a space (Ҳ, Ṭ, İ), the below sentences are rewards;

i. (Ҳ, Ṭ, İ) Is an İpg -Ṭ$_1$-space.
ii. For each elements$r_1 \neq r_2$, there are two İpg-closed sets F_1 and F_2, such that ($r_1 \in F_1$-F_2) and ($r_2 \in F_2$-F_1).

Remark 2.7: If {r} is İpg- closed set for each r in Ҳ, then (Ҳ, Ṭ, İ) is İpg-Ṭ$_1$-space.
Definition 2.8: A space (Ҳ, Ṭ, İ) is namely İ-pre-g- Ṭ$_2$-space (frugally, İpg-Ṭ$_2$-space), if for each elements$r_1 \neq r_2$, there are disjoint İpg-open sets Ų$_1$ and Ų$_2$ satisfies $r_1 \in$ Ų$_1$ and $r_2 \in$ Ų$_2$.
Clearly; if (Ҳ, Ṭ) is Ṭ$_2$-space implie that (Ҳ, Ṭ, İ) is İpg-Ṭ$_2$-space, for any ideal İ on Ҳ.
Remark 2.9: If the space (Ҳ, Ṭ, İ) is İpg-Ṭ$_2$-space then it is İpg-Ṭ$_1$-space.
The inverse meaning implied in Remark 2.9, does not valid, in general.
Example 2.10: The İpg-Ṭ$_1$-space (Ҳ, Ṭ, İ); such that Ҳ $= \{r_1, r_2, r_3\}$, Ṭ $= \{$Ҳ$, \emptyset\}$ and İ $= \{\phi, \{r_1\}, \{r_2\}, \{r_1, r_2\}\}$ is notİpg-Ṭ$_2$-space. Since, for the elements $r_1 \neq r_3$, there are no disjoints İpg-open sets Ų$_1$ and Ų$_2$ such that $r_1 \in$ Ų$_1$ and $r_3 \in$ Ų$_2$.
Remark 2.11: For a space (Ҳ, Ṭ, İ), the below sentences are rewards;

i. (Ҳ, Ṭ, İ) is an İpg -Ṭ$_2$-space.
ii. For each elements $r_1 \neq r_2$, there are disjoint İpg-closed sets F_1 and F_2, satisfies $r_1 \in F_1$ and $r_2 \in F_2$.

We have the truth that confirms that if (Ҳ, Ṭ) is a Ṭ$_i$-space ($i = 0,1$ and 2), then the ideal space (Ҳ, Ṭ, İ) is a İpg-Ṭ$_i$-space. But the inverse meaning implied may be invalid, as shown in the following diagram.



Example Below shows the relationships between the unlike classes of notions that presented previously.

Example 2.12: The Ipg-T_i-space (X, T, I) where $X = \{r_1, r_2, r_3\}$, $T = \{X, \emptyset\}$ and $I = P(X)$ is not T_i-space (where $i = 0, 1$ and 2).

3. Separation axioms by using some types of function

In this part, we will using some types of functions that we were offered it in Ahmed and Esmaeel [1]. And study the notions of new separation axioms under influence of these functions.

Definition 3.1: [1]. A function $f : (X, T, I) \to (Y, \mathcal{T}, J)$ is

i. I-pre-g-open function, symbolizes Ipgo-function, if $f(U)$ is Jpg-open set in Y whenever U is Ipg-open set in X.

ii. I^*-pre-g-open function, symbolizes I^*pgo-function, if $f(U)$ is Jpg-open set in Y whenever U is open set in X.

iii. I^{**}-pre-g-open function, symbolizes I^{**}pgo-function, if $f(U)$ is open in Y whenever U is Ipg-open set in X.

Proposition 3.2: If (X, T, I) is an Ipg-T_0-space (respectively, Ipg-T_1-space and Ipg-T_2-space) and $f : (X, T, I) \to (Y, \mathcal{T}, J)$ is surjective, Ipgo-function then $(Y, \mathcal{T}, J)$ is Jpg-$\mathcal{T}_0$-space (respectively, Jpg-$\mathcal{T}_1$-space and Jpg-$\mathcal{T}_2$-space).

