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$\alpha g_{Ị}$-open sets and $\alpha g_{Ị}$-functions

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Abstract.

The objective of this paper is to show modern class of open sets which is an $\alpha g_{Ị}$-open. Some functions via this concept and the relationships among continuous function strongly $\alpha g_{Ị}$-continuous function $\alpha g_{Ị}$-irresolute function $\alpha g_{Ị}$-continuous function are studied.

Keywords. $\alpha g_{Ị}$-closed set, $\alpha g_{Ị}O$-functions, $\alpha g_{Ị}C$-functions, $\alpha g_{Ị}$-continuous function, Strongly $\alpha g_{Ị}$-continuous function, $\alpha g_{Ị}$-irresolute function, ideal.

1. Introduction.

An α-open was studied in 1965 by O. Njastad, a subset Ç is α-open set if $Ç \subseteq int(cl(int(Ç)))$[1,2]. The notion of ideal was studied by Kuratowski[3,4],that Ị is an ideal on Ӽ, where Ị is a collection of all subsets of Ӽ an ideal have two properties (if $Ç, Đ \in Ị$, then $Ç \cup Đ \in Ị$) and (if $Ç \in Ị$ and $Đ \subseteq Ç$, then $Đ \in Ị$.

There are many types for the ideal[5-7]

i. $Ị_{\{\emptyset\}}$: the trivial ideal where Ị={∅}.

ii. $Ị_{n}$: the ideal of all nowhere dense sets

$Ị_n = \{Ç \subseteq Ӽ: int(cl(Ç)) = \{\emptyset\}\}$.

iii. $Ị_{f}$: the ideal of all finite subsets of Ӽ

$Ị_f = \{ Ç \subseteq Ӽ$: Ç is a finite set$\}$.

The collection of all α-open sets is denoted by" $ĩ_{\alpha}$" and the collection of all α-closed is denoted by" $ɟ_{\alpha}$".

In this paper, we introduce $\alpha g_{Ị}$-closed set, and the complement of $\alpha g_{Ị}$-open set. More functions have been introduced via these concepts, such as $\alpha g_{Ị}$-open, $\alpha g_{Ị}^{*}$-open, $\alpha g_{Ị}^{**}$-open, $\alpha g_{Ị}$-continuous, $\alpha g_{Ị}$-irresolute and Strongly $\alpha g_{Ị}$-function.

2- On $\alpha g_{Ị}$-closed set

Definition 1: In ideal topological space (Ҳ, ĩ, Ị), Let Ç ⊆ Ҳ. Ç is said Ị-α-g-closed set denoted by "$\alpha g_{Ị}$-closed" ,if Ç-Ơ ∈ Ị then, cl(Ç)-Ơ ∈ Ị where Ơ ⊆ Ҳ and Ơ is an α-open sets.

Now, $Ç^c$ is Ị-α-g-open sets denoted by "$\alpha g_{Ị}$-open". The collection of all $\alpha g_{Ị}$-closed sets, where $Ç^c$ ∈ Ҳ, is denoted by "$\alpha g_{Ị}C$(Ҳ). The collection of all $\alpha g_{Ị}$-open sets "$\alpha g_{Ị}O$(Ҳ)".

Example 2: Consider the space (Ҳ, ĩ, Ị) where Ҳ={w,v}, ĩ={Ҳ,∅,{w}} and Ị={∅,{v}}.Then $ĩ_\alpha$={Ҳ,∅,{w}} and $ɟ_\alpha$={Ҳ,∅,{v}}, so $\alpha g_{Ị}C$(Ҳ) = $\alpha g_{Ị}O$(Ҳ) = {Ҳ,∅,{w},{v}}.

Example 3: Consider the space (Ҳ, ĩ, Ị) where Ҳ={w,v,z}, ĩ={Ҳ,∅,{w}} and Ị={∅,{v}}. Then $ĩ_\alpha$={Ҳ,∅,{w},{w,v},{w,z}} $ɟ_\alpha$={Ҳ,∅,{v,z},{z,},{v}}, so $\alpha g_{Ị}C$(Ҳ)={Ҳ,∅,{v,z},{z},{w,z}} $\alpha g_{Ị}O$(Ҳ) = {Ҳ,∅,{w},{w,v},{v}}.

Remark 4:

i. For each closed set in (Ҳ , ĩ) is an $\alpha g_{Ị}$-closed in (Ҳ, ĩ, Ị).
ii. For each open set in (Ҳ , ĩ) is an $\alpha g_{Ị}$-open in (Ҳ, ĩ, Ị).

