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ωMc –functions and NMc-functions

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Abstract

In this paper, we presented new types of Mc-function by using ω-open and N-open sets some of them are weaker than Mc-function and some are stronger, which are ωMc-function, Mωc-function, ωMωc-function, NMc-function, MNc-function and NMNc-function, also we submitted new kinds of continuous functions and compact functions and we illustrated the relationships between these types. The purpose of this paper is to expand the study of Mc-function and to get results that we need to find the relationship with the types that have been introduced.

Keywords: ωMc-function, Mωc-function, ωMωc-function, NMc-function, NMNc-function.

1. Introduction

In (2011) Farhan, Ala'A. Mahmood [1]. Introduced the concept of new function which was named by Mc-function where he defined it as (A function f from any topological space$(X, \daleth_X)$ into any topological space $(Y, \daleth_Y)$ was called Mc-function iff $f^{-1}(\mathcal{W})$ was a closed set in X for any compact set $\mathcal{W}$ in Y). Moreover, in (2015) the researchers in [2]. introduced new types of Mc-functions by using g-closed sets which were M-gc, gM-c and gM-gc functions, where every Mc-function was M-gc function and gMc function, The notion of ω-open sets submitted at the first time by Hdeib in [3]. While the notion of N-open sets submitted by Al-Omari in [4]. Where they defined these sets as (Any set $\mathcal{W}$ in a topological space $(X, \daleth)$ was called ω-open (respectively. N-open) set, if for any element $a \in \mathcal{W}$ there was an open set $\mathcal{B}$ containing a, with $\mathcal{B}$-$\mathcal{W}$ was countable (respectively. finite) set. $\mathcal{W}^c$ was ω-closed set (respectively. N-closed set). After that many researchers used these sets and introduced new concepts in different kinds of topological spaces. In this paper, we used these two sets to introduce new types of Mc-function, namely ωMc-function, Mωc-function, ωMωc-function, NMc-function, MNc-function and NMNc-function and we illustrated the relation between them and their relation with Mc-function, also we connected between these functions and T_2-space, ωT_2-space, NT_2-space, Kc- space, and c-c space. We used the same sets to define new forms of continuous and compact functions such as strongly ω-continuous, strongly N-continuous, ω-irresolute, ω^*-compact, N^*-compact, ω^{**}-compact, N^{**}-compact function, and we presented some propositions, remarks and examples to support our work.

1-ωMc-functions and NMc-functions.

We recall the following facts which we need in this our work, after that we will introduce the definitions of new types of Mc-function and illustrate the relation between these types.

Definition (1.1):

A space (X, T) is called:-

1- A compact space if any open cover for X possesses a finite sub cover [5].

2- A ω-compact space if any ω-open cover for X possesses a finite sub cover [5].

3- An N-compact space if any N-open cover for X possesses a finite sub cover [6].

Remark (1.2):

Every closed set is ω-closed [7]. (respectively N-closed [6].) set. And every N-open set is ω-open set [8]. Also every N-closed set is ω-closed set [8]. As well as every ω-compact (respectively N-compact) set is compact [5, 9].

Example (1.3):

1-In the excluded point topological space (X, T_{EX}) where X is a finite set, the set $\{x\}_{x \neq x_\circ}$ is ω-closed and N-closed set but not closed set, in which T_{EX}=$\{U \subseteq X, x_\circ \notin U$ for some $x_\circ \in X\} \cup \{X\}$.

2-The set of irrational number Q^c in the co-finite topological space $(\mathcal{R}, \mathrm{T}_{cof})$ is ω-open set but not N-open set.

3-The set of rational number Q in the co-finite topological space $(\mathcal{R}, \mathrm{T}_{cof})$ is ω-closed set but not N-closed set.

4-The included point topological space $(\mathcal{Z}, \mathrm{T}_{In})$ where T_{In}= $\{U \subseteq X$, where $x_\circ \in U$, for some $x_\circ$ in $X\} \cup \{\emptyset\}$ is a compact space but not ω-compact.

Definition (1.4):

The space (X, T) is called:-

1- A T_2-space, if for each non-equal elements x, y in X, there are disjoint $\mathcal{W}_1, \mathcal{W}_2 \in \mathrm{T}$ with $x \in \mathcal{W}_1$ and $y \in \mathcal{W}_2$ [10].

2- A ωT_2-space, if for each non-equal elements x, y in X, there are disjoint ω-open sets $\mathcal{W}_1, \mathcal{W}_2$ in X with $x \in \mathcal{W}_1$ and $y \in \mathcal{W}_2$ [11].

3- An NT_2-space, if for each non-equal elements x, y in X, there are disjoint N-open sets $\mathcal{W}_1, \mathcal{W}_2$ with $x \in \mathcal{W}_1$ and $y \in \mathcal{W}_2$ [6].

Example (1.5)

1- $(\mathcal{R}, \mathrm{T}_D)$ is T_2-space.

2- (X, T_{In}) where X is a finite set and T_{In}= $\{\mathcal{W} \subseteq X \mid a_\circ \in \mathcal{W}$, for some $a_\circ \in X\} \cup \{\emptyset\}$, is ωT_2-space and NT_2-space.

Remark (1.6)

1- Every closed subset of a compact space is a compact set [6].

