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Applying Shrinkage Estimation Technique of P(Y<Max $X_1, X_2,\ldots, X_k$) in Case of Generalized Exponential Distribution

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Abstract

This paper concerned with estimation reliability (R_k) for K components parallel system of the stress-strength model with non-identical components which is subjected to a common stress, when the stress and strength follow the Generalized Exponential Distribution (GED) with unknown shape parameter α and the known scale parameter θ (θ=1) to be common. Different shrinkage estimation methods will be considered to estimate R_k depending on maximum likelihood estimator and prior estimates based on simulation using mean squared error (MSE) criteria. The study approved that the shrinkage estimation using shrinkage weight function was the best.

Keywords: Generalized Exponential Distribution (GED), Stress–Strength (S-S) models, Maximum likelihood estimator (MLE), Shrinkage estimator (Sh) and mean squares error (MSE).

1- Introduction

At a specific time, the reliability is defined as a probability that the intended functions under specified operational conditions and environments the item will be performed. The reliability function is a function of the lifetime which is monotonically decreasing function; it is a quantitative measure of the quality of the item. The item will eventually cease to perform its intended function if the service life of the item is allowed to proceed unlimitedly. The reliability over time exhibit decreasing in all mechanical systems and their materials degrade as they age because their components are not ideal [1].

The widest approach is the well-known application of reliability estimation is the model of stress–strength. This model is used in many applications of system collapse and engineering such as strength failure and physics. Some systems of engineering, which may have more than one component there may fail together or separately.

In the statistical approach, the model of stress-strength, most of the considerations depend on the assumption that the component strengths are independently and identically distributed (iid). The assumption of the components of a system are of the different structure identical strength distributions may not be quite realistic" in many practical situations [2,3].

The issue of finding the reliability in a model of stress-strength has been discussed extensively in the literature when X and Y have some specified distribution. Here are a few references to contributions towards these models

"Hangal in 1996 obtained the estimation of system reliability with k components either series or parallel of a stress-strength model" [4]. He assumed that the K^{th} components strength and the common stress were independent and they follow the Exponential distribution with two parameters. Ali H.M. in 2013 estimated the reliability of the stress-strength system with non-identical parallel component subject to a common stress using Lomax distribution [5]. Sezer and Kinaci in 2013 estimated the stress-strength parameter (R) for a parallel system with two components based on masked data by using Exponential distribution; [6]. In 2014, Karam and Ali considered estimation of system Reliability for two components parallel stress-strength system model with non-identical parallel component which was subjected to common stress using Lomax distribution [7]. Recently, Karam in 2016 studied the reliability function for a component which had two components parallel which was subjected to common stress via Gompertz distribution [8].

"The two-parameters Generalized Exponential Distribution (GED) has been introduced and studied quite extensively by Gupta & Kundu (1999, 2001, 2002, 2003, 2004), Raqab (2002), Raqab and Ahsanullah (2001), Zheng (2002) and Kundu, Gupta and Manglick (2004). The two-parameter GED was an alternative to the well-known two-parameter Gamma, two-parameter Weibull" [9].

The estimation of reliability in the statistical literature is a very common problem.

The aim of this paper is to make comparisons between different proposal shrinkage estimator for the reliability (R_k) of K parallel components system in stress-strength model (Rk=P(Y<Max. x1,x2,…xk)) when they are follows the Generalized Exponential Distribution (GED).

