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New Games via soft-$\mathcal{J}$-Semi-g-Separation axioms

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Abstract

In this article, the notions of soft closed sets are introduced by using soft ideal and soft semi-open sets, which are soft-$\mathcal{J}$-semi-g-closed sets "s$\mathcal{J}$sg-closed" where many of the properties of these sets are clarified. Some games by using soft- $\mathcal{J}$-semi, soft separation axioms: like $\c{S}\mathcal{G}(\mathcal{T}_0 \ ,\chi)$, $\c{S}\mathcal{G}(\mathcal{T}_0 \ ,\mathcal{J})$. Using many figures and proposition to study the relationships among these kinds of games with some examples are explained.

Keywords: Soft ideal, Soft-$\mathcal{T}_i$-*space* , Soft-$\mathcal{J}$-semi-g-$\mathcal{T}_i$-*space* , $\c{S}\mathcal{G}(\mathcal{T}_i,\chi)$ $\c{S}\mathcal{G}(\mathcal{T}_i,\mathcal{J})$. Where $i \ = \{0,1,2\}$.

1.Introduction

In 2011, Shaber [1] introduced soft topological spaces. Shaber have been introduced to study many topological properties by using soft set like derived sets, compactness, separation axioms and other properties. [2-4]. Also, Kandil used the soft ideal which is a family of soft sets that meet hereditary and finite additively property of χ to study the notion of soft logical function [5], which was the starting point for studying the properties of soft ideal topological spaces $(\chi\,,\mathcal{T},\mathcal{H},\mathcal{J})$ and defined new types of near open soft sets and studied their properties as [6-8].

2.Preliminaries.

Definition 2.1. [9] Let $\chi \neq \emptyset$ and $\mathcal{H}$ be a set of parameters. Such that is $\wp(\chi)$ the power set of χ and $\boldsymbol{\mathcal{P}} \subsetneqq \mathcal{H}$. A pair $(\Gamma,\mathcal{H})$ (briefly $\Gamma_{\mathcal{H}}$) is a soft set over χ where, Γ is a function given by $\Gamma : \mathcal{H} \to \wp(\chi)$. So, $\Gamma_{\mathcal{H}} = \{\, \Gamma(\hbar) : \hbar \in \mathcal{P} \subseteq \mathcal{H}, \Gamma : \mathcal{H} \to \wp(\chi) \,\}$. The family of all soft sets (Is denoted by $ŞŞ(\chi)_{\mathcal{H}}$).

Definition 2.2. [9] Let $(\Gamma,\mathcal{H}), (\mathcal{G},\mathcal{H}) \in ŞŞ(\chi)_{\mathcal{H}}$. Then $(\Gamma,\mathcal{H})$ is a soft subset of , $(\mathcal{G},\mathcal{H})$, (briefly$(\Gamma,\mathcal{H}) \;\tilde{\subseteq}, (\mathcal{G},\mathcal{H})$), if $\Gamma(\hbar) \tilde{\subseteq} \mathcal{G}(\hbar)$, for all $\hbar \in \mathcal{H}$. Now $(\Gamma,\mathcal{H})$ is a soft subset of , $(\mathcal{G},\mathcal{H})$ and ,$(\mathcal{G},\mathcal{H})$ is a soft super set of $(\Gamma,\mathcal{H})$, $(\Gamma,\mathcal{H}) \tilde{\subseteq} (\mathcal{G},\mathcal{H})$.

Definition 2.3. [10] The complement of a soft set $(\Gamma,\mathcal{H})$ (Is denoted by $(\Gamma,\mathcal{H})'$) and $(\Gamma,\mathcal{H})' = (\Gamma',\mathcal{H})$ where $\Gamma' : \mathcal{H} \to \wp(\chi)$ is a function such that $\Gamma'(\hbar) = \chi - \Gamma(\hbar)$, for each $\hbar \in \mathcal{H}$ and Γ' is a soft complement of Γ.

Definition 2.4. [1] Let $(\Gamma,\mathcal{H})$ be a soft over χ and $x \in \chi$. Then $x \tilde{\in} (\Gamma,\mathcal{H})$ whenever, $x \in \Gamma(\hbar)$ for each $\hbar \in \mathcal{H}$.

Definition 2.5. [1] $(\Gamma,\mathcal{H})\,\chi$ is a NULL soft set (briefly $\tilde{\emptyset}$ or $\emptyset_{\mathcal{H}}$) if for each $\hbar \in \mathcal{H}$, $\Gamma(\hbar) = \emptyset$ (null set).

Definition 2.6. [1] A soft set $(\Gamma,\mathcal{H})$ over χ is an absolute soft set (briefly $\tilde{\chi}$ or $\chi_{\mathcal{H}}$) If for each $\hbar \in \mathcal{H}$, $\Gamma(\hbar) = \chi$.

Definition 2.7. [1] Let $\mathcal{T}$ be a collection of soft sets over χ with same $\mathcal{H}$, then $\mathcal{T} \in ŞŞ(\chi)_{\mathcal{H}}$ is a soft topology on χ if;

- **i.** $\tilde{\chi}, \tilde{\emptyset} \in \mathcal{T}$ where, $\tilde{\emptyset}(\hbar) = \emptyset$ and $\tilde{\chi}(\hbar) = \chi$, for each $\hbar \in \mathcal{H}$,
- **ii.** $\tilde{\bigcup}_{\alpha\in\Lambda} (\mathcal{O}_\alpha, \mathcal{H}) \in \mathcal{T}$ whenever, $(\mathcal{O}_\alpha, \mathcal{H}) \in \mathcal{T} \;\; \forall\, \alpha \in \Lambda$,
- **iii.** $((\Gamma,\mathcal{H}) \tilde{\cap} (\mathcal{G},\mathcal{H})) \in \mathcal{T}$ for each $(\Gamma,\mathcal{H}), (\mathcal{G},\mathcal{H}) \in \mathcal{T}$.

$(\chi, \mathcal{T}, \mathcal{H})$ is a soft topological space if $(\mathcal{O},\mathcal{H}) \in \mathcal{T}$ then $(\mathcal{O},\mathcal{H})$ is an open soft set.

Definition 2.8. [11] Let $(\chi, \mathcal{T}, \mathcal{H})$ be a soft topological space. A soft set $(\Gamma,\mathcal{H})$ over χ is a soft closed set in χ, if its complement $(\Gamma,\mathcal{H})' \in \mathcal{T}$, the family of all soft closed sets (Is denoted by $ŞC(\chi)_{\mathcal{H}}$).

Definition 2.9. [11] For any $(\chi, \mathcal{T}, \mathcal{H})$. Let $(\Gamma,\mathcal{H})' \tilde{\in} \tilde{\chi}$, then the soft closure of $(\Gamma,\mathcal{H})'$, (briefly cl $(\Gamma,\mathcal{H})$) ,(Is defined as $cl((\Gamma,\mathcal{H})) = \tilde{\cap} \{ (\mathcal{G},\mathcal{H}) : (\mathcal{G},\mathcal{H}) \in ŞC(\chi)_{\mathcal{H}}, (\Gamma,\mathcal{H}) \tilde{\subseteq} (\mathcal{G},\mathcal{H}) \}$.

Definition 2.10. [11] For any $(\chi, \mathcal{T}, \mathcal{H})$. Let$(\Gamma,\mathcal{H}) \in ŞŞ(\chi)_{\mathcal{H}}$, then the soft interior of $(\Gamma,\mathcal{H})$,(briefly int$(\Gamma,\mathcal{H})$) ,(Is defined as $int((\Gamma,\mathcal{H})) = \tilde{\cup} \{ (\mathcal{G},\mathcal{H}) : (\mathcal{G},\mathcal{H}) \in \mathcal{T}, (\mathcal{G},\mathcal{H}) \tilde{\subseteq} (\Gamma,\mathcal{H}) \}$.

Definition 2.11. [2] Two soft sets $(\mathcal{Z},\mathcal{H})$, $(\mathcal{N},\mathcal{H})$ in $ŞŞ(\chi)_{\mathcal{H}}$. Are said to be soft disjoint, if $(\mathcal{Z},\mathcal{H}) \tilde{\cap} (\mathcal{N},\mathcal{H}) = \tilde{\emptyset}$ written $\mathcal{Z}(\hbar) \cap \mathcal{N}(\hbar) = \{\emptyset\}$, for each $\hbar \in \mathcal{H}$.

Definition 2.12. [2] Two soft point $\hbar_{\mathcal{M}}, \hbar_{\mathcal{N}} \tilde{\in} \tilde{\chi}$ are distinct, written $\hbar_{\mathcal{M}} \neq \hbar_{\mathcal{N}}$, if $\exists\, (\mathcal{M},\mathcal{H})$ and $(\mathcal{N},\mathcal{H})$ are two soft disjoint sets, such that $\hbar_{\mathcal{M}} \tilde{\in} (\mathcal{M},\mathcal{H})$ *and* $\hbar \tilde{\in} (\mathcal{N},\mathcal{H})$.

Definition 2.13. [5] Let $\mathcal{I}$ be a non-null family of soft sets over χ with parameter $\mathcal{H}$, then $\mathcal{I} \tilde{\subseteq}$ ȘȘ $(\chi)_{\mathcal{H}}$ is a soft ideal whenever,
(1) If $(\Gamma,\mathcal{H}) \tilde{\in} \mathcal{I}$ and $(\mathcal{G},\mathcal{H}) \tilde{\in} \mathcal{I}$ implies,$(\Gamma,\mathcal{H}) \tilde{\cup} (\mathcal{G},\mathcal{H}) \tilde{\in} \mathcal{I}$.
(2) If $(\Gamma,\mathcal{H}) \tilde{\in} \mathcal{I}$ and $(\mathcal{G},\mathcal{H}) \tilde{\subseteq} (\Gamma,\mathcal{H})$ implies $(\mathcal{G},\mathcal{H}) \tilde{\in} \mathcal{I}$.
Any $(\chi,\mathcal{T},\mathcal{H})$ with a soft ideal $\mathcal{I}$ is a soft ideal topological space (briefly $(\chi,\mathcal{T},\mathcal{H},\mathcal{I})$).

Definition 2.14. [5] Any $(\chi,\mathcal{T},\mathcal{H})$ with a soft ideal $\mathcal{I}$ is namelya soft ideal topological space (briefly $(\chi,\mathcal{T},\mathcal{H},\mathcal{I})$).
Definition 2.15. [12] For any $(\chi,\mathcal{T},\mathcal{H})$, then $(\Gamma,\mathcal{H})$ is a soft semi-open set (briefly ȘȘ-$open\ set$) if $(\Gamma,\mathcal{H}) \tilde{\subseteq}$ cl(int$(\Gamma,\mathcal{H})$) . A complement of a soft semi-open set is a soft semi-closed (briefly ss-closed et). The collection of each soft semi -open sets in $(\chi,\mathcal{T},\mathcal{H})$ (briefly ȘȘ$O(\chi)$) . The collection of each soft semi-closed sets (briefly ȘȘ$C(\chi)_{\mathcal{H}}$).

Definition 2.16. [2] A soft topological space $(\chi,\mathcal{T},\mathcal{H})$ over χ is a soft-$\mathcal{T}_0$-space if for each $h_{\mathcal{M}}, h_{\mathcal{N}} \tilde{\in} \tilde{\chi}$ such that $h_{\mathcal{M}} \neq h_{\mathcal{N}}$, there exists a soft open set $(\omega,\mathcal{H})$ such that $h_{\mathcal{M}} \tilde{\in} (\omega,\mathcal{H})$ and $h_{\mathcal{N}} \tilde{\notin} (\omega,\mathcal{H})$ or $h_{\mathcal{M}} \tilde{\notin} (\omega,\mathcal{H})$ and $h_{\mathcal{N}} \tilde{\in} (\omega,\mathcal{H})$.

Theorem 2.17. [2] A soft topological space $(\chi,\mathcal{T},\mathcal{H})$ over χ is a soft-$\mathcal{T}_0$-space if and only if for each $h_{\mathcal{M}}, h_{\mathcal{N}} \tilde{\in} \tilde{\chi}$ such that $h_{\mathcal{M}} \neq h_{\mathcal{N}}$, there exists a soft closed set $(\mathcal{V},\mathcal{H})$ such that $h_{\mathcal{M}} \tilde{\in} (\mathcal{V},\mathcal{H})$, $h_{\mathcal{N}} \tilde{\notin} (\mathcal{V},\mathcal{H})$ or $h \tilde{\notin} (\mathcal{V},\mathcal{H})$, $h_{\mathcal{N}} \tilde{\in} (\mathcal{V},\mathcal{H})$.

Definition 2.18. [2] A soft topological space $(\chi,\mathcal{T},\mathcal{H})$ over χ is a soft-$\mathcal{T}_1$-space if for each h , $h_{\mathcal{N}} \tilde{\in} \tilde{\chi}$ such that $h_{\mathcal{M}} \neq h_{\mathcal{N}}$ $\exists$ $(\mathcal{P},\mathcal{H})$, $(\omega,\mathcal{H}) \in \mathcal{T}$ whenever, $h_{\mathcal{M}} \tilde{\in} (\mathcal{P},\mathcal{H})$, $h_{\mathcal{N}} \tilde{\notin} (\mathcal{P},\mathcal{H})$ and $h_{\mathcal{M}} \tilde{\notin} (\omega,\mathcal{H})$, $h_{\mathcal{N}} \tilde{\in} (\omega,\mathcal{H})$.

Theorem 2.19. [2] A space $(\chi,\mathcal{T},\mathcal{H})$ is a soft-$\mathcal{T}_1$-space if and only if for all $h_{\mathcal{M}}, h_{\mathcal{N}} \tilde{\in} \tilde{\chi}$ such that $h_{\mathcal{M}} \neq h_{\mathcal{N}}$. $\exists$ $(\mathcal{P},\mathcal{H})$, $(\mathcal{V},\mathcal{H})$ are two soft closed sets whenever, $h_{\mathcal{M}} \tilde{\in} (\mathcal{P},\mathcal{H})$, $h_{\mathcal{N}} \tilde{\notin} (\mathcal{P},\mathcal{H})$ and $h_{\mathcal{M}} \tilde{\notin} (\mathcal{V},\mathcal{H})$, $h_{\mathcal{N}} \tilde{\in} (\mathcal{V},\mathcal{H})$.

