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A Study of Shigellosis Bacteria disease Model with Awareness Effects

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Abstract

In this paper, a mathematical model is proposed and studied to describe the spread of shigellosis disease in the population community. We consider it divided into four classes namely: the 1st class consists of unaware susceptible individuals, 2nd class of infected individuals, 3rd class of aware susceptible individuals and 4th class are people carrying bacteria. The solution existence, uniqueness as well as bounded-ness are discussed for the shigellosis model proposed. Also, the stability analysis has been conducted for all possible equilibrium points. Finally the proposed model is studied numerically to prove the analytic results and discussing the effects of the external sources for disease and media coverage on the dynamical behaviors of shigellosis disease.

Keywords: Shigellosis Disease, Awareness, Media Coverage, Stability, External Sources.

1. Introduction

In fact, the infectious diseases always have an impact on people's health, so it is necessary to study the mechanism by which the disease spread and the conditions of minor and major infections and learn how to control diseases. Recently, a global pandemic have spread in most countries of the world, which is the Corona epidemic (nCovid19), as it appeared in the State of in Wuhan and moved to most countries of the world, where the number of cases of this disease reached more than 6,048,844 million injuries and the number of deaths was more than 367,227 thousand.

Throughout history, infectious diseases have had a major impact on the population. The effects of epidemics are the most obvious and exciting. In the last decade, that was, from 2010 to 2020, many epidemics have spread, most notably new types of influenza such as the Middle East corona and swine flu, and preceded them in the previous decade. For the World Health Organization, swine flu was considered one of the most dangerous viruses. In June 2012, a report was published for a study of a group of doctors, researchers and agencies, announcing the death of 280,000 people, while in 2010, the World Health Organization announced that 18,000 people died as a result of the epidemic.

An epidemic has played a major role of infectious diseases in the formation of the invasion of the New World and trolls these epidemics are epidemiologists in coordination with others in the medical field and researchers.

We well know, the many infectious disease are spread by virus as COVID-19 or bacteria as Cholera it is caused by the bacterium Vibrio cholera and there were many researchers studied the Cholera epidemic. Ridha and Muhseen [1] proposed the epidemic disease model with general recovery function. Muhseen and Zhou [2] studied spread Cholera disease with nonlinear incidence rate. M. Al-arydah, et al [3] studied cholera disease with education and chlorination. F.J. Luquero, et al [4] used vibrio cholera vaccine in an outbreak in Guinea.
We depend here on one from the basic modeling to study and analyze the infection disease which is the SIR model of Kermack and Mckendrick [5-7. The mathematical modeling is an important interdisciplinary activity in the study of some aspects of various disciplines. Bailey [8] and several authors provided many models for the spread and control of infectious diseases [9-12]. Methods used to control epidemics are outreach programs driven by the media that can modify the behavior human towards the disease. These awareness campaigns may differ between the various groups at risk that help limit the spread of infection. We would like to mention those who have a clear evaluation of the impact of awareness programs [13-15]. There are studies on the effect of media and the effectiveness of using face masks to reduce the spread of the influenza epidemic [16-17]. Misra [18] presented a non-linear mathematical model that evaluated awareness programs and showed their control that the outbreak of infectious diseases can be reduced through media coverage. In this work, we have proposed the mathematical model of the shigella disease involving media programs effect to control the spread the disease. We displayed the full details of the mathematical modeling of the shigella disease in section 2. We have also discussed some basic properties equilibrium points in section 3. In section 4 the local stability analysis was studied with the support of Gresgorin theorem. Furthermore, we studied the using of Lyapunov function to show the global stability of the proposed model at all equilibrium points in section 5. Finally, the effect of media coverage have been done to awareness for the shigella disease and also the risk of direct and indirect contact with carrier individuals on the out breaking of the shigella disease in the population. This was done through a numerical simulation. A discussion of the results effects and limitations involved were concluded in this paper.

2. Model formulation:

In this section, the population in this work can divided into to four classes, namely unaware susceptible class denoted by (S_u), infected class denoted by (I), aware susceptible denoted by (S_a), people carrying bacteria denoted by (B) and the number of media campaigns in that region at time t denoted by (M). The model is given by the following differential equation.

$$\begin{aligned}
\dot{S_u} &= \theta - a_1 S_u B - a_2 S_u M - a_3 S_u I - a_4 S_u - \mu S_u \\
\dot{I} &= a_1 S_u B + a_3 S_u I + a_4 S_u - (\mu + d + \eta) I \\
\dot{S_a} &= a_2 S_u M - \mu S_a \\
\dot{B} &= rB\left(1 - \frac{B}{K}\right) - \mu B + \epsilon I \\
\dot{M} &= \alpha (S_u + I) - \gamma M
\end{aligned} \tag{1}$$

With initial conditions $S_u(0) > 0, I(0) \geq 0, S_a(0) \geq 0, B(0) \geq 0, M(0) \geq 0$. The natural birth into susceptible class by $\theta > 0$. $a_1 > 0$ is the incidence of the disease by direct contact with the carriers of the bacteria. $a_2 > 0$ is the awareness rate. $a_3 > 0$ is the contact rate between susceptible and infected. $a_4 \geq 0$ represents the number of cases of the disease due to the external sources such as (food, water,…etc). $\mu > 0$ is the natural death rate of the in population . $d > 0$ is the disease related death. $\eta > 0$ is the removal rate. r and k are respectively, the growth rate carrying capacity of shigella disease. $\epsilon > 0$ is the increasing of the shigella disease bacteria due to infected class. $\alpha > 0$ represents the implementation rate of awareness programs which is proportional to the number of unaware susceptible and infective individuals in the population. Finally the depletion rate of awareness programs due to ineffectiveness, social problems in the population and similar factors is represented by $\gamma > 0$.