The probability density function for a r. v. X follows the Generalized Exponential Distribution (GED) is [9].

$$f(x;\alpha,\theta) = \frac{\alpha}{\theta}\, e^{-\frac{x}{\theta}}\,(1-e^{-\frac{x}{\theta}})^{\alpha-1}, \qquad x>0,\ \alpha,\theta>0$$

Whereas α refer to shape parameter and θ refer to scale parameter and the c.d.f. is:

$$F(x;\alpha,\theta) = [1-e^{-\frac{x}{\theta}}]^{\alpha}$$

In this work, we employee the case when θ will be known ($\theta = 1$), this implies to:

$$f(x;\alpha) = \alpha e^{-x}(1-e^{-x})^{\alpha-1} \qquad x>0,\ \alpha>0 \qquad (1)$$

And the cumulative distribution function will be:

$$F(x;\alpha) = (1-e^{-x})^{\alpha} \qquad x>0,\ \alpha>0 \qquad (2)$$

Assume $x_1, x_2, \dots x_k$ arises strength have (GED) the distribution in (1) with parameter α_i, i=1,2,…k all the components are subjected to the stress Y that has the same distribution with parameter α_{k+1}, and assumes to be independent and not identical.

Now, the reliability of the reliability K component parallel system of the stress-strength model is defined as below[6].

$R_k = P(Y < \max x_1, x_2, \dots x_k)$

Suppose $Z = \max(x_1, x_2, \dots x_k)$

$R_k = \int_0^{\infty} \bar{F}_z(y) f(y) dy$

$F_z(z)=P(Z<z)$

$=P(x_1<z)P(x_2<z)\ldots P(x_k<z)$

$=(1-e^{-z})^{\alpha_1}(1-e^{-z})^{\alpha_2}\ldots(1-e^{-z})^{\alpha_k}$

$=(1-e^{-z})^{\sum_{i=1}^{k}\alpha_i}$

$\bar{F}_z(y)=1-(1-e^{-y})^{\sum_{i=1}^{k}\alpha_i}$

$$R_k = \int_0^{\infty} \left(1-(1-e^{-y})^{\sum_{i=1}^{k}\alpha_i}\right) \alpha_{k+1} e^{-y}(1-e^{-y})^{\alpha_{k+1}-1} dy$$

$$R_k = \frac{\sum_{i=1}^{k}\alpha_i}{\sum_{i=1}^{k+1}\alpha_i} \qquad (3)$$

2. Estimation Methods of R_k:

<u>2-1 Maximum Likelihood Estimator (MLE):</u>

Let k be a component of a system is put on the life–testing experiment. Suppose $x_{11}, x_{12},\ldots, x_{1n1}, x_{21}, x_{22},\ldots,x_{2n2},\ldots, x_{k1}, x_{k2},\ldots,x_{knk}$, and y_t; t=1,2,…,m. then the likelihood function of the mentioned system will be[7].

$$l = L(\alpha_i, 1; x_i, y) = \prod_{j=1}^{n_i}\left(\prod_{i=1}^{k} f(x_{ij})\right)\prod_{t=1}^{m} g(y_t) = \prod_{i=1}^{k}\alpha_i^{n_i}\prod_{j=1}^{n_i}\prod_{i=1}^{k} e^{-\sum_{j=1}^{n_i}\sum_{i=1}^{k}x_{ij}}$$
$$\prod_{j=1}^{n_i}\prod_{i=1}^{k}(1-e^{-x_{ij}})^{\alpha_i-1}(\alpha_{k+1})^m\prod_{t=1}^{m} e^{-\sum_{t=1}^{m}y_t}\prod_{t=1}^{m}(1-e^{-y_t})^{(\alpha_{k+1})-1}$$

Take the logarithm of both sides, we get:

$$\mathrm{Ln}(l)=n_i\sum_{i=1}^{k}ln\alpha_i+\sum_{j=1}^{n_i}\sum_{i=1}^{k}x_{ij}$$
$$+(\alpha_i-1)\sum_{j=1}^{n_i}\sum_{i=1}^{k}ln(1-e^{-x_{ij}})+m\,ln\alpha_{k+1}+\sum_{t=1}^{m}y_t+(\alpha_{k+1}-1)\sum_{t=1}^{m}\ln(1-e^{-y_t})$$