Proof:

i. Let Ç is any closed set in (Ҳ, ĩ, Ị) and Ơ be an α-open set such that Ç-Ơ ∈ Ị since cl(Ç) = Ç this implies that Ç is an $\alpha g_{Ị}$-closed set.
ii. Let Ơ ∈ Ҳ, then $Ơ^c$ is a closed set this implies that $Ơ^c$ is an $\alpha g_{Ị}$-closed set, so Ơ is an $\alpha g_{Ị}$-open set.

The reverse way of Remark 2.4 is wrong in general see Example 2.2.

Remark 5: A space (Ҳ, ĩ, Ị):

i. If Ị = Ᵽ(Ҳ) then $\alpha g_{Ị}C$(Ҳ) = $\alpha g_{Ị}O$(Ҳ) = Ᵽ(Ҳ).
ii. If ĩ = D then $\alpha g_{Ị}C$(Ҳ) = $\alpha g_{Ị}O$(Ҳ) = Ᵽ(Ҳ).

Remark 6: For any space (Ҳ, ĩ, Ị), then the two idea $\alpha g_{Ị}$-closed set and αg^*-closed set are the same, if Ị={∅}.

The following example display that the two notion $\alpha g_{Ị}$-closed set and αg^*-closed set are separate, in general.

Example 7:

i. The set {w} in Example 2.2 is an $\alpha g_{Ị}$-closed set but not αg^*-closed set, and {v} is an $\alpha g_{Ị}$-open set, but not αg^*-open set.
ii. For a space (Ҳ, ĩ, Ị), where Ҳ={ṟ,ş,w,v}, ĩ = {Ҳ,∅,{ṟ,ş},{w,v}} and Ị={∅,ṟ}. Then $ĩ_\alpha$= ĩ, leads to αg^*C(Ҳ) = Ᵽ(Ҳ) and αg^*O(Ҳ) = Ᵽ(Ҳ). It seems obvious that the set {ṟ} is αg^*-closed set but not $\alpha g_{Ị}$-closed.

Remark 8: For any set Ҳ, let ҳ ∈ Ҳ and ĩ = {Ҳ,∅,{ҳ}}, Ị=$Ị_n$={Ç ⊆ Ҳ: int(cl(Ç))={∅}} then $\alpha g_{Ị}C$(Ҳ) = Ᵽ(Ҳ).

Proof:

Let $Ị_n$ = { Ç ⊆ Ҳ: int(cl(Ç)) = {∅}}, Ҳ be any set and ĩ = {Ҳ, ∅, {ҳ}} such that ҳ ∈ Ҳ, $ĩ_\alpha$={Ơ ⊆ Ҳ; ҳ ∈ Ơ} ∪ {∅}, for any set Ç ⊆ Ҳ, and Ơ is α-open set, if Ç-Ơ ∈ $Ị_n$ this implies ҳ ∉ (Ç-Ơ), so

cl(Ç-Ơ) = Ҳ/{ҳ}, then $int(cl$(Ç-Ơ)) = ∅, then ҳ ∉ Ç and ҳ ∈ Ơ, since ҳ ∉ Ç this implies cl(Ç) = Ҳ/{ҳ}, thus (Ҳ/{ҳ}-Ơ)∈ $Ị_n$,if ҳ ∈ Ç and ҳ ∈ Ơ then ҳ ∉ (cl(Ç)-Ơ), so cl(Ç)-Ơ ∈ $Ị_n$, hence $\alpha g_{Ị}C$(Ҳ) = Ᵽ(Ҳ).

Theorem 9: Let Ç and Đ are two $\alpha g_{Ị}$-closed sets then Ç ∪ Đ is an $\alpha g_{Ị}$-closed.
Proof: Let Ç and Đ are two $\alpha g_{Ị}$-closed set in (Ҳ, ĩ, Ị) and Ơ ∈ $ĩ_\alpha$ subset of Ҳ, where (Ç ∪ Đ)-Ơ ∈ Ị, then Đ-Ơ ∈ Ị and Ç-Ơ ∈ Ị, so cl(Đ)-Ơ ∈ Ị and cl(Ç)-Ơ ∈ Ị therefore, (cl(Ç)-Ơ) ∪ (cl(Đ)-Ơ) ∈ Ị, *so* cl(Ç ∪ Đ)-Ơ ∈ Ị.Hence Ç ∪ Đ is an $\alpha g_{Ị}$-closed sets.

Corollary 10: Let Ç and Đ are two $\alpha g_{Ị}$-open sets then Ç ∩ Đ is an $\alpha g_{Ị}$-open.
Proof: Let Ç and Đ are two $\alpha g_{Ị}$-open sets in Ҳ then $Ç^c$, $Đ^c$ are two $\alpha g_{Ị}$-closed sets therefore, $Ç^c$ ∪ $Đ^c$ is an $\alpha g_{Ị}$-closed set by theorem 2.9. Hence$(Ç ∩ Đ)^c$ is an $\alpha g_{Ị}$-closed set so Ç ∩ Đ is an $\alpha g_{Ị}$-open set.