Theorem (1): The uniformly bounded of the any solutions are discussed in the following.

Proof: Let $(S_u(t), I(t), S_a(t), B(t), M(t))$ is the solution of the system (1) with positive initial condition $(S_u(0), I(0), S_a(0), B(0), M(0))$ which defines the function

$N(t) = S_u(t) + I(t) + S_a(t) + B(t)$ then take the time derivative of $N(t)$ along the solution of the system (1);this gives

$$\begin{aligned}
&\frac{dN}{dt} = \Theta + rB\left(1 - \frac{B}{N}\right) - q(S_u + I + S_a + B) \\
&\frac{dN}{dt} \leq H - qN \ ; \ H = \frac{rK}{4} + \Theta, \qquad q = min\{\mu, \mu - \epsilon\} \\
&\frac{dN}{dt} + qN \leq H
\end{aligned}$$

Clearly, by solving the above equation, we obtain

$$N(t) \leq \frac{H}{q} + \left(N_0 - \frac{H}{q}\right) e^{-qt}$$

Therefore, $N(t) \leq \frac{H}{q}, \qquad as\ t \to \infty$

$$\begin{aligned}
&\frac{dM}{dt} = \alpha\,(S_u + I) - \gamma M \\
&\frac{dM}{dt} \leq \widetilde{H} - \gamma M \ ; \ \widetilde{H} = \alpha\,(S_u + I)
\end{aligned}$$

By a similar way we get:

$$M(t) \leq \frac{\alpha H}{\gamma q}, \qquad as\ t \rightarrow \infty$$

We obtain that, the solution of system (1) is confined in the following region

$$\Omega = \left\{(S_u, I, S_a, B, M) \in R_+^5 : N \leq \frac{H}{q}, 0 \leq M \leq \frac{\alpha H}{\gamma q}\right\}$$

Thus, these solutions are uniformly bounded and the proof is complete. ■

3. The number of equilibrium points:

It is easy that aware susceptible S_a is related with variable $S_u(t)$ and $M(t)$ only. Hence for fixed values of, $S_u(t)$ and $M(t)$, the calculate value of S_a can be found simply by solving the system (1). In fact, we can determine the value of S_a by the following equation

$$S_a = \frac{a_2 \tilde{S}_u \tilde{M}}{\mu} \tag{2}$$

Consequently, we can reduce system (1) and rewrite it to the following system

$$\begin{aligned}
\dot{S_u} &= \theta - a_1 S_u B - a_2 S_u M - a_3 S_u I - a_4 S_u - \mu S_u \\
\dot{I} &= a_1 S_u B + a_3 S_u I + a_4 S_u - (\mu + d + \eta) I \\
\dot{B} &= rB\left(1 - \frac{B}{K}\right) - \mu B + \epsilon I \\
\dot{M} &= \alpha\,(S_u + I) - \gamma M
\end{aligned} \tag{3}$$

Clearly, there are only two equilibrium points of system (3) under the following conditions:

- The first equilibrium point exists when $I = 0$ (when $a_4 = 0$) and $B = 0$, and is called disease free equilibrium point which is denoted by $E_0 = (S_u^{\wedge}, 0, 0, M^{\wedge})$, where

$$M^{\wedge} = \frac{\alpha}{\gamma} S_u^{\wedge} \tag{4}$$

While S_u is a positive real root of the following quadratic equation

$$A_1 S_u^2 + A_2 S_u + A_3 = 0 \tag{5a}$$

Here

$$S_u = \frac{-\left(A_2 + \sqrt{A_2^2 - 4A_1A_3}\right)}{2A_1} \tag{5b}$$

$$A_1 = \frac{-a_2 \alpha}{\gamma}$$
$$A_2 = -\mu$$
$$A_3 = \theta$$

- The endemic equilibrium point, denoted by $E_1 = (S_u^*, I^*, B^*, M^*)$ where

$$I^* = \frac{B^*(\mu k + rB^* - rk)}{k\epsilon} \tag{6a}$$

$$M^* = \frac{\alpha(k\epsilon S_u^* + (\mu - r)kB^* + rB^{*2})}{k\epsilon\gamma} \tag{6b}$$

While (S_u^*, B^*) represents a positive intersection point of the following two isoclines:

$$f(S_u, B) = m_1 S_u B^2 + m_2 B^2 + m_3 S_u B + m_4 S_u + m_5 B = 0 \tag{7}$$

$$g(S_u, B) = n_1 S_u B^2 + n_2 S_u^2 + n_3 S_u B + n_4 S_u + n_5 = 0 \tag{8}$$

Here,

$$\begin{aligned}
&m_1 = a_3 r, m_2 = -(\mu + d + \eta), \mathrm{m}_3 = a_1 k\epsilon + a_3(\mu - r)k \\
&m_4 = a_4 k\epsilon, \quad m_5 = -(\mu + d + \eta)(\mu - r) \\
&\mathrm{n}_1 = -(a_2 \mathrm{r} + a_3 \gamma r), \qquad \mathrm{n}_2 = -a_2 \alpha k\epsilon \\
&\mathrm{n}_3 = -[a_1 k\epsilon\gamma + a_2 k(\mu - r) + a_3 \gamma k(\mu - r)] \\
&\mathrm{n}_4 = -k\epsilon\gamma(a_4 + \mu), \mathrm{n}_5 = k\epsilon\gamma\theta
\end{aligned}$$

Clearly as $B \to 0$ the first isoclines (7) intersects the $S_u -$ axis at zero

However when $B \to 0$ the second isoclines (8) will intersect the $S_u -$ axis at a unique positive point, say S_{u1}