2- Every ω-closed subset of an ω-compact space is an ω-compact set [12].

3- Every N-closed subset of an N-compact space is an N-compact set [6].

4- Every compact subset of a T_2-space is a closed set [12].

5- Every ω-compact subset of an ωT_2-space is an ω-closed set [12].

6- Every N-compact subset of an NT_2-space is an N-closed set [13].

7- Every T_2-space is ωT_2-space [14].

8- Every T_2-space is NT_2-space [6].

Definition (1.7)

A function $f:(X,\tau_X)\longrightarrow(Y,\tau_Y)$ is called Mωc-function if $f^{-1}(\mathcal{W})$ is a ω-closed set in X for any compact set $\mathcal{W}$ in Y.

Definition (1.8)

A function $f:(X,\tau_X)\longrightarrow(Y,\tau_Y)$ is called MNc-function if $f^{-1}(\mathcal{W})$ is an N-closed set in X for any compact set $\mathcal{W}$ in Y.

Example (1.9)

The identity function $I_X:(X,\tau_{ind})\longrightarrow(X,\tau_X)$ where X is a denumerable set (infinite countable set), is Mωc-function. But in case X is a finite set then f is MNc-function**.**

Definition (1.10):

A function $f:(X,\tau_X)\longrightarrow(Y,\tau_Y)$ is called ωMc-function if $f^{-1}(\mathcal{W})$ is a closed set in X for any ω-compact set $\mathcal{W}$ in Y.

Definition (1.11)

A function $f:(X,\tau_X)\longrightarrow(Y,\tau_Y)$ is called NMc-function if $f^{-1}(\mathcal{W})$ is a closed set in X for any N-compact set $\mathcal{W}$ in Y.

Example (1.12)

The identity function $I_{\mathcal{R}}:(\mathcal{R},\tau_D)\longrightarrow(\mathcal{R},\tau_{ind})$ is ωMc-function and NMc-function.

Definition (1.13)

A function $f:(X,\tau_X)\longrightarrow(Y,\tau_Y)$ is called ωMωc-function if $f^{-1}(\mathcal{W})$ is an ω-closed set in X for any ω-compact set $\mathcal{W}$ in Y.

Example (1.14)

The function $f:(N,\tau_{ind})\longrightarrow(\mathcal{R},\tau_{\mathcal{R}})$ is ωMωc-function.

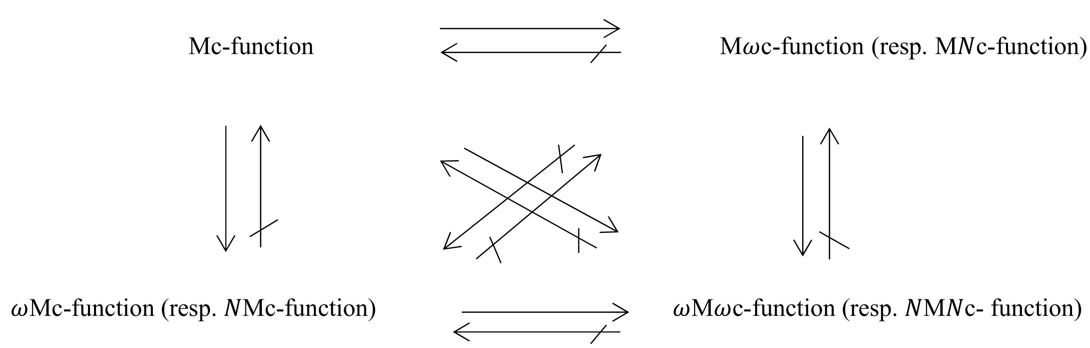
Definition (1.15)

The function $f:(X,\tau_X)\longrightarrow(Y,\tau_Y)$ is called NMNc-function if $f^{-1}(\mathcal{W})$ is an N-closed set in X for any N-compact set $\mathcal{W}$ in Y.

Example (1.16)

The function $f:(X,\tau_{ind})\longrightarrow(\mathcal{R},\tau_{\mathcal{R}})$ where X is a finite set, is NMNc-function.

The following scheme is helpful



Remark (1.17)

1- Every NMc-function is ωMc-function.

2- Every MNc-function is Mωc-function.

3- Every NMNc-function is ωMωc-function.

Example (1.18)

1- $I_{\mathcal{Z}}:(\mathcal{Z},\tau_{ind})\longrightarrow(\mathcal{Z},\tau_{cof})$ is Mωc-function but neither MNc-function nor Mc-function.

2- $I_X:(X,\tau_{ind})\longrightarrow(X,\tau_D)$ where X is a finite set, is MNc-function but not Mc-function.