Definition 2.20. [2] Let $(\chi,\mathcal{T},\mathcal{H})$ be a soft topological space over χ is said to be soft-$\mathcal{T}_2$-space if, for each h , $h_{\mathcal{N}} \tilde{\in} \tilde{\chi}$ such that $h_{\mathcal{M}} \neq h_{\mathcal{N}}$. $\exists$ $(\mathcal{P},\mathcal{H})$, $(\omega,\mathcal{H}) \in \mathcal{T}$ whenever, $h_{\mathcal{M}} \tilde{\in} (\mathcal{P},\mathcal{H}) h_{\mathcal{M}}$, $h_{\mathcal{N}} \tilde{\in} (\omega,\mathcal{H})$ and $(\mathcal{P},\mathcal{H}) \tilde{\cap} (\omega,\mathcal{H}) = \{\tilde{\emptyset}\}$.

Proposition 2.21. [2] For all soft- $\mathcal{T}_{i+1}$-space is a soft- $\mathcal{T}_i$-space and i $\in$ {0,1,2}
Proof. Obvious.
Note that for all soft- $\mathcal{T}_1$-space is a soft- $\mathcal{T}_0$-space and for all a soft- $\mathcal{T}_2$-space is a soft- $\mathcal{T}_1$-space. The converse is not true hold in general.

3. On soft ideal semi-g-closed set.

Definition 3.1: In soft ideal topological space $(\chi,\mathcal{T},\mathcal{H},\mathcal{I})$, let $(\Gamma,\mathcal{H}) \in$ ȘȘO(χ), then $(\Gamma,\mathcal{H})$ is a soft-$\mathcal{I}$-semi-g-closed set (briefly s$\mathcal{I}$sg-closed). If cl$(\Gamma,\mathcal{H}) - (\mho,\mathcal{H}) \in \mathcal{I}$ whenever, $(\Gamma,\mathcal{H}) - (\mho,\mathcal{H}) \in \mathcal{I}$ and $(\mho,\mathcal{H}) \in$ ȘȘO(χ). $\tilde{\chi} - (\Gamma,\mathcal{H})$ is a soft-$\mathcal{I}$-semi-g-open set (briefly s$\mathcal{I}$sg-open set). The family of each s$\mathcal{I}$sg- closed sets (briefly s$\mathcal{I}$sgc(χ)) .The family of each s$\mathcal{I}$sg-open soft sets (briefly s$\mathcal{I}$sgo$(\chi)_{\mathcal{H}}$).

Example 3.2: For any space $(\chi, \mathcal{T}, \mathcal{H}, \mathcal{I})$, where $\chi = \{1,2\}$, $\mathcal{H} = \{\hbar_1, \hbar_2\}$, $\mathcal{T} = \{\widetilde{\emptyset}, \widetilde{X}, \Gamma\}$, $\mathcal{I} = \{\widetilde{\emptyset}, \mathcal{K}\}$ such that $(\Gamma, \mathcal{H}) = \{(\hbar_1, \{2\}), (\hbar_2, \chi)\}$ and $(\mathcal{K}, \mathcal{H}) = \{(\hbar_1, \{\emptyset\}), (\hbar_2, \{1\})\}$ then $\c{S}\c{S}O(\chi) = \mathcal{T}$, s$\mathcal{I}$sg-c$(\chi)_{\mathcal{H}} = \{\widetilde{\emptyset}, \tilde{\chi}, (\mathcal{P}, \mathcal{H}), (\omega, \mathcal{H}), (\mathcal{Z}, \mathcal{H}), (\mathcal{D}, \mathcal{H}), (\mathcal{E}, \mathcal{H}), (\mathcal{N}, \mathcal{H}), (\mathcal{G}, \mathcal{H})\}$ such that $(\mathcal{P}, \mathcal{H}) = \{(\hbar_1, \{1\}), (\hbar_2, \{1\})\}$, $(\omega, \mathcal{H}) = \{(\hbar_1, \chi), (\hbar_2, \{\emptyset\})\}$, $(\mathcal{Z}, \mathcal{H}) = \{(\hbar_1, \chi), (\hbar_2, \{1\})\}$, $(\mathcal{D}, \mathcal{H}) = \{(\hbar_1, \chi), (\hbar_2, \{2\})\}$, $(\mathcal{E}, \mathcal{H}) = \{(\hbar_1, \{1\}), (\hbar_2, \{\emptyset\})\}$, $(\mathcal{N}, \mathcal{H}) = \{(\hbar_1, \{1\}), (\hbar_2, \{2\})\}$ and $(\mathcal{G}, \mathcal{H}) = \{(\hbar_1, \{1\}), (\hbar_2, \chi)\}$.

Remark 3.3: For any $(\chi, \mathcal{T}, \mathcal{H}, \mathcal{I})$ then

i. Each closed soft set is a s$\mathcal{I}$sg-closed.

ii. Each open soft set is a s$\mathcal{I}$sg-open.

Proof (i) Let $(\mathcal{P}, \mathcal{H})$ be any closed soft set in $(\chi, \mathcal{T}, \mathcal{H}, \mathcal{I})$ and $(\mho, \mathcal{H})$ be a soft semi-open set such that $(\mathcal{P}, \mathcal{H}) - (\mho, \mathcal{H}) \in \mathcal{I}$, but cl$(\mathcal{P}, \mathcal{H}) = (\mathcal{P}, \mathcal{H})$, since $(\mathcal{P}, \mathcal{H})$ is a closed soft set so, cl$(\mathcal{P}, \mathcal{H})$-$(\mho, \mathcal{H}) = (\mathcal{P}, \mathcal{H}) - (\mho, \mathcal{H}) \in \mathcal{I}$. This implies $(\mathcal{P}, \mathcal{H})$ is a soft-$\mathcal{I}$-semi-g-closed soft set.

(ii) Let $(\mho, \mathcal{H})$ be any open soft set in $(\chi, \mathcal{T}, \mathcal{H}, \mathcal{I})$ then $\tilde{\chi} - (\mho, \mathcal{H})$ is a closed soft set. By (i) $(\tilde{\chi} - (\mho, \mathcal{H}))$ is a s$\mathcal{I}$sg-closed set thus $(\mho, \mathcal{H})$ is a s$\mathcal{I}$sg-open soft set .

The converse of Remark 3.3 is not hold. See Example 3. 2

i. Let $(\mathcal{P}, \mathcal{H}) = \{(\hbar_1, \{1\}), (\hbar_2, \{1\})\}$ is a s$\mathcal{I}$sg-closed set, but $(\mathcal{P}, \mathcal{H})$ is not closed soft set.

ii. Let $(\mathcal{P}, \mathcal{H}) = \{(\hbar_1, \{2\}), (\hbar_2, \{2\})\}$ is a s$\mathcal{I}$sg-open set, but $(\mathcal{P}, \mathcal{H}) \notin \boldsymbol{\mathcal{T}}$.

4. Separation Axioms with soft-$\mathcal{I}$- Semi-g-open Sets

Definition 4.1. A space $(\chi, \mathcal{T}, \mathcal{H}, \mathcal{I})$ is a soft-$\mathcal{I}$-semi-g-$\mathcal{T}_0$-space (briefly $s\mathcal{I}sg$-$\mathcal{T}_0$-space) , if for each $\hbar_{\mathcal{M}} \neq \hbar_{\mathcal{N}}$ and $\hbar_{\mathcal{M}}, \hbar_{\mathcal{N}} \tilde{\in} \tilde{\chi}$, $\exists$ $(\mathfrak{D}, \mathcal{H}) \in$ s$\mathcal{I}$sg-o$(\chi)_{\mathcal{H}}$ whenever, $\hbar_{\mathcal{M}} \tilde{\in} (\mathfrak{D}, \mathcal{H})$, $\hbar_{\mathcal{N}} \tilde{\notin} (\mathfrak{D}, \mathcal{H})$ or $\hbar_{\mathcal{M}} \tilde{\notin} (\mathfrak{D}, \mathcal{H})$, $\hbar_{\mathcal{N}} \tilde{\in} (\mathfrak{D}, \mathcal{H})$.

Example 4.2. In $(\chi, \mathcal{T}, \mathcal{H}, \mathcal{I})$ Let $\chi = \{1,2,3\}$, $\mathcal{H} = \{\hbar_1, \hbar_2\}$, $\mathcal{T} = \{\widetilde{\emptyset}, \tilde{\chi}, (\boldsymbol{\mathcal{P}}, \mathcal{H}), (\omega, \mathcal{H})\}$ where, $(\boldsymbol{\mathcal{P}}, \mathcal{H}) = \{(\hbar_1, \{1\}), (\hbar_2, \{1\})\}$, $(\omega, \mathcal{H}) = \{(\hbar_1, \{1,2\}), (\hbar_2, \{1,2\})$ and $\mathcal{I} = \{\widetilde{\emptyset}\}$**.** Then $\c{S}\c{S}O(\chi)_{\mathcal{H}} = \{(\Gamma, \mathcal{H}) ; 1 \tilde{\in} (\Gamma, \mathcal{H})\}$. So, $s\mathcal{I}sg\text{-}c(\chi)_{\mathcal{H}} = \{\widetilde{\emptyset}, \tilde{\chi}, (\boldsymbol{\mathcal{P}}', \mathcal{H}), (\omega', \mathcal{H})\}$ and $s\mathcal{I}sg\text{-}o(\chi)_{\mathcal{H}} = \mathcal{T}$, hence $(\chi, \mathcal{T}, \mathcal{H}, \mathcal{I})$ is a $s\mathcal{I}sg$-$\mathcal{T}_0$-space. Since $\forall$ $\hbar_{\mathcal{M}} \neq \hbar$, $\exists$ $(\mathfrak{D}, \mathcal{H}) \in$ s$\mathcal{I}$sg-o$(\chi)_{\mathcal{H}}$ whenever, $\hbar_{\mathcal{M}} \tilde{\in} (\mathfrak{D}, \mathcal{H})$, $\hbar_{\mathcal{N}} \tilde{\notin} (\mathfrak{D}, \mathcal{H})$ or $\hbar_{\mathcal{M}} \tilde{\notin} (\mathfrak{D}, \mathcal{H})$, $\hbar_{\mathcal{N}} \tilde{\in} (\mathfrak{D}, \mathcal{H})$.

Proposition 4.3. If $(\chi, \mathcal{T}, \mathcal{H})$ is a soft-$\mathcal{T}_0$-space then $(\chi, \mathcal{T}, \mathcal{H}, \mathcal{I})$ is a $s\mathcal{I}sg$-$\mathcal{T}_0$-space.

Proof : Let $\hbar_{\mathcal{M}}, \hbar_{\mathcal{N}} \tilde{\in} \tilde{\chi}$ such that $\hbar_{\mathcal{M}} \neq \hbar_{\mathcal{N}}$ since $(\chi, \mathcal{T}, \mathcal{H})$ is a soft-$\mathcal{T}_0$-space, then $\exists$ $(\mathfrak{D}, \mathcal{H}) \in \mathcal{T}$ whenever, $\hbar_{\mathcal{M}} \tilde{\in} (\mathfrak{D}, \mathcal{H})$, $\hbar_{\mathcal{N}} \tilde{\notin} (\mathfrak{D}, \mathcal{H})$ or $\hbar_{\mathcal{M}} \tilde{\notin} (\mathfrak{D}, \mathcal{H})$, $\hbar_{\mathcal{N}} \tilde{\in} (\mathfrak{D}, \mathcal{H})$. By Remark 2.3, $(\mathfrak{D}, \mathcal{H})$ is a $s\mathcal{I}sg$-open set such that $\hbar_{\mathcal{M}} \tilde{\in} (\mathfrak{D}, \mathcal{H})$ and $\hbar_{\mathcal{N}} \tilde{\notin} (\mathfrak{D}, \mathcal{H})$ or $\hbar_{\mathcal{M}} \tilde{\notin} (\mathfrak{D}, \mathcal{H})$ and $\hbar_{\mathcal{N}} \tilde{\in} (\mathfrak{D}, \mathcal{H})$.

Theorem 4.4 $(\chi, \mathcal{T}, \mathcal{H}, \mathcal{I})$ is a $s\mathcal{I}sg$-$\mathcal{T}_0$-space if and only if for each $\hbar_{\mathcal{M}} \neq \hbar_{\mathcal{N}}$ there is a $s\mathcal{I}sg$-closed set $(V, \mathcal{H})$ such that $\hbar_{\mathcal{M}} \tilde{\in} (V, \mathcal{H})$, $\hbar_{\mathcal{N}} \tilde{\notin} (V, \mathcal{H})$ or $\hbar_{\mathcal{M}} \tilde{\notin} (V, \mathcal{H})$, $\hbar$ $\tilde{\in} (V, \mathcal{H})$.

Proof :($\Rightarrow$) Let $\hbar_{\mathcal{M}}, \hbar_{\mathcal{N}} \tilde{\in} \tilde{\chi}$ such that $\hbar_{\mathcal{M}} \neq \hbar_{\mathcal{N}}$ since χ is a $s\mathcal{I}sg$-$\mathcal{T}_0$-space, then $\exists$ $(\mathfrak{D}, \mathcal{H}) \in$ s$\mathcal{I}$sg-o$(\chi)_{\mathcal{H}}$ whenever, $\hbar_{\mathcal{M}} \tilde{\in} (\mathfrak{D}, \mathcal{H})$ and $\hbar_{\mathcal{N}} \tilde{\notin} ((\mathfrak{D}, \mathcal{H})$ or $\hbar_{\mathcal{M}} \tilde{\notin}$

$(\mathfrak{O},\mathcal{H})$ and $\hbar_{\mathcal{N}} \tilde{\in} (\mathfrak{O},\mathcal{H})$, then $\exists$ $(\mathrm{V},\mathcal{H}) \in s\mathcal{I}sg\text{-c}(\chi)_{\mathcal{H}}$ whenever, $\hbar_{\mathcal{M}} \tilde{\in} (\mathrm{V},\mathcal{H})$ and $\hbar_{\mathcal{N}} \tilde{\notin} (\mathrm{V},\mathcal{H})$ or $\hbar_{\mathcal{M}} \tilde{\notin} (\mathrm{V},\mathcal{H})$, $\hbar_{\mathcal{N}} \tilde{\in} (\mho,\mathcal{H})$ where, $(\tilde{\chi} - (\mho,\mathcal{H})) = (\mathrm{V},\mathcal{H})$.