Consequently, these two isoclines (7) and (8) have an intersection point in the interior of the positive quadrant of $S_u B$ – plane, namely (S_u^*, B^*),

Provided that the following conditions are satisfied

$$\left.\begin{array}{lcl}
\frac{\partial f}{dS_u} > 0 & and & \frac{\partial f}{dB} < 0 \\
 & or & \\
\frac{\partial f}{dS_u} < 0 & and & \frac{\partial f}{dB} > 0
\end{array}\right\} \tag{9a}$$

$$\left.\begin{array}{lcl}
\frac{\partial g}{dS_u} > 0 & and & \frac{\partial g}{dB} > 0 \\
 & or & \\
\frac{\partial g}{dS_u} < 0 & and & \frac{\partial g}{dB} < 0
\end{array}\right\} \tag{9b}$$

Therefore, we have the endemic equilibrium point $E_1 = (S_u^*, I^*, B^*, M^*)$ if the above conditions (9a)-(9b) hold and the following condition is satisfied too

$$rk < \mu k + rB^* \tag{10}$$

4. Local stability analysis:

In this section, the local stability conditions of system (3) near $E_i, i = 0,1$ are established in the following theorems.

Theorem (2): If the E_0 point exists, it is locally asymptotically stable if the following conditions are held

$$r < \mu \tag{11a}$$

$$\alpha < a_2 M^{\wedge} + \mu \tag{11b}$$

$$S_u^{\wedge} < min\left\{\frac{\mu + d + \eta - (\epsilon + \alpha)}{2a_3}, \frac{\mu - r}{2a_1}, \frac{\gamma}{a_2}\right\} \tag{11c}$$

Proof: Clearly the Jacobian matrix of system (3) at E_0 can be written as

$$J(E_0) = (w_{ij})_{4x4}$$

where

$$= \begin{pmatrix} -a_2 M^{\wedge} - \mu & -a_3 S_u^{\wedge} & -a_1 S_u^{\wedge} & -a_2 S_u^{\wedge} \\ 0 & a_3 S_u^{\wedge} - \mu - d - \eta & a_1 S_u^{\wedge} & 0 \\ 0 & \epsilon & r - \mu & 0 \\ \alpha & \alpha & 0 & -\gamma \end{pmatrix}$$

Now, by applying the condition in the Gersgorin theorem [19].

$$|w_{ii}| > \sum_{\substack{i=1 \\ i \neq j}}^{4} |w_{ij}| \qquad (12)$$

We get all the eigenvalues of above Jacobian exist in the region

$$\Omega = \cup \left\{ u^* \in C : |u^* - w_{ii}| < \sum_{\substack{i=1 \\ i \neq j}}^{4} |w_{ij}| \right\}$$

Then all the eigenvalues of $J(E_0)$ exist in the disc centered at w_{ii}. Thus if the diagonal elements are negative and the conditions (11a) and (11c) are held, all the eigenvalues will exist in the left half plane and the E_0 is locally asymptotically stable ■

Theorem (3): The $E_1 = (S_u^*, I^*, B^*, M^*)$ of system (3) is locally asymptotically stable under the following conditions held

$$S_u^* < min\left\{ \frac{\mu + d + \eta - (\epsilon + \alpha)}{2a_3}, \frac{2rB^* + k(\mu - r)}{2a_1 k}, \frac{\gamma}{a_2} \right\} \qquad (13a)$$

$$\alpha < a_2 M^* + \mu \qquad (13b)$$

Proof: It is easy from Jacobian matrix of system (3) at $E_1 = (S_u^*, I^*, B^*, M^*)$ that can be written as $J(E_1) = (d_{ij})_{4X4}$

Here,

$$d_{11} = -a_1 B^* - a_2 M^* - a_3 I^* - a_4 - \mu, d_{12} = -a_3 S_u^*, d_{13} = -a_1 S_u^*$$
$$d_{14} = -a_2 S_u^*, d_{21} = a_1 B^* + a_3 I^* + a_4, d_{22} = a_3 S_u^* - (\mu + d) - \eta$$
$$d_{23} = a_1 S_u^*, d_{32} = \epsilon, d_{33} = \frac{kr - 2rB^* - k\mu}{k}, d_{41} = \alpha, d_{42} = \alpha$$

$$d_{44} = -\gamma, d_{31} = d_{24} = d_{34} = d_{43} = 0$$

Now, by applying the condition in the Gersgorin theorem [19].

$$|d_{ii}| > \sum_{\substack{i=1 \\ i \neq j}}^{4} |d_{ij}| \qquad (14)$$

We get all the eigenvalues of above Jacobian exist in the region

$$\Omega = \cup \left\{ u^* \in C: |u^* - d_{ii}| < \sum_{\substack{i=1 \\ i \neq j}}^{4} |d_{ij}| \right\}$$

Then all the eigenvalues of $J(E_1)$ exist in the disc centered at d_{ii} . Thus if the diagonal elements are negative and the conditions (13a) and (13b) are held, all the eigenvalues will exist in the left half plane and the E_1 is locally asymptotically stable. ■

5. Global stability analysis:

In this section, we discuss the global stability conditions and determine the basin of attraction of these equilibrium points of system (3) that is presented as shown in the following theorems.