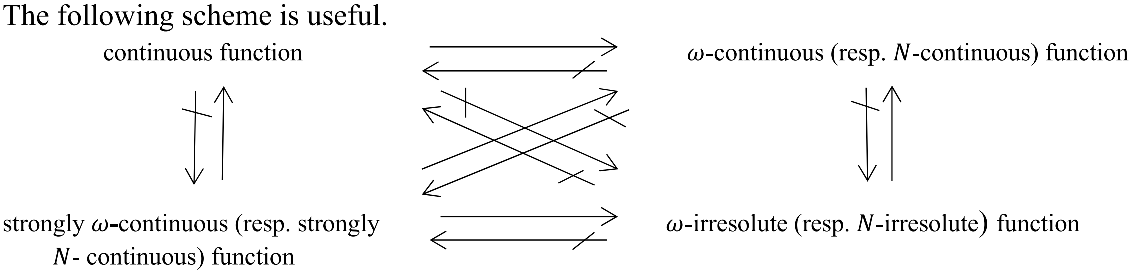
3- $f:(\mathcal{Z},\tau_{ind}) \longrightarrow (\mathcal{Z},\tau_{ind})$ is ωMωc-function but not ωMc-function.
4- $f:(X,\tau_{ind}) \longrightarrow (X,\tau_{ind})$ where X is a finite set, is NMNc-function but not NMc-function.
5- $I_X:(X,\tau_X) \longrightarrow (X,\tau)$ where X= {1, 2, 3} and τ= $\{\emptyset, X, \{1\}\}$ is ωMωc-function and NMNc-function, but not Mc-function.
6- $I_Z:(\mathcal{Z},\tau_{ind}) \longrightarrow (\mathcal{Z},\tau)$ is Mωc-function, but not ωMc-function.
7- $I_X:(X,\tau_{ind}) \longrightarrow (X,\tau)$ where X is a finite set, is MNc-function but not NMc-function.
8- $I_Z:(\mathcal{Z},\tau_{cof}) \longrightarrow (\mathcal{Z},\tau_{ind})$ is ωMc-function but not NMc-function.
9- $I_Z:(\mathcal{Z},\tau_{ind}) \longrightarrow (\mathcal{Z},\tau_{ind})$ is Mωc-function and ωMωc-function but neither NMNc-function nor MNc-function.

Definition (1.19)

A function $f:(X,\tau_X) \longrightarrow (Y,\tau_Y)$ is called:-
1- A continuous function If $f^{-1}(\mathcal{W})$ is open set in X for any open set $\mathcal{W}$ in the space Y [15].
2- An ω-continuous function if $f^{-1}(\mathcal{W})$ is ω-open set in X for each open set $\mathcal{W}$ in Y [16].
3- An N-continuous function if $f^{-1}(\mathcal{W})$ is N-open set in X for each open set $\mathcal{W}$ in Y [15].
4- A strongly ω-continuous function if $f^{-1}(\mathcal{W})$ is open set in X for each ω-open set $\mathcal{W}$ in Y.
5- A strongly N-continuous function if $f^{-1}(\mathcal{W})$ is open set in X for each N-open set $\mathcal{W}$ in Y.
6- An ω-irresolute function if $f^{-1}(\mathcal{W})$ is ω-open set in X for each ω-open set in Y.
7- An N-irresolute function if $f^{-1}(\mathcal{W})$ is N-open set in X for each N-open set $\mathcal{W}$ in Y [9].

Example (1.20)

1- $f:(X,\tau_D) \longrightarrow (Y,\tau_Y)$ is continuous function.
2- The function f from the excluded point space (X,τ_{EX}) where X is a countable set into any space (Y,τ_Y) is ω-continuous function, where $\tau_{EX} = \{\mathcal{W} \subseteq X, a_\circ \notin \mathcal{W}$ for some $a_\circ \in X\} \cup \{X\}$.
3- The function f from the included point space (X,τ_{In}) into the discrete space (X,τ_D) where X is a finite set, is N-continuous function.
4- $f:(X,\tau_X) \longrightarrow (X,\tau_{ind})$, where $f(a)=d$ for any $a \in X$, is strongly ω-continuous function.
5- The identity function from the co-countable space (X,τ_{coc}) into the same space where X is uncountable set, is strongly N-continuous function.
6- A function f from the included point space $(\mathcal{Z},\tau_{In})$ into the co-finite space $(\mathcal{Z},\mu_{cof})$ is ω-irresolute function.
7- The function f from elective topology on an infinite set to the indiscrete topology on the same set in which $f(a) = d$ for any a in the domain, is N-irresolute function.
The following scheme is useful.



Remark (1.21)

1- Every N-continuous function is ω-continuous.
2- Every strongly ω-continuous function is strongly N-continuous.
3- No relation between ω-irresolute function and N-irresolute.

Example (1.22)

1- $f: (\mathcal{Z}, \mathcal{T}_{ind}) \longrightarrow (\mathcal{Z}, \mathcal{T}_D)$ is ω-continuous but not N-continuous function.

2- $I_{\mathcal{Z}}: (\mathcal{Z}, \mathcal{T}_{cof}) \longrightarrow (\mathcal{Z}, \mathcal{T}_{ind})$ is strongly N-continuous but not strongly ω-continuous function.

3- $I_{\mathcal{Z}}: (\mathcal{Z}, \mathcal{T}_{cof}) \longrightarrow (\mathcal{Z}, \mathcal{T}_{ind})$ is ω-irresolute but not N-irresolute function.

Definition (1.23)

1- A space $(X, \mathcal{T})$ is called:-

1- c-c space if each closed set in X is compact and each compact set is closed [17].

2- Kc-space if each compact set in X is closed [18].

Example (1.24)

1- $(X, \mathcal{T}_D)$ where X is a finite set is c-c space.

2- $(\mathcal{R}, \mathcal{T}_u)$ is Kc-space.

Remark (1.25)

Every T_2-space is Kc-space.