Remark 11:
i. The union of any collection of $\alpha g_{Ị}$-closed sets is not necessarily $\alpha g_{Ị}$-closed.
ii. The intersection of collection of $\alpha g_{Ị}$-open sets is not necessarily $\alpha g_{Ị}$-open.

For example: Consider a space (Ҳ, ĩ, Ị), when Ҳ = Ɲ, the set of all natural numbers, ĩ = ĩ cof, is a topology of all sets that complement is a finite set and Ị=$Ị_{Ị_f}$={Ơ ⊆ Ɲ, Ơ is a finite set}, $ĩ_\alpha$={Ơ ⊆ Ɲ, Ơ is an infinite set}∪{∅}.
Clearly, {ŋ} is an $\alpha g_{Ị}$-closed set, ∀ ŋ ∈ $Ȩ^+$, where $Ȩ^+$ is the positive even numbers, but ∪{{ŋ}:ŋ∈ $Ȩ^+$}=$Ȩ^+$ which is not $\alpha g_{Ị}$-closed set. Similarly; $Ç_n$=Ɲ-{ŋ} is an of $\alpha g_{Ị}$-open set, ∀ ŋ ∈ $Ȩ^+$ but ∩{$Ç_n$: ŋ ∈ $Ȩ^+$}=$Ộ^+$, where $Ộ^+$ is the positive odd number ,$Ộ^+$ is not $\alpha g_{Ị}$-closed set.

Theorem 12: In (Ҳ, ĩ, Ị), let Ç ⊆ Ҳ. Ç is an $\alpha g_{Ị}$-open set if and only if (Ꞙ − int(Ç)) ∈ Ị, whenever (Ꞙ-Ç) ∈ Ị, ∀ Ꞙ ∈ $ɟ_\alpha$.
Proof: (→)Let Ç ⊆ Ҳ, where Ç be an $\alpha g_{Ị}$-open sets and (Ꞙ-Ç) ∈ Ị, Ꞙ ∈ $ɟ_\alpha$, since (Ҳ-Ç) is an $\alpha g_{Ị}$-closed set and (Ҳ-Ç)-Ơ ∈ Ị, Ơ ∈ $ĩ_\alpha$implies cl(Ҳ-Ç)-Ơ ∈ Ị, whenever (Ҳ-Ç)-Ơ ∈ Ị, for each Ơ ∈ $ĩ_\alpha$, cl(Ҳ-Ç)-Ơ = (Ҳ-Ơ)-(Ҳ-cl(Ҳ-Ç) since Ç-Đ = (Ҳ-Đ)-(Ҳ-Ç), thus (Ҳ-Ơ)-(Ҳ-(Ҳ-int(Ҳ-Ҳ-Ç))) = (Ҳ-Ơ)-int(Ç) = Ꞙ-int(Ç) ∈ Ị.

(←) Let Ꞙ-int(Ç) ∈ Ị, whenever Ꞙ-Ç ∈ Ị, for each Ꞙ ∈ $ɟ_\alpha$. Let (Ҳ-Ç)-Ơ ∈ Ị; Ơ ∈ $ĩ_\alpha$, (Ҳ-Ç)-Ơ = (Ҳ-Ơ)-Ç ∈ Ị, let Ҳ-Ơ = Ꞙ ∈ $ɟ_\alpha$ and Ꞙ-Ç ∈ Ị this implies Ꞙ-int(Ç) ∈ Ị, now Ꞙ-int(Ç) = cl(Ҳ-Ç)-(Ҳ-Ꞙ) = cl(Ҳ-Ç)-Ơ ∈ Ị, thus (Ҳ-Ç) is an $\alpha g_{Ị}$-closed set, hence Ç is an $\alpha g_{Ị}$-open set.

3-Open function

Definition 1: The function ƒ: (Ҳ, ĩ, Ị) → (Υ, ɟ, ʝ) is called;
i. $\alpha g_{Ị}$-open function, denoted by "$\alpha g_{Ị}$o-function" if ƒ(Ơ) is an $\alpha g_{ʝ}$-open set in Υ. Whenever Ơ is an $\alpha g_{Ị}$-open in Ҳ.
ii. $\alpha g^*_{Ị}$-open function, denoted by "$\alpha g^*_{Ị}$o-function" if ƒ(Ơ) is an $\alpha g_{ʝ}$-open set in Υ. Whenever Ơ ∈ ĩ.

iii. $\alpha g_{Ị}^{**}$-open function, denoted by "$\alpha g_{Ị}^{**}$o-function" if ƒ(Ơ) is an open set in Ƴ. Whenever Ơ is an $\alpha g_{Ị}$-open set in Ӽ.

