Ibn Al Haitham Journal for Pure and Applied Science

Journal homepage: http://jih.uobaghdad.edu.iq/index.php/j/index

Determine Most Stable Isobar for Nuclides with A= (15-30) & (101- 115)

Murtadha S. Nayyef **Naz T. Jaralla**

Department of Physics, College of Education for Pure Science (Ibn Al-Haitham), University of Baghdad, Iraq.

Mafiz1993@gmail.com naztalab2016@yahoo.com

Article history: Received 30 December 2019, Accepted 16 February 2020, Published in October 2020.

Doi: 10.30526/33.4.2520

Abstract

In this study the most stable isobar for some isobaric families (light and intermediate) nuclei with mass number (A) equals to (15-30) & (101- 115) have been determined. This determination of stable nuclide can help to determine the suitable nuclide, which can be used in different fields.

Most stable isobar can be determined by two means. First: plot mass parabolas (plotting the binding energy (B.E) as a function of the atomic number (Z)) for these isobaric families, in this method most stable isobars represent the lowest point in mass parabola (the nuclide with the highest value of binding energy).

Second: calculated the atomic number for most stable isobar (Z_A) value.

Our results show that there is only one stable nuclide for isobars with odd mass number (A) (one mass parabolas), while for nuclides with an even mass number (A) there is more than one stable nuclide (two mass parabola).

Also, our results show that nuclides representing the most stable isobars in the two methods, which used in this study practically, are the same nuclide.

Keywords: Binding Energy, Weizsäcker equation, Mass Parabolas, Isobaric nuclides.

1.Introduction

The minimum portion of the elements that the keeps the basic properties are the atom. An atom is made of a small massive central called the nucleus, this nucleus is surrounded by orbiting electrons[1].

These nucleons are amounts of neutrons and protons, which are grouped in two collections. These neutrons and protons are independently spread during sure energy statuses, and they are being together with their public reacts [2].

These protons and neutrons are much heavier than electrons, which move around the nucleus. Protons have a positive charge equal in quantity to the negative charge of the electron, while neutrons have practically the equivalent mass as a proton with no charge [3].

Isobars are nuclei, which have similar atomic mass number (A), but atomic numbers (Z) are different. An instance of a pair of isobars is:

$$_{1}H^{3} \;\&\; _{2}He^{3}$$

These isobars are with two neutrons, one proton for hydrogen nuclei, one neutron and two protons for helium nuclide respectively. Isobaric nuclei show a projecting part in beta decay and electron capture[4].

Beta particles (β) are electrons, which carry a negative or positive charge (e^- & e^+). In the situation of (β^-) decay atomic number (Z) growths by one unit. But in the case of (β^+) decay the atomic number (Z) will be decreased by one unit. Some nuclei go through a radioactive change by taking an atomic electron, normally from the K shell, releasing a neutrino as well as reductions atomic number (Z) by one unit[5].

2. Theoretical Part

In nuclear physics, one of the elementary and significant models is the liquid drop model (LDM). This model the nucleus assumes like a liquid drop collected with its related properties. In this model, binding energy (BE) of the nuclide has volume term (the react of nucleons with near nucleons regardless of reduction in interaction of external nucleons), surface term (the effect of the reduction in interactions of outward nucleons), Coulomb term (the react of coulomb repulsion amongst protons), asymmetry term (unlike quantity of energy in equivalent and unlike styles of neutrons and protons numbers), and parity term (additional stability and resultantly extra negative energy of the nucleus for pair-pair nuclei) [6].

2.1 Bethe– Weizsäcker formula

In 1935, Bethe and Weizsäcker projected an excellent parameterization of the binding energies of nuclei in their ground state. This formula depended on the liquid-drop analogy but also combined two quantum ingredients. Asymmetry energy is one, which is likely favors of same neutrons and proton numbers. A pairing energy is the next one which favors arrangements where two same fermions are paired[7].

The formula of Bethe and Weizsäcker is:

$$B.E(A,Z) = a_v A - a_s A^{2/3} - a_c Z(Z-1)A^{-1/3} - a_a(A-2Z)^2 A^{-1} \pm \delta A^{-1/2} + \eta \qquad (1)$$

where: a_v, a_s, a_c, a_a, δ and η are the energy constants that represent to the volume, surface, Coulomb, asymmetry, pairing and shell energy terms, respectively[8].

