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Some Aspects of Weighted Rayleigh Distribution

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Abstract

In this paper, we proposed a new class of weighted Rayleigh distribution based on two parameters, scale and shape parameters which are introduced in Rayleigh distribution. The main properties of this class are investigated and derived.

Keywords: weighted Rayleigh distribution, quintile function skew-ness, and kurtosis.

1. Introduction

The Rayleigh distribution is an important distribution as a lifetime sample modeling. Many researchers developed various generalizations of Rayleigh probability density functions to increase the flexibility in life time sample modeling [1]. Introduced a new class of density functions depending on shape parameter in the normal distribution, and then called it weighted normal distribution or (skew-normal distribution). [2]. Used the idea of Azzalini to find the shape parameter to an exponential distribution and called it weighted exponential distribution, as well as put the general mathematical formula to treat the weighted statistical distributions which are as follow:

$$f_w(X) = \frac{1}{Pr[x_2 < \theta x_1]} f(x_1)F(\theta\, x_1) \qquad (1)$$

Where $f_w(X)$ is weighted probability density function.

$f(x_1)$ isstandard probability density function for $r.v(x_1)$.

$f(\theta x_1)$ iscumulative distribution function with respect to weighted parameter θ for standard distribution.

$P_r(x_2 < \theta x_1)$ probability for $r.v(x_2)$ with respect to the $r.v(x_1)$ and weighted function(θ). [3]. Study weighed Weibull distribution by using the idea of Azzalini, and introduce the basic properties for this model. [4]. Study the skew-ness parameter of a gamma distribution by employing the idea of Azzalini, which creates a new class of weighed gamma distribution. [5]. Propose the extension of Weibull distribution weighted and (the main derived) main properties of this class are derived [6]. Estimate Linley's approximation method for weighted exponential distribution by using Monte Carlo simulation study. [7]. Introduce two shape parameters to the existing weighted exponential distribution to develop the Beta weighted exponential distribution using the legit of beta function.[8]. propose model named exponentiated weighted exponential distribution and some of the basic statistical properties of the proposed model are studied and provided. [9]. Focus on Bayes estimation of weighed exponential distribution with fuzzy data. [10]. Derive two parameters inverted weighted exponential distribution and its various statistical properties are established. [11]. Present a new generalization weighted Weibull distribution.The aim of this paper is to propose a new weighted Rayleigh distribution depending on the idea of Azzalini and apply the general formula of Gupta and Kundu in equation (1) to find the skew ness parameter in Rayleigh density function, then study some basic statistical properties of this distribution. The rest of this article is organized as follows: we include in section two the weighted Rayleigh distribution with application, Cumulative distribution function, reliability function and hazard function. In section three, the moment, mean, and variance. In section four, the moment generating functions. In section five, the factorial moment generating function, skew ness kurtosis, and characteristic-function and in section six, the quintile function, mode and median.

2. Weighted Rayleigh Distribution

In this section, we introduce of the probability density function and the cumulative distribution function of weighted Rayleigh distribution (WRD).

Lemma (1)

Suppose that x_1 and x_2 are random sample of size two of Rayleigh distribution with parameter θ, then the $pr[\alpha x_1 > x_2] = \frac{\alpha^2}{\alpha^2+1}$ where $\frac{\alpha^2+1}{\alpha^2}$ is weighted Rayleigh distribution.