Proposition 2: Let ƒ: (Ӽ, ῑ, Ị) → (Ƴ, ɟ, ʝ) is a function;

i. If ƒ is an open function then ƒ is $\alpha g_{Ị}^{*}$o-function

Proof: Let Ơ ∈ ῑ, since ƒ is an open function then ƒ(Ơ)∈ ɟ and since for each open sets is an $\alpha g_{Ị}$-open set then ƒ(Ơ) is an $\alpha g_{ʝ}$-open set in Ƴ, then ƒ is an $\alpha g_{Ị}^{*}$o-function.

ii. If ƒ is an $\alpha g_{Ị}^{**}$o-function then ƒ is an $\alpha g_{Ị}$-open function.

Proof: Let Ơ be an $\alpha g_{Ị}$-open set in Ӽ, since ƒ is an $\alpha g_{Ị}^{**}$o-function, then ƒ(Ơ)∈ ɟ, since for each open set is an $\alpha g_{Ị}$-open set, this implies that ƒ(Ơ) is an $\alpha g_{ʝ}$-open set in Ƴ, then ƒ is an $\alpha g_{Ị}$-open function.

iii. If ƒ is an $\alpha g_{Ị}$o-function then ƒ is an $\alpha g_{Ị}^{*}$o-function.

Proof: Let Ơ ∈ ῑ, since for each open set is an $\alpha g_{Ị}$-open set, then ƒ(Ơ) is an $\alpha g_{ʝ}$-open set in Ƴ, thus ƒ is an $\alpha g_{Ị}^{*}$o-function.

iv. If ƒ is an $\alpha g_{Ị}^{**}$o-function then ƒ is an open function.

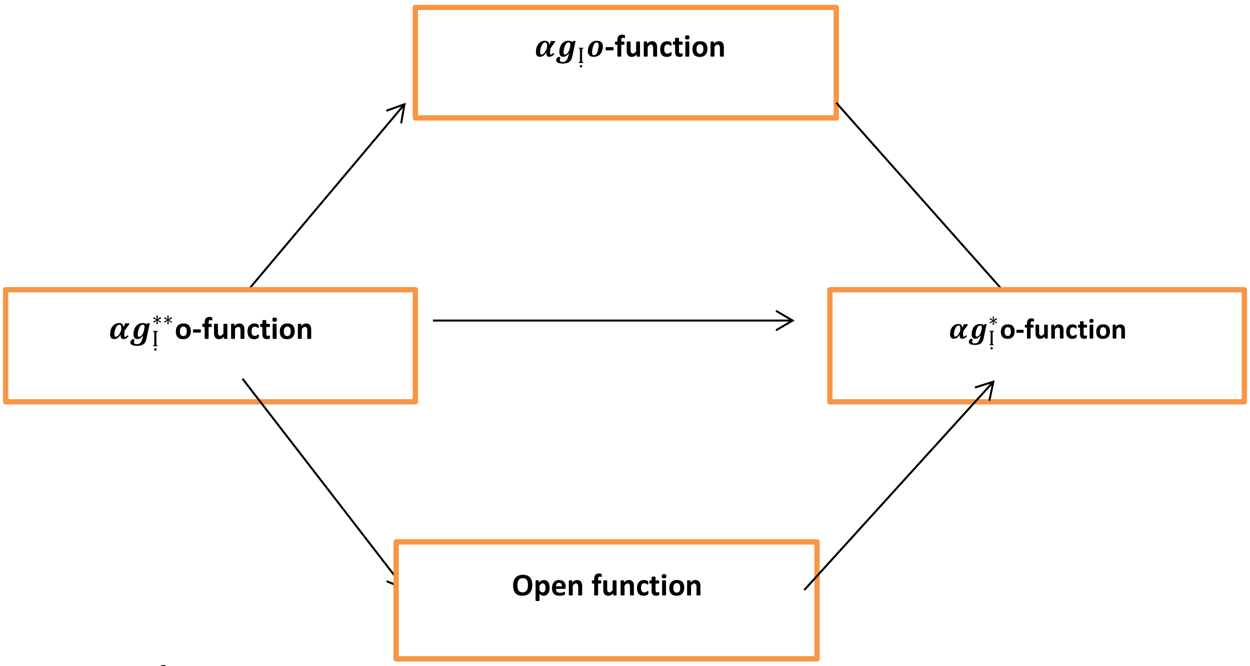
Proof: Let Ơ ∈ ῑ, since for each open set is an $\alpha g_{Ị}$-open set, then Ơ be an $\alpha g_{Ị}$-open set in Ӽ, since ƒ is an $\alpha g_{Ị}^{**}$o-function thus ƒ(Ơ) is an open set in Ƴ, then ƒ is an open function.

v. If ƒ is an $\alpha g_{Ị}^{**}$o-function then ƒ is an $\alpha g_{Ị}^{*}$o-function.