One set of this factors is[8]: a_v=15.8Mev, a_s=18.3Mev, a_c=0.72Mev, a_a=23.2MeV.

$$\text{And } \delta = \begin{cases} +11.2 \text{ MeV for (even N, even Z)}. \\ 0 \text{ for (even N odd Z, or even Z, odd N)}. \\ -11.2 \text{ MeV for (odd N, odd Z)}. \end{cases}$$

$$\text{And } \eta = \begin{cases} 3\text{ Mev (N and Z} = \text{magic number).} \\ 2\text{ Mev (N or Z} = \text{magic number and other is odd).} \\ 1\text{ Mev (N or Z} = \text{magic number and other is even).} \\ 0\text{ (N or Z} = \text{no magic number).} \end{cases}$$

2.2 Applications of Bethe– Weizsäcker Formula

2.2.1 Mass Parabola

One of the important uses of Bethe-Weizsäcker semi empirical formula is to limit the most stable isobar of an assumed mass number (A) against beta decay by plotting the binding energy as a function to the atomic number (Z) [9].

Mass parabolas for the different isobars fall into two clusters according to whether the mas number (A) is even or odd.[9]

The isobars are placed on the sides of the parabola unstable, so these nuclides will be decayed to be more stable and lower on the parabola. [10].

The mass number (A) of nuclei with ($Z > Z_A$) can decay by emitting positive Beta (β^+) and neutrino (ν) or electron capture (EC). While the mass number (A) of nucleus with ($Z < Z_A$) can decay by emitting negative beta (β^-) and antineutrino ($\bar{\nu}$)[11].

2.2.2 Atomic numbers for most stable isobar (Z_A) value.

The presence of the Coulomb term and the asymmetric term refers to a nucleus with the highest binding energy found by setting ($\partial B/\partial Z = 0$) for every mass number (A). The greatly bound nuclide has ($Z = N = A/2$) for nucleus with a low mass number (A) where the asymmetry term controls, but the Coulomb term favors ($N > Z$) for a large mass number (A) [7].

The neutron and proton number which agrees to the highest binding energy of the nucleus provides the value of atomic number Z (A) for the most bound isobars [12]:

$$\partial B/\partial Z = 0 \Rightarrow Z(A) = Z(A) = \frac{A}{2 + {a_c A^{2/3}}/{2a_a}} \quad (2)$$

$$Z(A) \sim \frac{A/2}{1 + 0.0077\, A^{2/3}} \quad (3)$$

This value of (Z) is near to, but not essentially same to the value of atomic number (Z) that takes the stable isobar for an assumed mass number (A).the reason for that (one must also take into account) is the difference of neutron–proton mass [7].

3.Results and Discussion

The results of this study will be presented and discussed in detail, whereas this study aims to determine the most stable isobar for (light and intermediate) nuclei with mass number (A) equals to (15-30) and (101- 115) .

The most stable isobar for isobars under this study has been determined by two different methods: first by plotting mass parabola (values of binding energy (B.E) which are plotted as a function to the atomic number (Z) for each isobar under this study and second by calculating the atomic number (Z_A) for most stable isobars.

3.1 determined the most stable isobar by mass parabola

Binding energy (B.E) for light and intermediate nuclei with mass number (A) equals to (15-30) and (101-115) were calculated by using equation (1).

These values of binding energy (B.E) increased with increasing the mass number (A) of nuclides, the values of binding energy (B.E) which were ranging between (46-255) MeV for nuclides with mass number (A= 15-30) and (810-978) MeV for nuclides with mass number (A= 101-115) which were plotted as a function to the atomic number (Z) for each isobar in isobaric family, so we get mass parabolas for different isobars as shown in the **Figures (1 & 2)**.