$(\Leftarrow)$ Let $\hbar$, $\hbar_{\mathcal{N}} \tilde{\in} \tilde{\chi}$ such that $\hbar_{\mathcal{M}} \neq \hbar_{\mathcal{N}}$ and there is a $s\mathcal{I}sg$-closed set $(\mathrm{V},\mathcal{H})$ such that $\hbar_{\mathcal{M}} \tilde{\in} (\mathrm{V},\mathcal{H})$, $\hbar_{\mathcal{N}} \tilde{\notin} (\mathrm{V},\mathcal{H})$ or $\hbar_{\mathcal{M}} \tilde{\notin} (\mathrm{V},\mathcal{H})$, $\hbar_{\mathcal{N}} \tilde{\in} (\mathfrak{O},\mathcal{H})$. Then there is $s\mathcal{I}sg$-open set $(\tilde{\chi} - (\mathrm{V},\mathcal{H})) = (\mathfrak{O},\mathcal{H})$ such that $\hbar_{\mathcal{M}} \tilde{\in} (\mathfrak{O},\mathcal{H})$, $\hbar_{\mathcal{N}} \tilde{\notin} (\mathfrak{O},\mathcal{H})$ or $\hbar_{\mathcal{M}} \tilde{\notin} (\mathfrak{O},\mathcal{H})$, $\hbar_{\mathcal{N}} \tilde{\in} (\mathfrak{O},\mathcal{H})$.

Definition 4.5. $(\chi,\mathcal{T},\mathcal{H},\mathcal{I})$ is a soft-$\mathcal{I}$-semi-g-$\mathcal{T}_1$-space (briefly $s\mathcal{I}sg$-$\mathcal{T}_1$-space),If for each $\hbar_{\mathcal{M}}$, $\hbar_{\mathcal{N}} \tilde{\in} \tilde{\chi}$ and $\hbar_{\mathcal{M}} \neq \hbar_{\mathcal{N}}$. Then there are $s\mathcal{I}sg$-open sets $(\mho_1,\mathcal{H})$, $(\mho_2,\mathcal{H})$ whenever, $\hbar_{\mathcal{M}} \tilde{\in} ((\mho_1,\mathcal{H}) - (\mho_2,\mathcal{H}))$ and $\hbar_{\mathcal{N}} \tilde{\in} ((\mho_2,\mathcal{H}) - (\mho_1,\mathcal{H}))$.

Example 4.6. A space $(\chi,\mathcal{T},\mathcal{H},\mathcal{I})$ when $\chi = \mathcal{H} = \mathbb{N}$ the set of all natural number $\mathcal{T} = \mathcal{T}_{\text{Scof}} = \{ \Gamma_{\mathcal{A}} : \Gamma'(\boldsymbol{\hbar})$ is finite set $\forall\, \boldsymbol{\hbar} \} \tilde{\cup} \{\tilde{\emptyset}\}$ and $\mathcal{I} = \{\tilde{\emptyset}\}$. So, $(\chi,\mathcal{T},\mathcal{H},\mathcal{I})$ is a $s\mathcal{I}sg$-$\mathcal{T}_1$-space. If for each $\hbar$, $\hbar_{\mathcal{N}} \tilde{\in} \tilde{\chi}$ and $\hbar_{\mathcal{M}} \neq \hbar_{\mathcal{N}}$. Then there are $s\mathcal{I}sg$-open sets $(\tilde{\chi} - \mathcal{U})$, $(\tilde{\chi} - \mathcal{V})$ such that $\mathcal{U} \subseteq \hbar_{\mathcal{M}}$, $\mathcal{V} \subseteq \hbar_{\mathcal{N}}$ and $\mathcal{U}$, $\mathcal{V}$ are two finite sets whenever, $\hbar_{\mathcal{M}} \tilde{\in} (\tilde{\chi} - \mathcal{V})$, $\hbar_{\mathcal{N}} \notin (\tilde{\chi} - \mathcal{V})$ and $\hbar_{\mathcal{M}} \notin (\tilde{\chi} - \mathcal{U})$, $\hbar_{\mathcal{N}} \tilde{\in} (\tilde{\chi} - \mathcal{U})$ and $(\tilde{\chi} - \mathcal{V}) \cap (\tilde{\chi} - \mathcal{U}) \neq \{\emptyset\}$.

Proposition 4.7. If $(\chi,\mathcal{T},\mathcal{H})$ is a soft-$\mathcal{T}_1$-space then $(\chi,\mathcal{T},\mathcal{H},\mathcal{I})$ is a soft-$\mathcal{I}$-semi-g-$\mathcal{T}_1$-space.

Proof : Let $\hbar_{\mathcal{M}}$, $\hbar_{\mathcal{N}} \tilde{\in} \tilde{\chi}$ such that $\hbar_{\mathcal{M}} \neq \hbar_{\mathcal{N}}$ since $(\chi,\mathcal{T},\mathcal{H})$ is a soft-$\mathcal{T}_1$-space, then $\exists$ $(\mho_1,\mathcal{H})$, $(\mho_2,\mathcal{H}) \in \mathcal{T}$such that $\hbar_{\mathcal{M}} \tilde{\in} ((\mho_1,\mathcal{H}) - (\mho_2,\mathcal{H}))$ and $\hbar_{\mathcal{N}} \tilde{\in} ((\mho_2,\mathcal{H}) - (\mho_1,\mathcal{H}))$. By Remark 3.3, $(\mho_1,\mathcal{H})$ and $(\mho_2,\mathcal{H})$ are $s\mathcal{I}sg$-open sets , and the proof is over .

Proposition 4.8. If $(\chi,\mathcal{T},\mathcal{H},\mathcal{I})$ is a $s\mathcal{I}sg$-$\mathcal{T}_1$-space then it is a $s\mathcal{I}sg$-$\mathcal{T}_0$-*space*.

Proof : Let $\hbar_{\mathcal{M}}$, $\hbar_{\mathcal{N}} \tilde{\in} \tilde{\chi}$ such that $\hbar_{\mathcal{M}} \neq \hbar_{\mathcal{N}}$ since $(\chi,\mathcal{T},\mathcal{H},\mathcal{I})$ is a $s\mathcal{I}sg$-$\mathcal{T}_1$-*space*, then $\exists$ $(\mho_1,\mathcal{H})$, $(\mho_2,\mathcal{H}) \in s\mathcal{I}sg\text{-o}(\chi)_{\mathcal{H}}$ such that, $\hbar_{\mathcal{M}} \tilde{\in} ((\mho_1,\mathcal{H}) - (\mho_2,\mathcal{H}))$ and $\hbar_{\mathcal{N}} \tilde{\in} ((\mho_2,\mathcal{H}) - (\mho_1,\mathcal{H}))$. Then $\exists$ $(\mho,\mathcal{H}) \in s\mathcal{I}sg\text{-o}(\chi)_{\mathcal{H}}$ -open set whenever, $\hbar_{\mathcal{M}} \tilde{\in} (\mho,\mathcal{H})$, $\hbar_{\mathcal{N}} \tilde{\notin} (\mho,\mathcal{H})$ or $\hbar_{\mathcal{M}} \tilde{\notin} (\mho,\mathcal{H})$, $\hbar_{\mathcal{N}} \tilde{\in} (\mho,\mathcal{H})$.

The conclusions in proposition 4.8, is not reversible by example 4.2. $(\chi,\mathcal{T},\mathcal{H},\mathcal{I})$ is a $s\mathcal{I}sg$-$\mathcal{T}_0$-space, but is not $s\mathcal{I}sg$-$\mathcal{T}_1$-space. Since $\exists$ $\hbar_{\mathcal{M}} \neq \hbar_{\mathcal{N}}$; $\hbar_{\mathcal{M}} = \{1,2\}$ and $\hbar_{\mathcal{N}} = \{3\}$ there is no $(\mathcal{U},\mathcal{H})$ and $(\mathcal{V},\mathcal{H})$ such that $\hbar_{\mathcal{M}} \tilde{\in} (\mathcal{U},\mathcal{H})$, $\hbar_{\mathcal{N}} \tilde{\notin} (\mathcal{U},\mathcal{H})$ and $\hbar_{\mathcal{N}} \tilde{\in} (\mathcal{V},\mathcal{H})$, $\hbar_{\mathcal{M}} \tilde{\notin} (\mathcal{V},\mathcal{H})$.

Theorem 4.9. A space $(\chi,\mathcal{T},\mathcal{H},\mathcal{I})$ is a $s\mathcal{I}sg$-$\mathcal{T}_1$-space if and only if for each $\hbar_{\mathcal{M}}$, $\hbar_{\mathcal{N}} \tilde{\in} \tilde{\chi}$ and $\hbar_{\mathcal{M}} \neq \hbar_{\mathcal{N}}$ there are two $s\mathcal{I}sg$-closed sets $(\mathcal{V}_1,\mathcal{H})$, $(\mathcal{V}_2,\mathcal{H})$ such that $\hbar_{\mathcal{M}} \tilde{\in} ((\mathcal{V}_1,\mathcal{H}) \cap (\mathcal{V}_2{}',\mathcal{H}))$ and $\hbar_{\mathcal{N}} \tilde{\in} ((\mathcal{V}_2,\mathcal{H}) \cap (\mathcal{V}_1{}',\mathcal{H}))$.

Proof :

$(\Rightarrow)$ Let $\hbar$, $\hbar_{\mathcal{N}} \tilde{\in} \tilde{\chi}$ such that $\hbar_{\mathcal{M}} \neq \hbar_{\mathcal{N}}$ since $(\chi,\mathcal{T},\mathcal{H},\mathcal{I})$ is a soft-$\mathcal{T}_1$-*space*, then $\exists$ $(\mho_1,\mathcal{H})$, $(\mho_2,\mathcal{H}) \in s\mathcal{I}sg\text{-}o(\chi)_{\mathcal{H}}$ whenever, $\hbar_{\mathcal{M}} \tilde{\in} ((\mho_1,\mathcal{H}) - (\mho_2,\mathcal{H}))$ and $\hbar_{\mathcal{N}} \tilde{\in} ((\mho_2,\mathcal{H}) - (\mho_1,\mathcal{H}))$. Then there is a $s\mathcal{I}sg$-closed sets $(\mathcal{V}_1,\mathcal{H})$, $(\mathcal{V}_2,\mathcal{H})$ whenever, $\hbar_{\mathcal{M}} \tilde{\in} ((\mathcal{V}_1,\mathcal{H}) - (\mathcal{V}_2,\mathcal{H}))$ and $\hbar_{\mathcal{N}} \tilde{\in} ((\mathcal{V}_2,\mathcal{H}) - (\mathcal{V}_1,\mathcal{H}))$ where, $(\tilde{\chi} - (\mho_2,\mathcal{H})) = (\mathcal{V}_2,\mathcal{H})$ and $(\tilde{\chi} - (\mho_1,\mathcal{H}))$

= $(\mathcal{V}_1, \mathcal{H})$. Then there are two $s\mathcal{I}sg$-closed sets $(\mathcal{V}_1, \mathcal{H})$, $(\mathcal{V}_2, \mathcal{H})$ such that $\hbar_{\mathcal{M}} \tilde{\in} ((\mathcal{V}_1, \mathcal{H}) \cap (\mathcal{V}_2', \mathcal{H}))$ and $\hbar_{\mathcal{N}} \tilde{\in} ((\mathcal{V}_2, \mathcal{H}) \cap (\mathcal{V}_1', \mathcal{H}))$.

($\Leftarrow$) Let $\hbar_{\mathcal{M}}$, $\hbar_{\mathcal{N}} \tilde{\in} \tilde{\chi}$ such that $\hbar_{\mathcal{M}} \neq \hbar_{\mathcal{N}}$ and there are two $s\mathcal{I}sg$-closed sets $(\mathcal{V}_1, \mathcal{H})$, $(\mathcal{V}_2, \mathcal{H})$ such that $\hbar_{\mathcal{M}} \tilde{\in} ((\mathcal{V}_1, \mathcal{H}) \tilde{\cap} (\mathcal{V}_2', \mathcal{H}))$ and $\hbar_{\mathcal{N}} \tilde{\in} ((\mathcal{V}_2, \mathcal{H}) \tilde{\cap} (\mathcal{V}_1', \mathcal{H}))$ .Then there are $s\mathcal{I}sg$-open sets $(\mho_1,\mathcal{H})$, $(\mho_2,\mathcal{H})$ whenever, $\hbar_{\mathcal{M}} \tilde{\in} ((\mho_1,\mathcal{H}) - (\mho_2,\mathcal{H}))$ and $\hbar_{\mathcal{N}} \tilde{\in} ((\mho_2,\mathcal{H}) - (\mho_1,\mathcal{H}))$ where, $(\tilde{\chi} - (\mathcal{V}_2, \mathcal{H})) = (\mho_2,\mathcal{H})$ and $(\tilde{\chi} - (\mathcal{V}_1, \mathcal{H})) = (\mho_1,\mathcal{H})$.

Definition 4.10. $(\chi, \mathcal{T}, \mathcal{H}, \mathcal{I})$ is a soft-$\mathcal{I}$-semi-g-$\mathcal{T}_2$-space (briefly $s\mathcal{I}sg$-$\mathcal{T}_2$-space).If for any $\hbar_{\mathcal{M}} \neq \hbar_{\mathcal{N}}$ there are $s\mathcal{I}sg$-open sets $(\mathfrak{O}_1,\mathcal{H})$, $(\mathfrak{O}_2,\mathcal{H})$ such that $\hbar_{\mathcal{M}} \tilde{\in} (\mathfrak{O}_1,\mathcal{H})$, $\hbar_{\mathcal{N}} \tilde{\in} (\mathfrak{O}_2,\mathcal{H})$ and $(\mathfrak{O}_1,\mathcal{H}) \cap (\mathfrak{O}_2,\mathcal{H}) = \{\tilde{\emptyset}\}$.