Proof:

$pr[\alpha x_1 > x_2] = pr[x_2 < \alpha x_1]$

$$pr[x_2 < \alpha x_1] = \int_{all x_1} \int_{all x_2} f(x_1, x_2)\, dx_2 dx_1$$

$$pr[x_2 < \alpha x_1] = \int_{\circ}^{\infty} \int_{\circ}^{\alpha x_1} \theta x_1\, e^{-\frac{\theta}{2}x_1^2}\, \theta x_2\, e^{-\frac{\theta}{2}x_2^2}\, dx_2 dx_1$$

$$pr[x_2 < \alpha x_1] = \int_{\circ}^{\infty} \theta x_1\, e^{-\frac{\theta}{2}x_1^2} \left[\int_{\circ}^{\alpha x_1} \theta x_2\, e^{-\frac{\theta}{2}x_2^2}\, dx_2 \right] dx_1$$

$$pr[x_2 < \alpha x_1] = \int_{\circ}^{\infty} \theta x_1\, e^{-\frac{\theta}{2}x_1^2} \left[- e^{-\frac{\theta}{2}x_2^2} \Big|_{\circ}^{\alpha x_1} \right] dx_1$$

$$pr[x_2 < \alpha\, x_1] = \int_{\circ}^{\infty} \theta x_1\, e^{-\frac{\theta}{2}x_1^2} \left[1 - \, e^{--\frac{\theta}{2}\alpha^2\, x_1^2} \right] dx_1$$

$$pr[x_2 < \alpha\, x_1] = \int_{\circ}^{\infty} \theta x_1\, e^{-\frac{\theta}{2}x_1^2} dx_1 - \int_{\circ}^{\infty} \theta x_1\, e^{-\frac{\theta}{2}x_1^2(\alpha^2+1)} dx_1$$

$$pr[x_2 < \alpha\, x_1] = 1 - \left[\frac{-1}{\alpha^2+1}\int_{\circ}^{\infty} \theta x_1\,(\alpha^2+1)e^{-\frac{\theta}{2}x_1^2(\alpha^2+1)}dx_1\right]$$

$$pr[x_2 < \alpha\, x_1] = 1 - \left[\frac{1}{\alpha^2+1}\right] = \frac{\alpha^2}{\alpha^2+1}$$

$$\frac{1}{pr[x_2 < \alpha x_1]} = \frac{1}{\frac{\alpha^2}{\alpha^2+1}} = \frac{\alpha^2+1}{\alpha^2}$$

Then the proposed new weighted Rayleigh distribution is defined as follows:

$$f(x;\alpha,\theta) = \frac{\alpha^2+1}{\alpha^2}\theta x\, e^{-\frac{\theta}{2}x^2}\left[1 - e^{-\frac{\theta}{2}x^2\alpha^2}\right] \quad (2)$$

Now, proving that the weighted Rayleigh distribution is probability density function as follows:

$$\int_{\circ}^{\infty} \frac{\alpha^2+1}{\alpha^2}\,\theta\, x e^{-\frac{\theta}{2}x^2}\left[1 - e^{-\frac{\theta}{2}x^2\alpha^2}\right]dx = 1$$

$$\frac{\alpha^2+1}{\alpha^2}\left\{\int_{\circ}^{\infty} \theta x e^{-\frac{\theta}{2}x^2}\;dx - \int_{\circ}^{\infty} \theta x e^{\frac{\theta}{2}x^2(\alpha^2+1)}\,dx\right\}$$

$$= \frac{\alpha^2+1}{\alpha^2}\left\{1 - \frac{1}{\alpha^2+1}\right\} = \frac{\alpha^2+1}{\alpha^2} * \frac{\alpha^2}{\alpha^2+1} = 1$$

Lemma (2)

The cumulative distribution function of weighted Rayleigh distribution is as follows:

Proof:

$$F(x;\ \alpha,\theta) = Pr\,[X \le x] = \int_{\circ}^{x} f(u)du$$

$$F(x;\ \alpha,\theta) = \int_{o}^{x}\frac{(\alpha^2+1)}{\alpha^2}\,\theta\, u e^{-\frac{\theta}{2}u^2}\left(1 - e^{-\frac{\theta}{2}u^2\alpha^2}\right)du$$

$$F(x;\ \alpha,\theta) = \frac{\alpha^2+1}{\alpha^2}\left\{\frac{(\alpha^2+1) - (\alpha^2+1)e^{-\frac{\theta}{2}x^2} + e^{-\frac{\theta}{2}x^2(\alpha^2+1)} - 1}{\alpha^2+1}\right\}$$