Derive the last equation with respect to the unknown shape parameters α_i (i=1,2,…,k+1) and equate the result to zero, we get:

$$\frac{d\mathrm{Ln}(l)}{d\alpha_i}=\frac{n_i}{\alpha_i}+\sum_{j=1}^{n_i}ln(1-e^{-x_{ij}})=0, \quad i=1,2,\ldots,k$$
$$\frac{d\mathrm{Ln}(l)}{d\alpha_{k+1}}=\frac{m}{\alpha_{k+1}}+\sum_{t=1}^{m}ln(1-e^{-y_t})=0$$

Thus, the maximum likelihood estimator of the parameter α_i (i=1,2,…,k+1) is as follows:

$$\hat{\alpha}_{i_{mle}}=\frac{-n_i}{\sum_{j=1}^{n_i}ln(1-e^{-x_{ij}})}\ ;i=1,2,\ldots,k \qquad (4)$$

$$\hat{\alpha}_{(k+1)_{mle}}=\frac{-m}{\sum_{t=1}^{m}ln(1-e^{-y_t})} \qquad (5)$$

To obtain the reliability estimation for K component parallel system of (S-S) model (R_k) using Maximum Likelihood method, we must substitute $\hat{\alpha}_{i_{mle}}$ and $\hat{\alpha}_{(k+1)_{mle}}$ in equation (3) then we get the following:

$$\hat{R}_{k_{mle}}=\frac{\sum_{i=1}^{k}\hat{\alpha}_{i_{mle}}}{\sum_{i=1}^{k+1}\hat{\alpha}_{i_{mle}}} \qquad (6)$$

<u>2-2 Shrinkage Estimation Method (Sh):</u>

Thompson in 1968 suggested the problem of shrink a usual estimator $\hat{\alpha}$ unbiased of the parameter α to prior information α_0 using shrinkage weight factor $\emptyset(\hat{\alpha})$, such that $0\le\emptyset(\hat{\alpha})\le1$. Thompson said that "We are estimating α and we believe α_0 is closed to the true value of α, that is mean something bad happens if $\alpha_0\approx\alpha$ and we do not use α_0". Thus, the form of shrinkage estimator of α denoted by $\hat{\alpha}_{sh}$ defined as below: see in [10-14].

$\hat{\alpha}_{sh} = \emptyset(\hat{\alpha})\hat{\alpha} + (1 - \emptyset(\hat{\alpha}))\alpha_0$ (7)

Now, we use the unbiased estimator $\hat{\alpha}_{ub}$ as a usual estimator and α_0 estimator as initial estimate (prior estimation) of α in equation (7).

Where $\emptyset(\hat{\alpha})$ denotes the shrinkage weight factor as we mentioned above such that as 0≤ $\emptyset(\hat{\alpha})$ ≤1, which may be a function of $\hat{\alpha}_{ub}$, a function of sample size (n_i, m) or may be a constant or can be found by minimizing the mean square error of $\hat{\alpha}_{sh}$.

Thus, the shrinkage estimator for the shape parameter α of GED will be as follows:

$\hat{\alpha}_{sh} = \emptyset(\hat{\alpha})\hat{\alpha}_{ub} + (1 - \emptyset(\hat{\alpha}))\alpha_0$ (8)

Where,

$\hat{\alpha}_{i_{ub}} = \frac{n_i-1}{n_i}\hat{\alpha}_{i_{mle}} = \frac{n_i-1}{-\sum_{j=1}^{n_i} ln(1-e^{-x_{ij}})}$, i=1,2,...,k

Implies,

$E(\hat{\alpha}_{i_{ub}}) = \alpha_i$ and $Var(\hat{\alpha}_{i_{ub}}) = \frac{\alpha_i^2}{n_i-2}$,

Also, $\hat{\alpha}_{(k+1)_{ub}} = \frac{m-1}{m}\hat{\alpha}_{(k+1)_{mle}} = \frac{m-1}{-\sum_{t=1}^{m} ln(1-e^{-y_t})}$

As well as, $E(\hat{\alpha}_{(k+1)_{ub}}) = \alpha_{k+1}$ and $Var(\hat{\alpha}_{(k+1)_{ub}}) = \frac{\alpha_{k+1}^2}{m-2}$.