Example (1.26):

$(\mathcal{R}, \mathcal{T}_{coc})$ is Kc-space but not T_2-space.

Proposition (1.27)

If $f: (X, \mathcal{T}_X) \longrightarrow (Y, \mathcal{T}_Y)$ is a ω-continuous function where Y is a T_2-space, then it is an Mωc-function.

Proof

Let $\mathcal{K}$ be a compact subset of Y, since Y is a T_2-space then $\mathcal{K}$ is closed (by remark (1.6)), but f is ω-continuous function, so $f^{-1}(\mathcal{K})$ is ω-closed set in X, therefore f is Mωc-function. By the same way we can prove the following proposition.

Proposition (1.28)

If $f: (X, \mathcal{T}_X) \longrightarrow (Y, \mathcal{T}_Y)$ is:-

1- An N-continuous function where Y is a T_2-space, then it is an Mωc-function (respectively MNc-function).

2- An ω-continuous function where Y is a Kc-space (respectively c-c space), then it is an Mωc-function.

3- An N-continuous function where Y is a Kc-space (respectively c-c space), then it is an MNc-function.

4- A continuous function where Y is a T_2-space (respectively Kc-space, or c-c space), then it is an Mωc-function, MNc-function, ωMc-function, NMc-function, ωMωc-function and NMNc-function.

5- A strongly ω-continuous function where Y is a ωT_2-space (respectively Kc-space, c-c space), then it is a ωMc-function and ωMωc-function.

6- A strongly ω-continuous function where Y is a NT_2-space (respectively Kc-space, c-c space), then it is an NMc-function and NMNc-function.

7- A strongly N-continuous function where Y is a NT_2-space, then it is an NMc-function.

8- An ω-irresolute function where Y is a ωT_2-space (respectively Kc-space, c-c space), then it is an ωMωc-function.

9- An N-irresolute function where Y is a NT_2-space (respectively Kc-space, c-c space), then it is an NMNc-function.

Proof:

1- Let K be a compact set in Y which is a T_2-space, so it is closed set (by remark (1.6)), so f^{-1}(K) is ω-closed (respectively N-closed) set (since f is an N-continuous function), so f is an Mωc-function and an MNc-function.

By the same way we can proof the rest.

Definition (1.29)

A function $f : X \longrightarrow Y$ is called:-

1- A compact function if the inverse image of any compact set in Y is a compact set in X [2].

2- An ω^*-compact (respectively N^*-compact) function if the inverse image of any compact set in Y is an ω-compact (respectively N-compact) set in X.

3- An ω-compact [12]. (respectively N-compact) function the inverse image of any ω-compact (respectively N-compact) set in Y is a compact set in X.

4- An ω^{**}-compact (respectively N^{**}-compact) function the inverse image of any ω-compact (respectively N-compact) set in Y is an ω-compact (respectively N-compact) set in X.

(1.30) Example

1- $f: (\mathcal{R}, \mathcal{T}_{cof}) \longrightarrow (\mathcal{R}, \mathcal{T})$ is compact, ω^*-compact, N^*-compact, ω-compact, N-compact, ω^{**}-compact and N^{**}-compact function.

Proposition (1.31):

If $f: X \longrightarrow Y$ is:-

1- ω-compact (respectively N-compact) function where X is a T_2-space, and then it is ωMc-function and ωMωc-function (respectively NMc-function and NMNc-function).

2- ω-compact (respectively N-compact) function where X is a Kc-space, and then it is ωMc-function and ωMωc-function (respectively NMc-function and NMNc-function).

3- ω-compact (respectively N-compact) function where X is a c-c space, and then it is ωMc-function and ωMωc-function (respectively NMc-function and NMNc-function).

4- Compact function where X is a T_2-space (respectively Kc-space, c-c space), and then it is ωMc-function and NMc-function.

5- ω^{**}-compact (respectively N^{**}-compact) function where X is a T_2-space, and then it is ωMωc-function (respectively NMNc-function).

6- ω^{**}-compact (respectively N^{**}-compact) function where X is a Kc-space, and then it is ωMωc- function (respectively NMNc-function).

7- ω^{**}-compact (respectively N^{**}-compact) function where X is a c-c space, and then it is ωMωc-function (respectively NMNc-function).

8- ω^*-compact (respectively N^*-compact) function where X is a ωT_2-space (respectively NT_2-space), and then it is Mωc-function (resp. MNc-function).

9- ω^*-compact (respectively N^*-compact) function where X is a Kc-space, and then it is Mωc-function (respectively MNc-function).

10- ω^*-compact (respectively N^*-compact) function where X is a c-c space, and then it is Mωc-function (respectively MNc-function).

11- ω^{**}-compact (respectively N^{**}-compact) function where X is a ωT_2-space (respectively NT_2-space), and then it is ωMωc-function (respectively NMNc-function).

Proof:

Suppose f is ω-compact (respectively N-compact) function and X is a T_2-space, let $\mathcal{W}$ be an ω-compact set in Y, so $f^{-1}(\mathcal{W})$ is compact set in X which is a T_2-space, thus $f^{-1}(\mathcal{W})$ is

closed set (by remark (1.6)), therefore f is ωMc-function (respectively NMc-function). Now, since every closed set is N-closed and ω-closed set, so f is ωMωc-function (respectively NMNc-function).
We can prove the rest by the same way.