Proof: By proposition 3.2-ii and proposition 3.2-iii, prove is over.

The following scheme explains the relationship between the various concepts presented in Definition 3.1.

Arrow chart
(3.1)



$\alpha g_{Ị}$-open function

The following are some examples showing that the opposite direction of the above schema is incorrect.

Example 3: A function $f: (X, \tau, I) \rightarrow (X, \tau, J)$, where $X = \{e_1, e_2, e_3\}$ such that $f(e_1) = (e_2)$, $f(e_2) = (e_1)$, $f(e_3) = (e_3)$, $\tau = \{X, \emptyset, \{e_1\}\}$, $I = \{\emptyset\}$ and $J = \{\emptyset, \{e_2\}, \{e_3\}, \{e_2, e_3\}\}$ then $\tau_\alpha = \{X, \emptyset, \{e_1\}, \{e_1, e_2\}, \{e_1, e_3\}\}$ then $\alpha g_I C(X) = \{X, \emptyset, \{e_2, e_3\}\}$ and $\alpha g_I O(X) = \{X, \emptyset, \{e_1\}\}$. So $\alpha g_J C(X) = P(X)$ and $\alpha g_J O(X) = P(X)$.

Then f is αg_Io-function and αg_I^*o-function which is not αg_I^{**}o-function and not an open function, since $\{e_1\}$ is an open set in X and αg_I-open set, but $f(e_1) = (e_2)$ which is not open.

Example 4: The function $f: (X, \tau, I) \rightarrow (X, \tau, I)$; where $X = \{e_1, e_2, e_3\}$ such that $f(e) = (e), \forall e \in X$, $\tau = \{X, \emptyset, \{e_1\}\}$, $I = \{\emptyset, \{e_2\}, \{e_3\}, \{e_2, e_3\}\}$ and $J = \{\emptyset\}$. Then $\tau_\alpha = \{X, \emptyset, \{e_1\}, \{e_1, e_2\}, \{e_1, e_3\}\}$ then $\alpha g_I C(X) = P(X)$ and $\alpha g_I O(X) = P(X)$. So $\alpha g_J C(X) = \{X, \emptyset, \{e_2, e_3\}\}$ and $\alpha g_J O(X) = \{X, \emptyset, \{e_1\}\}$.

It is easy to see that f is an open function and αg_I^*o-function but it is not αg_Io-function and not αg_I^{**}c-function, since $\{e_2\} \in \alpha g_I O(X)$ but $f(e_2) = (e_1)$ which is not open and not αg_J-open set.

Definition 5: The function $f: (X, \tau, I) \rightarrow (Y, \sigma, J)$ is said,

i. αg_I-closed function, denoted by "αg_Ic-function" if $f(O)$ is αg_J-closed in Y whenever O is an αg_I-closed in X.

ii. αg_I^*-closed function, denoted by "αg_I^*c-function", if $f(O)$ is αg_J-closed in Y whenever O is an closed in X.

iii. αg_I^{**}-closed function, denoted by "αg_I^{**}c-function", if $f(O)$ is closed in Y whenever O is an αg_I-closed in X.

Proposition 6: Let $f: (X, \tau, I) \rightarrow (Y, \sigma, J)$ is function,

i. If f is a closed function then f is an αg_I^*c-function.

ii. If f is an αg_I^{**}c-function then f is an αg_Ic-function.

iii. If f is an αg_I^{**}c-function then f is a closed function.