Theorem (4): The E_0 of system (3) is globally asymptotically in the sub region of R_+^4 under the following sufficient conditions

$$\left(a_2 - \frac{\alpha}{M}\right)^2 < 4\left(\frac{a_2 M^{\wedge} + \mu}{S_u}\right)\left(\frac{\gamma}{M}\right) \quad (15a)$$

$$S_u^{\wedge} < min\left\{\frac{\mu - r}{a_1}, \frac{(\mu + d + \eta - \alpha - \epsilon)M + \alpha M^{\wedge}}{a_3 M}\right\} \quad (15b)$$

Proof: consider the following positive definite function

$$V_0(S_u, I, B, M) = \left(S_u - S_u^{\wedge} - S_u^{\wedge} \ln \frac{S_u}{S_u^{\wedge}}\right) + I + B + \left(M - M^{\wedge} - M^{\wedge} \ln \frac{M}{M^{\wedge}}\right)$$

It is easy to see that $V_0(S_u, I, B, M) \in C^1(R_+^4, R)$ and $V_0(S_u^{\wedge}, 0,0, M^{\wedge}) = 0$, while $V_0(S_u, I, B, M) > 0$ $\forall (S_u, I, B, M) \in R_+^4$ and $(S_u, I, B, M) \neq (S_u^{\wedge}, 0,0, M^{\wedge})$.

$$\frac{dV_0}{dt} = \left(\frac{S_u - S_u^{\wedge}}{S_u}\right)\frac{dS_u}{dt} + \frac{dI}{dt} + \frac{dB}{dt} + \left(\frac{M - M^{\wedge}}{M}\right)\frac{dM}{dt}$$

$$\frac{dV_0}{dt} = \left(\frac{S_u - S_u^{\wedge}}{S_u}\right)[\theta - a_1 S_u B - a_2 S_u M - a_3 S_u I - a_4 S_u - \mu S_u]$$
$$+[a_1 S_u B + a_3 S_u I - (\mu + d + \eta)I] + \left[rB\left(1 - \frac{B}{k}\right) - \mu B + \epsilon I\right]$$
$$+\left(\frac{M - M^{\wedge}}{M}\right)[\alpha (S_u + I) - \gamma M]$$

Furthermore by taking the derivative and simplifying the resulting terms, we obtain

That

$$\frac{dV_0}{dt} = -\left[\left(\frac{a_2 M^{\wedge} + \mu}{S_u}\right)(S_u - S_u^{\wedge})^2 + \left(a_2 - \frac{\alpha}{M}\right)(M - M^{\wedge})(S_u - S_u^{\wedge}) + \frac{\gamma}{M}(M - M^{\wedge})^2\right]$$
$$-\left(\mu - (r + a_1 S_u^{\wedge})\right)B - \left[(\mu + d + \eta) + \frac{\alpha M^{\wedge}}{M} - (a_3 S_u^{\wedge} + \in + \alpha)\right]I$$

By using the above condition, we obtain that

$$\frac{dV_0}{dt} \leq -\left[\sqrt{\frac{a_2M^{\wedge}+\mu}{S_u}}(S_u - S_u^{\wedge}) + \sqrt{\frac{\gamma}{M}}(M - M^{\wedge})\right]^2 - \left(\mu - (r + a_1S_u^{\wedge})\right)B$$
$$-\left[(\mu + d + \eta) + \frac{\alpha M^{\wedge}}{M} - (a_3S_u^{\wedge} + \in + \alpha)\right]I$$

Clearly, $\dot{V}_0 = 0$ at $E_0 = (S_u^{\wedge}, 0,0, M^{\wedge})$, moreover $\dot{V}_0 < 0$ otherwise. Hence $\dot{V}_0$ is negative definite and then the solution starting from any initial point satisfies the conditions (15a) and (15b), will converge to E_0 point. ■

Theorem (5): The E_1 of system (3) is globally asymptotically stable under the following conditions held.

$$b_{12}^2 < \tfrac{4}{9}b_{11}b_{22} \qquad (16a)$$

$$b_{13}^2 < \tfrac{4}{6}b_{11}b_{33} \qquad (16b)$$

$$b_{14}^2 < \tfrac{4}{6}b_{11}b_{44} \qquad (16c)$$

$$b_{23}^2 < \tfrac{4}{6}b_{22}b_{33} \qquad (16d)$$

$$b_{24}^2 < \tfrac{4}{6}b_{22}b_{44} \qquad (16e)$$

$$a_3S_u < \mu + d + \eta \qquad (16f)$$

$$r < \tfrac{r}{k}(B + B^*) + \mu \qquad (16g)$$

Proof: Consider the following function

$$V_1(S_u, I, B, M) = \frac{1}{2}(S_u - S_u^*)^2 + \frac{1}{2}(I - I^*)^2 + \frac{1}{2}(B - B^*)^2 + \frac{1}{2}(M - M^*)^2$$

It is easy to see that $V_1 = (S_u, I, B, M) \in C^1(R_+^4, R)$ in addition $V_1(S_u^*, I^*, B^*, M^*) = 0$ while $V_1(S_u, I, B, M) > 0 \ \forall (S_u, I, B, M) \in R_+^4$ and $(S_u, I, B, M) \neq (S_u^*, I^*, B^*, M^*)$.

$$\frac{dV_1}{dt} = (S_u - S_u^*)\frac{dS_u}{dt} + (I - I^*)\frac{dI}{dt} + (B - B^*)\frac{dB}{dt} + (M - M^*)\frac{dM}{dt}$$
$$\frac{dV_1}{dt} = (S_u - S_u^*)[\theta - a_1S_uB - a_2S_uM - a_3S_uI - \mu S_u]$$
$$+(\mathrm{I} - \mathrm{I}^*)[a_1S_uB + a_3S_uI - (\mu + d + \eta)I]$$
$$+(B - B^*)\left[rB\left(1 - \frac{B}{k}\right) - \mu B + \epsilon I\right]$$
$$+(M - M^*)[\alpha(S_u + I) - \gamma M]$$