2-2-1 Shrinkage Estimator using Shrinkage Weight Function (sh1):

The shrinkage weight factor here is considered as a function of n_i and m respectively as the following:

$\emptyset_i(\hat{\alpha}) = e^{-n_i}$, and $\emptyset_{k+1}(\hat{\alpha}) = e^{-m}$; i=1,2,…,k

Then the shrinkage estimator of α which is defined in equation (8) using above shrinkage weight function will be as below, knowing that n_i and m are defined in subsection (2-1).

$\hat{\alpha}_{i_{sh1}} = \emptyset_i(\hat{\alpha})\hat{\alpha}_{i_{ub}} + (1 - \emptyset_i(\hat{\alpha}))\alpha_0$ for i=1,2,…,k+1 (9)

To obtain the reliability estimation for K component parallel system of (S-S) model (R_k) using shrinkage weight function, we must substitute equation (9) in equation (3) then we get the following:

$\hat{R}_{k_{sh1}} = \frac{\sum_{i=1}^{k}\hat{\alpha}_{i_{sh1}}}{\sum_{i=1}^{k+1}\hat{\alpha}_{i_{sh1}}}$ (10)

2-2-2 Constant Shrinkage Weight Factor (sh2):

We suggest in this subsection constant shrinkage weight factor $\varphi_i(\hat{\alpha}) = 0.3$; i=1,2,…,k+1. Therefore, the shrinkage estimator using specific constant weight factor will be as follows:

$\hat{\alpha}_{i_{sh2}} = \varphi_i(\hat{\alpha})\hat{\alpha}_{i_{ub}} + (1 - \varphi_i(\hat{\alpha}))\alpha_0$ for i=1,2,…,k+1 (11)

To find the reliability estimation for K component parallel system of (S-S) model using the constant shrinkage weight factor we must substitute equation (11) in equation (3) then we get the following:

$\hat{R}_{k_{sh2}} = \frac{\sum_{i=1}^{k}\hat{\alpha}_{i_{sh2}}}{\sum_{i=1}^{k+1}\hat{\alpha}_{i_{sh2}}}$ (12)

2-2-3 Modified Thompson Type Shrinkage Weight Function (th):

In this subsection, we introduce modifying the shrinkage weight factor considered by Thompson in 1968 as below.

$\gamma(\hat{\alpha}_i) = \frac{(\hat{\alpha}_{i_{ub}}-\alpha_0)^2}{(\hat{\alpha}_{i_{ub}}-\alpha_0)^2 + Var(\hat{\alpha}_{i_{ub}})}(0.01)$; for i =1,2,…,k+1.

Therefore, the shrinkage estimator of using above modified shrinkage weight factor will be:

$\hat{\alpha}_{i_{th}} = \gamma(\hat{\alpha}_i)\hat{\alpha}_{i_{ub}} + (1-\gamma(\hat{\alpha}_i))\alpha_0$; for i =1,2,…,k+1 (13)

To find the reliability estimation for K component parallel system of (S-S) model using modified Thompson type shrinkage we must substitute equation (13) in equation (3) then we get the following:

$$\hat{R}_{k_{th}} = \frac{\sum_{i=1}^{k}\hat{\alpha}_{i_{th}}}{\sum_{i=1}^{k+1}\hat{\alpha}_{i_{th}}} \quad (14)$$

3. Simulation Study

The performance and comparisons of the considered estimators of reliability are studied for numerical results and obtained using sample of different size =(10, 30, 50 and 100), based on 1000 repetition via MSE criteria. Monte Carlo simulation was used for this purpose as it steps below: -

step1: the random sample was generated following the uniform continuous distribution which is defined on the interval (0,1) as $u_{i1}, u_{i2}, \ldots, u_{in_i}$;i=1,2,…,k.