Proposition (1.32) [19].

1- If Y is a subspace of $(X, \text{Ţ})$ and $\mathcal{W}$ is ω-open set in X, then $\mathcal{W}$ is ω-open set in Y provided that $\mathcal{W} \subseteq Y$.

2- If Y is a subspace of $(X, \text{Ţ})$ and $\mathcal{W}$ is ω-closed set in X, then $\mathcal{W}$ is ω-closed set in Y provided that $\mathcal{W} \subseteq Y$.

Proposition (1.33)

If $f: X \longrightarrow Y$ is a ωMc-function (respectively NMc-function) and $\mathcal{W} \subseteq X$ then $f|_{\mathcal{W}}: \mathcal{W} \longrightarrow Y$ is a ωMc-function (respectively NMc-function).

Proof:

Let $\mathcal{G}$=$f|_{\mathcal{W}}: \mathcal{W} \longrightarrow Y$ and let K be an ω-compact (respectively N-compact) subset of Y, since f is an ωMc-function (respectively NMc-function), then $f^{-1}(K)$ is a closed subset of X. Now $\mathcal{G}^{-1}$(K) = f^{-1}(K)$\cap\mathcal{W}$, hence $\mathcal{G}^{-1}$(K) is a closed subset of $\mathcal{W}$. Therefore $\mathcal{G} = f|_{\mathcal{W}}: \mathcal{W} \longrightarrow Y$ is a ωMc-function (respectively NMc-function).
By the same way we can prove the following proposition.

Proposition (1.34)

If $\mathcal{W} \subseteq X$ and $f: X \longrightarrow Y$ is:-

1- An Mωc-function (respectively MNc-function), then $f|_{\mathcal{W}}: \mathcal{W} \longrightarrow Y$ is an Mωc-function (respectively MNc-function).

2- An ωMωc-function (respectively NMNc-function), then $f|_{\mathcal{W}}: \mathcal{W} \longrightarrow Y$ is a ωMωc-function (respectively NMNc-function).

Proposition (1.35)

If $f: X \longrightarrow Y$ is ωMc-function (respectively NMc-function) where X is a compact space, then it is ω-compact (respectively N-compact) function.

Proof: Suppose K is a ω-compact (respectively N-compact) subset of Y, so f^{-1}(K) is a closed subset of X(since f is ωMc-function (respectively NMc-function)), but X is compact, hence f^{-1}(K) is compact (by remark (1.6)), therefore f is ω-compact (respectively N-compact) function.
In a same manner, we can prove the following corollary.

Corollary (1.36)

If $f: X \longrightarrow Y$ is:-

1- ωMc-function (respectively NMc-function) where X is a c-c space, then it is ω-compact (respectively N-compact) function.

2- Mc-function where X is a compact space, then it is ω-compact (respectively N-compact) function.

3- Mc-function where X is a c-c space, then it is ω-compact (respectively N-compact) function.

4- Mc-function where X is an ω-compact (respectively N-compact) space, then it is ω^*-compact (respectively N^*-compact) function.

5- Mωc-function (respectively MNc-function) where X is an ω-compact (respectively N-compact) space, then it is ω^*-compact (respectively N^*-compact) function.

6- ωMωc-function (respectively NMNc-function) where X is an ω-compact (respectively N-compact) space, then it is ω^{**}-compact (respectively N^{**}-compact) function.
7- Mωc-function (respectively MNc-function) where X is an ω-compact (respectively N-compact) space, then it is ω^{**}-compact (respectively N^{**}-compact) function.
8- Mc-function where X is an ω-compact (respectively N-compact) space, then it is ω^{**}-compact (respectively N^{**}-compact) function.

Proof

1- Suppose K is a ω-compact (respectively N-compact) subset of Y, so f^{-1}(K) is a closed subset of X(since f is ωMc-function (respectively NMc-function)), but X is c-c space, hence f^{-1}(K) is compact, therefore f is ω-compact (respectively N-compact) function.
2- Suppose K is a ω-compact (respectively N-compact) subset of Y, so it is compact (by remark (1.2)), hence f^{-1}(K) is a closed subset of X(since f is Mc-function), but X is compact space, hence f^{-1}(K) is compact (by remark (1.6)), therefore f is ω-compact (respectively N-compact) function.
3- Suppose K is an ω-compact (respectively N-compact) subset of Y, so it is compact (by remark (1.2)), hence f^{-1}(K) is a closed subset of X(since f is Mc-function), but X is c-c space, hence f^{-1}(K) is compact, therefore f is ω-compact (respectively N-compact) function.
4- Suppose K is a compact subset of Y, hence f^{-1}(K) is a closed subset of X(since f is Mc-function), and by remark (1.2) it is ω-closed (respectively N-closed) subset of X which is ω-compact (respectively N-compact) space, hence f^{-1}(K) is ω-compact (respectively N-compact) (by remark (1.6)), therefore f is ω^{*}-compact (respectively N^{*}-compact) function.
5- Suppose K is a compact subset of Y, hence f^{-1}(K) is an ω-closed (respectively N-closed) subset of X (since f is Mωc-function (respectively MNc-function)), but X is ω-compact (respectively N-compact) space, hence f^{-1}(K) is ω-compact (respectively N-compact) (by remark (1.6)), therefore f is ω^{*}-compact (respectively N^{*}-compact) function.
6- Suppose K is an ω-compact (respectively N-compact) subset of Y, hence f^{-1}(K) is an ω-closed (respectively N-closed) subset of X (since f is ωMωc-function (respectively NMNc-function)), but X is an ω-compact (respectively N-compact) space, hence f^{-1}(K) is ω-compact (respectively N-compact) (by remark (1.6)), therefore f is ω^{**}-compact (respectively N^{**}-compact) function.
7- Suppose K is an ω-compact (respectively N-compact) subset of Y, so it is compact (by remark (1.2)), hence f^{-1}(K) is an ω-closed (respectively N-closed) subset of X (since f is Mωc-function (respectively MNc-function)), but X is ω-compact (respectively N-compact) space, hence f^{-1}(K) is ω-compact (respectively N-compact) (by remark (1.6)), therefore f is ω^{**}-compact (respectively N^{**}-compact) function.
8- Suppose K is an ω-compact (respectively N-compact) subset of Y, so it is compact (by remark (1.2)), hence f^{-1}(K) is closed subset of X (since f is Mc-function (respectively Mc-function)), and by remark (1.2)) it is an ω-closed (respectively N-closed), but X is ω-compact (respectively N-compact) space, hence f^{-1}(K) is ω-compact (respectively N-compact) (by remark (1.6)), therefore f is ω^{**}-compact (respectively N^{**}-compact) function.