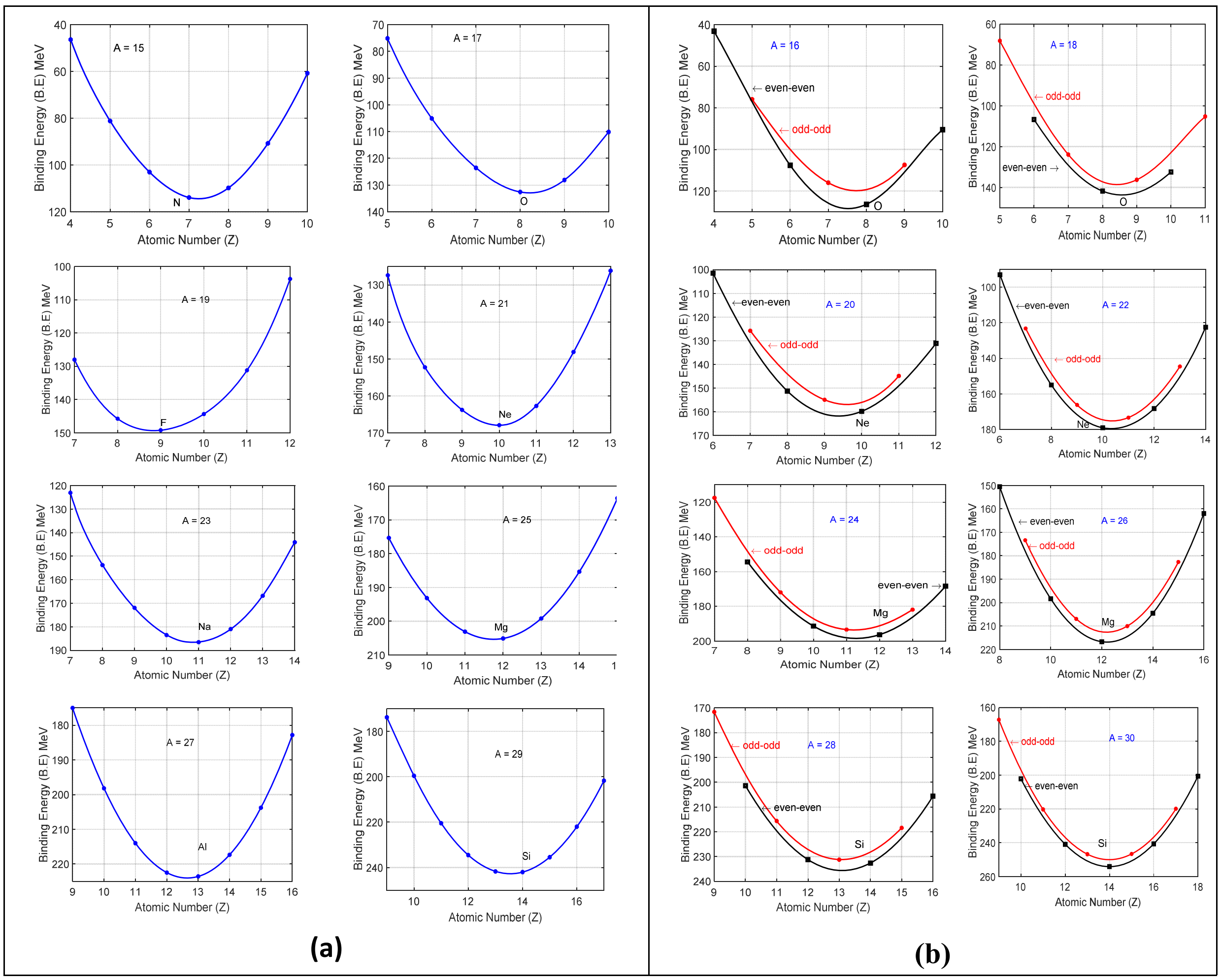

Figure 1. Mass parabolas for isobars with mass number (15-30) (a) odd (A) (b) even (A).