step2: we will generate the random sample follows the continuous uniform distribution defined on the interval (0, 1) as $w_1, w_2, \ldots, w_m$.

step3: the above uniform random samples transformed to GED using the cumulative distribution function (c. d. f.) as follow:

$F(x_{ij}) = (1-e^{-x_{ij}})^{\alpha_i}$

$U_{ij} = (1-e^{-x_{ij}})^{\alpha_i}$

; i=1,2…,k, j=1,2,...,n $x_{ij} = [-\ln(1-U_{ij}^{\frac{1}{\alpha_i}})]$

And, by the same method, we get

, t=1,2,...,m $y_t = [-\ln(1-W_t^{\frac{1}{\alpha_{k+1}}})]$

Step4: we compute the estimator of R_k in method of maximum likelihood using equation (6).

Step5: we compute the estimators of R_k in three shrinkage methods using equations (10), (12) and (14).

Step6: on (L=1000) repetition, we calculate the mean square error (MSE) as it comes:

$$MSE = \frac{1}{L}\sum_{i=1}^{L}(\hat{R}_{k_i} - R_{k_i})^2$$

Whereas $\hat{R}_{k_i}$ indicates the real value of Reliability R_k of the proposed estimators.

Table 1. The estimation value when R_k = 0.64705882 , α1=1.5, α2=1.9, α 3=2.1, α 4=3.

n1, n2, n3, m	$\hat{R}_{k_{mle}}$	$\hat{R}_{k_{sh1}}$	$\hat{R}_{k_{sh2}}$	$\hat{R}_{k_{th}}$
10, 10, 10, 10	0.64974127	0.64705883	0.64710777	0.64718198
10, 100, 50, 30	0.65146655	0.64705883	0.64732080	0.64728223
30, 10, 50, 100	0.65669301	0.64705886	0.64721804	0.64741435
30, 50, 100, 10	0.63666133	0.64705881	0.64715662	0.64652903
50, 100, 30, 30	0.64701349	0.64705882	0.64719754	0.64711634
50, 10, 30, 50	0.65380516	0.64705889	0.64706895	0.64721992
100, 10, 50, 30	0.64988329	0.64705885	0.64703872	0.64713598
100, 50, 30, 10	0.63613730	0.64705888	0.64723084	0.64677433

Table 2. The MSE value when R_k = 0.64705882 , α1=1.5, α2=1.9, α 3=2.1, α 4=3.

n1, n2, n3, m	$\hat{R}_{k_{mle}}$	$\hat{R}_{k_{sh1}}$	$\hat{R}_{k_{sh2}}$	$\hat{R}_{k_{th}}$	Best
10, 10, 10, 10	0.0068345086009	0.0000000000166	0.0000785103279	0.0000379840661	$\hat{R}_{k_{sh1}}$
10, 100, 50, 30	0.0023974820438	0.0000000000011	0.0000248009970	0.0000116520014	$\hat{R}_{k_{sh1}}$
30, 10, 50, 100	0.0015326690134	0.0000000000016	0.0000152722700	0.0000076319598	$\hat{R}_{k_{sh1}}$
30, 50, 100, 10	0.0061024701636	0.0000000000143	0.0000687911107	0.0000361160422	$\hat{R}_{k_{sh1}}$
50, 100, 30, 30	0.0020754099172	0.0000000000000	0.0000212700068	0.0000101099334	$\hat{R}_{k_{sh1}}$
50, 10, 30, 50	0.0021413961985	0.0000000000015	0.0000222919914	0.0000107131426	$\hat{R}_{k_{sh1}}$
100, 10, 50, 30	0.0026738779103	0.0000000000018	0.0000287629381	0.0000144133293	$\hat{R}_{k_{sh1}}$
100, 50, 30, 10	0.0056015578911	0.0000000000113	0.0000584342389	0.0000268982293	$\hat{R}_{k_{sh1}}$

