Abstract
The notions $I$-semi-$g$-closedness and $I$-semi-$g$-openness were used to generalize and introduced new classes of separation axioms in ideal spaces. Many relations among several sorts of these classes are summarized, also.


1. Introduction
Abdel Karim and Nasir [1]. Introduced the notion $I$-semi-$g$-closed set in ideal spaces. A set $A$ of $(X, T, I)$ is $I$-semi-$g$-closed set (artlessly, $Isg$-closed), when prerequisite $A \cup U \in I$ and $U$ is semi-open, leads to $cl(A) \cup U \in I$. So, the set $A \subseteq X$ is nameable $I$-semi-$g$-open (artlessly, $Isg$-open) whenever $X - A$ is $Isg$-closed set. The collection of all $Isg$-closed (respectively, $Isg$-open) sets in $(X, T, I)$ notate as $Isg-C(X)$ (respectively, $Isg-O(X)$) . Every open set in $(X, T)$ is $Isg$-open in $(X, T, I)$[1]. And the reverse of this statement is incorrect [1].

The idea of ideals on a nonempty set was started by Kuratowski in 1933[2]. The ideal $I$ on $X$ where $I = P(X)$ that is valid the preconditions finite additivity ($A$ and $B \in I$ implies $A \cup B \in I$) and heredity ($A \subseteq B$ and $B \in I$ implies $A \in I$) [2].

In 1945, Vaidynathaswamy [3]. Was the first mathematician who uses the idea of ideal in science of topology, which used to create a new generalization of topological spaces, which is called ideal topological space and notate as $(X, T, I)$[4].

Since then, more authors are interested in this scope of the study. Many results have been achieved [5-8].

The concept of "semi-open set" was firstly offered by N.Levine [9]. A set $A$ in $(X, \mathcal{O})$ is called semi-open if $A \subseteq cl(int(A))$ [9].

Recently, the main goal of our work is to construct generalizations of the notion separation axioms in topological space by using the notion of $Isg$-open set.
2. Separation Axioms via $\mathcal{I}$-open Sets.

Here, we will present new type of separation axioms in ideal spaces via $\mathcal{I}$-open set. Also, relations between these sorts are discussed. The following definition is for the research who did this research.

**Definition 2.1.** The triple $(X, \mathcal{T}, \mathcal{I})$ is called $\mathcal{I}$-semi-$g$-$T_0$-space (shortly, $\mathcal{I}$-$g$-$T_0$-space), if for any element $c_1 \neq c_2$, there is an $\mathcal{I}$-open set $U$ containing only one of them.

**Remark 2.2.** If $(X, \mathcal{T})$ is $T_0$-space then $(X, \mathcal{T}, \mathcal{I})$ is $\mathcal{I}$-$g$-$T_0$-space.

**Remark 2.3.** The below phrases are rewards;

i. $X$ is $\mathcal{I}$-$g$-$T_0$-space.

ii. For each element $c_1 \neq c_2$, there is an $\mathcal{I}$-open set $V$ containing only one of them.

**Definition 2.4.** $(X, c, \mathcal{I})$ is called $\mathcal{I}$-semi-$g$-$T_1$-space (artlessly, $\mathcal{I}$-$g$-$T_1$-space), if for any elements $c_1 \neq c_2$, there are $\mathcal{I}$-open sets, $V$ whenever $(c_1 \in U \cap V)$, $(c_2 \in V \cap U)$.

**Remark 2.5.** If $(X, \mathcal{T})$ is $T_1$-space then $(X, \mathcal{T}, \mathcal{I})$ is $\mathcal{I}$-$g$-$T_1$-space.

**Remark 2.6.** $(X, \mathcal{T}, \mathcal{I})$ is $\mathcal{I}$-$g$-$T_0$-space whenever it is an $\mathcal{I}$-$g$-$T_1$-space.

The conclusions in Remark 2.6, is not reversible by example below.

**Example 2.7.** The $\mathcal{I}$-$g$-$T_0$-space $(X, \mathcal{T}, \mathcal{I})$ where $X = \{c_1, c_2, c_3\}$, $\mathcal{C} = \{X, \phi, \{c_1\}, \{c_2\}, \{c_1, c_2\}\}$ and $\mathcal{I} = \{\phi\}$, is not $\mathcal{I}$-$g$-$T_1$-space.

**Remark 2.8.** The below phrases are rewards;

i. $X$ is $\mathcal{I}$-$g$-$T_1$-space.

ii. For each elements $c_1 \neq c_2$, there are two $\mathcal{I}$-open sets $A$ and $B$ satisfy $(c_1 \in A \cap B^c)$ and $(c_2 \in B \cap A^c)$.