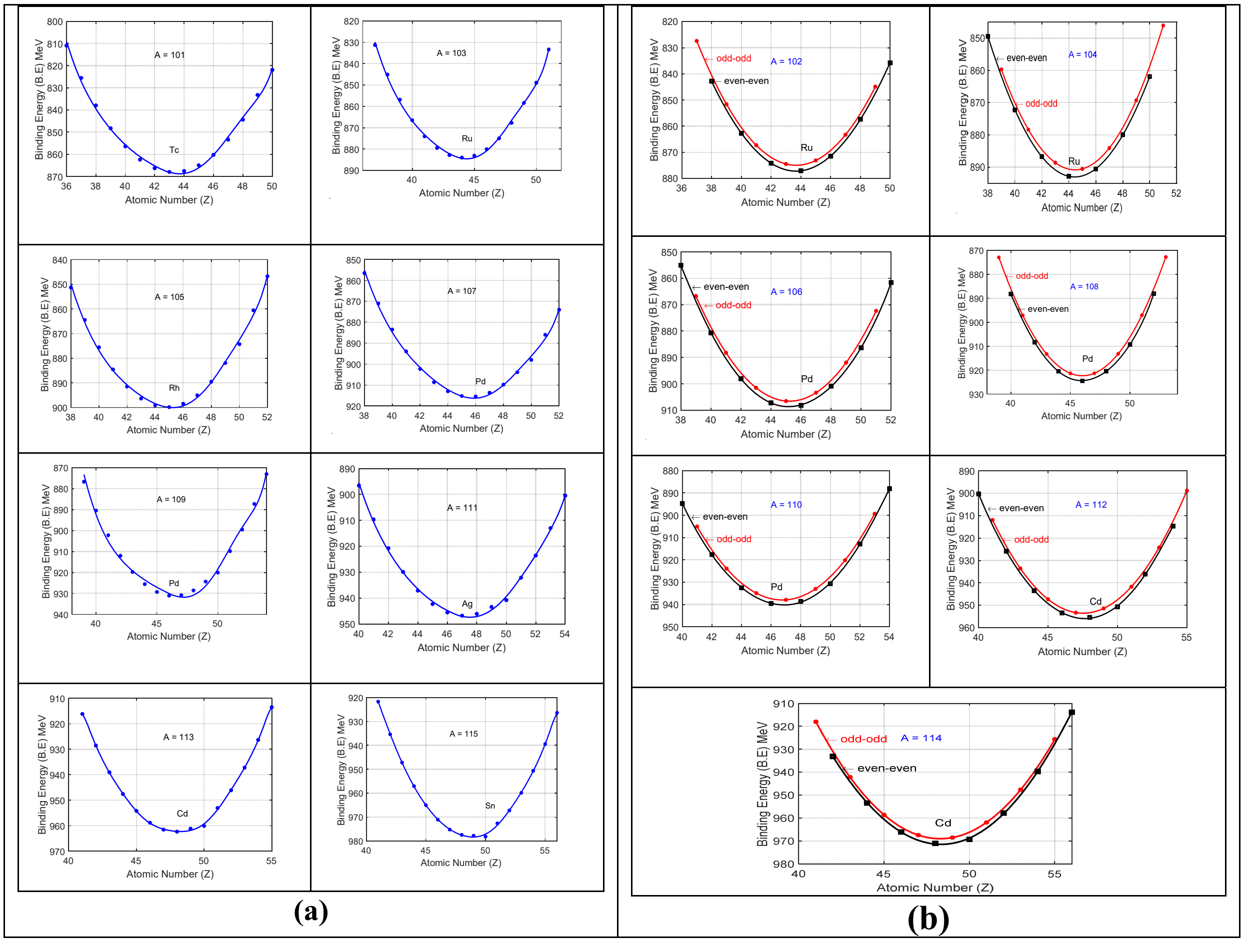

Figure 2. Mass parabolas for isobars with mass number (101-115) (a) odd (A) (b) even (A).

Here, mass parabolas for isobars with odd and even mass number (A) are drawn separately because we have only one mass parabola for nuclei with an odd mass number (A) which are coming from (even- odd) or (odd-even) nuclide, also the term of pairing equivalents to zero in these isobars.

While there are two mass parabolas for nuclei with an even mass number (A); one of these mass parabola for (odd-odd) nuclei and second for (even-even) nuclide which located under the parabola of (odd-odd) nuclide, for these nuclide the pairing term (is positive for even-even nuclei and negative for odd-odd nucleus) in the binding energy formula .

So, we can see clearly from **Figures (1(a) & 2(a))** of mass parabola for an odd mass number (A) that we have only one mass parabola as well as one stable isobar.

While **Figures (1(b) & 2(b))** of mass parabola for an even mass number (A) show that they are more than one stable isobar.

The most stable isobars for nuclide with odd and even mass number (A) have the highest value of binding energy (B.E.) and lie at or near the bottom of the mass parabola.

Some of most stable isobars which are determined for isobaric family with mass number A= [(15-30) & (101-115)] are the same in the case of isobars with odd mass number and even mass number, such as (Oxygen) which represents the stable nuclide for odd isobar with mass number equal to (A=17) as well as for even isobar with mass number equals to (A=16 & 18) in first range, and (Ruthenium) nuclide which represent the stable isobar in the case of odd mass number (A=103) and even mass number (A=102 &104) for second range.

The reason for this behavior is that these nuclides have high value of binding energy (B.E) comparison with adjacent nuclide.

3.2 determined the atomic number (Z_A) value for most stable isobars

The most stable isobar for isobars families under this study are also determined by calculating the (Z_A) from equation (3). The results are presented in column (2) of **Tables (1 &2).**

Then the atomic number of the most stable isobar (Z_A) is plotted as a function to the mass number (A) as shown in **Figures (3 & 4)**.

Table 1. Mass number (A), the atomic number for most stable isobar (Z_A) calculated from equation (3) for isobar with (A=15-30).