Table 3. The estimation value when R_k = 0.6785714 , α1=1.1, α2=1.2, α 3=1.5, α 4=1.8.

n1, n2, n3, m	$\hat{R}_{k_{mle}}$	$\hat{R}_{k_{sh1}}$	$\hat{R}_{k_{sh2}}$	$\hat{R}_{k_{th}}$
10, 10, 10, 10	0.67866259	0.67857132	0.67836930	0.67852404
10, 100, 50, 30	0.71429325	0.67856884	0.68137355	0.68053712
30, 10, 50, 100	0.68400211	0.67857136	0.67839122	0.67867854
30, 50, 100, 10	0.85657436	0.67857813	0.69362884	0.6904753
50, 100, 30, 30	0.59101645	0.67857142	0.67160744	0.67225224
50, 10, 30, 50	0.46316784	0.67856860	0.66490300	0.66659916
100, 10, 50, 30	0.68193435	0.67857142	0.67868409	0.67870308
100, 50, 30, 10	0.94405220	0.67858039	0.69525541	0.69315286

Table 4. The MSE value when R_k = 0.6785714, α1=1.1, α2=1.2, α 3=1.5, α4=1.8.

n1, n2, n3, m	$\hat{R}_{k_{mle}}$	$\hat{R}_{k_{sh1}}$	$\hat{R}_{k_{sh2}}$	$\hat{R}_{k_{th}}$	Best
10, 10, 10, 10	0.006413473894	0.000000000016	0.000076589118	0.000039866197	$\hat{R}_{k_{sh1}}$
10, 100, 50, 30	0.0021457805724	0.0000000000066	0.0000125077396	0.0000079031698	$\hat{R}_{k_{sh1}}$
30, 10, 50, 100	0.0012709588070	0.0000000000011	0.0000127008484	0.0000058904503	$\hat{R}_{k_{sh1}}$
30, 50, 100, 10	0.031954410612	0.0000000000450	0.0002301025638	0.0001437716440	$\hat{R}_{k_{sh1}}$
50, 100, 30, 30	0.009892213008	0.000000000000	0.0000670207090	0.0000491702945	$\hat{R}_{k_{sh1}}$
50, 10, 30, 50	0.0479186501084	0.0000000000079	0.0001973440002	0.0001483221229	$\hat{R}_{k_{sh1}}$
100, 10, 50, 30	0.002481844844	0.0000000000012	0.0000261782216	0.0000129859998	$\hat{R}_{k_{sh1}}$
100, 50, 30, 10	0.0704976787510	0.000000000080	0.0002800972547	0.0002136971265	$\hat{R}_{k_{sh1}}$

Table 5. The estimation value when R_k = 0.68965517, α1=1, α2=2, α 3=3, α 4=2.7.

n1, n2, n3, m	$\hat{R}_{k_{mle}}$	$\hat{R}_{k_{sh1}}$	$\hat{R}_{k_{sh2}}$	$\hat{R}_{k_{th}}$
10, 10, 10, 10	0.80040930	0.68965568	0.69089509	0.69099792
10, 100, 50, 30	0.6884277	0.68965515	0.68950451	0.68959020
30, 10, 50, 100	0.69469071	0.68965516	0.68945879	0.68974836
30, 50, 100, 10	0.67838443	0.68965516	0.68974088	0.68913416
50, 100, 30, 30	0.68716399	0.68965517	0.68953574	0.68950827
50, 10, 30, 50	0.69398643	0.68965514	0.68954182	0.68976537
100, 10, 50, 30	0.68908654	0.68965516	0.68934005	0.68950500
100, 50, 30, 10	0.67662788	0.68965513	0.68951009	0.68918936

Table 6. The MSE value when R_k = 0.68965517, α1=1, α2=2, α 3=3, α 4=2.7.