Example 4.11. A space $(\chi, \mathcal{T}, \mathcal{H}, \mathcal{I})$; $\chi = \{1 ,2 ,3\}$, $\mathcal{T} = \{\tilde{\emptyset}, \tilde{\chi}\}$ and $\mathcal{I} = ȘȘ(\chi)_{\mathcal{H}}$ .Then $ȘȘO(\chi)_{\mathcal{H}} = \mathcal{T}$. So, $s\mathcal{I}sg\text{-}c(\chi)_{\mathcal{H}} = s\mathcal{I}sg\text{-}o(\chi)_{\mathcal{H}} = ȘȘ(\chi)_{\mathcal{H}}$. Then $(\chi, \mathcal{T}, \mathcal{H}, \mathcal{I})$ is a $s\mathcal{I}sg$-$\mathcal{T}_2$-space.

Remark 4.12. If $(\chi, \mathcal{T}, \mathcal{H})$ is a soft-$\mathcal{T}_2$-$space$, then $(\chi, \mathcal{T}, \mathcal{H}, \mathcal{I})$ is a $s\mathcal{I}sg$-$\mathcal{T}_2$-$space$.
Proof : Let $\hbar_{\mathcal{M}}$, $\hbar_{\mathcal{N}} \tilde{\in} \tilde{\chi}$ whenever, $\hbar_{\mathcal{M}} \neq \hbar_{\mathcal{N}}$ since $(\chi, \mathcal{T}, \mathcal{H}, \mathcal{I})$ is a soft-$\mathcal{T}_2$-$space$, then $\exists$ $(\mathfrak{O}_1,\mathcal{H}),(\mathfrak{O}_2,\mathcal{H}) \in \mathcal{T}$ such that $\hbar_{\mathcal{M}} \tilde{\in} (\mathfrak{O}_1,\mathcal{H})$, $\hbar_{\mathcal{N}} \tilde{\in} (\mathfrak{O}_2,\mathcal{H})$ and $(\mathfrak{O}_1,\mathcal{H}) \tilde{\cap} (\mathfrak{O}_2,\mathcal{H}) = \{\tilde{\emptyset}\}$, by remark 3.3, there are $s\mathcal{I}sg$-open sets $(\mathfrak{O}_1,\mathcal{H}),(\mho_2,\mathcal{H})$, such that $\hbar_{\mathcal{M}} \tilde{\in} (\mathfrak{O}_1,\mathcal{H})$, $\hbar_{\mathcal{N}} \tilde{\in} (\mathfrak{O}_2,\mathcal{H})$ and $(\mathfrak{O}_1,\mathcal{H}) \tilde{\cap} (\mathfrak{O}_2,\mathcal{H}) = \{\tilde{\emptyset}\}$.

Remark 4.13. If $(\chi, \mathcal{T}, \mathcal{H}, \mathcal{I})$ is a $s\mathcal{I}sg$-$\mathcal{T}_2$-$space$ then it is a $s\mathcal{I}sg$-$\mathcal{T}_1$-$space$.
Proof : Let $\hbar_{\mathcal{M}}$, $\hbar_{\mathcal{N}} \tilde{\in} \tilde{\chi}$ whenever, $\hbar_{\mathcal{M}} \neq \hbar_{\mathcal{N}}$ since $(\chi, \mathcal{T}, \mathcal{H}, \mathcal{I})$ is a $s\mathcal{I}sg$-$\mathcal{T}_2$-$space$,then there are $s\mathcal{I}sg$-open sets $(\mho_1,\mathcal{H})$, $(\mho_2,\mathcal{H})$ such that $\hbar_{\mathcal{M}} \tilde{\in} (\mho_1,\mathcal{H})$, $\hbar_{\mathcal{N}} \tilde{\in} (\mho_2,\mathcal{H})$ and $(\mho_1,\mathcal{H}) \cap (\mho_2,\mathcal{H}) = \{ \tilde{\emptyset} \}$. Implies, $\hbar_{\mathcal{M}} \tilde{\in} ((\mho_1,\mathcal{H}) - (\mho_2,\mathcal{H}))$ and $\hbar_{\mathcal{N}} \tilde{\in} ((\mho_2,\mathcal{H}) - (\mho_1,\mathcal{H}))$.

The conclusions in Remark 4.13, is not reversible by example 3.6.

A space $(\chi, \mathcal{T}, \mathcal{H}, \mathcal{I})$ is a $s\mathcal{I}sg$-$\mathcal{T}_1$-space. If for each $\hbar$, $\hbar_{\mathcal{N}} \tilde{\in} \tilde{\chi}$ and $\hbar_{\mathcal{M}} \neq \hbar_{\mathcal{N}}$. Then there are $s\mathcal{I}sg$-open sets $(\tilde{\chi} - \mathcal{U})$, $(\tilde{\chi} - \mathcal{V})$ whenever, , $\hbar_{\mathcal{M}} \tilde{\in} (\tilde{\chi} - \mathcal{V})$, $\hbar_{\mathcal{N}} \notin (\tilde{\chi} - \mathcal{V})$ and $\hbar_{\mathcal{M}} \notin (\tilde{\chi} - \mathcal{U})$, $\hbar_{\mathcal{N}} \tilde{\in} (\tilde{\chi} - \mathcal{U})$ and $(\tilde{\chi} - \mathcal{V}) \cap (\tilde{\chi} - \mathcal{U}) \neq \{\emptyset\}$.Which is not $s\mathcal{I}sg$- $\mathcal{T}_2$-space. Since for any two $s\mathcal{I}sg$-open sets $(\mho_1,\mathcal{H})$, $(\mho_2,\mathcal{H})$ such that $\hbar_{\mathcal{M}} \tilde{\in} (\mho_1,\mathcal{H})$, $\hbar_{\mathcal{N}} \tilde{\in} (\mho_2,\mathcal{H})$ then $(\mho_1,\mathcal{H}) \cap (\mho_2,\mathcal{H}) \neq \tilde{\emptyset}$. We have previously noted that χ is a $s\mathcal{I}sg$- $\mathcal{T}_i$-$space$ whenever it is a $\mathcal{T}_{i+1}$-$space$ ($\forall\ i = 0 , 1\ and\ 2$).

The opposite is not generally achieved by example below.

Example 4.14. $(\chi, \mathcal{T}, \mathcal{H}, \mathcal{I})$ is a $s\mathcal{I}sg$-$\mathcal{T}_i$-$space$ ($i \in \{0,1,2\}$) , where, $\chi = \{1,2,3\}$, $\mathcal{T} = \{\tilde{\emptyset}, \tilde{\chi}\}$ and $\mathcal{I} = ȘȘ(\chi)_{\mathcal{H}}$. since, $s\mathcal{I}sg\text{-}c(\chi)_{\mathcal{H}} = s\mathcal{I}sg\text{-}o(\chi)_{\mathcal{H}} = ȘȘ(\chi)_{\mathcal{H}}$. But the space $(\chi, \mathcal{T}, \mathcal{H})$ is not soft- $\mathcal{T}_i$-$space$ ($i \in \{0,1,2\}$).

The following chart shows the relationships among the various types of notions of our previously mentioned.

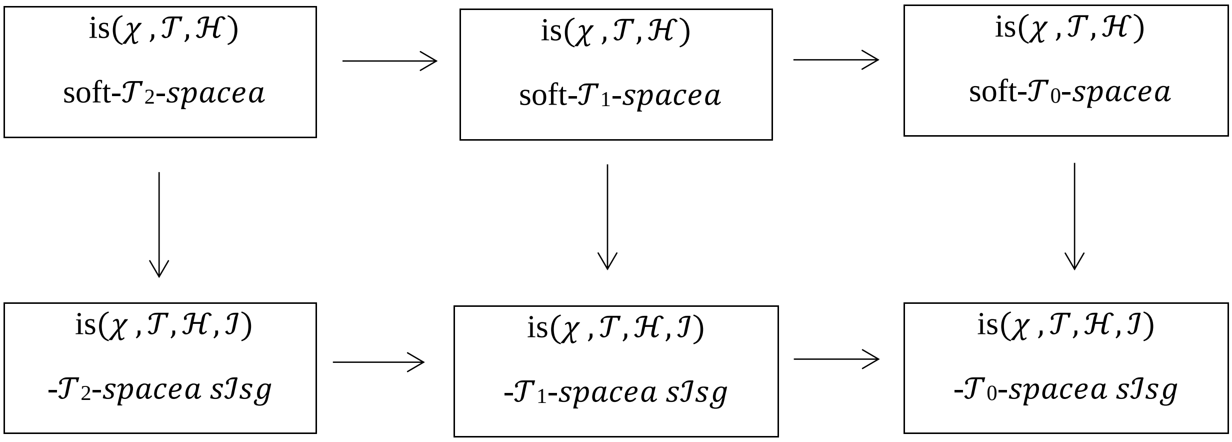


Figure 1: soft- $\mathcal{T}_i$-$space$

5. Games in soft ideal topological spaces

In this section, a new game by linking them with soft separation axioms via open (respectively, s$\mathcal{J}$sg-open) sets was inserted.

Definition 5.1. For a soft ideal space $(\chi,\mathcal{T},\mathcal{H},\mathcal{J})$, determane a game $Ş\mathcal{G}(\mathcal{T}_0,\chi)$ (respectively $Ş\mathcal{G}(\mathcal{T}_0,\mathcal{J})$) as follows:
Player I and Player II are play an inning for each positive integer numbers in the r-th inning:
The first step, Player I Choose $(\hbar_{\mathcal{M}})_r \neq (\hbar_{\mathcal{N}})_r$ where, $(\hbar_{\mathcal{M}})_r$, $(\hbar_{\mathcal{N}})_r \tilde{\in} \tilde{\chi}$.

In the second step, Player II Chooses $\mathcal{B}_r$ a soft open (respectively s$\mathcal{J}$sg-open set) containing only one of the two elements $(\hbar_{\mathcal{M}})_r$, $(\hbar_{\mathcal{N}})_r$.
Then Player II wins in the soft game $Ş\mathcal{G}(\mathcal{T}_0,\chi)$ (respectively $S\mathcal{G}(\mathcal{T}_0,\mathcal{J})$ if $\mathcal{B} = \{ \mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3, \ldots \mathcal{B}_r, \ldots\}$ be a collection of a soft open set (respectively, s$\mathcal{J}$sg-open) set in χ such that ∀ , $(\hbar_{\mathcal{M}})_r$, $(\hbar_{\mathcal{N}})_r \tilde{\in} \chi$, $\exists\, \mathcal{B}_r \in \mathcal{B}$ containing only one of two element $(\hbar_{\mathcal{M}})_r$, $(\hbar_{\mathcal{N}})_r$.
Otherwise, Player I wins.

Example 5.2. Let $Ş\mathcal{G}(\mathcal{T}_0,\chi)$ (respectively $Ş\mathcal{G}(\mathcal{T}_0,\mathcal{J})$) be a soft game where, $\chi = \{1,2,3\}$, $\mathcal{H} = \{ \hbar_1, \hbar_2\}$, $\mathcal{T} = \{\tilde{\emptyset}, \tilde{\chi}, (\mathcal{P},\mathcal{H}), (\omega,\mathcal{H}), (\mathcal{Z},\mathcal{H})\}$ where, $(\mathcal{P},\mathcal{H}) = \{(\hbar_1,\{1\}),(\hbar_2,\{1\})\}$, $(\omega,\mathcal{H}) = \{(\hbar_1,\{3\}),(\hbar_2,\{3\})\}$, $(\mathcal{Z},\mathcal{H}) = \{(\hbar_1,\{1,3\}),(\hbar_2,\{1,3\})\}$ and $\mathcal{J} = \{\tilde{\emptyset}\}$. Then $ŞŞO(\chi) = \{ \{1\} \tilde{\in} (\Gamma,\mathcal{H})$ and $\{3\} \tilde{\notin} \Gamma(\hbar)\ \forall \hbar$, $\{3\} \tilde{\in} (\Gamma,\mathcal{H})$ and $\{1\} \tilde{\notin} \Gamma(\hbar)\ \forall \hbar$, $\{1,3\} \tilde{\in} (\Gamma,\mathcal{H})\} \cup \{\tilde{\emptyset}\}$, then $s\mathcal{J}sgc(\chi)_{\mathcal{H}} = SC(\chi)_{\mathcal{H}}$ and $s\mathcal{J}sgo(\chi)_{\mathcal{H}} = \mathcal{T}$.

Then in the first inning:
The first step, Player I Choose $\hbar_{\mathcal{M}} \neq \hbar_{\mathcal{N}}$ where, $\hbar$, $\hbar_{\mathcal{N}} \tilde{\in} \tilde{\chi}$ such that $\hbar_{\mathcal{M}} = \{1\}$ and $\hbar_{\mathcal{N}} = \{2\}$.
In the second step, Player II Choose $(\mathcal{P},\mathcal{H}) = \{(\hbar_1,\{1\}),(\hbar_2,\{1\})\}$ a soft open (respectively, s$\mathcal{J}$sg-open set)).

In the second inning:
The first step, Player I Chooses $\hbar_{\mathcal{M}} \neq \hbar_{O}$ where, $\hbar_{\mathcal{M}}$, $\hbar_{O} \tilde{\in} \tilde{\chi}$ such that $\hbar_{\mathcal{M}} = \{1\}$ and $\hbar_{O} = \{3\}$.
In the second step, Player II Choose $(\omega,\mathcal{H}) = \{(\hbar_1,\{3\}),(\hbar_2,\{3\})\}$ which is a soft open (Respectively, s$\mathcal{J}$sg-open set).

In the third inning: The first step, Player I choose $\hbar_{\mathcal{N}} \neq \hbar_{O}$ where, $\hbar$, $\hbar_{O} \tilde{\in} \tilde{\chi}$ such that $\hbar_{\mathcal{N}} = \{2\}$ and $\hbar_{O} = \{3\}$.
In the second step, Player II Choose $(\omega, \mathcal{H})$ = {($\hbar_1$,{3}),($\hbar_2$,{3})} which is a soft open (respectively, s$\mathcal{I}$sg-open set)).

In the fourth inning: The first step, Player I Choose $\hbar_{\mathcal{M}} \neq \hbar_{\mathcal{R}}$ where, $\hbar$, $\hbar_{\mathcal{R}} \tilde{\in} \tilde{\chi}$ such that $\hbar_{\mathcal{M}} = \{1\}$ and $\hbar_{\mathcal{R}} = \{2,3\}$.
In the second step, Player II Choose $(\mathcal{P}, \mathcal{H})$ = {($\hbar_1$,{1}),($\hbar_2$,{1})} which is a soft open (respectively, s$\mathcal{I}$sg-open set)).

