Ibn Al Haitham Journal for Pure and Applied Science

Journal homepage: http://jih.uobaghdad.edu.iq/index.php/j/index

Some Games with Soft -ḷ-Pre-Generalized Open Sets

Hammood A.A. **Esmaeel R.B.**

Department of Mathematics, College of Education for Pure Science (Ibn Al-Haitham), University of Baghdad,Iraq.

Abd.Ali113a@ihcoedu.uobaghdad.edu.iq ranamumosa@yahoo.com

Article history: Received 29 , March, 2021, Accepted 11,April, 2021, Published in October 2021.

Doi: 10.30526/34.4.2702

Abstract

In this paper, the concept of soft closed groups is presented using the soft ideal pre-generalized open and soft pre-open, which are *soft*-ḷ-*pre*-*g*-closed sets "*s*ḷ*pg*-closed", Which illustrating several characteristics of these groups. We also use some games and *soft pre-open* Separation Axiom, such as Şǥ(Ʈ₀, Ҳ, ḷ) that use many tables and charts to illustrate this. Also, we put some proposals to study the relationship between these games and give some examples.

Keywords: Soft ideal, Soft-$Ʈ_i$-*space* , Soft-ḷ-*pre*-*g*-$Ʈ_i$-*space* , Şǥ($Ʈ_i$, Ҳ, ḷ). Where i= {0,1,2}

1.Introduction

Shaber [1] established the introduced soft topological space in 2011. Through the use of soft sets, such as derived sets, compactness, separation axioms and other characteristics, various studies are introduced to study many topological characteristics. [2-4]. In addition, usesoft ideals as a group of soft sets to study the concept of soft logic functions [5]. This is the starting point for studying the properties of soft ideal topological spaces (Ҳ, Ʈ, Đ, ḷ), and defining new type of near-open soft sets and studies their properties as [6-8]. In this paper, we will present new types of games Şǥ($Ʈ_0$, Ҳ, ḷ), Şǥ($Ʈ_1$, Ҳ, ḷ), Şǥ($Ʈ_2$, Ҳ, ḷ)) and determine the winning and losing strategies for any two players.

2.Preliminaries

Some basic of soft space (Ҳ, Ʈ, Đ) with soft ideal are presented.

Definition 2.1: [9] Let Ҳ ≠ ∅ and Đ be a set of *parameters*, were $\wp$(Ҳ) the collection of Ҳ and $P \neq \emptyset$ such that $P \subseteq$ Đ. (F, Đ) (Briefly F$_{Đ}$) is a soft set over Ҳ whenever, F is a function such that F: Đ → $\wp$(Ҳ). So, F$_{Đ}$ = { F(d): $d \in P \subseteq$ Đ , F : Đ → $\wp$(Ҳ) }. The *collection* of all soft sets is (briefly ŞŞ(Ҳ)$_{Đ}$).

Definition 2.2: [9] Let $(F, Đ), (\mathcal{Z}, Đ) \in ŞŞ(X)_{Đ}$. Then $(F, Đ)$ is a soft subset of $(\mathcal{Z}, Đ)$, (briefly $(F, Đ) \tilde{\subseteq} (\mathcal{Z}, Đ)$), if $F(d) \tilde{\subseteq} \mathcal{Z}(d)$, for all $d \in Đ$. Now $(F, Đ)$ is a soft subset of $(\mathcal{Z}, Đ)$and $(\mathcal{Z}, Đ)$ is a soft super set of $(F, Đ)$, $(F, Đ) \tilde{\subseteq} (\mathcal{Z}, Đ)$.

Definition 2.3: [10] The complement of $(F, Đ)$ (briefly $(F, Đ)'$) $(F, Đ)' = (F', Đ)$, $F': Đ \to \wp(X)$ is a function such that $F'(d) = X - F(d)$, for all $d \in Đ$ and F' is namely the soft complement of F.

Definition 2.5: [1] $(F, Đ)$ *is a NULL soft set* (briefly $\tilde{\emptyset}$ or $\emptyset_{Đ}$) whenever, $\forall d \in Đ, F(d) = \emptyset$.

Definition 2.6: [1] $(F, Đ)$ *is an absolute soft set* (briefly $\tilde{X}$ or $X_{Đ}$) whenever, $\forall d \in Đ$, $F(d) = X$.

Definition 2.7: [1] *Let* $\mathcal{T}$ is the set of soft sets on X with the same Đ, then $\mathcal{T} \in ŞŞ(X)_{Đ}$ is a soft topology on X if;

i. $\tilde{X}, \tilde{\emptyset} \in \mathcal{T}$ where, $\tilde{\emptyset}(d) = \emptyset$ and $\tilde{X}(d) = X$, for each $d \in Đ$

ii. $\cup_{\alpha \in \Lambda} (\eta_\alpha, Đ) \in \mathcal{T}$ whenever, $(\eta_\alpha, Đ) \in \mathcal{T}$ $\forall \alpha \in \Lambda$,

iii. $((F, Đ) \tilde{\cap} (\mathcal{Z}, Đ)) \in \mathcal{T}$ for each $(F, Đ), (\mathcal{Z}, Đ) \in \mathcal{T}$.

The triple $(X, \mathcal{T}, Đ)$ i*s a soft topological spa*ce if $(\eta, Đ) \in \mathcal{T}$, then $(\eta, Đ)$ is an open soft set.

Definition 2.8: [11] Let $(X, \mathcal{T}, Đ)$ be a soft topological space. A soft set $(F, Đ)$ over X is a soft closed set in X, if $(F, Đ)' \in \mathcal{T}$, the collection of each *soft* closed sets (briefly $ŞC(X)_{Đ}$).

Definition 2.9: [11] *For any soft* space $(X, \mathcal{T}, Đ)$. Let $(F, Đ)' \in ŞŞ(X)_{Đ}$, then *the soft closure* of $(F, Đ)'$, (briefly $cl(F, Đ)$), $cl((F, Đ)) = \tilde{\cap} \{ (\mathcal{M}, Đ) : (\mathcal{M}, Đ) \in ŞC(X)_{Đ}, (F, Đ) \tilde{\subseteq} (\mathcal{M}, Đ) \}$.

Definition 2.10: [11] For any $(X, \mathcal{T}, Đ)$. Let $(F, Đ) \in ŞŞ(X)_{Đ}$,*then the soft interior* of $(\mathcal{Z}, Đ)$, (briefly $int(\mathcal{Z}, Đ)$), $int(\mathcal{Z}, Đ) = \tilde{\cup} \{ (\mathcal{M}, Đ) : (\mathcal{M}, Đ) \in \mathcal{T}, (\mathcal{M}, Đ) \tilde{\subseteq} (\mathcal{Z}, Đ) \}$.

Definition 2.11: [5] *Let* $\mathfrak{l} \neq \emptyset$, *then* $\mathfrak{l} \tilde{\subseteq} ŞŞ(X)_{Đ}$ is a soft ideal whenever,

i. If $(F, Đ) \tilde{\in} \mathfrak{l}$ and $(\mathcal{Z}, Đ) \tilde{\in} \mathfrak{l}$ implies, $(F, Đ) \tilde{\cup} (\mathcal{Z}, Đ) \tilde{\in} \mathfrak{l}$.

ii. If $(F, Đ) \tilde{\in} \mathfrak{l}$ and $(\mathcal{Z}, Đ) \tilde{\subseteq} (F, Đ)$ implies, $(\mathcal{Z}, Đ) \tilde{\in} \mathfrak{l}$.