n1, n2, n3, m	$\hat{R}_{k_{mle}}$	$\hat{R}_{k_{sh1}}$	$\hat{R}_{k_{sh2}}$	$\hat{R}_{k_{th}}$	Best
10, 10, 10, 10	0.01229792159789	0.00000000000026	0.0000015438839	0.0000018105906	$\hat{R}_{k_{sh1}}$
10, 100, 50, 30	0.0020793896045	0.0000000000003	0.0000212660377	0.0000100571815	$\hat{R}_{k_{sh1}}$
30, 10, 50, 100	0.0013116339341	0.0000000000011	0.0000135119636	0.0000065066202	$\hat{R}_{k_{sh1}}$
30, 50, 100, 10	0.0055564329633	0.0000000000126	0.0000609082708	0.0000319882618	$\hat{R}_{k_{sh1}}$
50, 100, 30, 30	0.002072936381	0.0000000000000	0.000022190642	0.000011232268	$\hat{R}_{k_{sh1}}$
50, 10, 30, 50	0.0020661841322	0.0000000000013	0.0000219388121	0.0000106684559	$\hat{R}_{k_{sh1}}$
100, 10, 50, 30	0.0022984507903	0.0000000000012	0.0000240920439	0.0000111287847	$\hat{R}_{k_{sh1}}$
100, 50, 30, 10	0.0056989431761	0.0000000000118	0.0000604307903	0.0000308152485	$\hat{R}_{k_{sh1}}$

Table 7. The estimation value when R_k = 0.8000000, α1=2, α2=3, α 3=3, α 4=2.

n1, n2, n3, m	$\hat{R}_{k_{mle}}$	$\hat{R}_{k_{sh1}}$	$\hat{R}_{k_{sh2}}$	$\hat{R}_{k_{th}}$
10, 10, 10, 10	0.79732879	0.79999992	0.79981682	0.79994799
10, 100, 50, 30	0.80021734	0.79999996	0.79999223	0.80000082
30, 10, 50, 100	0.80422188	0.79999999	0.79992073	0.80014540
30, 50, 100, 10	0.78689067	0.79999990	0.79980236	0.79949901
50, 100, 30, 30	0.79698163	0.79999999	0.79987571	0.79986822
50, 10, 30, 50	0.80392120	0.80000001	0.80000297	0.80016313
100, 10, 50, 30	0.80006170	0.79999996	0.79987705	0.79999486
100, 50, 30, 10	0.79144712	0.80000007	0.80024377	0.79978967

Table 8. The MSE value when R_k = 0.8000000 , α1=2, α2=3, α 3=3, α 4=2.

n1, n2, n3, m	$\hat{R}_{k_{mle}}$	$\hat{R}_{k_{sh1}}$	$\hat{R}_{k_{sh2}}$	$\hat{R}_{k_{th}}$	Best
10, 10, 10, 10	0.0036930321488	0.0000000000088	0.0000416870241	0.0000218510918	$\hat{R}_{k_{sh1}}$
10, 100, 50, 30	0.0011807809520	0.0000000000003	0.0000119623355	0.0000055447844	$\hat{R}_{k_{sh1}}$
30, 10, 50, 100	0.0007760955807	0.0000000000009	0.0000082061304	0.0000039207273	$\hat{R}_{k_{sh1}}$
30, 50, 100, 10	0.0035662706436	0.0000000000068	0.0000348598683	0.0000183278607	$\hat{R}_{k_{sh1}}$
50, 100, 30, 30	0.0011429353051	0.0000000000000	0.0000115756637	0.0000058533095	$\hat{R}_{k_{sh1}}$
50, 10, 30, 50	0.0010247431814	0.0000000000009	0.0000110864286	0.0000055558376	$\hat{R}_{k_{sh1}}$
100, 10, 50, 30	0.0014165597010	0.0000000000009	0.0000148389019	0.0000071877688	$\hat{R}_{k_{sh1}}$
100, 50, 30, 10	0.0031290741177	0.0000000000059	0.0000309491744	0.0000151569884	$\hat{R}_{k_{sh1}}$