**Remark 2.9.** For $(X, c, \mathcal{I})$, if $\mathcal{C}$ is $\mathcal{I}$-closed set $\forall c \in X$, then $X$ is $\mathcal{I}$-$g$-$T_1$-space.

The conclusions in Remark 2.9, is not reversible by example below.

**Example 2.10.** Given $(X, \mathcal{T}, \mathcal{I})$; $(X = \{c_1, c_2, c_3\}, \mathcal{C} = \{\phi, X\}$ and $\mathcal{I} = \{X, \phi, \{c_1\}\}$) is $\mathcal{I}$-$g$-$T_1$-space, while $\{c_1\}$ is not $\mathcal{I}$-closed set.

**Definition 2.11.** $(X, \mathcal{T}, \mathcal{I})$ is called $\mathcal{I}$-semi-$g$-$T_2$-space (artlessly, $\mathcal{I}$-$g$-$T_2$-space), if for any elements $c_1 \neq c_2$, there are $\mathcal{I}$-open sets $\mathcal{V}$ and $\mathcal{U}$ satisfy $c_1 \in \mathcal{U} \setminus \mathcal{V}$ and $c_2 \in \mathcal{V} \setminus \mathcal{U}$.

**Remark 2.12.** If $(X, \mathcal{T})$ is $T_2$-space, then $(X, \mathcal{T}, \mathcal{I})$ is $\mathcal{I}$-$g$-$T_2$-space.

**Remark 2.13.** A space $(X, \mathcal{T}, \mathcal{I})$ is $\mathcal{I}$-$g$-$T_1$-space whenever it is an $\mathcal{I}$-$g$-$T_2$-space.

The conclusions in Remark 2.13, is not reversible by example below.

**Example 2.14.** A space $(X, \mathcal{T}, \mathcal{I})$ when $X = \mathcal{N}$ the set of all natural numbers, $\mathcal{T} = \mathcal{T}_{cog}$ is the cofinite topology on $X$ and $\mathcal{I} = \{\phi\}$. Clearly, that $(X, \mathcal{T}, \mathcal{I})$ is $\mathcal{I}$-$g$-$T_1$-space which is not $\mathcal{I}$-$g$-$T_2$-space.

We have previously noted that $X$ is $\mathcal{I}$-$g$-$T_1$-space whenever it is $T_1$-space ($\forall i = 0, 1$ and $2$). The opposite is not generally achieved by example below.

**Example 2.15.** $(X, \mathcal{T}, \mathcal{I})$ is $\mathcal{I}$-$g$-$T_1$-space ($\forall i = 0, 1$ and $2$), where $X = \{c_1, c_2, c_3\}$, $\mathcal{T} = \{X, \phi\}$ and $\mathcal{I}$ is the class of all sets in $X$. But the space $(X, \mathcal{T})$ is not $T_1$-space.

The following chart shows the relationships among the various types of notions of our previously mentioned.
3. Separation Axioms via Some Types of Functions

In this portion we will review some sorts of functions which presented by Abdel Karim and Nasir [1]. And study their impact on the notions of separated axioms.

**Definition 3.1:**[1] \( \mathcal{P}: (X, \mathcal{T}, \mathcal{I}) \rightarrow (Y, \mathcal{J}, j) \) is:

i. \( l\)-semi-\(g\)-open function, notate as \( \text{Isg-function} \), if the precondition \( U \) is \( \text{Isg-open} \) in \( X \), leads to \( \mathcal{P}(U) \) is \( \text{jsg-open} \) in \( Y \).

ii. \( l^*\)-semi-\(g\)-open function, notate as \( l^*\text{sgo-function} \), if the precondition \( U \) is open in \( X \), leads to \( \mathcal{P}(U) \) is \( \text{jsg-open} \) in \( Y \).

iii. \( l^{**}\)-semi-\(g\)-open function, notate as \( l^{**}\text{sgo-function} \), if the precondition \( U \) is \( \text{Isg-open} \) in \( X \), leads to \( \mathcal{P}(U) \) is \( \text{Isg-open} \) in \( Y \).

**Proposition 3.2:** \( X \) is \( \text{Isg-T}_1 \)-space \((\forall i = 0, 1, 2) \) and \( \mathcal{P} \) is a surjective \( \text{Isg-function} \) from \((X, \mathcal{T}, \mathcal{I}) \) to \((Y, \mathcal{J}, j) \) implies \( Y \) is \( \text{jsg-T}_1 \)-space.

**Proof:** Follows from \( \mathcal{P}(U) \) is \( \text{jsg-open} \) in \( Y \) for all \( \text{Isg-open} \) set \( U \) in \( X \).