Furthermore, by the derivative and simplifying the

$$\frac{dV_1}{dt} = -\left[\frac{b_{11}}{3}(S_u - S_u^*)^2 + b_{12}(I - I^*)(S_u - S_u^*) + \frac{b_{22}}{3}(I - I^*)^2\right]$$
$$-\left[\frac{b_{11}}{3}(S_u - S_u^*)^2 + b_{13}(B - B^*)(S_u - S_u^*) + \frac{b_{33}}{2}(B - B^*)^2\right]$$
$$-\left[\frac{b_{11}}{3}(S_u - S_u^*)^2 + b_{14}(M - M^*)(S_u - S_u^*) + \frac{b_{44}}{2}(M - M^*)^2\right]$$
$$-\left[\frac{b_{22}}{3}(I - I^*)^2 - b_{23}(B - B^*)(I - I^*) + \frac{b_{33}}{2}(B - B^*)^2\right]$$
$$-\left[\frac{b_{22}}{3}(I - I^*)^2 - b_{24}(M - M^*)(I - I^*) + \frac{b_{44}}{2}(M - M^*)^2\right]$$

Therefore according to the condition (16a) and (16g) we obtain that:

$$\frac{dV_1}{dt} < -\left[\sqrt{\frac{b_{11}}{3}}(S_u - S_u^*) + \sqrt{\frac{b_{22}}{3}}(I - I^*)\right]^2 - \left[\sqrt{\frac{b_{11}}{3}}(S_u - S_u^*) + \sqrt{\frac{b_{33}}{2}}(B - B^*)\right]^2$$
$$-\left[\sqrt{\frac{b_{11}}{3}}(S_u - S_u^*) + \sqrt{\frac{b_{44}}{2}}(M - M^*)\right]^2 - \left[\sqrt{\frac{b_{22}}{3}}(I - I^*) - \sqrt{\frac{b_{33}}{2}}(B - B^*)\right]^2$$
$$-\left[\sqrt{\frac{b_{22}}{3}}(I - I^*) - \sqrt{\frac{b_{44}}{2}}(M - M^*)\right]^2$$

Where

$$b_{11} = a_1B^* + a_2M^* + a_3I^* + a_4 + \mu\,, \quad b_{12} = a_3S_u - a_1B^* - a_3I^* - a_4$$
$$b_{22} = \mu + d + \eta - a_3S_u, \quad b_{23} = a_1S_u + \epsilon, \quad b_{13} = a_1S_u$$
$$b_{33} = \frac{r}{k}(B + B^*) - (r - \mu), \quad b_{14} = a_2S_u - \alpha, \quad b_{24} = \alpha, \quad b_{44} = \gamma$$

Clearly, $\frac{dV_1}{dt} < 0$, and then V_1 is a Lyapunov function provided that the given condition (16a) and (16g) held . Therefore, E_1 is globally asymptotically stable. ■

6. Numerical Simulation:

In this present section, the spread and control of shigellosis disease are investigated by numerically simulation for many sets of initial values and different sets of parameters values. The objectives of this section are determined by the effect of contact rate, media rate and external sources as well confirm our obtained results. It is observed that through choosing the following data

$$\begin{aligned} &\theta = 1.2\,,\ a_1 = 1.5\,,\ a_2 = 1.19\,,\ a_3 = 0.15\,,\ a_4 = 1.15\,,\ \mu = 0.5 \\ &d = 0.3\,,\ \eta = 0.5\,,\ r = 0.3\,,\ k = 0.1\,,\ \epsilon = 0.02\,,\ \alpha = 0.1\,,\ \gamma = 0.3 \end{aligned} \qquad (17)$$

The dynamical behaviors of system (1) converge to the $E_1 = (0.54, 0.53, 0.46, 0.03, 0.35)$ and the investigation of the global stability, are shown in **Figure 1.** starting from different sets of initial points.