Any $(X,\mathcal{T},Đ)$ with a soft ideal $\mathfrak{l}$is a soft ideal topological space (briefly $(X,\mathcal{T},Đ, \mathfrak{l})$).

Definition 2.12: [5] The space $(X, \mathcal{T}, Đ)$ with a soft ideal $\mathfrak{l}$ can be defined as $(X,\mathcal{T},Đ, \mathfrak{l})$ a soft topological space.

Definition 2.13: [12] For any $(X, \mathcal{T}, Đ)$, then $(F, Đ)$ is a *soft pre*-open set (briefly $Şp$-open set if $(F, Đ) \tilde{\subseteq} int(cl(F, Đ))$. a *soft pre*-closed set (briefly $(F, Đ)'$).The family of each pre *soft*-open sets in $(X, \mathcal{T}, Đ)$ (briefly $ŞpO(X)$). The *collection* of each *soft pre-closed* sets (*briefly* $ŞpC(X)$).

Definition 2.14: [2] Let (Ӿ, Ƭ, Đ) be a soft topological space over Ӿ is a soft- $Ƭ_0$-space if for all, $ɖ_{\mathcal{M}}$, $ɖ_{\mathcal{N}}$ $\tilde{\in}$ $\tilde{Ӿ}$ such that $ɖ_{\mathcal{M}} \neq ɖ_{\mathcal{N}}$. *If there exist soft open set* (Ƞ, Đ) such that $ɖ_{\mathcal{M}}$ $\tilde{\in}$ (Ƞ, Đ) , $ɖ_{\mathcal{N}}$ $\tilde{\notin}$ (Ƞ, Đ) or $ɖ_{\mathcal{M}}$ $\tilde{\notin}$ (Ƞ, Đ) , $ɖ_{\mathcal{N}}$ $\tilde{\in}$ (Ƞ, Đ).

Definition 2.15: [2] Let (Ӿ, Ƭ, Đ) be a soft topological space over Ӿ is a *soft* $Ƭ_1$-space if for all, $ɖ_{\mathcal{N}}$ $\tilde{\in}$ $\tilde{Ӿ}$ *such that* $ɖ_{\mathcal{M}} \neq ɖ_{\mathcal{N}}$. ∃ (F, Đ), (Ƞ, Đ) ∈ Ƭ whenever, ɖ $\tilde{\in}$ (F, Đ), $ɖ_{\mathcal{N}}$ $\tilde{\notin}$ (F, Đ) and $ɖ_{\mathcal{M}}$ $\tilde{\notin}$ (Ƞ, Đ), $ɖ_{\mathcal{N}}$ $\tilde{\in}$ (Ƞ, Đ).

Definition 2.16: [2] *Let* (Ӿ, Ƭ, Đ) be a *soft* topological *space over* Ӿ is *said* to be *soft*-$Ƭ_2$-space if, for each, $ɖ_{\mathcal{N}}$ $\tilde{\in}$ $\tilde{Ӿ}$ such that $ɖ_{\mathcal{M}} \neq ɖ_{\mathcal{N}}$. ∃ (F, Đ), (Ƞ, Đ) ∈ Ƭ whenever, $ɖ_{\mathcal{M}}$ $\tilde{\in}$ (F, Đ), $ɖ_{\mathcal{N}}$ $\tilde{\in}$ (Ƞ, Đ) and (F, Đ) $\tilde{\cap}$ (Ƞ, Đ) = {$\tilde{\emptyset}$}.

Proposition 2.17: [2] for all soft- $Ƭ_{i+1}$-space is a soft- $Ƭ_i$-space and i ∈ {0,1,2}

Definition 2.18:[13] for a soft ideal space (Ӿ, Ƭ, Đ, Ɉ), determane a game Ș$ɡ$($Ƭ_0$, Ӿ) as follows:

P Ⅰ and P Ⅱ play an inning for each positive integer numbers in the *z-th* inning:

The first step, P Ⅰ chooses$(ɖ_{\mathcal{M}})_z \neq (ɖ_{\mathcal{N}})_z$ where, $(ɖ_{\mathcal{M}})_z$, $(ɖ_{\mathcal{N}})_z$ $\tilde{\in}$ $\tilde{Ӿ}$ .In the second step, P Ⅱ chooses $Ƀ_z$ a open-soft containing only one of the two elements $(ɖ_{\mathcal{M}})_z$, $(ɖ_{\mathcal{N}})_z$.

Then P Ⅱ wins in the soft game Ș$ɡ$($Ƭ_0$, Ӿ) if Ƀ = { $Ƀ_1$, $Ƀ_2$, $Ƀ_3$,… $Ƀ_z$,…..} is a collection of an open-soft set in Ӿ such that ∀ , $(ɖ_{\mathcal{M}})_z$, $(ɖ_{\mathcal{N}})_z$ $\tilde{\in}$ Ӿ , ∃ $Ƀ_z$ ∈ Ƀ containing only one of two element $(ɖ_{\mathcal{M}})_z$, $(ɖ_{\mathcal{N}})_z$. Otherwise, P Ⅰ wins.

Definition 2.19:[13] for a soft ideal space (Ӿ, Ƭ, Đ, Ɉ), determine a game Ș$ɡ$($Ƭ_1$, Ӿ) as follows:

P Ⅰ and P Ⅱ are play an inning with each positive integer numbers in the *z-th* inning: The first step, P Ⅰ choose $(ɖ_{\mathcal{M}})_z \neq (ɖ_{\mathcal{N}})_z$ where, $(ɖ_{\mathcal{M}})_z$, $(ɖ_{\mathcal{N}})_z$ $\tilde{\in}$ $\tilde{Ӿ}$. In the second step, P Ⅱ chooses ($Ƌ_z$, Đ), ($Ư_z$, Đ) are two open-soft sets such that $(ɖ_{\mathcal{M}})_z$ $\tilde{\in}$ (($Ƌ_z$, Đ) − ($Ư_z$, Đ)) and $(ɖ_{\mathcal{N}})_z$ $\tilde{\in}$ (($Ư_z$, Đ) − ($Ƌ_z$, Đ)).Then, P Ⅱ wins in the soft game Ș$ɡ$($Ƭ_1$, Ӿ) if Ƀ = {{($Ƌ_1$,Đ), ($Ư_1$,Đ)}, {($Ƌ_2$,Đ), ($Ư_2$,Đ)}, … , {($Ƌ_z$,Đ), ($Ư_z$,Đ)}, … } is a collection of an open-soft sets in Ӿ such that ∀ $(ɖ_{\mathcal{M}})_z \neq (ɖ_{\mathcal{N}})_z$ such that, $(ɖ_{\mathcal{M}})_z$, $(ɖ_{\mathcal{N}})_z$ $\tilde{\in}$ $\tilde{Ӿ}$, ∃{($Ƌ_z$, Đ), ($Ư_z$, Đ)} ∈ Ƀ such that $(ɖ_{\mathcal{M}})_z$ $\tilde{\in}$ (($Ƌ_z$, Đ) − ($Ư_z$, Đ))and $(ɖ_{\mathcal{N}})_z$ $\tilde{\in}$ (($Ư_z$, Đ)− ($Ƌ_z$, Đ)). Otherwise, P Ⅰ wins in the soft game Ș$ɡ$($Ƭ_1$, Ӿ).