Proposition (1.37)

If X is an ω-compact and ωT_2-space (respectively N-compact and NT_2-space), then a function $f: X \longrightarrow Y$ is an ω^{**}-compact (respectively N^{**}-compact) function iff it is ωMωc-function (respectively NMNc-function).

Proof:

Let X be an ω-compact and ωT_2-space (respectively N-compact and NT_2-space), let f be an ω^{**}-compact (respectively N^{**}-compact). To prove f is an ωMωc-function (respectively NMNc-function). Let $\mathcal{M}$ be an ω-compact (respectively N-compact) set in Y, then $f^{-1}(\mathcal{M})$ is ω-compact (respectively N-compact) set in X (since f is ω^{**}-compact (respectively N^{**}-compact)), and since X is ωT_2-space (respectively NT_2-space), so $f^{-1}(\mathcal{M})$ is ω-closed (respectively N-closed) set in X, hence f is an ωMωc-funtion (respectively NMNc-function). Conversely, let f be an ωMωc-function (respectively NMNc-function). To prove f is an ω^{**}-compact (respectively N^{**}-compact). Let $\mathcal{M}$ be an ω-compact (respectively N-compact) set in Y, so $f^{-1}(\mathcal{M})$ is ω-closed (respectively N-closed) set in X (because f is ωMωc-function (respectively NMNc-function)), but X is ω-compact (respectively. N-compact) space, hence $f^{-1}(\mathcal{M})$ is ω-compact (respectively N-compact) set, therefore f is ω^{**}-compact (respectively N^{**}-compact) function.

Corollary (1.38)

1- If X is a Kc-space, and $f: X \longrightarrow Y$ is an ω^{**}-compact (respectively N^{**}-compact) function, then it is an ωMωc-function (respectively NMNc-function).

2- If X is an ω-compact (respectively N-compact) space and Kc-space, then a function $f: X \longrightarrow Y$ is an ω^{**}-compact (respectively N^{**}-compact) function iff it is ωMωc-function (respectively NMNc-function).

3- If X is a c-c space, and $f: X \longrightarrow Y$ is an ω^{**}-compact (respectively N^{**}-compact) function, then it is an ωMωc-function (respectively NMNc-function).

4- If X is an ω-compact (respectively N-compact) and c-c space, then a function $f: X \longrightarrow Y$ is an ω^{**}-compact (respectively N^{**}-compact) function iff it is ωMωc-function (respectively NMNc-function).

5- If X is an ω-compact (respectively N-compact) and T_2-space, then a function $f: X \longrightarrow Y$ is an ω^{**}-compact (respectively N^{**}-compact) function iff it is ωMωc-function (respectively NMNc-function).

Proof:

1- Let K be an ω-compact (respectively N-compact) subset of Y, so f^{-1}(K) is ω-compact (respectively N-compact) subset of X (since f is an ω^{**}-compact (respectively N^{**}-compact) function)), and then it is compact subset of X (by remark (1.2)) which is Kc-space, so f^{-1}(K) is closed subset of X, and by remark (1.2)) it is ω-closed (respectively N-closed), so f is an ωMωc-function (respectively NMNc-function).

2- Let f be an ω^{**}-compact (respectively N^{**}-compact) function, and K be an ω-compact (respectively N-compact) subset of Y, so f^{-1}(K) is ω-compact (respectively N-compact) subset of X (since f is an ω^{**}-compact (respectively N^{**}-compact) function)), and then it is compact subset of X (by remark (1.2)) which is Kc-space, so f^{-1}(K) is closed subset of X, and by remark (1.2)) it is ω-closed (respectively N-closed), so f is an ωMωc-function (respectively NMNc-function). Conversely, let K be an ω-compact (respectively N-compact) subset of Y so f^{-1}(K) is ω-closed (respectively N-closed) subset of X (since f is an ωMωc-function (respectively NMNc-function)), but X is an ω-compact (respectively N-compact) space, so f^{-1}(K) is an ω-compact (respectively N-compact) subset of X, therefore f is an ω^{**}-compact (respectively N^{**}-compact) function.