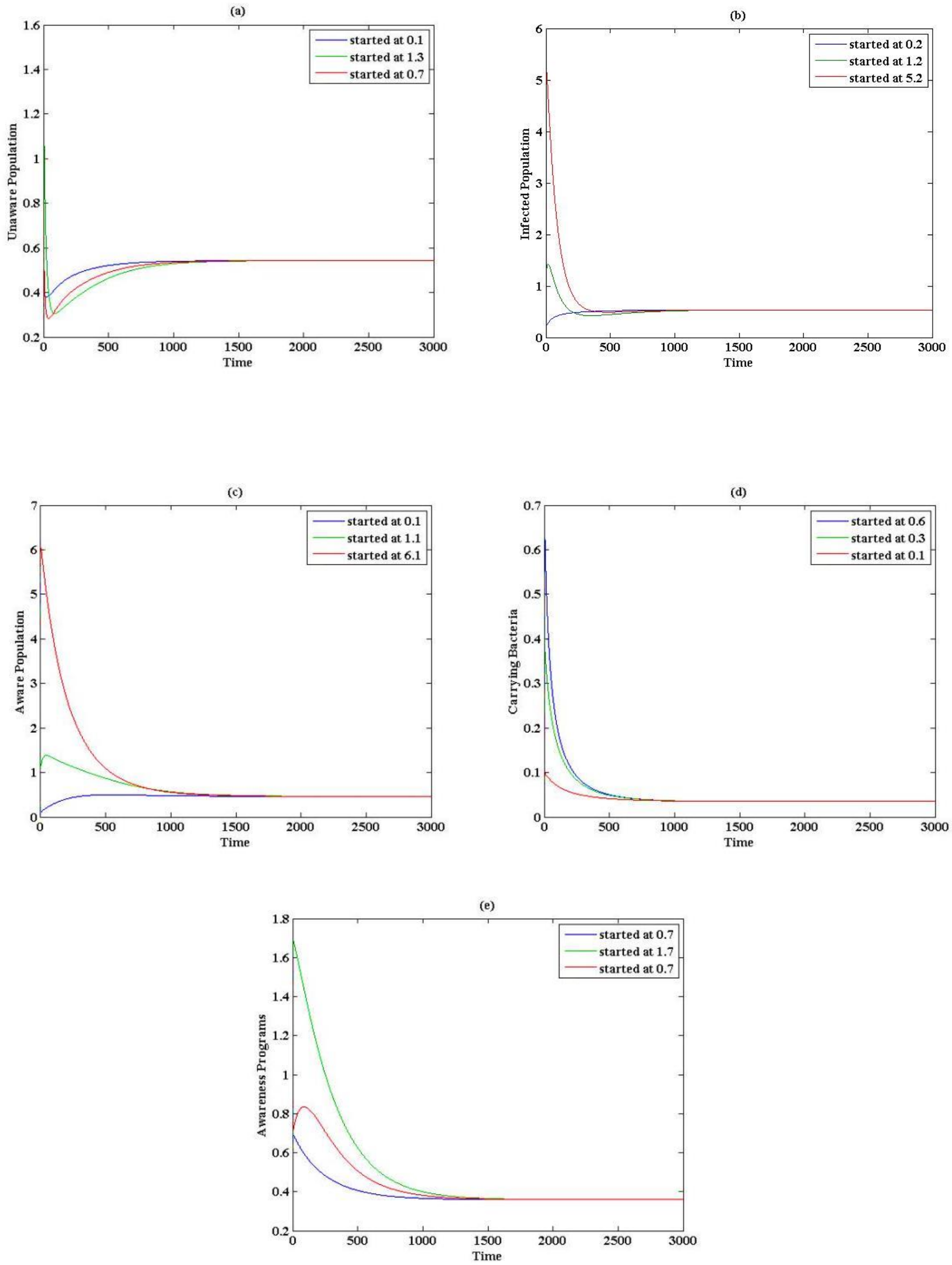

Figure 1. The trajectory of system (1) approaches asymptotically to a globally stable to endemic equilibrium point of system (1) for the parameter set in eq. (17), started from different sets of initial point. (a) for $S_u(t)$, (b) for $I(t)$, (c) for $S_a(t)$, (d) for $B(t)$, (e) for $M(t)$.

Clearly **Figure 1.** confirms our obtained analytical results regarding the existence of globally asymptotically stable endemic equilibrium point.

However, for the same data by equation (17) with $a_4 = 0$, $k = 0.01$ the solution of system (1) converge to the disease free equilibrium point is shown in the following **Figure 2.**