**Proposition 3.3:** \( X \) is \( \text{T}_1 \)-space \((\forall i = 0, 1, 2) \) and \( \mathcal{P} \) is a surjective \( l^*\text{sgo-function} \) from \((X, \mathcal{T}, \mathcal{I}) \) to \((Y, \mathcal{J}, j) \) implies \( Y \) is \( \text{jsg-T}_1 \)-space.

**Proof:** Follows from \( \mathcal{P}(U) \) is \( \text{jsg-open} \) in \( Y \) for all open set \( U \) in \( X \).

**Proposition 3.4:** \( X \) is \( \text{Isg-T}_1 \)-space \((\forall i = 0, 1, 2) \) and \( \mathcal{P} \) is a surjective \( l^{**}\text{sgo-function} \) from \((X, \mathcal{T}, \mathcal{I}) \) to \((Y, \mathcal{J}, j) \) implies \( Y \) is \( \text{T}_1 \)-space.

**Proof:** Follows from \( \mathcal{P}(U) \) is open in \( Y \) for all \( \text{Isg-open} \) set \( U \) in \( X \).

**Remark 3.5:** If \( \mathcal{P} \) is bijective open function from \((X, \mathcal{T}) \) to \((Y, \mathcal{J}) \) and \( X \) is \( \text{T}_i \)-space \((\forall i = 0, 1, 2) \), then \( (Y, \mathcal{J}, j) \) is \( \text{jsg-T}_i \)-space, for any ideal \( j \) on \( Y \).

**Definition 3.6:**[1] The function \( \mathcal{P} \) from \((X, \mathcal{T}, \mathcal{I}) \) to \((Y, \mathcal{J}, j) \) is:

i. \( l\)-semi-\(g\)-continuous function, notate as \( \text{Isg-continuous function} \), if \( \mathcal{P}^{-1}(U) \in \text{Isg-O}(X) \) for every \( U \in \mathcal{J} \).

ii. \( l^*\)-semi-\(g\)-continuous function, notate as \( \text{strongly-Isg-continuous function} \), if \( \mathcal{P}^{-1}(U) \in \mathcal{T} \) for every \( U \in \text{Isg-O}(Y) \).

iii. \( l^{**}\)-semi-\(g\)-irresolute function, notate as \( \text{Isg-irresolute function} \), if \( \mathcal{P}^{-1}(U) \in \text{Isg-O}(X) \) for every \( U \in \text{jsg-O}(Y) \).

**Proposition 3.7:** If \( Y \) is \( \text{T}_1 \)-space \((\forall i = 0, 1, 2) \) and \( \mathcal{P}: (X, \mathcal{T}, \mathcal{I}) \rightarrow (Y, \mathcal{J}, j) \) is an injective \( \text{Isg-continuous function} \) then \( X \) is \( \text{Isg-T}_1 \)-space.

**Proof:** Since \( \mathcal{P}^{-1}(U) \in \text{Isg-O}(X) \) for all \( U \in \mathcal{J} \).
Corollary 3.8: If \( Y \) is \( T_i \)-space (\( \forall i = 0,1 \) and 2) and \( \mathcal{P}: (X,T,\bar{I}) \rightarrow (Y,\mathcal{I},\bar{J}) \) is an injective continuous function then \( \bar{Y} \) is \( js-g-T_i \)-space.

Proof: Since, every continuous function is \( js-g \)-continuous [1]. And then Proposition 3.7 is applicable.

Proposition 3.9: If \( Y \) is \( js-g-T_i \)-space (\( \forall i = 0,1 \) and 2) and \( \mathcal{P}: (X,T,\bar{I}) \rightarrow (Y,\mathcal{I},\bar{J}) \) is an injective strongly-\( js-g \)-continuous function then \( X \) is \( T_i \)-space.

Proof: Follows from, \( \mathcal{P}^{-1}(\mathcal{U}) \in \mathcal{T} \) for all \( \mathcal{U} \in js-g(O(Y)) \).

Proposition 3.10: If \( Y \) is \( js-g-T_i \)-space (\( \forall i = 0,1,2 \) and \( \mathcal{P}: (X,T,\bar{I}) \rightarrow (Y,\mathcal{I},\bar{J}) \) is an injective \( js-g \)- irresolute function then \( X \) is \( js-g-T_i \)-space.

Proof: Since \( \mathcal{P}^{-1}(\mathcal{U}) \in js-g(O(X)) \) for all \( \mathcal{U} \in js-g(O(Y)) \).

4. I- Semi-g-convergence sequence

Sequences are one of important topics in all branches of mathematics, especially mathematical analysis and topology. So the convergence is the important property for the sequence [10, 11].

In this paragraph, we will use the notion of \( js-g \)-open set to create new class of convergence.