$$F(x;\ \alpha,\theta) = \frac{\alpha^2+1}{\alpha^2} - \frac{(\alpha^2+1)e^{-\frac{\theta}{2}x^2}}{\alpha^2} + \frac{e^{-\frac{\theta}{2}x^2(\alpha^2+1)}}{\alpha^2} - \frac{1}{\alpha^2}$$

$$F(x;\ \alpha,\theta) = 1 - \left[\frac{(\alpha^2+1)e^{-\frac{\theta}{2}x^2} - e^{-\frac{\theta}{2}x^2(\alpha^2+1)}}{\alpha^2}\right] \quad (3)$$

The reliability and hazard functions are as follows:

$$R(x) = 1 - F(x)$$

$$R(x) = \frac{(\alpha^2+1)e^{-\frac{\theta}{2}x^2} - e^{-\frac{\theta}{2}x^2(\alpha^2+1)}}{\alpha^2} \quad (4)$$

$$h(x) = \frac{f(x)}{R(x)}$$

$$h(x) = \frac{\theta x(\alpha^2+1)\left[1-e^{-\frac{\theta}{2}x^2 d^2}\right]}{\left[(\alpha^2+1)-e^{-\frac{\Theta}{2}x^2\alpha^2}\right]} \tag{5}$$

3. Moment Method

Now, we derived the moment a bout zero, then we find the mean and the variance of the Weighted Rayleigh distribution which is as follows:

$$E(X^r) = \int_o^\infty x^r \frac{\alpha^2+1}{\alpha^2}\theta x\, e^{-\frac{\theta}{2}x^2}\left[1-e^{-\frac{\theta}{2}x^2\alpha^2}\right]dx$$

$$E(X^r) = \frac{\alpha^2+1}{\alpha^2}\left\{\int_o^\infty x^{r+1}\,\theta e^{-\frac{\theta}{2}x^2}\,dx - \int_o^\infty x^{r+1}\,\theta e^{-\frac{\theta}{2}x^{2(\alpha^2+1)}}\,dx\right\}$$

$$let\ y = \frac{\theta}{2}x^2\ ,\ x = \left(\frac{2}{\theta}\right)^{\frac{1}{2}} y^{\frac{1}{2}}\ ,\ dx = \left(\frac{2}{\theta}\right)^{\frac{1}{2}}\frac{1}{2\sqrt{y}}dy$$

$$lt\ u = \frac{\theta}{2}x^2\,(\alpha^2+1)\ ,\ x = \left(\frac{2}{\theta(\alpha^2+1)}\right)^{\frac{1}{2}} u^{\frac{1}{2}}\ ,\ dx = \left(\frac{2}{\theta(\alpha^2+1)}\right)^{\frac{1}{2}}\frac{1}{2\sqrt{u}}du$$

$$E(x^r) = \frac{\alpha^2+1}{\alpha^2}\left\{\int_o^\infty \left[\left(\frac{2}{\theta}\right)^{\frac{1}{2}} y^{\frac{1}{2}}\right]^{r+1} \theta\, e^{-y}\left(\frac{2}{\theta}\right)^{\frac{1}{2}}\frac{1}{2\sqrt{y}}dy \right.$$
$$\left. - \int_o^\infty \left[\left(\frac{2}{\theta(\alpha^2+1)}\right)^{\frac{1}{2}} u^{\frac{1}{2}}\right]^{r+1} \theta\, e^{-u}\left(\frac{2}{\theta(\alpha^2+1)}\right)^{\frac{1}{2}}\frac{1}{2\sqrt{u}}du\right\}$$