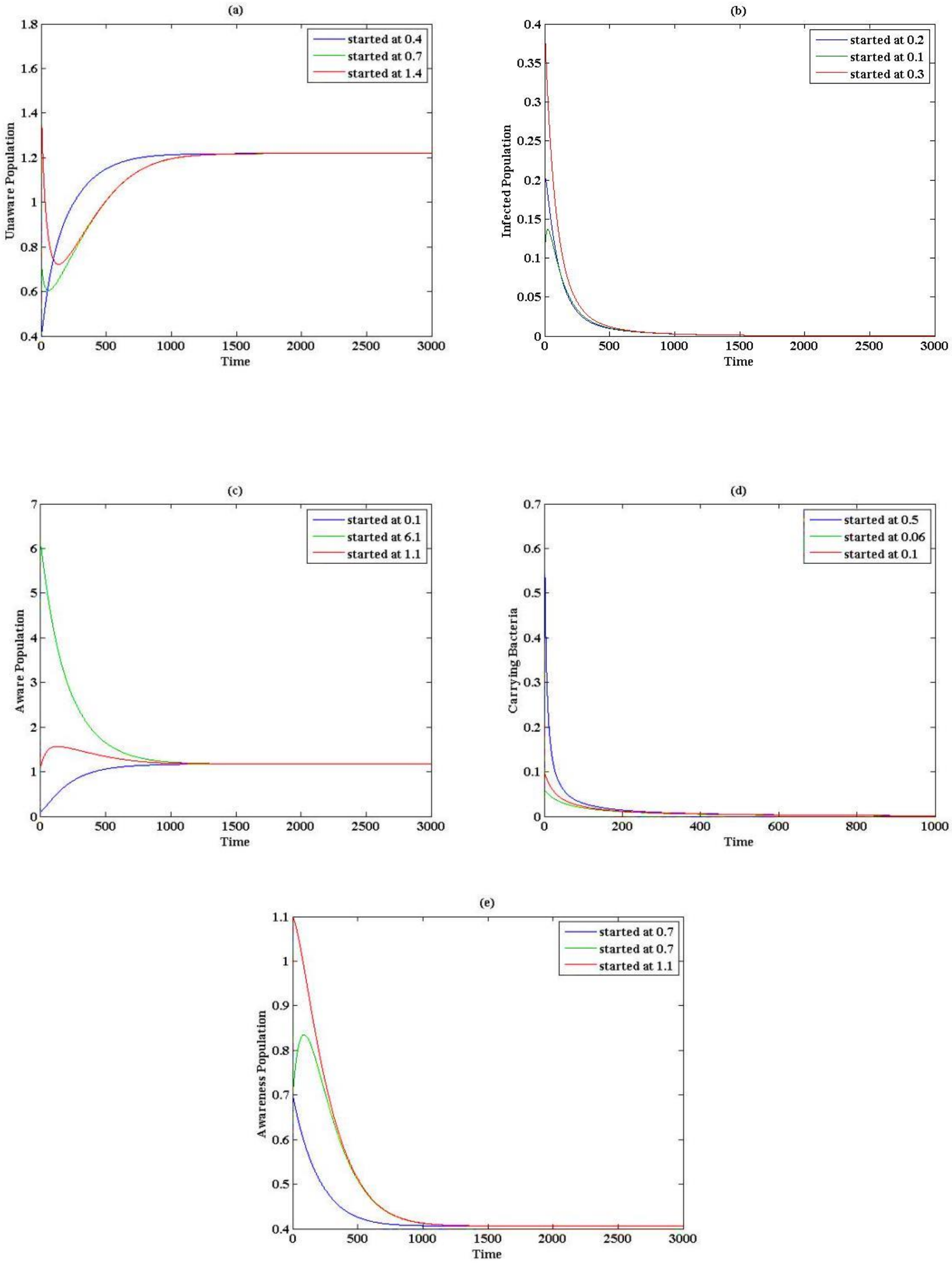

Figure 2. The trajectory of system (1) approaches asymptotically to a globally stable to $E_0 = (1.21,0,1.18,0,0.40)$, started from different sets of initial point. (a) for $S_u(t)$, (b) for $I(t)$, (c) for $S_a(t)$, (d) for $B(t)$, (e) for $M(t)$.

It is easy by keeping fixed the parameters values given in Eq. (17) with putting μ *and* k in the range $0.5 \leq \mu \leq 16.8, 0.001 \leq k \leq 0.01$ and $a_4 = 0$, the solution of the system (1) converge to $E_0 = (1.21,0,1.18,0,0.40)$ is shown in the typical figure given by **Figure 3.** below.

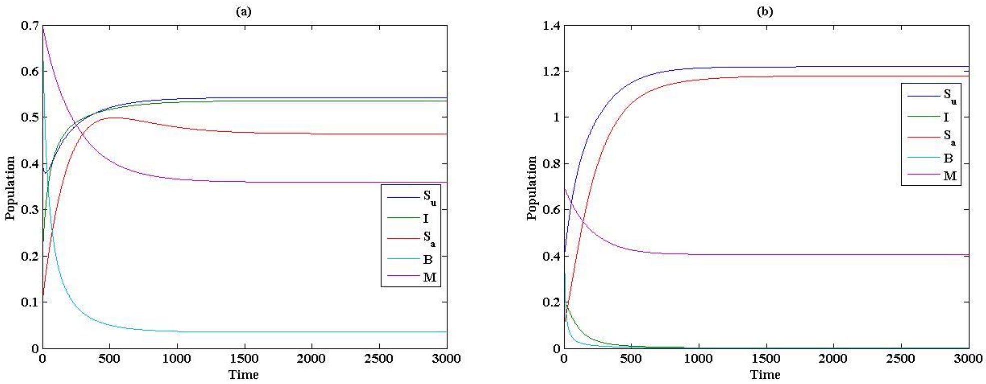

Figure 3. The solution of system (1) for the data (17) (a) for $\mu = 0.5$ and $k = 0.1$ (b) for $\mu = 0.7$ and $k = 0.01$.

According to Figure 3, it is clear that the dynamical behavior of system (1), transmission from endemic point to disease free point that is mean the endemic point became unstable.

Now, the dynamical behavior of system (1) under the effect of varying the contact infected rate value a_3 is investigated. System (1) is solved numerically by choosing the parameters values given by equation (17) with $a_3 = 3.15, 6.15, 9.15$ respectively and then the trajectories of the system (1) are shown in **Figure 4.**

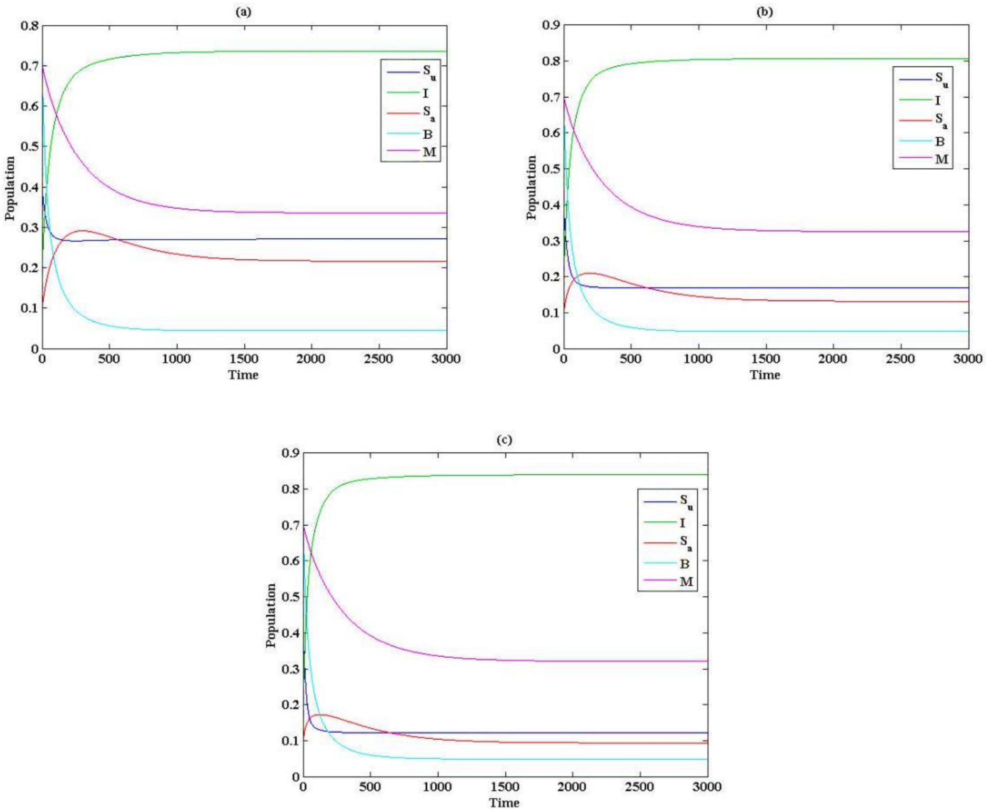

Figure 4. The solutions of the system (1) (a) for $a_3 = 3.15$ (b) for $a_3 = 6.15$ (c) $a_3 = 9.15$

Clearly, from figure 4. We see that the solution of system (1) is still a converge to the endemic equilibrium point. In addition it is observed that the number of asymptomatic susceptible unaware and aware population decreases while the number of infected population increases.