In the fifth inning: The first step, Player I Choose $\hbar_{\mathcal{N}} \neq \hbar_{\mathcal{S}}$ where, $\hbar$, $\hbar_{\mathcal{S}} \tilde{\in} \tilde{\chi}$ such that $\hbar_{\mathcal{N}} = \{2\}$ and $\hbar_{\mathcal{S}} = \{1,3\}$.
In the second step, Player II Choose $(\mathcal{Z}, \mathcal{H})$ = {($\hbar_1$,{1,3}),($\hbar_2$,{1,3})} which is a soft open (respectively, s$\mathcal{I}$sg-open set)).

In the sixth inning: The first step, Player I Choose $\hbar_{O} \neq \hbar_{\mathcal{L}}$ where, $\hbar$, $\hbar_{\mathcal{L}} \tilde{\in} \tilde{\chi}$ such that $\hbar_{O} = \{3\}$ and $\hbar_{\mathcal{L}} = \{1,2\}$.
In the second step, Player II Choose $(\omega, \mathcal{H})$ = {($\hbar_1$,{3}),($\hbar_2$,{3})} which is a soft open (respectively, s$\mathcal{I}$sg-open set)).

Then $\mathcal{B}$ = {,$(\mathcal{P}, \mathcal{H})$,$(\omega, \mathcal{H})$,$(\mathcal{Z}, \mathcal{H})$} is the winning strategy for Player II in $\c{S}\mathcal{G}(\mathcal{T}_0, \chi)$ (respectively $\c{S}\mathcal{G}(\mathcal{T}_0, \mathcal{I})$). Hence Player $II \uparrow \c{S}\mathcal{G}(\mathcal{T}_0, \chi)$ (respectively $\c{S}\mathcal{G}(\mathcal{T}_0, \mathcal{I})$).

Example 5.3. Let $\c{S}\mathcal{G}(\mathcal{T}_0, \chi)$ (respectively $\c{S}\mathcal{G}(\mathcal{T}_0, \mathcal{I})$) is a game where, $\chi = \{1,2,3\}$, $\mathcal{H} = \{\hbar_1, \hbar_2\}$, $\mathcal{T} = \{\tilde{\emptyset}, \tilde{\chi}, (\omega, \mathcal{H})\}$ where, $(\omega, \mathcal{H})$ = {($\hbar_1$,{3}),($\hbar_2$,{3})} and $\mathcal{I} = \{\tilde{\emptyset}\}$ then s$\mathcal{I}$sgc$(\chi)_{\mathcal{H}}$ = SC$(\chi)_{\mathcal{H}}$ and s$\mathcal{I}$sgo$(\chi)_{\mathcal{H}} = \mathcal{T}$.

In the first inning: The first step, Player I Choose $\hbar_{\mathcal{M}} \neq \hbar_{\mathcal{N}}$ where, $\hbar$, $\hbar_{\mathcal{N}} \tilde{\in} \tilde{\chi}$ since $\hbar_{\mathcal{M}} = \{1\}$ and $\hbar_{\mathcal{N}} = \{2\}$.

In the second step, Player II cannot find $(\mho, \mathcal{H})$ which is a soft open (Respectively, s$\mathcal{I}$sg-open set)) containing one of $\hbar_{\mathcal{M}}$, $\hbar_{\mathcal{N}}$. Hence Player $I \uparrow \c{S}\mathcal{G}(\mathcal{T}_0, \chi)$ (respectively $\c{S}\mathcal{G}(\mathcal{T}_0, \mathcal{I})$).

Remark 5.4. For a space $(\chi, \mathcal{T}, \mathcal{H}, \mathcal{I})$:

- **i.** If Player $II \uparrow \c{S}\mathcal{G}(\mathcal{T}_0, \chi)$ then Player $II \uparrow \c{S}\mathcal{G}(\mathcal{T}_0, \mathcal{I})$.
- **ii.** If Player $I \uparrow \c{S}\mathcal{G}(\mathcal{T}_0, \chi)$ then Player $I \uparrow \c{S}\mathcal{G}(\mathcal{T}_0, \mathcal{I})$.

Remark 5.5. For a space $(\chi, \mathcal{T}, \mathcal{H}, \mathcal{I})$, if Player $II \downarrow \c{S}\mathcal{G}(\mathcal{T}_0, \chi)$ then Player $II \downarrow \c{S}\mathcal{G}(\mathcal{T}_0, \mathcal{I})$.

Theorem 5.6. A space $(\chi, \mathcal{T}, \mathcal{H})$ (respectively $(\chi, \mathcal{T}, \mathcal{H}, \mathcal{I})$) is $\mathcal{T}_0$-space (respectively, s$\mathcal{I}$sg-$\mathcal{T}_0$-space) if and only if Player $II \uparrow \c{S}\mathcal{G}(\mathcal{T}_0, \chi)$ (respectively, $\c{S}\mathcal{G}(\mathcal{T}_0, \mathcal{I})$).

Proof: ($\Rightarrow$) in the r-th inning Player in $\c{S}\mathcal{G}(\mathcal{T}_0, \chi)$ (respectively, $\c{S}\mathcal{G}(\mathcal{T}_0, \mathcal{I})$) Choose $(\hbar_{\mathcal{M}})_r \neq (\hbar_{\mathcal{N}})_r$ where, $(\hbar_{\mathcal{M}})_r$, $(\hbar_{\mathcal{N}})_r \tilde{\in} \tilde{\chi}$, Player in II in $\c{S}\mathcal{G}(\mathcal{T}_0, \chi)$ (respectively, $\c{S}\mathcal{G}(\mathcal{T}_0, \mathcal{I})$) choose $(\mho_r, \mathcal{H})$ is a soft open (respectively, s$\mathcal{I}$sg-open set) containing only one of the two elements $(\hbar_{\mathcal{M}})_r$, $(\hbar_{\mathcal{N}})_r$. Since $(\chi, \mathcal{T}, \mathcal{H})$ is a soft $\mathcal{T}_0$-space (respectively, s$\mathcal{I}$sg-$\mathcal{T}_0$-space). Then if $\mathcal{B}$ = { $(\mho_1, \mathcal{H})$, $(\mho_2, \mathcal{H})$, $(\mho_3, \mathcal{H})$,…,$(\mho_r, \mathcal{H})$,...} is the winning strategy for Player in II in $\c{S}\mathcal{G}(\mathcal{T}_0, \chi)$ (respectively, $\c{S}\mathcal{G}(\mathcal{T}_0, \mathcal{I})$). Hence Player $II \uparrow \c{S}\mathcal{G}(\mathcal{T}_0, \chi)$ (respectively, $\c{S}\mathcal{G}(\mathcal{T}_0, \mathcal{I})$).
($\Leftarrow$) Clear.

Corollary 5.7. For a space $(\chi, \mathcal{T}, \mathcal{H})$:

i- Player $\mathcal{II} \uparrow \varsigma\mathcal{G}(\mathcal{T}_0, \chi)$ if and only if $\forall\, \hbar_{\mathcal{M}} \neq \hbar_{\mathcal{N}}$ where, $\hbar_{\mathcal{M}}, \hbar_{\mathcal{N}} \tilde{\in} \chi\ \exists\, (\mathcal{A}, \mathcal{H})$ is a $closed$ set where $\hbar_{\mathcal{M}} \tilde{\in} (\mathcal{A}, \mathcal{H})$ and $\hbar_{\mathcal{N}} \tilde{\notin} (\mathcal{A}, \mathcal{H})$.

ii- Player $\mathcal{II} \uparrow \varsigma\mathcal{G}(\mathcal{T}_0, \mathcal{I})$ if and only if $\forall\, \hbar_{\mathcal{M}} \neq \hbar_{\mathcal{N}}$ where, $\hbar_{\mathcal{M}}, \hbar_{\mathcal{N}} \tilde{\in} \chi\ \exists\, (\mathcal{B}, \mathcal{H})$ is a s$\mathcal{I}$sg-closed set where $\hbar_{\mathcal{M}} \tilde{\in} (\mathcal{B}, \mathcal{H})$ and $\hbar_{\mathcal{N}} \tilde{\notin} (\mathcal{B}, \mathcal{H})$.

Proof:

i. (⟹) Let $\hbar_{\mathcal{M}} \neq \hbar_{\mathcal{N}}$ where, $\hbar_{\mathcal{M}}, \hbar_{\mathcal{N}} \tilde{\in} \chi$. Since Player $\mathcal{II} \uparrow \varsigma\mathcal{G}(\mathcal{T}_0, \chi)$, then by Theorem 5.6, the space $(\chi, \mathcal{T}, \mathcal{H})$ is a soft- $\mathcal{T}_0$-space. Then Theorem 1.17, is applicable.
(⟸) By Theorem 2.17, the space $(\chi, \mathcal{T}, \mathcal{H})$ is a soft- $\mathcal{T}_0$-space. Then Theorem 4.6, is applicable.

ii. (⟹) Let $\hbar_{\mathcal{M}} \neq \hbar_{\mathcal{N}}$ where, $\hbar_{\mathcal{M}}, \hbar_{\mathcal{N}} \tilde{\in} \chi$. Since Player $\mathcal{II} \uparrow \varsigma\mathcal{G}(\mathcal{T}_0, \mathcal{I})$, then by Theorem 4.1.6, the space $(\chi, \mathcal{T}, \mathcal{H})$ is a s$\mathcal{I}$sg-$\mathcal{T}_0$-space. Then Theorem 4.4, is applicable.
(⟸) By Theorem 4.4, the space $(\chi, \mathcal{T}, \mathcal{H})$ is a s$\mathcal{I}$sg-$\mathcal{T}_0$-space. Then Theorem 4.6, is applicable.

Corollary 5.8.

i- A space $(\chi, \mathcal{T}, \mathcal{H})$ is a soft-$\mathcal{T}_0$-space if and only if Player $\mathcal{I} \not\uparrow \varsigma\mathcal{G}(\mathcal{T}_0, \chi)$.

ii- A space $(\chi, \mathcal{T}, \mathcal{H}, \mathcal{I})$ is a s$\mathcal{I}$sg-$\mathcal{T}_0$-space if and only if Player $\mathcal{I} \not\uparrow \varsigma\mathcal{G}(\mathcal{T}_0, \mathcal{I})$.

Proof: By Theorem 5.6, the proof is over.

Theorem 5.9. For a space $(\chi, \mathcal{T}, \mathcal{H}, \mathcal{I})$:

i- A space $(\chi, \mathcal{T}, \mathcal{H})$ is not soft-$\mathcal{T}_0$-space if and only if Player $\mathcal{I} \uparrow \varsigma\mathcal{G}(\mathcal{T}_0, \chi)$.

ii- A space $(\chi, \mathcal{T}, \mathcal{H}, \mathcal{I})$ is not s$\mathcal{I}$sg-$\mathcal{T}_0$-space if and only if Player $\mathcal{I} \uparrow \varsigma\mathcal{G}(\mathcal{T}_0, \mathcal{I})$.

Proof:

i- (⟹) in the r-th inning Player $\mathcal{I}$ in $\varsigma\mathcal{G}(\mathcal{T}_0, \chi)$ choose $(\hbar_{\mathcal{M}})_r \neq (\hbar_{\mathcal{N}})_r$ where, $(\hbar_{\mathcal{M}})_r$, $(\hbar_{\mathcal{N}})_r \tilde{\in} \tilde{\chi}$, Player $\mathcal{II}$ in $\varsigma\mathcal{G}(\mathcal{T}_0, \chi)$ cannot find $(\mho_r, \mathcal{H})$ is a soft $open$ set $(\hbar_{\mathcal{M}})_r \tilde{\in} (\mho_r, \mathcal{H})$, $(\hbar_{\mathcal{N}})_r \tilde{\notin} (\mho_r, \mathcal{H})$ or $(\hbar_{\mathcal{M}})_r \tilde{\notin} (\mho_r, \mathcal{H})$, $(\hbar_{\mathcal{N}})_r \tilde{\in} (\mho_r, \mathcal{H})$.
$(\hbar_{\mathcal{M}})_r$, $(\hbar_{\mathcal{N}})_r$, because $(\chi, \mathcal{T}, \mathcal{H})$ is not soft-$\mathcal{T}_0$-space. Hence Player $\mathcal{I} \uparrow \varsigma\mathcal{G}(\mathcal{T}_0, \chi)$.
(⟸) Clear.

ii- (⟹) in the r-th inning Player $\mathcal{I}$ in $\varsigma\mathcal{G}(\mathcal{T}_0, \mathcal{I})$ choose $(\hbar_{\mathcal{M}})_r \neq (\hbar_{\mathcal{N}})_r$ where, $(\hbar_{\mathcal{M}})_r$, $(\hbar_{\mathcal{N}})_r \tilde{\in} \tilde{\chi}$, Player $\mathcal{II}$ in $\varsigma\mathcal{G}(\mathcal{T}_0, \mathcal{I})$ cannot find $(\mho_r, \mathcal{H})$ is a s$\mathcal{I}$sg $open$ set $(\hbar_{\mathcal{M}})_r \tilde{\in} (\mho_r, \mathcal{H})$, $(\hbar_{\mathcal{N}})_r \tilde{\notin} (\mho_r, \mathcal{H})$ or $(\hbar_{\mathcal{M}})_r \tilde{\notin} (\mho_r, \mathcal{H})$, $(\hbar_{\mathcal{N}})_r \tilde{\in} (\mho_r, \mathcal{H})$, because $(\chi, \mathcal{T}, \mathcal{H})$ is not s$\mathcal{I}$sg-$\mathcal{T}_0$-space. Hence Player $\mathcal{I} \uparrow \varsigma\mathcal{G}(\mathcal{T}_0, \mathcal{I})$.
(⟸) Clear.

Corollary 5.10.

i- A space $(\chi, \mathcal{T}, \mathcal{H})$ is not soft-$\mathcal{T}_0$-space if and only if Player $\mathcal{II} \not\uparrow \varsigma\mathcal{G}(\mathcal{T}_0, \chi)$.

ii- A space $(\chi, \mathcal{T}, \mathcal{H}, \mathcal{I})$ is not s$\mathcal{I}$sg-$\mathcal{T}_0$-space if and only if Player $\mathcal{II} \not\uparrow \varsigma\mathcal{G}(\mathcal{T}_0, \mathcal{I})$.