Proof: Since $f(U)$ is Jpg-open in Y whenever U is Ipg-open set in X.

Proposition 3.3: If a space (X, T) is T_0-space (respectively, T_1-space and T_2-space) and $f : (X, T, I) \to (Y, \mathcal{T}, J)$ is surjective, I^*pgo-function then $(Y, \mathcal{T}, J)$ is Jpg-$\mathcal{T}_0$-space (respectively, Jpg-$\mathcal{T}_1$-space and Jpg-$\mathcal{T}_2$-space).

Proof: Since $f(U)$ is Jpg-open set in Y whenever U is open in X.

Proposition 3.4: If a space (X, T, I) is Ipg-T_0-space (respectively, Ipg-T_1-space and Ipg-T_2-space) and $f : (X, T, I) \to (Y, \mathcal{T}, J)$ is surjective, I^{**}pgo-function then $(Y, \mathcal{T})$ is $\mathcal{T}_0$-space (respectively, $\mathcal{T}_1$-space and $\mathcal{T}_2$-space).

Proof: Since $f(U)$ is open in Y whenever U is Ipg-open set in X.

Remark 3.5: If $f : (X, T) \to (Y, \mathcal{T})$ is a bijective open function and a space (X, T) is a T_0-space (respectively, $\mathcal{T}_1$-space and $\mathcal{T}_2$-space), then the space $(Y, \mathcal{T}, J)$ is a Jpg-$\mathcal{T}_0$-space (respectively, Jpg-$\mathcal{T}_1$-space and Jpg-$\mathcal{T}_2$-space), for any ideal J on Y.

Definition 3.6: [2]. A function $f : (X, T, I) \to (Y, \mathcal{T}, J)$ is;

i. I-pre-g-continuous function, symbolizes Ipg-continuous, if $f^{-1}(v) \in I\text{pgO}(X)$ for all $v \in \mathcal{T}$.

ii. Strongly-I-pre-g-continuous function, Symbolizes strongly-Ipg-continuous, if $f^{-1}(v) \in T$, for all $v \in J\text{pgO}(Y)$.

iii. I-pre-g-irresolute function, symbolizes Ipg-irresolute, if $f^{-1}(v) \in I\text{pgO}(X)$ for all $v \in J\text{pgO}(Y)$.

Proposition 3.7: If $(Y, \mathcal{T})$ is T_0-space (respectively, T_1-space and T_2-space) and $f : (X, T, I) \to (Y, \mathcal{T}, J)$ is injective, Ipg- continuous function, then (X, T, I) is an Ipg-T_0-space(respectively, Ipg-T_1-space and Ipg-T_2-space).

Proof: Since $f^{-1}(v) \in I\text{pgO}(X)$ for all $v \in \mathcal{T}$.

Corollary 3.8: If a space $(Y, \mathcal{T})$ is T_0-space (respectively, T_1-space and T_2-space) and $f : (X, T, I) \to (Y, \mathcal{T}, J)$ is injective, continuous function then (X, T, I) is an Ipg- T_0-space (respectively, Ipg-T_1-space and Ipg-T_2-space).

Proof: Clearly, the continuity leads to Ipg- continuity [2]. So Proposition 3.7 is valid.

Proposition 3.9: If (Y, σ, J) is jpg-σ_0-space (respectively, jpg-σ_1-space and jpg-σ_2-space) and $f : (X, T, I) \to (Y, \sigma, J)$ is injective,strongly-İpg-continuous then the space (X, T) is T_0-space(respectively, T_1-space and T_2-space).

Proof: follows by the result $f^{-1}(V) \in T$, for all $V \in$ jpgO(Y).

Proposition 3.10: If a space (Y, σ, J) is a jpg-σ_0-space (respectively, jpg-σ_1-space and jpg-σ_2-space) and $f : (X, T, I) \to (Y, \sigma, J)$ is an injective, İpg-irresolute function, then (X, T, I) is an İpg-T_0-space(respectively, İpg-T_1-space and İpg-T_2-space).

Proof: follows by the result if $f^{-1}(V) \in$ İpgO(X) for all $V \in$ jpgO(Y).