Definition2.20:[13] For a soft ideal space (Ӿ, Ƭ, Đ, Ɉ), determine a game Ș$ɡ$($Ƭ_2$, Ӿ) as follows:

P Ⅰ and P Ⅱ are play an inning with each positive integer numbers in the *z-th* inning: The first step, P / Choose $(ɖ_{\mathcal{M}})_z \neq (ɖ_{\mathcal{N}})_z$ whenever, $(ɖ_{\mathcal{M}})_z$, $(ɖ_{\mathcal{N}})_z$ $\tilde{\in}$ $\tilde{Ӿ}$. In the second step, P Ⅱ choose ($Ƌ_z$, Đ), ($Ư_z$, Đ) are two open-soft sets such that $(ɖ_{\mathcal{M}})_z$ $\tilde{\in}$ ($Ƌ_z$, Đ) , $(ɖ_{\mathcal{N}})_z$ $\tilde{\in}$ ($Ư_z$, Đ) and ($Ƌ_z$, Đ) $\tilde{\cap}$ ($Ư_z$, Đ) = {$\tilde{\emptyset}$}. Then P Ⅱ wins in the game Ș$ɡ$($Ƭ_2$, Ӿ) if Ƀ = {{(Ƌ, Đ), (Ư, Đ)}, {(Ư, Đ), (Ć, Đ)}, {(Ƌ, Đ), (Ć, Đ)}} be a collection of a open-soft sets in Ӿ such that ∀ $(ɖ_{\mathcal{M}})_z \neq (ɖ_{\mathcal{N}})_z$ whenever, $(ɖ_{\mathcal{M}})_z$, $(ɖ_{\mathcal{N}})_z$ $\tilde{\in}$ $\tilde{Ӿ}$, ∃{($Ƌ_z$, Đ), ($Ư_z$, Đ) } ∈ Ƀ such that $(ḓ_{\mathcal{M}})_z \tilde{\in} ((\text{Ƌ}_z, \text{Đ})$ and $(ḓ_{\mathcal{N}})_z \tilde{\in} ((\text{Ư}_z, \text{Đ})$ and $(\text{Ƌ}_z, \text{Đ}) \tilde{\cap} (\text{Ư}_z, \text{Đ}) = \{\tilde{\emptyset}\}$. Otherwise, P Ⅰ wins in the game Ȿg̡(Ţ2, Ẋ).

3.On Soft- ȴ -*pre* - *g*- closed Set

Definition 3.1: for the soft ideal topological space (Ẋ, Ţ, Đ, ȴ), let (F, Đ) ∈ ȿȿ(Ẋ)Đ then, (F , Đ) is a soft-ȴ-*pre*-*g*-closed set (briefly sȴ*pg*-closed). If (*F*, Đ) - (Ƞ, Đ) ∈ ȴ then, cl(F, Đ) − (Ƞ, Đ) ∈ ȴ for each (Ƞ, Đ) ∈ ȿ*p*O(Ẋ), and $\tilde{\text{Ẋ}}$ − (F, Đ) is a *soft*-ȴ-*pre*-*g*-open set (briefly *s*ȴ*pg*-open set). The family of all *s*ȴ*pg*- closed sets (briefly *s*ȴ*pg*-C(Ẋ)) and the family of all sȴ*pg*-open soft sets (briefly *s*ȴ*pg*-O(Ẋ)).

Example 3.2: For a space (Ẋ, Ţ, Đ, ȴ), whenever Ẋ = {e̡, m̡}, Đ ={d1, d2}, Ţ={$\tilde{\emptyset}$,$\tilde{\text{Ẋ}}$,(F, Đ), (Ƞ, Đ)}, ȴ = {$\tilde{\emptyset}$, $\mathcal{M}$} such *that* (F, Đ)={(d1, {∅}), (d2, {e̡})}, and (Ƞ, Đ)={(d1,{e̡}),(d2,{e̡})} and (*M*, Đ)={(d1, {∅}) , (d2, {e̡})} then, ȿ*p*O(Ẋ) = {$\tilde{\emptyset}$, $\tilde{\text{Ẋ}}$, (*F*, Đ), (Ƞ, Đ), ($\mathcal{Z}$, Đ) , ($\mathcal{H}$, Đ), ($\mathcal{E}$, Đ), ($\mathcal{N}$, Đ), ($\mathcal{G}$, Đ)}, sȴ*pg*-*C*(Ẋ)={ $\tilde{\text{Ẋ}}$, $\tilde{\emptyset}$, (F′, Đ), (Ƞ′, Đ)} ; (F′, Đ))={(d1, {Ẋ}), (d2, {m̡})}, (Ƞ′, Đ)={(d1,{m̡}),(d2,{m̡})} such that, ($\mathcal{Z}$, Đ)={(d_1,∅),(d_2, Ẋ)}, ($\mathcal{H}$, Đ)= {(d1, {e̡}), (d2, Ẋ)}, ($\mathcal{E}$, Đ) ={(d_1,{m̡}),(d_2,{e̡})}, ($\mathcal{N}$, Đ)={(d_1,{m̡}),(d_2, Ẋ)} and ($\mathcal{G}$, Đ) = {(d_1, Ẋ),(d_2,{e̡})}, and *s*ȴ*pg*-O(Ẋ) =Ţ.

Remark 3.3: For any (Ẋ, Ţ, Đ, ȴ) then

i. Every closed *soft set* is a sȴ*pg*-closed.

ii. Every *open soft set* is a sȴ*pg*-open.

Proof *(i)* Let ($\mathcal{M}$, Đ) be any closed soft set in (Ẋ, Ţ, Đ, ȴ) and (Ƞ, Đ) be a soft-*pre*-open set such that ($\mathcal{M}$, Đ) − (Ƞ, Đ) ∈ ȴ , but cl($\mathcal{M}$, Đ) = ($\mathcal{M}$, Đ), since ($\mathcal{M}$, Đ) is a closed soft set so, cl($\mathcal{M}$, Đ)- (Ƞ, Đ) = ($\mathcal{M}$, Đ)– (Ƞ, Đ) ∈ ȴ; this implies ($\mathcal{M}$, Đ) is a soft-ȴ-*pre*-*g*-closed soft set.

(ii) Let (Ƞ, Đ) be any open soft set in (Ẋ, Ţ, Đ, ȴ) then $\tilde{\text{Ẋ}}$ – (Ƞ, Đ)is a closed soft set this implies by (i) ($\tilde{\text{Ẋ}}$ - ($\mathcal{M}$, Đ))is a sȴ*pg*-closed set; thus ($\mathcal{M}$, Đ)is a sȴ*pg*-open soft set .

The converse of Remark 3.3 is not hold. See Example 3.4

Example 3.4: *Consider* Ẋ = {e̡, m̡}, Đ ={d_1, d2},Ţ={$\tilde{\emptyset}$,$\tilde{\text{ẋ}}$, F(d) = {e̡} ∀d}then ȿ*p*O(Ẋ)= {(∅)˜, Ẋ˜, (*M*, Đ), (Ƞ, Đ), (*Z*, Đ), (*H*, Đ), (*E*, Đ), (*N*, Đ), (*G*, Đ), (₵, Đ), (*ω*, Đ), (Ƌ, Đ), (*α*, Đ)}, such that ($\mathcal{M}$, Đ)={(d_1,∅),(d2,{e̡})},(Ƞ, Đ)={(d_1,∅),(d2,{Ẋ})}, ($\mathcal{Z}$, Đ)={(d_1,{e̡}),(d2,{ ∅})}, ($\mathcal{H}$, Đ) ={(d_1,{e̡}),(d2,{e̡})}, ($\mathcal{E}$, Đ)={(d_1,{ e̡ }),(d2,{m̡})}, ($\mathcal{N}$, Đ)={(d_1, {e̡}),(d2,{Ẋ})}, ($\mathcal{G}$, Đ)={(d_1,{m̡}),(d2,{e̡})}, (₵, Đ)={(d_1,{m̡}),(d2,{Ẋ})}, (*ω*, Đ)={(d_1,Ẋ),(d2,{∅})}, (Ƌ, Đ)={(d_1,Ẋ),(d2,{e̡})}, (*α*, Đ)={(d_1,Ẋ),(d2,{m̡})}, ȴ = ȿȿ(Ẋ)Đ, *s*ȴ*pg*-c(Ẋ)= *s*ȴ*pg*-o(Ẋ)= ȿȿ(Ẋ)Đ.

i. Let ($\mathcal{E}$, Đ) = {(d1, {e̡}), (d2, {m̡})} is a sȴ*pg*-closed set, but ($\mathcal{E}$, Đ) is not *closed* softset.

ii. Let ($\mathcal{G}$, Đ)= {(d1, {m̡}), (d2, {e̡})} is a sȴ*pg*-open set, but ($\mathcal{G}$, Đ) ∉ Ţ.