Definition 4.1. Given a space \((X,T,\bar{I})\) where \( c_0 \in X \) and the sequence \((S_n)_{n \in N}\) in \( X \). Then \((S_n)_{n \in N}\) is called \( I \)-semi-g-convergence to \( c_0 \) (artlessly, \( S_n \leftrightarrow c_0 \)) if for every \( js-g \)-open set \( \mathcal{U} \) contained \( c_0, \exists \kappa \in N \) where \( S_n \in \mathcal{U} \forall n \geq \kappa \).

A sequence \((S_n)_{n \in N}\) is nameable \( I \)-semi-g-divergence if it is not \( I \)-semi-g-convergence.

Proposition 4.2. If \((X,T,\bar{I})\) is \( js-g-T_2 \)-space then every \( I \)-semi-g-convergence sequence in \( X \) has only one limit point.

Proof: If we consider \((S_n)_{n \in N}\) is a seq. in \( X \) and \( S_n \leftrightarrow \epsilon_1 \) so \( S_n \leftrightarrow \epsilon_2 \) where \( \epsilon_1, \epsilon_2 \in X \). Since \( X \) is \( js-g-T_2 \)-space, then there are disjoint \( js-g \)-open sets \( \mathcal{U} \) and \( \mathcal{V} \) such that \( \epsilon_1 \in \mathcal{U} \) and \( \epsilon_2 \in \mathcal{V} \). Now, since \( S_n \leftrightarrow \epsilon_1 \) and \( \epsilon_1 \in \mathcal{U} \) implies \( \exists \kappa_1 \in N; S_n \in \mathcal{U} \forall n \geq \kappa_1 \). So, \( S_n \leftrightarrow \epsilon_2 \) and \( \epsilon_2 \in \mathcal{V} \) implies \( \exists \kappa_2 \in N; S_n \in \mathcal{V} \forall n \geq \kappa_2 \). Leads to \( \mathcal{U} \cap \mathcal{V} \neq \emptyset \), where that contradiction.

The precondition that a space \( X \) is \( js-g-T_2 \)-space is very requisite to make Proposition 4.2, is valid.

Example 4.3. Consider \((X,T,\bar{I})\) where \( X = \{\epsilon_1, \epsilon_2, \epsilon_3\}, T = \{\emptyset, X\} \) and \( \bar{I} = \{\emptyset\} \).

Obviously; the sequence \((S_n)_{n \in N}\) in \( X \), where \( S_n = \epsilon_1 \) for all \( n \), has more than one limit point; that \( \epsilon_1 \leftrightarrow \epsilon_1, S_n \leftrightarrow \epsilon_2 \) and \( S_n \leftrightarrow \epsilon_3 \).

Proposition 4.4. If a sequence \((S_n)_{n \in N}\) is \( I \)-semi-g-convergent to \( c_0 \) in an ideal space \( X \), then it is convergent to \( c_0 \).

Proof. Since every open set in \((X,T,\bar{I})\) is \( js-g \)-open, then the proof is over.

The meaningfulness in proposition 4.4, is not reversible, in general.

Example 4.5. For an ideal space \((X,T,\bar{I})\), where \( X = \mathcal{N}, T = \{\emptyset, X\} \) and \( \bar{I} = \mathcal{P}(X) \). The sequence \((S_n)_{n \in N}, \) where \( S_n = n \forall n \in \mathcal{N}, \) is convergent to \( n = 1 \) which is non-I-semi-g-convergence.

Proposition 4.6. Let \( \mathcal{P} \) be \( js-g \)- irresolute function from \( X \) to \( Y \) and \((S_n)_{n \in N}\) be a sequence in \( X \), \( A(S_n) \leftrightarrow A(c_0) \) in \( Y \) whenever \( S_n \leftrightarrow c_0 \) in \( X \).
Proposition 4.7. Let $\mathcal{P}$ be Isg-continuous function from $\mathcal{X}$ to $\mathcal{Y}$ and $(S_n)_{n \in \mathbb{N}}$ be a sequence in $\mathcal{X}$. $\mathcal{P}(S_n)$ is convergent to $\mathcal{P}(c_0)$ in $\mathcal{Y}$ whenever $S_n \rightarrow c_0$ in $\mathcal{X}$.

Proposition 4.8. Let $\mathcal{P}$ be strongly-Isg-continuous function from $\mathcal{X}$ to $\mathcal{Y}$ and $(S_n)_{n \in \mathbb{N}}$ be a sequence in $\mathcal{X}$, $\mathcal{P}(S_n) \rightarrow \mathcal{P}(c_0)$ in $\mathcal{Y}$ whenever $S_n$ is convergent to $c_0$ in $\mathcal{X}$.

5. Conclusion
The separation axioms appeared in new types and generalizations by used I-semi-g-openness, in addition a new types of convergence in ideal spaces were studied such as I-semi-g-convergence, and the relationships between them was clarified by using several example.

References