$$E(X^r) = \frac{\alpha^2+1}{\alpha^2}\left\{\int_o^\infty \left(\frac{2}{\theta}\right)^{\frac{r}{2}} y^{\frac{r}{2}}\left(\frac{2}{\theta}\right)^{\frac{1}{2}} y^{\frac{1}{2}}\,\theta\, e^{-y}\left(\frac{2}{\theta}\right)^{\frac{1}{2}}\frac{1}{2}y^{-\frac{1}{2}}\,dy \right.$$
$$\left. - \int_o^\infty \left(\frac{2}{\theta(\alpha^2+1)}\right)^{\frac{r}{2}}\left(\frac{2}{\theta(\alpha^2+1)}\right)^{\frac{1}{2}} u^{\frac{r}{2}}\,u^{\frac{1}{2}}\,\theta\, e^{-u}\left(\frac{2}{\theta(\alpha^2+1)}\right)^{\frac{1}{2}}\frac{1}{2}u^{-\frac{1}{2}}du\right\}$$

$$E(X^r) = \frac{\alpha^2+1}{\alpha^2}\left\{\left(\frac{2}{\theta}\right)^{\frac{r}{2}}\int_o^\infty \frac{2}{\theta}y^{\frac{r}{2}}\,\theta\ e^{-y}\frac{1}{2}dy - \left(\frac{2}{\theta(\alpha^2+1)}\right)^{\frac{r}{2}}\int_o^\infty \frac{2}{\theta(\alpha^2+1)}u^{\frac{r}{2}}\,\theta\, e^{-u}\frac{1}{2}du\right.$$

$$E(X^r) = \frac{\alpha^2+1}{\alpha^2}\left\{\left(\frac{2}{\theta}\right)^{\frac{r}{2}}\Gamma(\frac{r}{2}+1) - \left(\frac{2}{\Theta}\right)^{\frac{r}{2}}\frac{1}{(\alpha^2+1)^{\frac{r}{2}+1}}\Gamma(\frac{r}{2}+1)\right\}$$

$$E(X^r) = \frac{\alpha^2+1}{\alpha^2}\left(\frac{2}{\theta}\right)^{\frac{r}{2}}\Gamma(\frac{r}{2}+1)\left[1-\frac{1}{(\alpha^2+1)^{\frac{r}{2}+1}}\right] \tag{6}$$

When $r = 1\,, then$

$$E(X) = \frac{\alpha^2+1}{\alpha^2}\left(\frac{2}{\theta}\right)^{\frac{1}{2}}\ \Gamma(\frac{3}{2})\left[1-\frac{1}{(\alpha^2+1)^{3/2}}\right]$$

$$E(X)=\frac{\alpha^2+1}{\alpha^2}\left(\frac{2}{\theta}\right)^{\frac{1}{2}}\frac{1}{2}\sqrt{\pi}\left[1-\frac{1}{(\alpha^2+1)^{\frac{3}{2}}}\right]$$

$$E(X)=\frac{\alpha^2+1}{\alpha^2}\left[\frac{\pi}{2\theta}\right]^{\frac{1}{2}}\left[1-\frac{1}{(\alpha^2+1)^{3/2}}\right] \quad (7)$$

$when\ r=2$, then

$$E(X^2)=\frac{\alpha^2+1}{\alpha^2}\left(\frac{2}{\theta}\right)\ \Gamma(2)\left[1-\frac{1}{(\alpha^2+1)^2}\right]$$

$$E(X^2)=\frac{2(\alpha^2+2)}{\theta(\alpha^2+1)} \quad (8)$$

The variance:

$$var\ (X)=E(X^2)-[E(X)]^2$$

$$var(X)=\frac{2(\alpha^2+2)}{\theta(\alpha^2+1)}-\frac{(\alpha^2+1)^2}{\propto^4}\frac{\pi}{2\theta}\left[1-\frac{1}{(\alpha^2+1)^{\frac{3}{2}}}\right]^2$$

$$var(X)=\frac{4\alpha^4(\alpha^2+2)-\pi\left[(\alpha^2+1)^{\frac{3}{2}}-1\right]^2}{2\theta\alpha^4(\alpha^2+1)} \quad (9)$$