Finally, the dynamical behavior of system (1) under the effect of varying the incidence rate of the disease by direct contact with the carriers of the bacteria. So, we can chose the same set of parameters values given by equation (17) with $a_1 = 5.5, 20.5, 35.5$ respectively and then the trajectories of the system (1) are shown in the **Figure 5.**

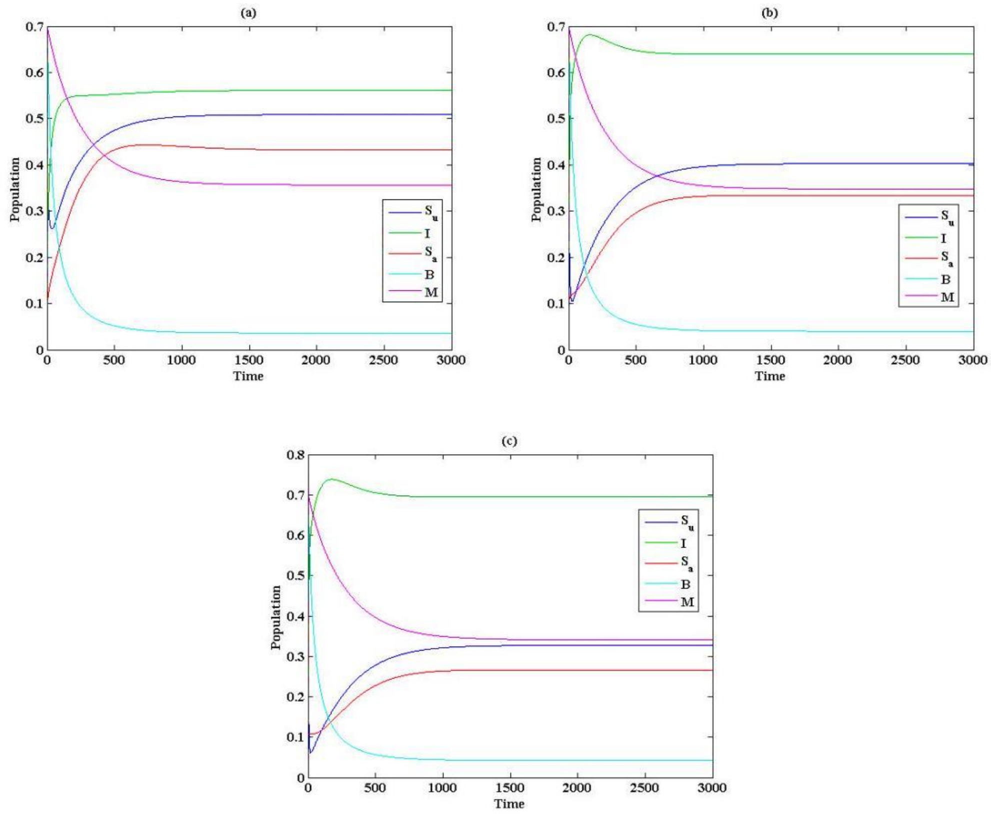

Figure 5. Time series of solutions of the system (1) (a) for $a_1 = 5.5$ (b) for $a_1 = 20.5$ (c) $a_1 = 35.5$

Clearly, system (1) has an asymptotically stable to the endemic equilibrium point. In addition it is observed that there is a slight change in the system.

7. Conclusion and Discussion:

Awareness programs that control the spread of the epidemic strong steps should be taken regarding their implementation of diseases such as smallpox, measles, influenza, and others. Behavioral changes caused by awareness programs have the ability to control the size of the epidemic and then predict the future course of the outbreak to guide public health policy.

This is a different thing because outbreak of the disease involves the environment, and (direct and indirect) multiple ways of transmission how to cover the media, the awareness programs will be affected the dynamics of shigellosis.

In this work, we proposed and analyzed a model to study the effect of awareness programs on shigellosis disease. The model included four ordinary differential equations describing four different class: unaware susceptible individuals S_u, infected individuals I, aware susceptible individuals S_a and people carrying bacteria B. System (1) has only two equilibrium points. The conditions for existence, stability for each equilibrium points are obtained. Further, it is observed that the disease free equilibrium point (E_0) exists when $a_4 = 0$ and $I = B = 0$ and locally stable if the conditions (11a-11c) are held, and then it is globally stable if the conditions (15a-15b) are held. The endemic equilibrium point (E_1) exists if the conditions

(9a-9b) are held and locally stable if the conditions (13a-13b) are held more than it is globally stable if the conditions (16a-16g) are held. Finally, to understand the effect of varying each parameter on the global system (1) and confirm our above analytical results, system (1) has been solved numerically for different sets of initial points and different sets of parameter given by equation (17), and the following observation are made:

1. For the set of hypothetical parameters values given by Eq. (17), the system (1) approached asymptotically to the global stable endemic equilibrium point, E_1 .
2. Equation (17), when we set the following values $a_4 = 0$ and $k = 0.01$ the system (1) converge to disease free equilibrium point E_0 and satisfied the global stability.
3. If the contact rates a_3 and a_1 increase, the solution of system (1) still converge to the endemic equilibrium point.

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