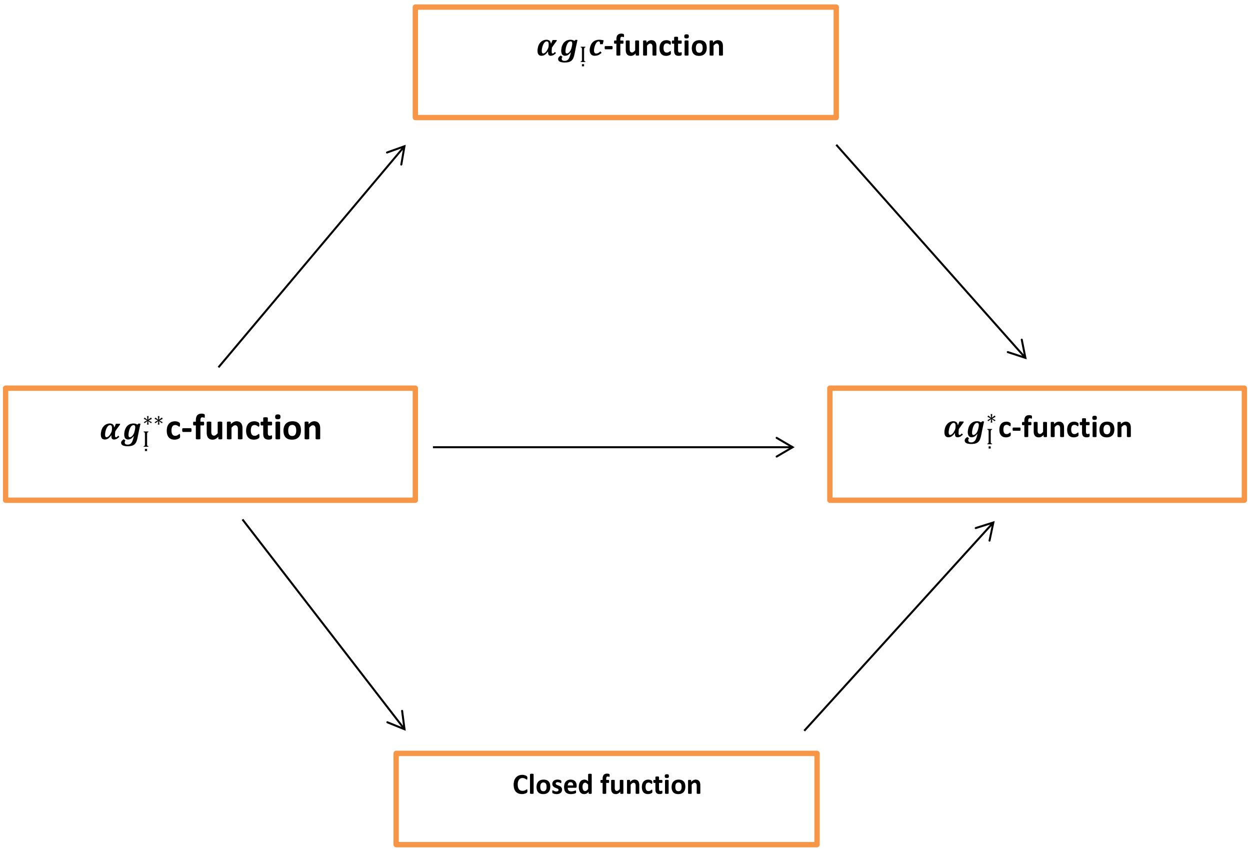
iv. If f is an αg_Ic-function then f is an αg_I^*c-function.

v. If f is an αg_I^{**}c-function then f is an αg_I^*c-function.

Proof: By Remark 2.4 and Definition 3.5.

The follow Diagram shows the relationships between the different concepts that are inserted in Definition 3.5

Arrow chart
(3.2)



$\alpha g_{Ị}$-closed function

Example 3.3 and 3.4 show that the opposite direction of the above chart is incorrect.

Remark 7: If ƒ is onto function then:

i. $\alpha g_{Ị}$o-function and $\alpha g_{Ị}$c-function are the same.
ii. $\alpha g_{Ị}^{*}$o-function and $\alpha g_{Ị}^{*}$c-function are the same.
iii. $\alpha g_{Ị}^{**}$o-function and $\alpha g_{Ị}^{**}$c-function are the same.

Proof: since ƒ is an onto function then the prove is easy by using Definition 3.1 and Definition 3.5

4- Near continuous function

Definition 1: A function ƒ: (Ӽ, ĩ, Ị) → (ϒ, ƫ, ʝ) is called;

i. Ị-α-g-continuous function, denoted by "$\alpha g_{Ị}$-continuous function", if $ƒ^{-1}$(Ơ) is an $\alpha g_{Ị}$-open set in Ӽ, where Ơ ∈ ƫ.
ii. Strongly Ị-α-g-continuous function, denoted by "Strongly $\alpha g_{Ị}$-continuous function" if $ƒ^{-1}$(Ơ) ∈ ĩ, whenever Ơ is an $\alpha g_{ʝ}$-open set in ϒ.
iii. Ị-α-g-irresolute function, denoted by "$\alpha g_{Ị}$-irresolute function", if $ƒ^{-1}$(Ơ) is an $\alpha g_{Ị}$-open set in Ӽ, where Ơ is an $\alpha g_{ʝ}$-open set in ϒ.