4-The Moment Generated Function

We derived the moment generating function for weighted Rayleigh distribution which is as follows:

$$m.g.f.=E[e^{tX}]=\int_{\circ}^{\infty} e^{tx}\ f(x)dx$$

By using Taylor series:

$$E(e^{tX})=\int_{\circ}^{\infty}\left(1+\frac{tx}{1!}+\frac{tx^2}{2!}+\frac{tx^3}{3!}+\cdots+\frac{tx^r}{r!}\right)f(x)dx$$

$$m.g.f=\sum_{r=0}^{\infty}\frac{t^r}{r!}\ E(x^r)$$

$$m.g.f=\sum_{r=0}^{\infty}\frac{t^r}{r!}\ \ \frac{\alpha^2+1}{\alpha^2}\left(\frac{2}{\theta}\right)^{\frac{r}{2}}\Gamma\left(\frac{r}{2}+1\right)\left[1-\frac{1}{(\alpha^2+1)^{r/2+1}}\right]$$

$$m.g.f=\frac{\alpha^2+1}{\alpha^2}\sum_{r=0}^{\infty}\frac{t^r}{r!}\ \left(\frac{2}{\theta}\right)^{\frac{r}{2}}\Gamma\left(\frac{r}{2}+1\right)\left[1-\frac{1}{(\alpha^2+1)^{r/2+1}}\right] \quad (10)$$

5- Factorial Moment Generating Function:

$$M_x(t)=E(t^x)=\int_{\circ}^{\infty} t^x\,f(x)dx$$

$$M_x(t)=\int_{\circ}^{\infty} e^{\ell n t^x}f(x)dx$$

$$M_x(t) = \int_{\circ}^{\infty} \left(1 + x\,\ell nt + \frac{x^2(\ell nt)^2}{2!} + \cdots + \frac{x^r(\ell nt)^r}{r!} + \cdots \ldots\ldots\right) f(x)dx$$

$$M_x(t) = \sum_{r=0}^{\infty} \frac{(\ell nt)^r}{r!} E(x^r)$$

$$M_x(t) = \frac{\alpha^2+1}{\alpha^2} \sum_{r=0}^{\infty} \frac{(\ell nt)^r}{r!} \left(\frac{2}{\theta}\right)^{\frac{r}{2}} \Gamma\left(\frac{r}{2}+1\right)\left[1 - \frac{1}{(\alpha^2+1)^{\frac{r}{2}+1}}\right] \quad (11)$$

Then the Coefficient of skew ness of this distribution is as follows:

$$C.S = \frac{M_3}{(M_2)^{\frac{3}{2}}}\ ,\ \ M_3 = E(X - E(X)))^3,\ \ M_2 = E(X - E(X)))^2$$

$$C.S = \frac{E(X-E(X)))3,}{E(X-E(X)))3,} = 1 \quad (12)$$

$$C.K = \frac{M_4}{(M_2)^2} = \frac{E(X-E(X)))^4}{(E(X-E(X)))\ )^2} \quad (13)$$

The characteristic function of this distribution is as follow:

$$Q_X\,(t) = E\left(e^{itx}\right) = \int_{\circ}^{\infty} e^{itx}\, f\,(x)dx$$

$$Q_X\,(t) = \int_{\circ}^{\infty} \left(1 + itx + \frac{(it)^2x^2}{2!} + \cdots \frac{(it)^r x^r}{r!}\right) f(x)dx$$

$$Q_X\,(t)\ =\ \sum_{r=0}^{\infty} \frac{(it)^r}{r!} E(X^r)$$

$$Q_X\,(t) = \sum_{r=0}^{\infty} \frac{(it)^r}{r!} \frac{\alpha^2+1}{\alpha^2} \left(\frac{2}{\theta}\right)^r \Gamma(\frac{r}{2}+1)\ \left[1 - \frac{1}{(\alpha^2+1)^{\frac{r}{2}+1}}\right]$$

$$Q_X\,(t) = \frac{\alpha^2+1}{\alpha^2} \sum_{r=0}^{\infty} \frac{(it)^r}{r!} \left(\frac{2}{\theta}\right)^r \Gamma\left(\frac{r}{2}+1\right)\left[1 - \frac{1}{(\alpha^2+1)^{\frac{r}{2}+1}}\right] \quad (14)$$