1. Separation Axioms with soft-ȴ- pre-g-open Sets.

Definition 4.1. A space (Ẋ, Ţ, Đ, ȴ) is a soft-ȴ-*pre*-*g*-Ţ0-space (briefly sȴ*pg*-Ţ0-space), if for each $ḓ_{\mathcal{M}} \neq ḓ_{\mathcal{N}}$ and $ḓ_{\mathcal{M}}, ḓ_{\mathcal{N}} \tilde{\in}$ Ẋ, ∃ (*U*, Đ) ∈ *s*ȴ*pg*-O(Ẋ) whenever, $ḓ_{\mathcal{M}} \tilde{\in}$ (*U*, Đ) ∧ $ḓ_{\mathcal{N}} \tilde{\notin}$ (*U*, Đ) or $ḓ_{\mathcal{M}} \tilde{\notin}$ (*U*, Đ) ∧ $ḓ_{\mathcal{N}} \tilde{\in}$ (*U*, Đ).

Example 4.2. In (Ꭓ, Ʈ, Đ, ꝉ) Let Ꭓ= { ꬲ, ɱ, ɽ}, Đ= {ꝱ₁ , ꝱ₂}, Ʈ= { $\tilde{Ꭓ}$, $\tilde{\emptyset}$, (Ꞩ, Đ), ($\mathcal{Z}$, Đ)} where, ((Ꞩ, Đ) = {(ꝱ₁,{ꬲ}) , (ꝱ₂ ,{ꬲ})}, ($\mathcal{Z}$, Đ) = {(ꝱ₁,{ꬲ , ɱ}), (ꝱ₂ ,{ꬲ , ɱ })} and ꝉ= {$\tilde{\emptyset}$} **.** Then $ŞpO$(Ꭓ)= { (F, Đ) ; ꬲ∈ (F, Đ) for some ꝱ ∈ Đ}. So, sꝉpg-C(Ꭓ) = { $\tilde{\emptyset}$, $\tilde{\chi}$,(Ꞩ′, Đ) , ($\mathcal{Z}'$, Đ) } and sꝉpg-$O(\chi)$ = Ʈ, hence, ((Ꭓ, Ʈ, Đ, ꝉ)) is a sꝉpg-$Ʈ_0$-space. Since ∀ ꝱ$_{\mathcal{M}}$ ≠ ꝱ , ∃ (Ƞ, Đ) ∈ sꝉpg-O(Ꭓ) whenever, ꝱ$_{\mathcal{M}}$ $\tilde{\in}$ (Ƞ, Đ,) ∧ ꝱ$_{\mathcal{N}}$ $\tilde{\notin}$ (Ƞ, Đ) or ꝱ$_{\mathcal{M}}$ $\tilde{\notin}$ (Ƞ, Đ) ∧ ꝱ$_{\mathcal{N}}$ $\tilde{\in}$ (Ƞ, Đ).

Proposition 4.3. If (Ꭓ, Ʈ, Đ) is a soft-$Ʈ_0$-space then (Ꭓ, Ʈ, Đ, ꝉ) is a sꝉpg-$Ʈ_0$-space.
***Proof* :** Let ꝱ$_{\mathcal{M}}$, ꝱ$_{\mathcal{N}}$ $\tilde{\in}$ $\tilde{Ꭓ}$ such that ꝱ$_{\mathcal{M}}$ ≠ ꝱ$_{\mathcal{N}}$ since (Ꭓ, Ʈ, Đ) is a soft-$Ʈ_0$-space, then ∃ (Ƞ, Đ) ∈ Ʈ whenever, ꝱ$_{\mathcal{M}}$ $\tilde{\in}$ (Ƞ, Đ), ꝱ$_{\mathcal{N}}$ $\tilde{\notin}$ (Ƞ, Đ) or ꝱ$_{\mathcal{M}}$ $\tilde{\notin}$ (Ƞ, Đ), ꝱ$_{\mathcal{N}}$ $\tilde{\in}$ (Ƞ, Đ) . By *Remark* 3.3, (Ƞ, Đ) is a sꝉpg-open set *such that* ꝱ$_{\mathcal{M}}$ $\tilde{\in}$ (Ƞ, Đ) and ꝱ$_{\mathcal{N}}$ $\tilde{\notin}$ (Ƞ, Đ) or ꝱ$_{\mathcal{M}}$ $\tilde{\notin}$ (Ƞ, Đ) and ꝱ$_{\mathcal{N}}$ $\tilde{\in}$ (Ƞ, Đ).

Definition 4.4. (Ꭓ, Ʈ, Đ, ꝉ) is a soft-ꝉ-pre-g-$Ʈ_1$-space (briefly sꝉpg-$Ʈ_1$-space),If for each ꝱ$_{\mathcal{M}}$, ꝱ$_{\mathcal{N}}$ $\tilde{\in}$ $\tilde{\tilde{Ꭓ}}$ and ꝱ$_{\mathcal{M}}$ ≠ ꝱ$_{\mathcal{N}}$. Then there are $s\mathcal{I}sg$-open sets ($Ƞ_1$,Đ), ($Ƞ_2$,Đ) whenever, Đ$_{\mathcal{M}}$ $\tilde{\in}$ (($Ƞ_1$,Đ) – ($Ƞ_2$,Đ)) and ꝱ$_{\mathcal{N}}$ $\tilde{\in}$ (($Ƞ_2$,Đ) – ($Ƞ_1$,Đ)).

Example 4.5. A topological space (Ꭓ, Ʈ, Đ, ꝉ) when Ꭓ= ℕ *the set* of all *tural numbers*, Ʈ = $Ʈ_{Scof}$ = { F$_{\mathcal{A}}$: F′(ꝱ) is finite set ∀ ꝱ } $\tilde{\cup}$ {$\tilde{\emptyset}$ } and ꝉ = {$\tilde{\emptyset}$}. so (Ꭓ, Ʈ, Đ, ꝉ) is a sꝉpg-$Ʈ_1$-space. If ꝱ$_{\mathcal{M}}$, ꝱ$_{\mathcal{N}}$ $\tilde{\in}$ $\tilde{Ꭓ}$ and ꝱ$_{\mathcal{M}}$ ≠ ꝱ$_{\mathcal{N}}$. Then there are sꝉpg-*open* sets ($\tilde{Ꭓ}$ – Ƅ$_{\mathcal{N}}$) , ($\tilde{Ꭓ}$ – Ƅ$_{\mathcal{M}}$) whenever, Ƅ$_{\mathcal{N}}$ and Ƅ$_{\mathcal{M}}$ are two *finite* sets such that Ƅ$_{\mathcal{N}}$⊆ ꝱ$_{\mathcal{N}}$, Ƅ$_{\mathcal{M}}$⊆ ꝱ$_{\mathcal{M}}$ such that ꝱ$_{\mathcal{M}}$ $\tilde{\in}$ ($\tilde{Ꭓ}$ – Ƅ$_{\mathcal{N}}$) and ꝱ$_{\mathcal{N}}$ $\tilde{\in}$ ($\tilde{Ꭓ}$ – Ƅ$_{\mathcal{M}}$) and ($\tilde{Ꭓ}$ – Ƅ$_{\mathcal{N}}$) $\tilde{\cap}$ ($\tilde{Ꭓ}$ – Ƅ$_{\mathcal{M}}$) ≠ {∅}.