4. On İpg- convergence

In this part we will use the notion İpg- openness to erection some class of convergence in ideal spaces namely İpg-convergence. So, the action of some sorts of functions are discussed like, İpg- open function, İpg-continuous-function, İpg-irrisolute- function and strongly-İpg-continuous function [1].

Definition 4.1: Let (X, T, I) be an ideal space, $x \in X$ and $(s_n)_{n\in\aleph}$ be a sequence in X. Then $(s_n)_{n\in\aleph}$ is namely İpg-convergence to x (frugally, $S_n \rightsquigarrow x$) if for every İpg- open set U contained x_0, $\exists k \in \aleph$ such that $s_n \in U \; \forall \; n \geq k$.

A sequence $(s_n)_{n\in\aleph}$ is namely İpg-divergence if it is not İpg-convergence.

Proposition 4.2: If (X, T, I) is İpg-T_2space then every İpg-convergence sequence in X has a unique limit point.

Proof: Let $(s_n)_{n\in\aleph}$ be a sequence in X where $s_n \rightsquigarrow x$ and $s_n \rightsquigarrow y$; $x \neq y$ where $x, y \in X$. Since (X, T, I) is İpg-T_2- space, then $\exists U, V \in$ İpgO(X) such that $x \in U$ and $y \in V$, where $U \cap V = \emptyset$. Since $s_n \rightsquigarrow x$ and $x \in U \in$ İpgO(X) leads to $\exists k_1 \in \aleph$; $s_n \in U \; \forall \; n \geq k_1$. So $s_n \rightsquigarrow y$ and $y \in V \in$ İpgO(X) leads to $\exists k_2 \in \aleph$; $s_n \in V \; \forall n \geq k_2$. Hence, $U \cap V \neq \emptyset$, that is contradiction.

The precondition that a space X is İpg-T_2- space is very requisite to make Proposition 4.2, is valid.

Example 4.3: For a space (X, T, I) where $X = \{x_1, x_2, x_3\}$, $T = \{X, \emptyset\}$ and $I = \{\emptyset\}$.

Obviously; the sequence $(s_n)_{n\in\aleph}$ in X, where $s_n = x_1$ for all n, has three limit points; that $s_n \rightsquigarrow x_1$, $s_n \rightsquigarrow x_2$ and $s_n \rightsquigarrow x_3$.

In mathematics, convergence sequence was an important subject [10, 11]. The following proposition explains the relationships between convergence and İpg -convergence to x_0.

Proposition 4.4: If a sequence $(s_n)_{n\in\aleph}$ is İpg -convergence to x_0 in(X, T, I), then it is convergence to x_0.

Proof: Since every open set in $(X, T, I)$is İpg-open, then the proof is over.

The meaningfulness in Proposition 4.4, cannot be inverting, in general.

Example 4.5: For a space (X, T, I), where $X = \aleph$ set of all neutral numbers, $T = \{X, \emptyset\}$ and $I = P(X)$. The sequence $(s_n)_{n\in\aleph}$, where $s_n = n \; \forall \; n \in \aleph$, is convergence to $n = 1$ which is not İpg -convergence.

Proposition 4.6: Let $f : (X, T, I) \to (Y, \sigma, J)$ be İpg-irresolute function and $(s_n)_{n\in\aleph}$ be a sequence in X. If $s_n \rightsquigarrow x_0$ in X, then $f(s_n) \longrightarrow f(x_0)$ in Y.

Proposition 4.7: Let $f : (X, T, I) \to (Y, \sigma, J)$ be İpg-continuous function and $(s_n)_{n\in\aleph}$ be a sequence in X. If $s_n \rightsquigarrow x_0$ in X, then $f(s_n) \longrightarrow f(x_0)$ in Y.

Proposition 4.8: Let $\mathfrak{f} : (X, T, \dot{I}) \to (Y, T', J)$ be strongly-İsg-continuous function and $(s_n)_{n \in \aleph}$ be a sequence in X. If $s_n \rightsquigarrow x_0$ in X, then $\mathfrak{f}(s_n) \longrightarrow \mathfrak{f}(x_0)$ in Y.

5. Conclusion

The notion İ-*pre-g- openness*, was use to offered new classes of separation axioms and new type of convergence in ideal spaces. Some relations and examples among several types of separation axioms that offered were explained.

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