Proof: By Theorem 5.9, the proof is over.

Definition 5.11. For a soft ideal space $(\chi, \mathcal{T}, \mathcal{H}, \mathcal{I})$, determine a game $\varsigma\mathcal{G}(\mathcal{T}_1, \chi)$ (respectively, $\varsigma\mathcal{G}(\mathcal{T}_1, \mathcal{I})$) as follows:

Player $\mathcal{I}$ and Player $\mathcal{II}$ are play an inning with each positive integer numbers in the $r\ th$ inning: The first step, Player $\mathcal{I}$ Choose $(\hbar_{\mathcal{M}})_r \neq (\hbar_{\mathcal{N}})_r$ where, $(\hbar_{\mathcal{M}})_r$, $(\hbar_{\mathcal{N}})_r \tilde{\in} \tilde{\chi}$.
In the second step, Player $\mathcal{II}$ Choose $(\mathcal{A}_r, \mathcal{H}), (\mathcal{B}_r, \mathcal{H})$ are two soft open (respectively, s$\mathcal{I}$sg-$open$) sets such that $(\hbar_{\mathcal{M}})_r \tilde{\in} ((\mathcal{A}_r, \mathcal{H}) - (\mathcal{B}_r, \mathcal{H}))$and $(\hbar_{\mathcal{N}})_r \tilde{\in} ((\mathcal{B}_r, \mathcal{H}) - (\mathcal{A}_r, \mathcal{H}))$.
Then Player $\mathcal{II}$ wins in the soft game $\varsigma\mathcal{G}(\mathcal{T}_1, \chi)$ (respectively, $\varsigma\mathcal{G}(\mathcal{T}_1, \mathcal{I})$)

if $\mathcal{B} = \{\{(\mathcal{A}_1,\mathcal{H}),(\mathcal{B}_1,\mathcal{H})\},\{(\mathcal{A}_2,\mathcal{H}),(\mathcal{B}_2,\mathcal{H})\},\ldots,\{(\mathcal{A}_r,\mathcal{H}),(\mathcal{B}_r,\mathcal{H})\},\ldots\}$ be a collection of a soft open (respectively, $s\mathcal{I}$sg-$open$) sets in χ such that $\forall\ (\hbar_{\mathcal{M}})_r \neq (\hbar_{\mathcal{N}})_r$ where, $(\hbar_{\mathcal{M}})_r$, $(\hbar_{\mathcal{N}})_r \tilde{\in} \tilde{\chi}$, $\exists\{(\mathcal{A}_r,\mathcal{H}),(\mathcal{B}_r,\mathcal{H})\} \in \mathcal{B}$ such that $(\hbar_{\mathcal{M}})_r \tilde{\in} ((\mathcal{A}_r,\mathcal{H}) - (\mathcal{B}_r,\mathcal{H}))$ and $(\hbar_{\mathcal{N}})_r \tilde{\in} ((\mathcal{B}_r,\mathcal{H}) - (\mathcal{A}_r,\mathcal{H}))$. Otherwise, Player I wins in the soft game $\c{S}\mathcal{G}(\mathcal{T}_1, \chi)$ (respectively, $\c{S}\mathcal{G}(\mathcal{T}_1,\mathcal{I})$).

Example 5.12. Let a game $\c{S}\mathcal{G}(\mathcal{T}_1, \chi)$(respectively, $\c{S}\mathcal{G}(\mathcal{T}_1,\mathcal{I})$) be a game where, $\chi = \{1,2,3\}$, $\mathcal{H} = \{\hbar_1, \hbar_2\}$, $\mathcal{T} = \c{S}\c{S}(\chi)_{\mathcal{H}}$, $\mathcal{I} = \{\tilde{\emptyset}\}$. Then $\c{S}\c{S}O(\chi) = s\mathcal{I}\text{sg-}c(\chi)_{\mathcal{H}} = s\mathcal{I}\text{sg-}o(\chi)_{\mathcal{H}} = \c{S}\c{S}(\chi)_{\mathcal{H}}$.

In the first inning: The first step, Player I Chooses $\hbar_{\mathcal{M}} \neq \hbar_{\mathcal{N}}$ where, $\hbar_{\mathcal{M}}, \hbar_{\mathcal{N}} \tilde{\in} \tilde{\chi}$ such that $\hbar_{\mathcal{M}} = \{1\}$ and $\hbar_{\mathcal{N}} = \{2\}$

In the second step, Player II Choose $(\mathcal{A},\mathcal{H}),(\mathcal{B},\mathcal{H})$ such that $\mathcal{A}(\hbar) = \{1\}, \mathcal{B}(\hbar) = \{2\}$ $\forall\ \hbar$ which are soft open (respectively, $s\mathcal{I}$sg-$open$) sets.

In the second inning: The first step, Player I Choose $\hbar_{\mathcal{M}} \neq \hbar_{\mathcal{O}}$ where, $\hbar$, $\hbar_{\mathcal{O}} \tilde{\in} \tilde{\chi}$ such that $\hbar_{\mathcal{M}} = \{2\}$ and $\hbar_{\mathcal{O}} = \{3\}$.

In the second step, Player II Choose $(\mathcal{B},\mathcal{H}),(\mathcal{C},\mathcal{H})$ such that $\mathcal{B}(\hbar) = \{2\}, \mathcal{C}(\hbar) = \{3\}$ $\forall\ \hbar$ which are soft open (respectively, $s\mathcal{I}$sg-$open$) sets.

In the third inning: The first step, Player I Choose $\hbar_{\mathcal{N}} \neq \hbar_{\mathcal{O}}$ where, $\hbar$, $\hbar_{\mathcal{O}} \tilde{\in} \tilde{\chi}$ such that $\hbar_{\mathcal{N}} = \{1\}$ and $\hbar_{\mathcal{O}} = \{3\}$.

In the second step, Player II Choose $(\mathcal{A},\mathcal{H}),(\mathcal{C},\mathcal{H})$ such that $\mathcal{A}(\hbar) = \{1\}, \mathcal{C}(\hbar) = \{3\}$ $\forall\ \hbar$ which are soft open (respectively, $s\mathcal{I}$sg-$open$) sets.

In the fourth inning: The first step, Player I Choose $\hbar_{\mathcal{M}} \neq \hbar_{\mathcal{R}}$ where, $\hbar$, $\hbar_{\mathcal{R}} \tilde{\in} \tilde{\chi}$ such that $\hbar_{\mathcal{M}} = \{1\}$ and $\hbar_{\mathcal{R}} = \{2,3\}$.

In the second step, Player II Choose $(\mathcal{A},\mathcal{H}),(\mathcal{D},\mathcal{H})$ such that $\mathcal{A}(\hbar) = \{1\}, \mathcal{D}(\hbar) = \{2,3\}$ $\forall\ \hbar$ which are soft open (respectively, $s\mathcal{I}$sg-$open$) sets.

In the fifth inning: The first step, Player I Choose $\hbar_{\mathcal{N}} \neq \hbar_{\mathcal{S}}$ where, $\hbar$, $\hbar_{\mathcal{S}} \tilde{\in} \tilde{\chi}$ such that $\hbar_{\mathcal{N}} = \{2\}$ and $\hbar_{\mathcal{S}} = \{1,3\}$.

In the second step, Player II Choose $(\mathcal{B},\mathcal{H}),(\mathcal{E},\mathcal{H})$ such that $\mathcal{B}(\hbar) = \{2\}, \mathcal{E}(\hbar) = \{1,3\}$ $\forall\ \hbar$ which are soft open (respectively, $s\mathcal{I}$sg-$open$) sets.

In the sixth inning: The first step, Player I Choose $\hbar_{\mathcal{O}} \neq \hbar_{\mathcal{L}}$ where, $\hbar$, $\hbar_{\mathcal{L}} \tilde{\in} \tilde{\chi}$ such that $\hbar_{\mathcal{O}} = \{3\}$ and $\hbar_{\mathcal{L}} = \{1,2\}$.

In the second step, Player II Choose $(\mathcal{C},\mathcal{H}),(\mathcal{F},\mathcal{H})$ such that $\mathcal{C}(\hbar) = \{3\}, \mathcal{F}(\hbar) = \{1,2\}$ $\forall\ \hbar$ which are soft open (respectively, $s\mathcal{I}$sg-$open$) sets.

Then $\mathcal{B} = \{\{(\mathcal{A},\mathcal{H}),(\mathcal{B},\mathcal{H})\},\{(\mathcal{B},\mathcal{H}),(\mathcal{C},\mathcal{H})\},\{(\mathcal{A},\mathcal{H}),(\mathcal{C},\mathcal{H})\},\{(\mathcal{A},\mathcal{H}),(\mathcal{D},\mathcal{H})\},$ $\{(\mathcal{B},\mathcal{H}),(\mathcal{E},\mathcal{H})\},\{(\mathcal{C},\mathcal{H}),(\mathcal{F},\mathcal{H})\}\}$ is the winning strategy for Player II in $\c{S}\mathcal{G}(\mathcal{T}_1, \chi)$ (respectively, $\c{S}\mathcal{G}(\mathcal{T}_1,\mathcal{I})$). Hence Player $II \uparrow \c{S}\mathcal{G}(\mathcal{T}_1, \chi)$(respectively, $\c{S}\mathcal{G}(\mathcal{T}_1,\mathcal{I})$).

By the same way in Example 4.3, Player $I \uparrow \c{S}\mathcal{G}(\mathcal{T}_1, \chi)$ and Player $I \uparrow \c{S}\mathcal{G}(\mathcal{T}_1,\mathcal{I})$.

Remark 5.13. For a space $(\chi, \mathcal{T}, \mathcal{H}, \mathcal{I})$:

i- If Player $II \uparrow \c{S}\mathcal{G}(\mathcal{T}_1, \chi)$ then Player $II \uparrow \c{S}\mathcal{G}(\mathcal{T}_1,\mathcal{I})$.

ii- If Player $I \uparrow \c{S}\mathcal{G}(\mathcal{T}_1,\mathcal{I})$then Player $I \uparrow \c{S}\mathcal{G}(\mathcal{T}_1, \chi)$.

Remark 5.14. For a space $(\chi, \mathcal{T}, \mathcal{H}, \mathcal{I})$, if Player $II \downarrow \c{S}\mathcal{G}(\mathcal{T}_1, \chi)$ then Player $II \downarrow \c{S}\mathcal{G}(\mathcal{T}_1,\mathcal{I})$.

Theorem 5.15. A space $(\chi,\mathcal{T},\mathcal{H})$ (respectively, $(\chi,\mathcal{T},\mathcal{H},\mathcal{I})$) is a soft-$\mathcal{T}_1$ $space$ (respectively, s$\mathcal{I}$sg-$\mathcal{T}_1$-space) if and only if Player $II \uparrow \c{S}\mathcal{G}(\mathcal{T}_1,\chi)$ (respectively, $\c{S}\mathcal{G}(\mathcal{T}_1,\mathcal{I})$).

Proof: $(\Longrightarrow)$ in the r-th inning Player I in $\c{S}\mathcal{G}(\mathcal{T}_1,\chi)$ (respectively, $\c{S}\mathcal{G}(\mathcal{T}_1,\mathcal{I})$) choose $\forall (\hbar_{\mathcal{M}})_r \neq (\hbar_{\mathcal{N}})_r$ where, $(\hbar_{\mathcal{M}})_r$, $(\hbar_{\mathcal{N}})_r \,\tilde{\in}\, \tilde{\chi}$, Player II in $\c{S}\mathcal{G}(\mathcal{T}_1,\chi)$ (respectively, $\c{S}\mathcal{G}(\mathcal{T}_1,\mathcal{I})$) choose $(\mathcal{A}_r,\mathcal{H}),(\mathcal{B}_r,\mathcal{H})$ are two soft open (respectively, s$\mathcal{I}$sg-$open$) sets such that $(\hbar_{\mathcal{M}})_r \,\tilde{\in}\, ((\mathcal{A}_r,\mathcal{H}) - (\mathcal{B}_r,\mathcal{H}))$ and $(\hbar_{\mathcal{N}})_r \,\tilde{\in}\, ((\mathcal{B}_r,\mathcal{H}) - (\mathcal{A}_r,\mathcal{H}))$. Since $(\chi,\mathcal{T},\mathcal{H})$ a soft-$\mathcal{T}_1$ $space$ (respectively,s$\mathcal{I}$sg-$\mathcal{T}_1$-space).Then $\mathcal{B} = \{\{(\mathcal{A}_1,\mathcal{H}),(\mathcal{B}_1,\mathcal{H})\},\{(\mathcal{A}_2,\mathcal{H}),(\mathcal{B}_2,\mathcal{H})\}$ $,\ldots,\{(\mathcal{A}_r,\mathcal{H}),(\mathcal{B}_r,\mathcal{H})\},\ldots\}$ is the winning strategy for Player II in $\c{S}\mathcal{G}(\mathcal{T}_1,\chi)$ (respectively, $\c{S}\mathcal{G}(\mathcal{T}_1,\mathcal{I})$). Hence Player $II \uparrow \c{S}\mathcal{G}(\mathcal{T}_1,\chi)$ (respectively, $\c{S}\mathcal{G}(\mathcal{T}_1,\mathcal{I})$).

$(\Longleftarrow)$ Clear.