Proposition 4. 6. If (Ꭓ, Ʈ, Đ) is a $soft$-$Ʈ_1$-space , then, (Ꭓ, Ʈ, Đ, ꝉ) is a soft-ꝉ-pre-g-$Ʈ_1$-space.
Proof : Let ꝱ$_{\mathcal{M}}$, ꝱ$_{\mathcal{N}}$ $\tilde{\in}$ $\tilde{Ꭓ}$ such that ꝱ$_{\mathcal{M}}$ ≠ꝱ$_{\mathcal{N}}$ since (Ꭓ, Ʈ, Đ) is a soft-$Ʈ_1$-space, then ∃ ($Ƞ_1$,Đ), ($Ƞ_2$,Đ) ∈ Ʈ such that ꝱ$_{\mathcal{M}}$ $\tilde{\in}$ (($Ƞ_1$,Đ) – ($Ƞ_2$,Đ)) and ꝱ$_{\mathcal{N}}$ $\tilde{\in}$ (($Ƞ_2$,Đ) – ($Ƞ_1$,Đ)). *By Remark* 3.3, ($Ƞ_1$,Đ) and ($Ƞ_2$,Đ) are sꝉpg-open sets, and the proof is over.

Proposition 4.7. If (Ꭓ, Ʈ, Đ, ꝉ) is a sꝉpg-$Ʈ_1$-space then it is a sꝉpg-$Ʈ_0$-*space*.
Proof: Let ꝱ , ꝱ$_{\mathcal{N}}$ $\tilde{\in}$ $\tilde{Ꭓ}$ such that ꝱ$_{\mathcal{M}}$ ≠ ꝱ$_{\mathcal{N}}$ since (Ꭓ, Ʈ, Đ, ꝉ) is a sꝉpg-$Ʈ_1$-*space*, then ∃($Ƞ_1$, Đ), ($Ƞ_2$, Đ) ∈ sꝉpg-O(Ꭓ) such that, ꝱ$_{\mathcal{M}}$ $\tilde{\in}$ (($Ƞ_1$, Đ) – ($Ƞ_2$, Đ)) and ꝱ$_{\mathcal{N}}$ $\tilde{\in}$ (($Ƞ_2$, Đ) – ($Ƞ_1$, Đ)). Then ∃ (Ƞ , Đ) ∈ sꝉpg-O(Ꭓ)-open set *whenever*, ꝱ$_{\mathcal{M}}$$\tilde{\in}$ (Ƞ , Đ) , ꝱ$_{\mathcal{N}}$ $\tilde{\notin}$ (Ƞ , Đ) or ꝱ$_{\mathcal{M}}$ $\tilde{\notin}$ (Ƞ , Đ), ꝱ$_{\mathcal{N}}$ $\tilde{\in}$ (Ƞ , Đ) .

The conclusions in Proposition 4.7, is not *reversible* by *example* 4.8

Example 4.8. In the space (Ꭓ, Ʈ, Đ, ꝉ) ; Ꭓ= {ꬲ , ɱ , ɽ}, Ʈ= {$\tilde{Ꭓ}$,$\tilde{\emptyset}$, (Ƞ, Đ)} such that (Ƞ, Đ) ={(ꝱ₁ ,{ꬲ , ɱ}),(ꝱ₂ ,{ꬲ , ɱ})} and ꝉ =ŞŞ({ɱ, ɽ})$_Đ$. Then ŞpO(Ꭓ)={ŞŞ(Ꭓ)$_Đ${(Ƞ′, Đ), ($\mathcal{Z}$, Đ), ($\mathcal{M}$, Đ)}} such that ($\mathcal{Z}$, Đ) ={(ꝱ₁,{∅}), (ꝱ₂,{ɽ})}, and ($\mathcal{M}$,Đ)={(ꝱ₁,{ɽ}),(ꝱ₂,{∅})}. So, sꝉpg-C(Ꭓ) ={ $\tilde{\emptyset}$, $\tilde{Ꭓ}$, (Ƞ′, Đ), ($\mathcal{Z}$, Đ), ($\mathcal{M}$, Đ)} and sꝉpg-O(Ꭓ) = {$\tilde{\emptyset}$, $\tilde{Ꭓ}$,(Ƞ, Đ), ($\mathcal{Z}'$, Đ),($\mathcal{M}'$, Đ)}. Implies (Ꭓ, Ʈ, Đ, ꝉ) is a soft- $Ʈ_0$-space, which is not sꝉpg- $Ʈ_1$-space .

Definition 4.9. $(Ӽ, Ʈ, Ð, ɭ)$ is a soft-ɭ-*pre-g*-Ʈ$_2$-space (briefly *sɭpg*-Ʈ$_2$-space). If for any two *different elements* ȡ$_{\mathcal{M}}$ ≠ ȡ$_{\mathcal{N}}$ there are *sɭpg-open* sets (Ɑ$_1$,Ð), (Ɑ$_2$,Ð) such that ȡ$_{\mathcal{M}}$ $\tilde{\in}$ (Ɑ$_1$,Ð), ȡ$_{\mathcal{N}}$ $\tilde{\in}$ (Ɑ$_2$,Ð)and (Ɑ$_1$,Ð) ∩ (Ɑ$_2$,Ð) = {$\tilde{\emptyset}$}.

Example 4.10. A *topological* space (Ӽ, Ʈ, Ð, ɭ); Ӽ = {ȩ, ɱ, ɽ}, Ʈ= {$\tilde{Ӽ}$, $\tilde{\emptyset}$} and ɭ= ȘȘ(Ӽ)$_Ð$. Then Ș*pO*(Ӽ)= ȘȘ(Ӽ)$_Ð$. So, *sɭpg*-*C*(Ӽ)=*sɭpg*-*O*(Ӽ)=ȘȘ(Ӽ)$_Ð$. Then (Ӽ, Ʈ, Ð, ɭ) is a *sɭpg*-Ʈ$_2$-*space*.

Remark 4.11. If (Ӽ, Ʈ, Ð) is a soft-Ʈ$_2$-*space*, then (Ӽ, Ʈ, Ð, ɭ) is a *sɭpg*-Ʈ$_2$-*space*.
Proof : Let ȡ$_{\mathcal{M}}$, ȡ$_{\mathcal{N}}$ $\tilde{\in}$ $\tilde{Ӽ}$ whenever, ȡ$_{\mathcal{M}}$ ≠ ȡ$_{\mathcal{N}}$ since (Ӽ, Ʈ, Ð, ɭ) is a soft-Ʈ$_2$-*space* , then ∃ (Ɑ$_1$,Ð),(Ɑ$_2$,Ð) ∈ Ʈ such that ȡ$_{\mathcal{M}}$ $\tilde{\in}$ (Ɑ$_1$,Ð), ȡ$_{\mathcal{N}}$ $\tilde{\in}$ (Ɑ$_2$,Ð) and (Ɑ$_1$,Ð) $\tilde{\cap}$(Ɑ$_2$,Ð) = {$\tilde{\emptyset}$}.
By Remark 3.3, there are *sɭpg*-open sets (Ɑ$_1$,Ð), (Ɑ$_2$,Ð), such that ȡ$_{\mathcal{M}}$ $\tilde{\in}$ (Ɑ$_1$,Ð), ȡ$_{\mathcal{N}}$ $\tilde{\in}$ (Ɑ$_2$,Ð) and (Ɑ$_1$,Ð) $\tilde{\cap}$ (Ɑ$_2$,Ð) = {$\tilde{\emptyset}$}.