6-Quintile Function

The quintile of this distribution is denoted by $x = F^{-1}\,(v)$, v= $F\,(x)$:

$$F(x) = 1 - \left[\frac{(\alpha+1)e^{-\frac{\theta}{2}x^2} - e^{-\frac{\theta}{2}x^2(\alpha^2+1)}}{\alpha^2}\right]$$

$$V = 1 - \left[\frac{(\alpha^2+1)e^{-\frac{\theta}{2}x^2} - e^{-\frac{\theta}{2}x^2(\alpha^2+1)}}{\alpha^2}\right]$$

$$e^{-\frac{\theta}{2}x^2}\left[(\alpha^2+1)-\ e^{-\frac{\theta}{2}x^2\alpha^2}\right]-\alpha^2\,(1-v)=0 \tag{15}$$

The equation (15) is difficult to solve, and then, to solve it is by using the numerical methods after compensation the estimate of θ, α in it. To find the mode of this distribution we apply $f'(x)=0$ which is as follows:

$$Let\ \ K=\frac{\alpha^2+1}{\alpha^2}$$

$$\ell nf\,(x)=\ \ell n\,(K)+\ \ell n\theta+\ \ell n\,x-\ \tfrac{\theta}{2}x^2+\ \ell n\left[1-e^{-\frac{\theta}{2}x^2\alpha^2}\ \right]$$

$$\frac{d\ell nf\,(x)}{dx}=\frac{1}{x}-\ \theta x+\ \frac{\theta x\alpha^2e^{-\frac{\theta}{2}x^2\alpha^2}}{1-e^{-\frac{\theta}{2}x^2\alpha^2}}$$

$$\frac{d\ell nf\,(x)}{dx}=0\ ,\qquad \frac{1}{x}-\ \theta x+\ \frac{\theta x\alpha^2e^{-\frac{\theta}{2}x^2\alpha^2}}{1-e^{-\frac{\theta}{2}x^2\alpha^2}}=0$$

$$e^{-\frac{\theta}{2}x^2\alpha^2}\,[1-\theta x^2-\theta x^2\alpha^2]-(1-\theta x^2)=0 \tag{16}$$

The equation (16) can be solved using numerical methods because it very difficult.

To find the median of this distribution, we apply $F(x)=\frac{1}{2}$ which is as follows:

$$F(x)=\ \frac{1}{2}$$

$$1-\left[\frac{(\alpha^2+1)e^{-\frac{\theta}{2}x^2}\ -\ e^{-\frac{\theta}{2}x^2(\alpha^2+1)}}{\alpha^2}\right]=\frac{1}{2}$$

$$e^{-\frac{\theta}{2}x^2}\left[(\alpha^2+1)-\ e^{-\frac{\theta}{2}x^2(\alpha^2+1)}\right]-0.5\alpha^2=0 \tag{17}$$

The equation (17) solves it by using numerical methods.

Conclusion

Weighted distributions are employed mostly in study of reliability, bio-medicine, meta-analysis, econometrics, survival analysis, renewal processes, physics, ecology and branching processes. A weighted model based on the Rayleigh distribution is proposed in this work and the statistical properties of this model are presented. Some non-Bayesian and Bayesian methods may be used to estimate the parameter of proposed model in the future work. Furthermore, the proposed model may be employed for a real data depending on accurate goodness-of-fit test

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