Table 9. The estimation value when R_k =0.6296296, α1=2, α2=3, α 3=3, α 4=2.

n1, n2, n3, m	$\hat{R}_{k_{mle}}$	$\hat{R}_{k_{sh1}}$	$\hat{R}_{k_{sh2}}$	$\hat{R}_{k_{th}}$
10, 10, 10, 10	0.63076360	0.62962956	0.62951463	0.62964048
10, 100, 50, 30	0.63172841	0.62962958	0.62951579	0.62962435
30, 10, 50, 100	0.63790618	0.62962956	0.62958384	0.62986333
30, 50, 100, 10	0.620838632	0.62962972	0.62995512	0.62935807
50, 100, 30, 30	0.62658135	0.62962962	0.62943595	0.62939530
50, 10, 30, 50	0.63765206	0.62962964	0.62974108	0.62990918
100, 10, 50, 30	0.63290386	0.62962961	0.62962576	0.62977961
100, 50, 30, 10	0.62207442	0.62962977	0.63004524	0.62929717

Table 10. The MSE value when R_k = 0.6296296 , α1=2, α2=3, α 3=3, α 4=2.

n1, n2, n3, m	$\hat{R}_{k_{mle}}$	$\hat{R}_{k_{sh1}}$	$\hat{R}_{k_{sh2}}$	$\hat{R}_{k_{th}}$	Best
10, 10, 10, 10	0.0069000376819	0.0000000000169	0.0000799910406	0.0000388496851	$\hat{R}_{k_{sh1}}$
10, 100, 50, 30	0.0026648513273	0.0000000000014	0.0000282562972	0.0000140228606	$\hat{R}_{k_{sh1}}$
30, 10, 50, 100	0.0019060448282	0.0000000000020	0.0000199887466	0.0000100559705	$\hat{R}_{k_{sh1}}$
30, 50, 100, 10	0.0057617706197	0.0000000000126	0.0000630168023	0.0000304912831	$\hat{R}_{k_{sh1}}$
50, 100, 30, 30	0.0025093087038	0.0000000000000	0.0000262623068	0.0000129639110	$\hat{R}_{k_{sh1}}$
50, 10, 30, 50	0.0022561911791	0.0000000000023	0.0000244149758	0.0000123993849	$\hat{R}_{k_{sh1}}$
100, 10, 50, 30	0.0031588200355	0.0000000000023	0.0000338946302	0.0000168170931	$\hat{R}_{k_{sh1}}$
100, 50, 30, 10	0.0059428138792	0.0000000000136	0.0000667599379	0.0000335650838	$\hat{R}_{k_{sh1}}$

Numerical results:

In this work, when we notice for any n_i =(10,30,50 and 100) and all m=(10,30,50 and 100) the minimum mean square error (MSE) for the reliability estimator of K components parallel system of (S-S) model with the Generalized Exponential Distribution held using the shrinkage estimator as function of n_i and m based on weight factor of shrinkage.

This implies that shrinkage weight factor as function of n_i and m is the best and it is follow by using modified Thompson type shrinkage estimator($\hat{R}_{th}$). For all n_i and all m, the third order best estimator is the shrinkage estimator based on constant shrinkage weight function $(\hat{R}_{sh2})$ after ($\hat{R}_{sh1}$) and ($\hat{R}_{th}$) .

4. Conclusion

Of the previous item, the proposal can be found that the shrinkage estimation method using shrinkage weight factor as function of n_i and m ($\hat{R}_{sh1}$) which depends on prior estimate and unbiased estimator as a linear combination is the best estimator and performs good behavior than the others in the sense of MSE when estimate reliability for system contains K^{th} non-identical strength parallel components which are subjected to a common stress in stress-strength model and when the stress and strength follow the Generalized Exponential Distribution (GED).

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