Remark 4.12. If (Ӽ, Ʈ, Ð, ɭ) is a *sɭpg*-Ʈ$_2$-*space* then it is a *sɭpg*-Ʈ$_1$-*space*.

Proof: Let ȡ$_{\mathcal{M}}$, ȡ$_{\mathcal{N}}$ $\tilde{\in}$ $\tilde{Ӽ}$ whenever, ȡ$_{\mathcal{M}}$ ≠ ȡ$_{\mathcal{N}}$ since (Ӽ, Ʈ, Ð, ɭ) is a *sɭpg*-Ʈ$_2$-*space* ,then there are *sɭpg*-open sets (Ɑ$_1$,Ð) , (Ɑ$_2$,Ð) such that ȡ$_{\mathcal{M}}$ $\tilde{\in}$ (Ɑ$_1$,Ð), ȡ$_{\mathcal{N}}$ $\tilde{\in}$ (Ɑ$_2$,Ð) and (Ɑ$_1$,Ð) ∩(Ɑ$_2$,Ð) = { $\tilde{\emptyset}$ }. Implies, d$_{\mathcal{M}}$ $\tilde{\in}$ ((Ɑ$_1$,Ð) – (Ɑ$_2$,Ð)) and d$_{\mathcal{N}}$ $\tilde{\in}$ ((Ɑ$_2$,Ð) – (Ɑ$_1$,Ð)).
The *conclusions* in Remark 4.12 are not *reversible* by example 4.5.
A space (Ӽ, Ʈ, Ð, ɭ) is a *sɭpg*-Ʈ$_1$-space. If for each, ȡ$_{\mathcal{M}}$, ȡ$_{\mathcal{N}}$ $\tilde{\in}$ $\tilde{Ӽ}$ and ȡ$_{\mathcal{M}}$ ≠ ȡ$_{\mathcal{N}}$. Then there are *sɭpg-open* sets ($\tilde{Ӽ}$ – ɓ$_{\mathcal{N}}$), ($\tilde{Ӽ}$ – ɓ$_{\mathcal{M}}$) whenever, ɓ$_{\mathcal{N}}$ and ɓ$_{\mathcal{M}}$ are two *finite* sets such that ɓ$_{\mathcal{N}}$ ⊆ ȡ$_{\mathcal{N}}$, ɓ$_{\mathcal{M}}$ ⊆ ȡ$_{\mathcal{M}}$ such that ȡ$_{\mathcal{M}}$ $\tilde{\in}$ ($\tilde{Ӽ}$ – ɓ$_{\mathcal{N}}$) and ȡ$_{\mathcal{N}}$ $\tilde{\in}$ ($\tilde{Ӽ}$ – ɓ$_{\mathcal{M}}$) and ($\tilde{Ӽ}$ – ɓ$_{\mathcal{N}}$) $\tilde{\cap}$ ($\tilde{Ӽ}$ – ɓ$_{\mathcal{M}}$) ≠ {∅}. So, (Ӽ, Ʈ, Ð, ɭ) is not *sɭpg*-Ʈ$_2$-*space*.
We have *previously noted* that Ӽ is a *sɭpg*- Ʈ$_i$-*space* whenever, it is a Ʈ$_{i+1}$-*space* ($\forall\, i = 0, 1$ *and* 2).
The opposite is not necessarily true by the following example:
Example 4.13. (Ӽ, Ʈ, Ð, ɭ) *is a sɭpg*-Ʈ$_i$-*space* ($i \in \{0,1,2\}$) *, where,* Ӽ= {ȩ ,ɱ ,ɽ}, Ʈ= {$\tilde{\emptyset}$, $\tilde{Ӽ}$} *and* ɭ= ȘȘ(Ӽ)$_Ð$ *So, sɭpg*-*C*(Ӽ) = *sɭpg*-*O*(Ӽ) = ȘȘ(Ӽ)$_Ð$. *But the space* (Ӽ, Ʈ, Ð) *is not soft*- Ʈ$_i$-*space (i* ∈{0,1,2})
.
The following chart shows the relationships among the various types of notions of our previously mentioning

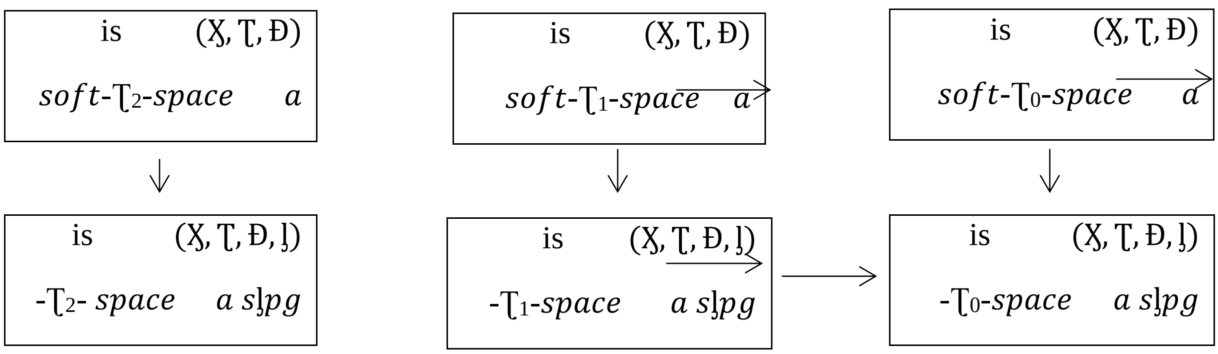


Figure 1. Separation Axioms with soft-ɭ- pre-g-open Sets

5. Games in soft -ļ-Pre-generalized Open sets topological spaces

In this section, a new game that connects them with soft separation axioms through sļpg open sets is inserted.

Definition 5.1. In the space (Ӽ, Ʈ, Ð, ļ), define a game Şg̦(Ʈ$_0$, Ӽ, ļ) as follows:

P Ⅰ and P Ⅱ are play an inning for every natural number in the z-th inning:

The first step, P Ⅰ Choose $(ḑ_{\mathcal{M}})_z \neq (ḑ_{\mathcal{N}})_z$ whenever, $(ḑ_{\mathcal{M}})_z$, $(ḑ_{\mathcal{N}})_z \widetilde{\in} \tilde{Ӽ}$. In the second step, P Ⅱ chooses(Ƀ$_z$,Ð) is a sļpg-O set s.t $ḑ_{\mathcal{M}} \widetilde{\in}$ Ƀ$_z$ ∧ $ḑ_{\mathcal{N}} \widetilde{\notin}$ Ƀ$_z$ or $ḑ_{\mathcal{M}} \widetilde{\notin}$ Ƀ$_z$ ∧ $ḑ_{\mathcal{N}} \widetilde{\in}$ Ƀ$_z$.Then P Ⅱ wins in the game Şg̦(Ʈ$_0$, Ӽ, ļ) if Ƀ = {(Ƀ$_1$,Ð), (Ƀ$_2$,Ð),… (Ƀ$_z$,Ð)…..} is a collection of a soft-ļ- pre open set in Ӽsuch that ∀ $(ḑ_{\mathcal{M}})_z$, $(ḑ_{\mathcal{N}})_z \widetilde{\in}$ Ӽ, ∃ (Ƀ$_z$,Ð) ∈ Ƀ s.t $(ḑ_{\mathcal{M}})_z \widetilde{\in}$ (Ƀ$_z$,Ð) , $(ḑ_{\mathcal{N}})_z \widetilde{\notin}$ (Ƀ$_z$,Ð) or $(ḑ_{\mathcal{M}})_z \widetilde{\notin}$ (Ƀ$_z$,Ð), $(ḑ_{\mathcal{N}})_z \widetilde{\in}$ (Ƀ$_z$,Ð) . Otherwise, P Ⅰ wins.