Mass number (A)	Atomic number for stable isobar (Z_A)
15	7.162028
16	7.624391
17	8.085258
18	8.544668
19	9.002655
20	9.459252
21	9.91449
22	10.3684
23	10.821
24	11.27232
25	11.72239
26	12.17123
27	12.61886
28	13.06529
29	13.51056
30	13.95467

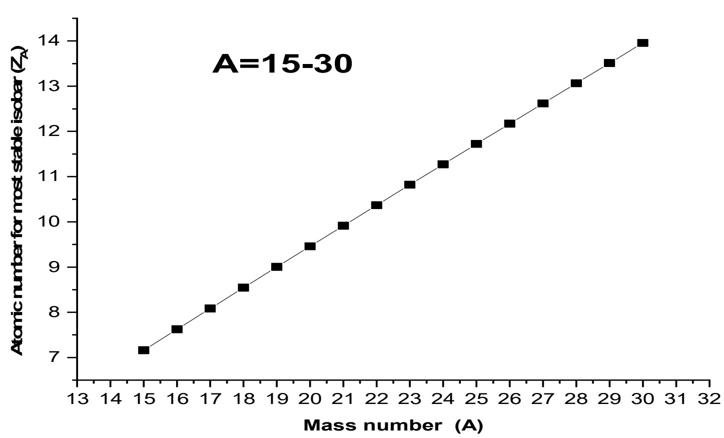

Figure 3. The relationship between atomic number for most stable isobar (Z_A) and mass number (A) for (A=15-30).

Table 2. Mass number (A), the atomic number for most stable isobar (Z_A) calculated from equation (3) for isobar with (A=101-115).

Mass number (A)	Atomic number for stable isobar (Z_A)
101	43.22642
102	43.61301
103	43.99901
104	44.38441
105	44.76922
106	45.15344
107	45.53708
108	45.92014
109	46.30263
110	46.68454
111	47.06589
112	47.44666
113	47.82688
114	48.20653
115	48.58563

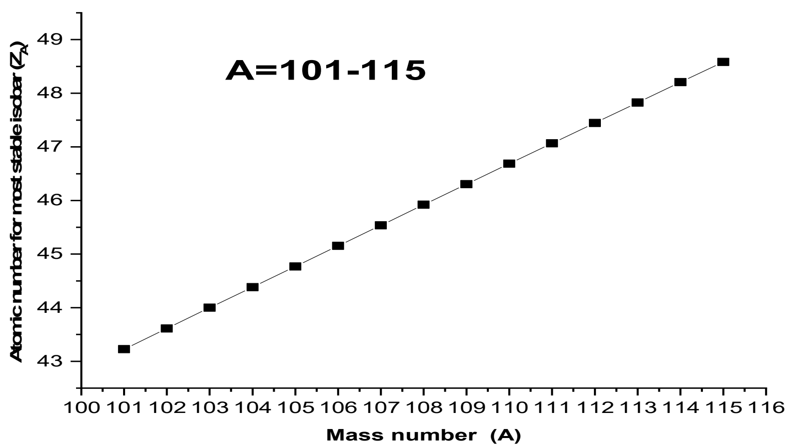

Figure 4. The relationship between atomic number for most stable isobar (Z_A) and mass number (A) for (A=101-115).

Figures (3&4) show a linear relationship among atomic number for most stable isobar (Z_A) and mass number.

3.3 comparison between most stable isobar determined by two methods (mass parabola and atomic number (Z_A)).

A Comparison has been done between the most stable isobars determined by the two methods which are described above (the lowest point of the mass parabola for each isobars and these which are determined from the calculating of the atomic number for most stable isobars (Z_A)) .This comparison is explained in tables (3 & 4) for isobaric family (A) = (15-30 & 101-115) respectively.

Table 3 . comparison between the most stable nuclide determined from mass parabola and atomic number for the most stable isobar (Z_A) value from the equation (3) for nuclides with (A=15-30).

Mass number (A)	Nuclide –Atomic number for stable isobar from mass parabola	Atomic number for stable isobar (Z_A) from eq.(3)
15	N-7	7.162028
16	N-7, O-8	7.624391
17	O-8	8.085258
18	O-8, F-9	8.544668
19	F-9	9.002655
20	F-9, Ne-10	9.459252
21	Ne-10	9.91449
22	Ne-10, Na-11	10.3684
23	Na-11	10.821
24	Na-11, Mg-12	11.27232
25	Mg-12	11.72239
26	Mg-12,Al-13	12.17123
27	Al-13	12.61886
28	Al-13, Si-14	13.06529
29	Si-14	13.51056
30	Al-13, Si-14	13.95467

Table 4.comparison between the most stable nuclide determined from mass parabola and atomic number for the most stable isobar (Z_A) value from the equation (3) for nuclides with (A=101-115).