3- Let K be an ω-compact (respectively N-compact) subset of Y, so f^{-1}(K) is ω-compact (respectively N-compact) subset of X (since f is an ω^{**}-compact (resp. N^{**}-compact) function)), and then it is compact subset of X (by remark (1.2)) which is c-c space, so f^{-1}(K) is closed subset of X, and by remark (1.2)) it is ω-closed (respectively N-closed) subset of X, so f is an ωMωc-function (respectively NMNc-function).

4- Let f be an ω^{**}-compact (respectively N^{**}-compact) function, and K be an ω-compact (respectively N-compact) subset of Y, so f^{-1}(K) is ω-compact (respectively N-compact) subset of X (since f is an ω^{**}-compact (resp. N^{**}-compact) function)), and then it is compact subset of X (by remark (1.2)) which is c-c space, so f^{-1}(K) is closed subset of X, and by remark (1.2)) it is ω-closed (respectively N-closed) subset of X, so f is an ωMωc-function (respectively NMNc-function). Conversely, let K be an ω-compact (respectively N-compact) subset of Y, so f^{-1}(K) is ω-closed (respectively N-closed) subset of X (since f is an ωMωc-function (respectively NMNc-function) which is an ω-compact (respectively N-compact) space, so f^{-1}(K) is an ω-compact (respectively N-compact) subset of X(by remark (1.6)) , so f be an ω^{**}-compact (respectively N^{**}-compact) function.

5- Let f be a ω^{**}-compact (respectively N^{**}-compact) function, and K be an ω-compact (respectively N-compact) subset of Y, so f^{-1}(K) is ω-compact (respectively N-compact) subset of X (since f is an ω^{**}-compact (resp. N^{**}-compact) function)), and then it is compact subset of X (by remark (1.2)) which is a T_2-space, so f^{-1}(K) is closed subset of X (by remark (1.6)), and by remark (1.2)) it is ω-closed (respectively N-closed) subset of X, so f is an ωMωc-function (respectively NMNc-function). Conversely, let K be a ω-compact (respectively N-compact) subset of Y, so f^{-1}(K) is ω-closed (respectively N-closed) subset of X (since f is an ωMωc-function (respectively NMNc-function) which is an ω-compact (respectively N-compact) space, so f^{-1}(K) is a ω-compact (respectively N-compact) subset of X(by remark (1.6)) , so f be an ω^{**}-compact (respectively N^{**}-compact) function.

Proposition (1.39)

The composition between:

1- Strongly ω-continuous function and Mωc-function (respectively MNc-function) is Mc-function.

2- ω-continuous (respectively N-continuous) function and ωMc-function (respectively NMc-function) is ωMωc-function (respectively NMNc-function).

3- Strongly ω-continuous (respectively strongly N-continuous) function and ωMωc-function (respectively NMNc-function) is ωMc-function (respectively NMc-function).

4- Strongly ω-continuous (respectively strongly N-continuous) function and Mc-function is Mc-function.

5- Strongly ω-continuous (respectively strongly N-continuous) function and ωMc-function (respectively NMc-function) is ωMc-function (respectively NMc-function).

6- Strongly ω-continuous (respectively strongly N-continuous) function and Mωc-function (respectively MNc-function) is Mc-function.

Proof:

1- Let $f: X \longrightarrow Y$ be strongly ω-continuous function and $\mathcal{G}: Y \longrightarrow \mathcal{Z}$ be Mωc-function (resp. MNc-function), and let K be a compact set in $\mathcal{Z}$, so $\mathcal{G}^{-1}$(K) is ω-closed (respectively N-closed) set in Y, so f^{-1} ($\mathcal{G}^{-1}$(K))=$(\mathcal{G} \circ f)^{-1}$(K) is closed set in X(since every N-closed set is ω-closed), so $\mathcal{G} \circ f$ is Mc-function.