Corollary 5.16. For a space $(\chi,\mathcal{T},\mathcal{H},\mathcal{I})$:

i- Player $II \uparrow \c{S}\mathcal{G}(\mathcal{T}_1,\chi)$ if $\forall\, \hbar_{\mathcal{M}} \neq \hbar_{\mathcal{N}}$ where $\hbar_{\mathcal{M}}, \hbar_{\mathcal{N}} \,\tilde{\in}\, \tilde{\chi}$, $\exists\, (\mathcal{A},\mathcal{H}),(\mathcal{B},\mathcal{H})$ are two closed sets such that $\hbar_{\mathcal{M}} \,\tilde{\in}\, ((\mathcal{A},\mathcal{H}) - (\mathcal{B},\mathcal{H}))$ and $\hbar_{\mathcal{N}} \,\tilde{\in}\, ((\mathcal{B},\mathcal{H}) - (\mathcal{A},\mathcal{H}))$.

ii- Player $II \uparrow \mathcal{G}(\mathcal{T}_1,\mathcal{I})$ if $\forall\, \hbar_{\mathcal{M}} \neq \hbar_{\mathcal{N}}$ where $\hbar_{\mathcal{M}}, \hbar_{\mathcal{N}} \,\tilde{\in}\, \tilde{\chi}$, $\exists\, (\mathcal{A},\mathcal{H}),(\mathcal{B},\mathcal{H})\}$ are two s$\mathcal{I}$sg-closed sets where, $\hbar_{\mathcal{M}} \,\tilde{\in}\, ((\mathcal{A},\mathcal{H}) - (\mathcal{B},\mathcal{H}))$ and $\hbar_{\mathcal{N}} \,\tilde{\in}\, ((\mathcal{B},\mathcal{H}) - (\mathcal{A},\mathcal{H}))$.

Proof:

i. $(\Longrightarrow)$ Let $\hbar_{\mathcal{M}} \neq \hbar_{\mathcal{N}}$ where $\hbar_{\mathcal{M}}, \hbar_{\mathcal{N}} \,\tilde{\in}\, \tilde{\chi}$. Since Player $II \uparrow \c{S}\mathcal{G}(\mathcal{T}_1,\chi)$, then by Theorem 4.1.15, the space $(\chi,\mathcal{T},\mathcal{H})$ is a soft-$\mathcal{T}_1$ $space$. Then Theorem 2.19, is applicable.
$(\Longleftarrow)$ By Theorem 2.19, the space $(\chi,\mathcal{T},\mathcal{H})$ is a soft-$\mathcal{T}_1$ $space$. Then Theorem 5.15, is applicable.

ii. $(\Longrightarrow)$Let $\hbar_{\mathcal{M}} \neq \hbar_{\mathcal{N}}$ where $\hbar_{\mathcal{M}}, \hbar_{\mathcal{N}} \,\tilde{\in}\, \tilde{\chi}$. Since Player $II \uparrow \c{S}\mathcal{G}(\mathcal{T}_1,\mathcal{I})$, then by Theorem 5.15, the space $(\chi,\mathcal{T},\mathcal{H})$ is a s$\mathcal{I}$sg-$\mathcal{T}_1$-space. Then Theorem 4.9, is applicable.
$(\Longleftarrow)$ By Theorem 4.9, the space $(\chi,\mathcal{T},\mathcal{H})$ is a s$\mathcal{I}$sg-$\mathcal{T}_1$-space. Then Theorem 5.15, is applicable.

Corollary 5.17.

i- A space $(\chi,\mathcal{T},\mathcal{H})$ is a soft-$\mathcal{T}_1$-space if and only if Player $I \not\uparrow \c{S}\mathcal{G}(\mathcal{T}_1,\chi)$.

ii- A space $(\chi,\mathcal{T},\mathcal{H},\mathcal{I})$ is a s$\mathcal{I}$sg-$\mathcal{T}_1$-space if and only if Player $I \not\uparrow \c{S}\mathcal{G}(\mathcal{T}_1,\mathcal{I})$.

Proof: By Theorem 5.15, the proof is over.

Theorem 5.18. For a space $(\chi,\mathcal{T},\mathcal{H},\mathcal{I})$:

i- A space $(\chi,\mathcal{T},\mathcal{H})$ is not soft-$\mathcal{T}_1$-space if and only if Player $I \uparrow \c{S}\mathcal{G}(\mathcal{T}_1,\chi)$.

ii- A space $(\chi,\mathcal{T},\mathcal{H},\mathcal{I})$ is not s$\mathcal{I}$sg-$\mathcal{T}_1$-space if and only if Player $I \uparrow \c{S}\mathcal{G}(\mathcal{T}_1,\mathcal{I})$.

Proof:

i. $(\Longrightarrow)$ in the r-th inning Player I in $\c{S}\mathcal{G}(\mathcal{T}_1,\chi)$ choose $(\hbar_{\mathcal{M}})_r \neq (\hbar_{\mathcal{N}})_r$ where, $(\hbar_{\mathcal{M}})_r$, $(\hbar_{\mathcal{N}})_r \,\tilde{\in}\, \tilde{\chi}$, Player II in $\c{S}\mathcal{G}(\mathcal{T}_1,\chi)$ cannot find $(\mathcal{A}_r,\mathcal{H}),(\mathcal{B}_r,\mathcal{H})$ are two soft open sets such that $(\hbar_{\mathcal{M}})_r \,\tilde{\in}\, ((\mathcal{A}_r,\mathcal{H}) - (\mathcal{B}_r,\mathcal{H}))$ and $(\hbar_{\mathcal{N}})_r \,\tilde{\in}\, ((\mathcal{B}_r,\mathcal{H}) - (\mathcal{A}_r,\mathcal{H}))$, because $(\chi,\mathcal{T},\mathcal{H})$ is not soft-$\mathcal{T}_1$-space. Hence Player $I \uparrow \c{S}\mathcal{G}(\mathcal{T}_1,\chi)$.
$(\Longleftarrow)$ Clear.

ii. $(\Longrightarrow)$ in the r-th inning Player I in $\c{S}\mathcal{G}(\mathcal{T}_1,\mathcal{I})$ choose $(\hbar_{\mathcal{M}})_r \neq (\hbar_{\mathcal{N}})_r$ where, $(\hbar_{\mathcal{M}})_r$, $(\hbar_{\mathcal{N}})_r \,\tilde{\in}\, \tilde{\chi}$, Player II in $\c{S}\mathcal{G}(\mathcal{T}_1,\mathcal{I})$ cannot find $(\mathcal{A}_r,\mathcal{H}),(\mathcal{B}_r,\mathcal{H})$ are two s$\mathcal{I}$sg-$open$ sets such that $(\hbar_{\mathcal{M}})_r \,\tilde{\in}\, ((\mathcal{A}_r,\mathcal{H}) - (\mathcal{B}_r,\mathcal{H}))$ and $(\hbar_{\mathcal{N}})_r \,\tilde{\in}\, ((\mathcal{B}_r,\mathcal{H}) - (\mathcal{A}_r,\mathcal{H}))$, because $(\chi,\mathcal{T},\mathcal{H})$ is not soft- $\mathcal{T}_1$-space. Hence Player $I \uparrow \c{S}\mathcal{G}(\mathcal{T}_1,\mathcal{I})$.

(⟸) Clear.

Corollary 5.19.

i- If a space $(\chi, \mathcal{T}, \mathcal{H})$ is not soft-$\mathcal{T}_1$-space if and only if Player $\mathit{II} \uparrow ŞG(\mathcal{T}_1, \chi)$.

ii- If a space $(\chi, \mathcal{T}, \mathcal{H}, \mathcal{I})$ is not s$\mathcal{I}$sg-$\mathcal{T}_1$-space if and only if Player $\mathit{II} \uparrow ŞG(\mathcal{T}_1, \mathcal{I})$.

Proof: Similar way of proof Theorem 5.18.

Definition 5.20. For a soft ideal space $(\chi, \mathcal{T}, \mathcal{H}, \mathcal{I})$, determine a game $ŞG(\mathcal{T}_2, \chi)$ (respectively, $ŞG(\mathcal{T}_2, \mathcal{I})$) as follows:

Player I and Player II are play an inning with each positive integer numbers in the r th inning: The first step, Player I Choose $(\hbar_{\mathcal{M}})_r \neq (\hbar_{\mathcal{N}})_r$ where, $(\hbar_{\mathcal{M}})_r, (\hbar_{\mathcal{N}})_r \tilde{\in} \tilde{\chi}$.

In the second step, Player II Choose $(\mathcal{A}_r, \mathcal{H}), (\mathcal{B}_r, \mathcal{H})$ are two soft open (respectively, s$\mathcal{I}$sg-$open$) sets such that $(\hbar_{\mathcal{M}})_r \tilde{\in} (\mathcal{A}_r, \mathcal{H})$, $(\hbar_{\mathcal{N}})_r \tilde{\in} (\mathcal{B}_r, \mathcal{H})$ and $(\mathcal{A}_r, \mathcal{H}) \tilde{\cap} (\mathcal{B}_r, \mathcal{H}) = \{\tilde{\emptyset}\}$. Then Player II wins in the game $ŞG(\mathcal{T}_2, \chi)$ (respectively, $ŞG(\mathcal{T}_2, \mathcal{I})$) if $\mathcal{B} = \{\{(\mathcal{A}, \mathcal{H}), (\mathcal{B}, \mathcal{H})\}, \{(\mathcal{B}, \mathcal{H}), (\mathcal{C}, \mathcal{H})\}, \{(\mathcal{A}, \mathcal{H}), (\mathcal{C}, \mathcal{H})\}\}$ be a collection of a soft open (respectively, s$\mathcal{I}$sg-$open$) sets in χ such that $\forall$ $(\hbar_{\mathcal{M}})_r \neq (\hbar_{\mathcal{N}})_r$ where, $(\hbar_{\mathcal{M}})_r, (\hbar_{\mathcal{N}})_r \tilde{\in} \tilde{\chi}$, $\exists\{(\mathcal{A}_r, \mathcal{H}), (\mathcal{B}_r, \mathcal{H})\} \in \mathcal{B}$ such that $(\hbar_{\mathcal{M}})_r \tilde{\in} (\mathcal{A}_r, \mathcal{H})$ and $(\hbar_{\mathcal{N}})_r \tilde{\in} (\mathcal{B}_r, \mathcal{H})$ and $(\mathcal{A}_r, \mathcal{H}) \tilde{\cap} (\mathcal{B}_r, \mathcal{H}) = \{\tilde{\emptyset}\}$. Otherwise, Player I wins in the game $ŞG(\mathcal{T}_2, \chi)$ (respectively, $ŞG(\mathcal{T}_2, \mathcal{I})$).

By example 5.12. $\forall$ $\hbar_{\mathcal{M}} \neq \hbar_{\mathcal{N}}$ where, $\hbar_{\mathcal{M}}, \hbar_{\mathcal{N}} \tilde{\in} \tilde{\chi}$ there exist $(\mathcal{M}, \mathcal{H}), (\mathcal{N}, \mathcal{H})$ are soft $\hbar_{\mathcal{M}} \tilde{\in} (\mathcal{M}, \mathcal{H})$ and $\hbar_{\mathcal{N}} \tilde{\in} (\mathcal{N}, \mathcal{H})$ such that $(\mathcal{M}, \mathcal{H}) \tilde{\cap} (\mathcal{N}, \mathcal{H}) = \{\tilde{\emptyset}\}$. So, then $\mathcal{B} = \{\{(\mathcal{A}, \mathcal{H}), (\mathcal{B}, \mathcal{H})\}, \{(\mathcal{B}, \mathcal{H}), (\mathcal{C}, \mathcal{H})\}, \{(\mathcal{A}, \mathcal{H}), (\mathcal{C}, \mathcal{H})\}, \{(\mathcal{A}, \mathcal{H}), (\mathcal{D}, \mathcal{H})\}, \{(\mathcal{B}, \mathcal{H}), (\mathcal{E}, \mathcal{H})\}, \{(\mathcal{C}, \mathcal{H}), (\mathcal{F}, \mathcal{H})\}\}$.

Is the winning strategy for Player II in $ŞG(\mathcal{T}_2, \chi)$ (respectively, $ŞG(\mathcal{T}_2, \mathcal{I})$). Hence Player $\mathit{II} \uparrow ŞG(\mathcal{T}_2, \chi)$ (respectively, $ŞG(\mathcal{T}_2, \mathcal{I})$).

By the same way in Example 5.3, Player $I \uparrow ŞG(\mathcal{T}_2, \chi)$ and Player $I \uparrow ŞG(\mathcal{T}_2, \mathcal{I})$.

Remark 5.21. For a space $(\chi, \mathcal{T}, \mathcal{H}, \mathcal{I})$:

i- If Player $\mathit{II} \uparrow ŞG(\mathcal{T}_2, \chi)$ then Player $\mathit{II} \uparrow ŞG(\mathcal{T}_2, \mathcal{I})$.

ii- If Player $I \uparrow ŞG(\mathcal{T}_2, \mathcal{I})$ then Player $I \uparrow ŞG(\mathcal{T}_2, \chi)$.

Remark 5.22. For a space (($\chi, \mathcal{T}, \mathcal{H}, \mathcal{I})$, if Player $\mathit{II} \downarrow ŞG(\mathcal{T}_2, \chi)$ then Player $\mathit{II} \downarrow ŞG(\mathcal{T}_2, \mathcal{I})$.

Theorem 5.23. A space $(\chi, \mathcal{T}, \mathcal{H})$ (respectively, $(\chi, \mathcal{T}, \mathcal{H}, \mathcal{I})$) is a soft-$\mathcal{T}_2$-$space$ (respectively, $s\mathcal{I}sg$-$\mathcal{T}_2$-$space$) if and only if Player $\mathit{II} \uparrow ŞG(\mathcal{T}_2, \chi)$ (respectively, $ŞG(\mathcal{T}_2, \mathcal{I})$).

Proof: (⟹) in the r-th inning Player I in $ŞG(\mathcal{T}_2, \chi)$ (respectively, $ŞG(\mathcal{T}_2, \mathcal{I})$) choose $(\hbar_{\mathcal{M}})_r \neq (\hbar_{\mathcal{N}})_r$ where, $(\hbar_{\mathcal{M}})_r$, $(\hbar_{\mathcal{N}})_r \tilde{\in} \tilde{\chi}$, Player II in $ŞG(\mathcal{T}_2, \chi)$ (respectively, $ŞG(\mathcal{T}_2, \mathcal{I})$) choose $(\mathcal{A}_r, \mathcal{H}), (\mathcal{B}_r, \mathcal{H})$ are two soft open (respectively, s$\mathcal{I}$sg-$open$) sets such that $(\hbar_{\mathcal{M}})_r \tilde{\in} ((\mathcal{A}_r, \mathcal{H})$ and $(\hbar_{\mathcal{N}})_r \tilde{\in} ((\mathcal{B}_r, \mathcal{H})$ and $(\mathcal{A}_r, \mathcal{H}) \tilde{\cap} (\mathcal{B}_r, \mathcal{H}) = \{\tilde{\emptyset}\}$. Since $(\chi, \mathcal{T}, \mathcal{H})$ a soft-$\mathcal{T}_2$ $space$ (respectively,s$\mathcal{I}$sg-$\mathcal{T}_1$-space).Then $\mathcal{B} = \{\{(\mathcal{A}_1, \mathcal{H}), (\mathcal{B}_1, \mathcal{H})\}, \{(\mathcal{A}_2, \mathcal{H}), (\mathcal{B}_2, \mathcal{H})\}, \ldots, \{(\mathcal{A}_r, \mathcal{H}), (\mathcal{B}_r, \mathcal{H})\}, \ldots\}$ is the winning strategy for Player II in $ŞG(\mathcal{T}_2, \chi)$ (respectively, $ŞG(\mathcal{T}_2, \mathcal{I})$). Hence Player $\mathit{II} \uparrow ŞG(\mathcal{T}_2, \chi)$ (respectively, $ŞG(\mathcal{T}_2, \mathcal{I})$).