Example 5.2. Let Ӽ= { e̡, m̡, r̡}, Let Şg̦(Ʈ$_0$, Ӽ, ļ) be a soft game and Ð= {ḑ$_1$, ḑ$_2$}, Ʈ= { $\tilde{Ӽ}$, $\widetilde{\emptyset}$, (Ƌ, Ð), ($\mathcal{Z}$, Ð) } where, ((Ƌ, Ð) = {(ḑ$_1$,{e̡}) , (ḑ$_2$,{e̡})}, ($\mathcal{Z}$, Ð) = {(ḑ$_1$,{e̡ , m̡ }) , (ḑ$_2$,{e̡ , m̡ })} and ļ= {$\widetilde{\emptyset}$} **.** Then, ŞpO(Ӽ)= { (F, Ð) ; e̡ ∈ (F, Ð) for some ḑ ∈ Ð}. So, sļpg-C(Ӽ) = { $\widetilde{\emptyset}$, $\tilde{Ӽ}$,(Ƌ′, Ð) , ($\mathcal{Z}'$, Ð) } and sļpg-O(Ӽ) = Ʈ.

Then in the first inning:

The first step, P Ⅰ chooses $ḑ_{\mathcal{M}} \neq ḑ_{\mathcal{N}}$ whenever, ḑ , $ḑ_{\mathcal{N}} \widetilde{\in} \tilde{Ӽ}$ s.t $ḑ_{\mathcal{M}}$ = {e̡} and $ḑ_{\mathcal{N}}$ = {m̡}.

In the second step, P Ⅱ chooses (Ƌ, Ð) = {(ḑ$_1$,{e̡}),(ḑ$_2$,{e̡})} is a sļpg-O set.

In the second inning:

The first step, P Ⅰ chooses $ḑ_{\mathcal{M}} \neq ḑ_{\mathcal{O}}$ whenever, ḑ , $ḑ_{\mathcal{O}} \widetilde{\in} \tilde{Ӽ}$ s.t $ḑ_{\mathcal{M}}$ = {e̡} and $ḑ_{\mathcal{O}}$ = {r̡}.

In the second step,

P Ⅱ chooses (Ƌ, Ð) = {(ḑ$_1$,{e̡}),(ḑ$_2$,{e̡})} which is a sļpg-O set.

In the third inning:

The first step, P Ⅰ chooses $ḑ_{\mathcal{N}} \neq ḑ_{\mathcal{O}}$ whenever, ḑ , $ḑ_{\mathcal{O}} \widetilde{\in} \tilde{Ӽ}$ s.t $ḑ_{\mathcal{N}}$ = {m̡} and $d_{\mathcal{O}}$ = {r̡}.

In the second step,

P Ⅱ choose$s$ ($\mathcal{Z}$, Ð) = {(ḑ$_1$,{e̡ , m̡ }) , (ḑ$_2$,{e̡ , m̡ })} which is a sļpg-O set.

In the fourth inning:

The first step, P Ⅰ chooses $ḑ_{\mathcal{M}} \neq ḑ_{\mathcal{R}}$ whenever, ḑ , $ḑ_{\mathcal{R}} \widetilde{\in} \tilde{Ӽ}$ s.t $ḑ_{\mathcal{M}}$ = {e̡} and $ḑ_{\mathcal{R}}$ = {m̡, r̡}.

In the second step,

P Ⅱ choose$s$ (Ƌ, Ð) = {(ḑ$_1$,{e̡}),(ḑ$_2$,{e̡})} which is a sļpg-O set.

In the fifth inning:

The first step, P Ⅰ Choose $ḑ_{\mathcal{O}} \neq ḑ_{\mathcal{S}}$ whenever, ḑ , $ḑ_{\mathcal{S}} \widetilde{\in} \tilde{Ӽ}$ s.t $ḑ_{\mathcal{O}}$ = {r̡} and $ḑ_{\mathcal{S}}$ = {e̡,m̡}.

In the second step,

P Ⅱ Choose ($\mathcal{Z}$, Ð) = {(ḑ$_1$,{e̡ , m̡ }) , (ḑ$_2$,{e̡ , m̡ })} which is a sļpg-O set.

In the sixth inning:

The first step, P Ⅰ Choose ḑ𝓜 ≠ ḑ𝓛 whenever, ḑ𝓜, ḑ𝓛 ⋐̃ X̃ such that ḑ𝓜 = {ɱ} and ḑ𝓛 = {ȩ, ɽ}.

In the second step,

P Ⅱ Choose$(\mathcal{Z}, Đ)$ = {(ḑ$_1$,{ȩ, ɱ}) , (ḑ$_2$,{ȩ, ɱ})}which is a $s\rfloor pg\text{-}O$ set.

Then Ƀ = {(Ⅎ, Đ), $(\mathcal{Z}, Đ)$} is the winning strategy for P // in Ȿg(Ţ$_0$, X̧, ỉ). Hence P Ⅱ ↑ Ȿg(Ţ$_0$, X̧, ỉ).

Remark 5.4. In the space (X̧, Ţ, Đ, ỉ):

i. If P Ⅱ ↑ Ȿg(Ţ$_0$, X̧) then P Ⅱ ↑ Ȿg(Ţ$_0$, X̧ , ỉ).

ii. If P Ⅰ ↑ Ȿg(Ţ$_0$, X̧ , ỉ) then P Ⅰ ↑ Ȿg (Ţ$_0$, X̧).

Remark 5.5. In the space (X̧, Ţ, Đ, ỉ), if P // ↓ Ȿg(Ţ$_0$, X̧) then P // ↓ Ȿg(Ţ$_0$, X̧ , ỉ)

Theorem 5.6. A space (X̧, Ţ, Đ, ỉ) is $\mathcal{T}_0$-space if and only if P // ↑ Ȿg(Ţ$_0$, X̧, ỉ).

Proof: (⇒) in the $z\text{-}th$ inning P Ⅰ in Ȿg(Ţ$_0$, X̧, ỉ) Choose $(ḑ_{\mathcal{M}})_z \neq (ḑ_{\mathcal{N}})_z$ whenever, $(ḑ_{\mathcal{M}})_z$, $(ḑ_{\mathcal{N}})_z$ ⋐̃ X̃, P Ⅱ in Ȿg(Ţ$_0$, X̧, ỉ) Choose $(Ⅎ_z, Đ)$ is a $s\rfloor pg$-open set s.t ḑ𝓜 ⋐̃ $(Ⅎ_z, Đ)$ ∧ ḑ𝓝 ⋐̸̃ $(Ⅎ_z, Đ)$ or ḑ𝓜 ⋐̸̃ $(Ⅎ_z, Đ)$ ∧ ḑ𝓝 ⋐̃ $(Ⅎ_z, Đ)$. Since (X̧, Ţ, Đ, ỉ) is a $s\rfloor pg$-Ţ$_0$-space. Hence P Ⅱ ↑ Ȿg(Ţ$_0$, X̧, ỉ).