Proposition 2: Let ƒ: (Ҳ, ĩ, Ị) → (Ύ, ɟ, ʝ) is a function;

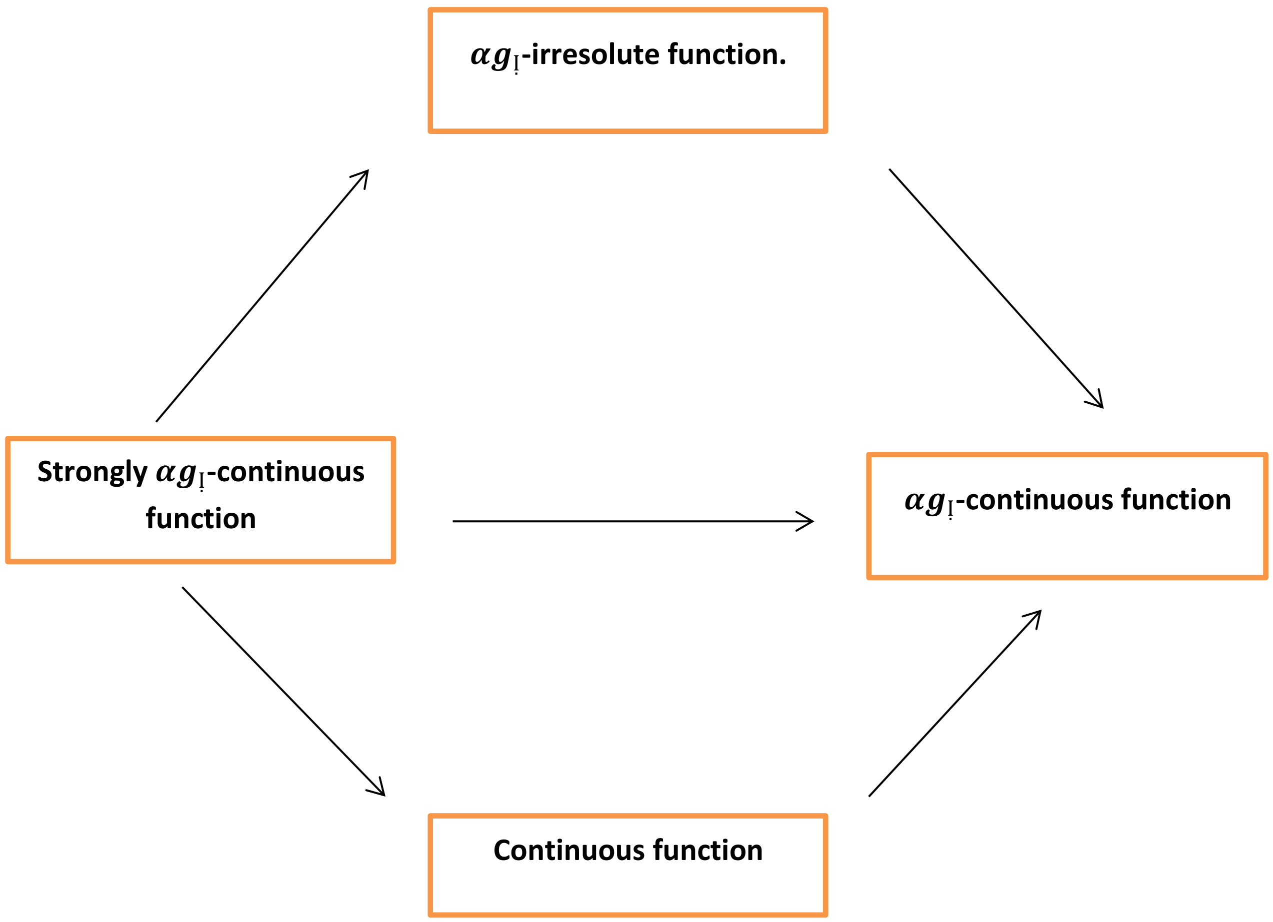
i. If ƒ is a continuous function, then ƒ is an αg_{I}-continuous function.
ii. If ƒ is Strongly αg_{I}-continuous function, then ƒ is a continuous function.
iii. If ƒ is an αg_{I}-irresolute function, then ƒ is an αg_{I}-continuous function.
iv. If ƒ is Strongly αg_{I}-continuous function, then ƒ is an αg_{I}-irresolute function.
v. If ƒ is Strongly αg_{I}-continuous function, then ƒ is an αg_{I}-continuous function.