(⟸) Clear.

Corollary 5.24.

i- A space $(\chi, \mathcal{T}, \mathcal{H})$ is a soft-$\mathcal{T}_2$ $space$ if and only if Player $I \uparrow ŞG(\mathcal{T}_2, \chi)$.

ii- A space $(\chi, \mathcal{T}, \mathcal{H}, \mathcal{I})$ is a s$\mathcal{I}$sg-$\mathcal{T}_2$-space if and only if Player I ⇞ $\text{Ş}\mathcal{G}(\mathcal{T}_2, \mathcal{I})$.

Proof: By Theorem 4.23, the proof is over.

Theorem 5. 25. For a space $(\chi, \mathcal{T}, \mathcal{H}, \mathcal{I})$:

i- A space $(\chi, \mathcal{T}, \mathcal{H})$ is not soft-$\mathcal{T}_2$-space if and only if Player $I \uparrow \text{Ş}\mathcal{G}(\mathcal{T}_2, \chi)$.

ii- A space $(\chi, \mathcal{T}, \mathcal{H}, \mathcal{I})$ is not a s$\mathcal{I}$sg-$\mathcal{T}_2$-space if and only if Player $I \uparrow \text{Ş}\mathcal{G}(\mathcal{T}_2, \mathcal{I})$.

Proof:

i- ($\Longrightarrow$) in the r-th inning Player I in $\text{Ş}\mathcal{G}(\mathcal{T}_2, \chi)$ choose $(\hbar_\mathcal{M})_r \neq (\hbar_\mathcal{N})_r$ where, $(\hbar_\mathcal{M})_r$, $(\hbar_\mathcal{N})_r \widetilde{\in} \tilde{\chi}$, Player II in $\text{Ş}\mathcal{G}(\mathcal{T}_2, \chi)$ cannot find $(\mathcal{A}_r, \mathcal{H}), (\mathcal{B}_r, \mathcal{H})$ are two soft-*open* sets such that $(\hbar_\mathcal{M})_r \widetilde{\in} (\mathcal{A}_r, \mathcal{H})$, $(\hbar_\mathcal{N})_r \widetilde{\in} ((\mathcal{B}_r, \mathcal{H})$ and $(\mathcal{A}_r, \mathcal{H}) \widetilde{\cap} (\mathcal{B}_r, \mathcal{H}) = \{\widetilde{\emptyset}\}$, because $(\chi, \mathcal{T}, \mathcal{H})$ is not soft-$\mathcal{T}_2$-space. Hence Player $I \uparrow \text{Ş}\mathcal{G}(\mathcal{T}_2, \chi)$.

($\Longleftarrow$) Clear.

ii- ($\Longrightarrow$) in the r-th inning Player I in $\text{Ş}\mathcal{G}(\mathcal{T}_2, \mathcal{I})$ choose $(\hbar_\mathcal{M})_r \neq (\hbar_\mathcal{N})_r$ where, $(\hbar_\mathcal{M})_r$, $(\hbar_\mathcal{N})_r \widetilde{\in} \tilde{\chi}$, Player II in $\text{Ş}\mathcal{G}(\mathcal{T}_2, \mathcal{I})$ cannot find $(\mathcal{A}_r, \mathcal{H}), (\mathcal{B}_r, \mathcal{H})$ are two s$\mathcal{I}$sg-*open* sets such that $(\hbar_\mathcal{M})_r \widetilde{\in} (\mathcal{A}_r, \mathcal{H})$, $(\hbar_\mathcal{N})_r \widetilde{\in} (\mathcal{B}_r, \mathcal{H})$ and $(\mathcal{A}_r, \mathcal{H}) \widetilde{\cap} (\mathcal{B}_r, \mathcal{H}) = \{\widetilde{\emptyset}\}$, because $(\chi, \mathcal{T}, \mathcal{H})$ is a not soft-$\mathcal{T}_2$ *space*. Hence Player $I \uparrow \text{Ş}\mathcal{G}(\mathcal{T}_2, \mathcal{I})$.

($\Longleftarrow$) Clear.

Corollary 5.26.

i- A space $(\chi, \mathcal{T}, \mathcal{H})$ is a not soft-$\mathcal{T}_2$ *space* if and only if Player II ⇞ $\text{Ş}\mathcal{G}(\mathcal{T}_2, \chi)$.

ii- A space $(\chi, \mathcal{T}, \mathcal{H}, \mathcal{I})$ is not a s$\mathcal{I}$sg-$\mathcal{T}_2$-space if and only if Player II ⇞ $\text{Ş}\mathcal{G}(\mathcal{T}_2, \mathcal{I})$.

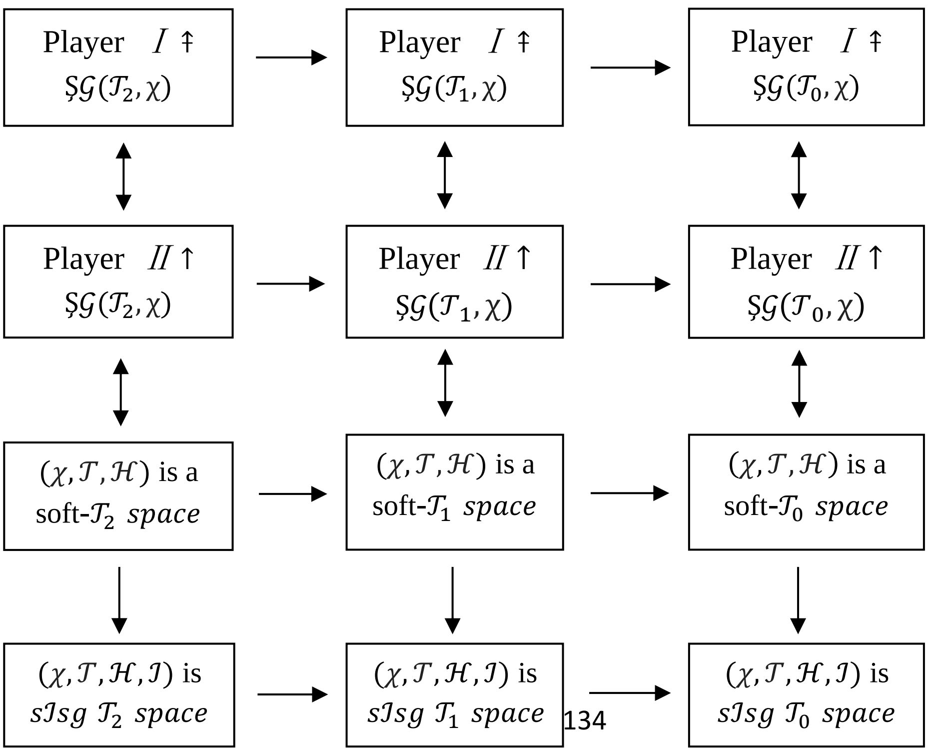
Proof: By Theorem 5.25, the proof is over.

Remark 5.27. For a space $(\chi, \mathcal{T}, \mathcal{H}, \mathcal{I})$:

i. If Player $II \uparrow \text{Ş}\mathcal{G}(\mathcal{T}_{i+1}, \chi)$ (respectively, $\text{Ş}\mathcal{G}(\mathcal{T}_{i+1}, \mathcal{I})$) then Player $II \uparrow \text{Ş}\mathcal{G}(\mathcal{T}_i, \chi)$ (respectively, $\text{Ş}\mathcal{G}(\mathcal{T}_i, \mathcal{I})$), where $i = \{0,1\}$.

ii. If Player $II \uparrow \text{Ş}\mathcal{G}(\mathcal{T}_i, \chi)$; then Player $II \uparrow \text{Ş}\mathcal{G}(\mathcal{T}_i, \mathcal{I})$, where $i = \{0,1,2\}$.

The following (figure) clarifies a relationships in Theorem 5.6, Theorem 5.15, Theorem 5.23 and Remark 5.27.



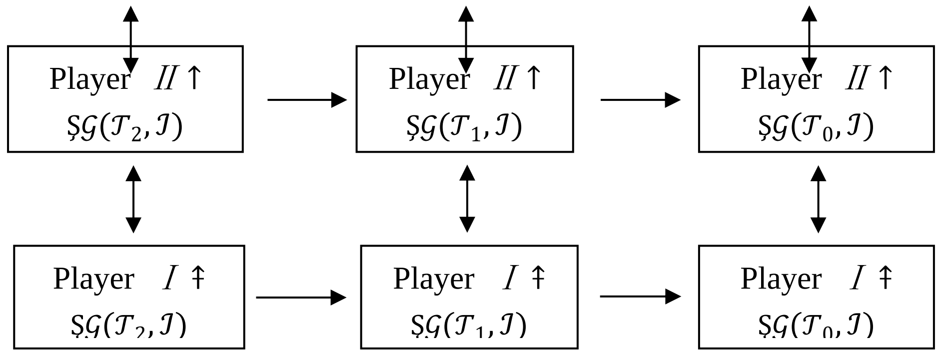


Figure 2: The winning strategy for Player *II*

Remark 5.28. For a space $(\chi, \mathcal{T}, \mathcal{J})$:

i- If Player $I \uparrow \c{S}\mathcal{G}(\mathcal{T}_i, \chi)$ (respectively, $\c{S}\mathcal{G}(\mathcal{T}_i, \mathcal{J})$) then Player $I \uparrow \c{S}\mathcal{G}(\mathcal{T}_{i+1}, \chi)$ (respectively, $\c{S}\mathcal{G}(\mathcal{T}_{i+1}, \mathcal{J})$), where $i = \{0,1\}$.

ii- If Player $I \uparrow \c{S}\mathcal{G}(\mathcal{T}_i, \mathcal{J})$ then Player $I \uparrow \c{S}\mathcal{G}(\mathcal{T}_i, \chi)$, where $i = \{0,1,2\}$.

The following (figure) clarifies a relationships in Theorem 5.9, Theorem 5.18, Theorem 5.26 and Remark 5.28.

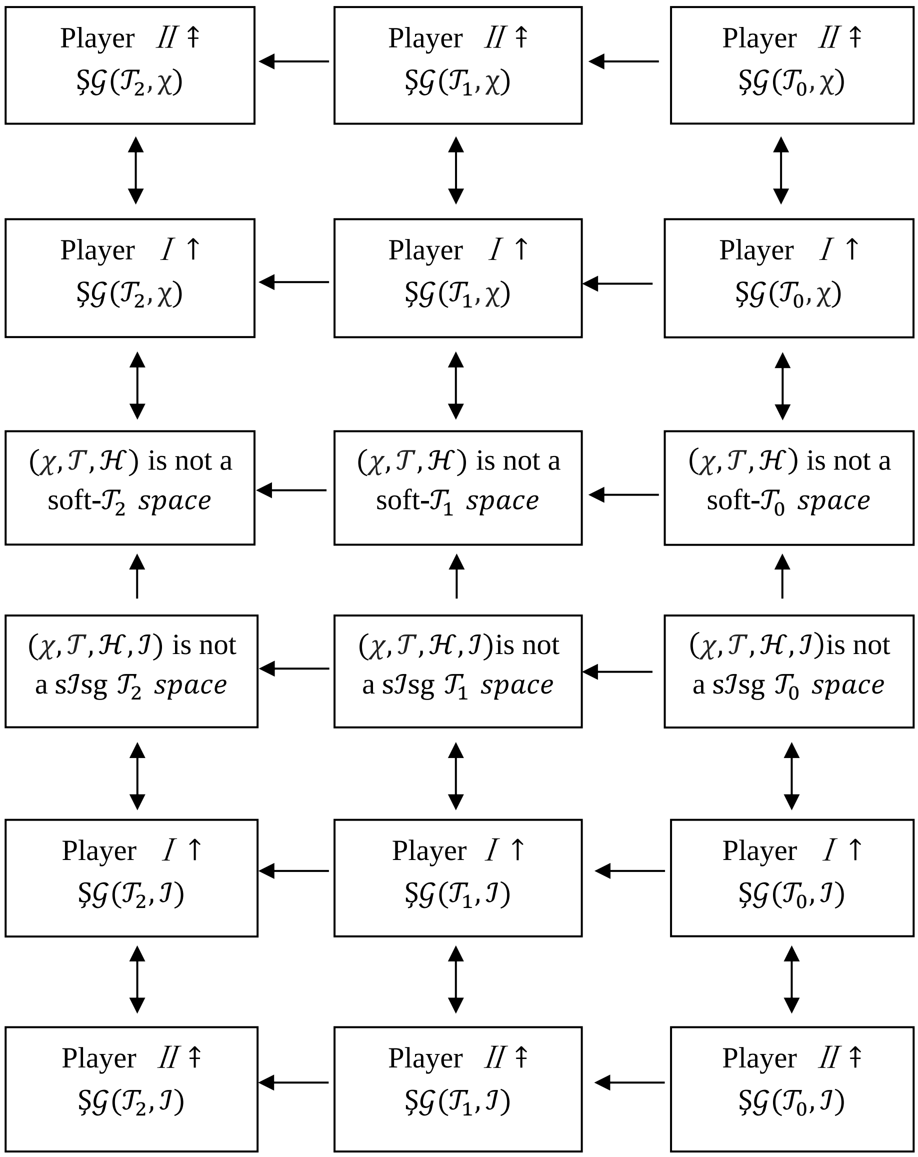


Figure 3: The winning strategy for Player *I*

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