2- Let $f: X \longrightarrow Y$ be an ω-continuous (respectively N-continuous) function and $\mathcal{G}: Y \longrightarrow \mathcal{Z}$ be ωMc-function (respectively NMc-function), and let K be an ω-compact (respectively N-compact) set in $\mathcal{Z}$, so $\mathcal{G}^{-1}$(K) is closed set in Y, and then f^{-1} ($\mathcal{G}^{-1}$(K))=$(\mathcal{G} \circ f)^{-1}$(K) is ω-closed (respectively N-closed) set in X, so $\mathcal{G} \circ f$ is ωMωc-function (resp. NMNc-function).
3- Let $f: X \longrightarrow Y$ be strongly ω-continuous (respectively strongly N-continuous) function and $\mathcal{G}: Y \longrightarrow \mathcal{Z}$ be an ωMωc-function (respectively NMNc-function), and let K be an ω-compact (respectively N-compact) set in $\mathcal{Z}$, so $\mathcal{G}^{-1}$(K) is ω-closed (respectively N-closed) set in Y, so f^{-1} ($\mathcal{G}^{-1}$(K))=$(\mathcal{G} \circ f)^{-1}$(K) is closed set in X(since every N-closed set is ω-closed), so $\mathcal{G} \circ f$ is ωMc-function (respectively NMc-function).
4- Let $f: X \longrightarrow Y$ be strongly ω-continuous (respectively strongly N-continuous) function and $\mathcal{G}: Y \longrightarrow \mathcal{Z}$ be Mc-function, and let K be a compact set in $\mathcal{Z}$, so $\mathcal{G}^{-1}$(K) is closed set in Y, and by remark (1.2) it is ω-closed (respectively N-closed) set in Y, so f^{-1} ($\mathcal{G}^{-1}$(K))=$(\mathcal{G} \circ f)^{-1}$(K) is closed set in X, so $\mathcal{G} \circ f$ is Mc-function.
5- Let $f: X \longrightarrow Y$ be strongly ω-continuous (respectively strongly N-continuous) function and $\mathcal{G}: Y \longrightarrow \mathcal{Z}$ be ωMc-function (respectively NMc-function), and let K be an ω-compact (respectively N-compact) set in $\mathcal{Z}$, so $\mathcal{G}^{-1}$(K) is closed set in Y, and by remark (1.2) it is ω-closed (respectively N-closed) set in Y, so f^{-1} ($\mathcal{G}^{-1}$(K))=$(\mathcal{G} \circ f)^{-1}$(K) is closed set in X, so $\mathcal{G} \circ f$ is ωMc-function (resp. NMc-function).
6- Let $f: X \longrightarrow Y$ be strongly ω-continuous (respectively strongly N-continuous) function and $\mathcal{G}: Y \longrightarrow \mathcal{Z}$ be Mωc-function (respectively MNc-function), and let K be a compact set in $\mathcal{Z}$, so $\mathcal{G}^{-1}$(K) is ω-closed (respectively N-closed) set in Y, so f^{-1} ($\mathcal{G}^{-1}$(K))=$(\mathcal{G} \circ f)^{-1}$(K) is closed set in X (since every N-closed set is ω-closed), so $\mathcal{G} \circ f$ is Mc-function.

Proposition (1.40)

If $f: X \longrightarrow Y$ and $\mathcal{G}: Y \longrightarrow \mathcal{Z}$ are two functions:-
1- If f is Mωc-function (respectively MNc-function) and $\mathcal{G}$ is strongly ω-continuous function where Y is a compact space, then $\mathcal{G} \circ f$ is an ω-irresolute (respectively N-irresolute) function.
2- If f is ωMc-function (respectively NMc-function) and $\mathcal{G}$ is ω-continuous (respectively N-continuous) function where Y is a ω-compact (respectively N-compact) space, then $\mathcal{G} \circ f$ is a continuous function.
3- If f is ωMωc-function (respectively NMNc-function) and $\mathcal{G}$ is strongly ω-continuous (respectively strongly N-continuous) function where Y is an ω-compact (respectively N-compact) space, then $\mathcal{G} \circ f$ is an ω-irresolute (respectively N-irresolute) function.
4- If f is Mc-function and $\mathcal{G}$ is ωMc-function (respectively NMc-function) function where Y is a compact space, then $\mathcal{G} \circ f$ is ωMc-function (respectively NMc-function).
5- If f is a compact function and $\mathcal{G}$ is ωMc-function (respectively NMc-function) where Y is a compact space, then $\mathcal{G} \circ f$ is a ω-compact (respectively N-compact) function
6- If f is Mc-function and $\mathcal{G}$ is strongly ω-continuous (respectively strongly N-continuous) function where Y is a c-c space, then $\mathcal{G} \circ f$ is strongly ω-continuous (respectively strongly N-continuous) function.
7- If f is Mωc-function (respectively MNc-function) and $\mathcal{G}$ is strongly ω-continuous function where Y is a c-c space, then $\mathcal{G} \circ f$ is an ω-irresolute (respectively N-irresolute) function.
And there are many other cases.

Proof:

1- Let f be an Mωc-function (respectively MNc-function) and $\mathcal{G}$ be a strongly ω-continuous function, and let $\mathcal{W}$ be ω-open set in $\mathcal{Z}$, so $\mathcal{G}^{-1}(\mathcal{W})$ is open set in Y, so $(\mathcal{G}^{-1}(\mathcal{W}))^c$ is closed set in Y which is compact space, so $(\mathcal{G}^{-1}(\mathcal{W}))^c$ is compact set in Y, then $f^{-1}[(\mathcal{G}^{-1}(\mathcal{W}))^c]=[(\mathcal{G} \circ f)^{-1}(\mathcal{W})]^c$ is ω-closed (respectively N-closed) set in X (since $f^{-1}[(\mathcal{G}^{-1}(\mathcal{W}))^c] = f^{-1}(Y - \mathcal{G}^{-1}(\mathcal{W})) = X - f^{-1}(\mathcal{G}^{-1}(\mathcal{W})) = X - (\mathcal{G} \circ f)^{-1}(\mathcal{W}) = [(\mathcal{G} \circ f)^{-1}(\mathcal{W})]^c$), and then $(\mathcal{G} \circ f)^{-1}(\mathcal{W})$ is ω-open (respectively N-open) set in X, hence $\mathcal{G} \circ f$ is ω-irresolute (respectively N-irresolute) function.

By the same way we can prove the rest.

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