Proof:

i. Let Ơ ∈ ɟ. Since ƒ is a continuous function, then $ƒ^{-1}$(Ơ) ∈ ĩ. $ƒ^{-1}$(Ơ) is an αg_{I}-open set in Ҳ By Remark 2.4. Hence ƒ is an αg_{I}-continuous function.
ii. Let Ơ ∈ ɟ. By Remark 2.4, Ơ is an αg_{j}-open set in Ύ. Since ƒ is Strongly αg_{I}-continuous function, then $ƒ^{-1}$(Ơ) ∈ ĩ. Hence ƒ is a continuous function.
iii. Let Ơ ∈ ɟ, this implies to Ơ is αg_{j}-open set in Ύ. Since ƒ is an αg_{I}-irresolute function then $ƒ^{-1}$(Ơ) is an αg_{I}-open set in Ҳ. Then ƒ is an αg_{I}-continuous function
iv. Let Ơ is an αg_{j}-open set in Ҳ. Since ƒ is a Strongly αg_{I}-continuous function, then $ƒ^{-1}$(Ơ) ∈ ĩ. By Remark 2.4, ƒ(Ơ) is αg_{I}-open set in Ҳ. This implies ƒ is an αg_{I}-irresolute function.
v. Let Ơ ∈ ɟ this implies Ơ is an αg_{j}-open set and since ƒ is a Strongly αg_{I}-continuous function, thus $ƒ^{-1}$(Ơ) is open set in Ҳ by Remark 2.4 $ƒ^{-1}$(Ơ) is an αg_{I}-open set, so ƒ is an αg_{I}-continuous function.

The follow scheme shows the relation between the variant notions were presented in Definition 4.1.

Arrow chart
(4.1)



Ị-α-g-continuous function

The following are some examples showing that the opposite direction of the above schema is incorrect.

Example 3: The function ƫ: (X̧, ĩ, Ị) → (X̧, ĩ, ȷ̣), where X̧={ė$_1$, ė$_2$, ė$_3$} such that ƫ(ė$_1$)=(ė$_1$), ƫ(ė$_2$)=(ė$_2$), ƫ(ė$_3$)=(ė$_3$), ĩ = {X̧,∅,{ė$_1$}}, Ị={∅} and ȷ̣={∅,{ė$_2$},{ė$_3$},{ė$_2$,ė$_3$}} then ĩ$_\alpha$={X̧,∅,{ė$_1$},{ė$_1$, ė$_2$},{ė$_1$, ė$_3$}} then $\alpha g_{Ị}C$(X̧) = {X̧,∅,{ė$_2$,ė$_3$}} and $\alpha g_{Ị}O$(X̧) = {X̧,∅,{ė$_1$}}. So $\alpha g_{ȷ̣}C$(X̧) = Ᵽ(X̧) and $\alpha g_{ȷ̣}O$(X̧) = Ᵽ(X̧).

It is possible to see clearly that ƫ is continuous and $\alpha g_{Ị}$-continuous function but not $\alpha g_{Ị}$-irresolute function since {ė$_3$} is an $\alpha g_{ȷ̣}$-open set in Y but ƫ$^{-1}$(ė$_3$) = ė$_3$ is not an $\alpha g_{Ị}$-open set in X̧.

Example 4: The function ƫ: (X̧, ĩ, Ị) → (X̧, ĩ, ȷ̣), where X̧={ė$_1$, ė$_2$, ė$_3$} such that ƫ(ė$_1$)=(ė$_1$), ƫ(ė$_2$)=(ė$_2$), ƫ(ė$_3$)=(ė$_3$), ĩ = {X̧,∅,{ė$_1$}}, ȷ̣={∅} and Ị={∅,{ė$_2$},{ė$_3$},{ė$_2$,ė$_3$}} then ĩ$_\alpha$={X̧,∅,{ė$_1$},{ė$_1$, ė$_2$},{ė$_1$, ė$_3$}} then $\alpha g_{ȷ̣}C$(X̧) = {X̧,∅,{ė$_2$,ė$_3$}} and $\alpha g_{ȷ̣}O$(X̧) = {X̧,∅,{ė$_1$}}. So $\alpha g_{Ị}C$(X̧) = Ᵽ(X̧) and $\alpha g_{Ị}O$(X̧) = Ᵽ(X̧).

It is possible to see clearly that ƫ is $\alpha g_{Ị}$-continuous function but not continuous function since {ė$_1$}∈ ĩ but ƫ$^{-1}$(ė$_1$) = ė$_2$ is not open in X̧, and not Strongly $\alpha g_{Ị}$-continuous function since {ė$_1$} ∈ $\alpha g_{ȷ̣}O$(X̧) but ƫ$^{-1}$(ė$_1$) = ė$_2$ is not open in X̧.

5- Conclusion

The concept of closed and open sets was used with the ideal concept to introduce new notions from these categories; $\alpha g_{Ị}$-closed set, $\alpha g_{Ị}$-open set. And we introduce a new functions like: $\alpha g_{Ị}$-open function, $\alpha g_{Ị}^{*}$-open function, $\alpha g_{Ị}^{**}$-open function, $\alpha g_{Ị}$-closed function, $\alpha g_{Ị}^{*}$-closed function and $\alpha g_{Ị}^{**}$-closed function with near continuous functions.

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