Mass number (A)	Nuclide –Atomic number for stable isobar from mass parabola	Atomic number for stable isobar (Z_A) from eq.(3)
101	Tc-43	43.22642
102	Tc-43, Ru-44	43.61301
103	Ru-44	43.99901
104	Ru-44, Rh-45	44.38441
105	Rh-45	44.76922
106	Rh-45, Pd-46	45.15344
107	Pd-46	45.53708
108	Rh-45, Pd-46	45.92014
109	Pd-46	46.30263
110	Pd-46, Ag-47	46.68454
111	Ag-47	47.06589
112	Ag-47, Cd-48	47.44666
113	Cd-48	47.82688
114	Cd-48, In-49	48.20653
115	In-49	48.58563

From these **Tables,** we can note that the stable isobars are always the same nuclide for isobars with an odd mass number in the two methods, but in cases of even isobars which have two stable isobars (in mass parabola method) only one of them is same stable isobars in comparison to second method.

4. Conclusions

1. Most stable isobars located at the bottom of mass parabolas and have the highest value of binding energy (B.E).
2. In the figures of mass parabolas, the value of binding energy increased by increasing the atomic number (left side of mass parabola) until reaching to the most stable isobar, while in the (right side of mass parabola) it decreased with increasing the distance from the most stable isobar (bottom of mass parabolas).
3. Nuclides with an odd mass number (A) have only one mass parabola as well as one stable isobar, while nuclides with an even mass number (A) have two mass parabolas so they have more than one stable isobar.
4. The nuclides representing the most stable isobars in two methods, which are used in this study, are almost the same nuclide.

References

1. De Sanctis, E.; Monti, S.; Ripani, M. Energy from Nuclear Fission: An Introduction, Undergraduate Lecture Notes in Physics, *Springer International Publishing Switzerland*, **2016**, ISBN 978-3-319-30649-0.
2. Huda, A, Marid.; Naz, T, Jarallh.; The relationship of Nuclear Decay Methods (alpha and beta) Particles with the Nuclear Deformation for Nuclei inUranium-238 and Thorium -232 Series, *Energy Procedia, Elsevier Ltd*, **2019,** *157,* 270–275, doi: 10.1016/j.egypro.**2018**.*11.190*.
3. King, George, C. Physics of energy sources, (First edition*). Hoboken, New Jersey : John Wiley Sons, Inc*, **2018**, ISBN: 9781119961673.
4. Choppin, G.; Liljenzin, JO.; Rydberg, J. Radiochemistry and nuclear chemistry, *Butterworth-Heineman,* **2002**, ISBN:0750674636.
5. James, E, Turner. Atoms, radiation and radiation protection, *WILEY-VCH Verlag GmbH & Co.KGaA, Weinheim*, **2007**.
6. Mirzaei, Mahmoud, Abadi, Vahid1.; Mirhabibi Mohsen.; Askari Mohammad Bagher. Estimation of Semi-Empirical Mass Formula Coefficients, *Nucl. Sci*, **2017**, *2, 1, 11–15*.
7. Basdevant, JL.; Rich, J.; Spiro, M. Fundamentals in nuclear physics: From nuclear structure to cosmology, *Springer Science & Business Media*, **2005**.
8. Norman, D, Cook. Models of the atomic nucleus Unification through a lattice of nucleons, (second Edition),*Germany: Springer*, **2010**, http://dx.doi.org/10.1007/978-3-642-14737-1
9. Rajan, MK.; Biju, RK.; Santhosh, KP.; Empirical formula for the most stable isobar against beta decay, *In DAE Symp. Nucl. Phys*, **2017**, *62,* 440-441.
10. Tian, J.; Yuan. D, Han, S.; Huang, Y, Wang, N. Nuclear mass parabola and its applications, **2017**, arXiv preprint arXiv:1709.05451.
11. Cottingham, WN.; Greenwood, DA.; Greenwood, DA. An introduction to nuclear physics, *Cambridge University Press*, **2001**.
12. K, Heyde. Basic Ideas and Concepts in Nuclear Physics: An Introductory Approach, (Third Edition),*CRC Press*, **2004**, ISBN:1420054945.