(⇐) Clear.

Corollary 5.7.

A space (X̧, Ţ, Đ, ỉ) is a $s\rfloor pg$-Ţ$_0$-space if and only if P Ⅰ ↑̸ Ȿg (Ţ$_0$, X̧, ỉ).

Proof: By Theorem 5.6, the proof is over.

Theorem 5.8. In the space (X̧, Ţ, Đ, ỉ) :

A space (X̧, Ţ, Đ, ỉ) is not $s\rfloor pg$-Ţ$_0$-space if and only if P Ⅰ ↑ Ȿg (Ţ$_0$, X̧, ỉ).

Proof:(⟹)in the z-th inning P Ⅰ in Ȿg (Ţ$_0$, X̧, ỉ)choose $(ḑ_{\mathcal{M}})_z \neq (ḑ_{\mathcal{N}})_z$ whenever, $(ḑ_{\mathcal{M}})_z$, $(ḑ_{\mathcal{N}})_z$ ⋐̃ X̃, P Ⅱ in Ȿg (Ţ$_0$, X̧, ỉ)cannot find $(Ⅎ_z, Đ)$ is a $s\rfloor pg\text{-}open$ set $(ḑ_{\mathcal{M}})_z$ ⋐̃ $(Ⅎ_z, Đ)$, $(ḑ_{\mathcal{N}})_z$ ⋐̸̃ $(Ⅎ_z, Đ)$ or $(ḑ_{\mathcal{M}})_z$ ⋐̸̃ $(Ⅎ_z, Đ)$, $(ḑ_{\mathcal{N}})_z$ ⋐̃ $(Ⅎ_z, Đ)$, because (X̧, Ţ, Đ, ỉ) is not $s\rfloor pg$-Ţ$_0$-space. Hence P Ⅰ ↑ Ȿg (Ţ$_0$, ỉ).

(⟸) Clear.

Corollary 5.9.

A space (X̧, Ţ, Đ, ỉ) is not $s\rfloor pg$-Ţ$_0$-space if and only if P Ⅱ ↑̸ Ȿg (Ţ$_0$, X̧, ỉ).

Proof: By Theorem 5.8, the proof is over.

Definition 5.10. In the space (X̧, Ţ, Đ, ỉ), define a game Ȿg(Ţ$_1$, X̧, ỉ) as follows:

P Ⅰ and P Ⅱ are play an inning for every natural number in the $z\text{-}th$ inning:

The first step, P Ⅰ Choose $(ḑ_{\mathcal{M}})_z \neq (ḑ_{\mathcal{N}})_z$ whenever, $(ḑ_{\mathcal{M}})_z$, $(ḑ_{\mathcal{N}})_z$ ⋐̃ X̃.

In the second step, P ll Choose (Ƌ$_z$, Ð), (Ư$_z$, Ð) are two $sʇpg$-Open soft sets s.t (ɖ$_{\mathcal{M}}$)$_z$ $\tilde{\in}$ (Ƌ$_z$, Ð) ⋀ (ɖ$_{\mathcal{N}}$)$_z$ $\tilde{\notin}$ (Ƌ$_z$, Ð) and (ɖ$_{\mathcal{M}}$)$_z$ $\tilde{\notin}$ (Ư$_z$, Ð) ⋀ (ɖ$_{\mathcal{N}}$)$_z$ $\tilde{\in}$ (Ư$_z$, Ð). Then P ll wins in the game Ȿg(Ʈ$_0$, Ӿ, ʇ) if Ƀ = {{(Ƌ$_1$, Ð), (Ư$_1$, Ð)},{(Ƌ$_2$, Ð), (Ư$_2$, Ð)},…{ (Ƌ$_z$, Ð), (Ư$_z$, Ð)}…...} is a collection of a soft-ʇ- pre open sets in Ӿ s.t ∀ (ɖ$_{\mathcal{M}}$)$_z$, (ɖ$_{\mathcal{N}}$)$_z$ $\tilde{\in}$ Ӿ,
∃ (Ƌ$_z$, Ð), (Ư$_z$, Ð) ∈ Ƀ s.t(ɖ$_{\mathcal{M}}$)$_z$ $\tilde{\in}$ (Ƌ$_z$, Ð) ⋀ (ɖ$_{\mathcal{N}}$)$_z$ $\tilde{\notin}$ (Ƌ$_z$, Ð) and (ɖ$_{\mathcal{M}}$)$_z$ $\tilde{\notin}$ (Ư$_z$, Ð) ⋀ (ɖ$_{\mathcal{N}}$)$_z$ $\tilde{\in}$ (Ư$_z$, Ð). Otherwise, P l wins in the game Ȿg(Ʈ$_1$, Ӿ, ʇ).

Example 5.11. Let Ȿg(Ʈ$_1$, Ӿ, ʇ) be a game whenever, Ӿ = {ȩ, ɱ, ɼ} , Ʈ= ȿȿ(Ӿ)$_Ð$, ʇ = {$\tilde{\emptyset}$}, Ð ={ ɖ$_1$, ɖ$_2$}. Then Ȿpo(Ӿ) = $sʇpg$-C(Ӿ) = $sʇpg$-O(Ӿ) = ȿȿ(Ӿ)$_Ð$.
In the first inning:
The first step, P l Choose ɖ$_{\mathcal{M}}$ ≠ ɖ$_{\mathcal{N}}$ whenever, ɖ$_{\mathcal{M}}$, ɖ$_{\mathcal{N}}$ $\tilde{\in}$ $\tilde{Ӿ}$ s.t ɖ$_{\mathcal{M}}$ = {ȩ} and ɖ$_{\mathcal{N}}$ ={ɱ}
In the second step,
P ll Choose (Ƌ,Ð), (Ư,Ð) s.t Ƌ(ɖ) = {ȩ}∀ ɖ, Ư(ɖ) = {ɱ} ∀ ɖ which are $sʇpg$-$open$ sets.
In the second inning:
The first step, P l Choose ɖ$_{\mathcal{N}}$ ≠ ɖ whenever, ɖ$_{\mathcal{N}}$, ɖ$_{\mathcal{O}}$ $\tilde{\in}$ $\tilde{Ӿ}$ s.t ɖ$_{\mathcal{N}}$= {ɱ} and ɖ$_{\mathcal{O}}$ = {ɼ}.
In the second step,
P ll Choose (Ư,Ð), (Ƈ,Ð) s.t Ư(ɖ) = {ɱ}∀ ɖ, Ƈ(ɖ) = {ɼ} ∀ ɖ which are $sʇpg$-$open$ sets.
In the third inning:
The first step, P l Choose ɖ$_{\mathcal{M}}$ ≠ ɖ$_{\mathcal{O}}$ whenever, ɖ , ɖ$_{\mathcal{O}}$ $\tilde{\in}$ $\tilde{Ӿ}$ s.t ɖ$_{\mathcal{M}}$ = {ȩ} and ɖ$_{\mathcal{O}}$ = {ɼ}.
In the second step,
P ll chooses (Ƌ,Ð), (Ƈ,Ð) s.t Ƌ(ɖ) = {ȩ}∀ ɖ, Ƈ(ɖ) = {ɼ} ∀ ɖ which are $sʇpg$-$open$ sets.
In the fourth inning:
The first step, P l chooses ɖ$_{\mathcal{M}}$ ≠ ɖ$_{\mathcal{R}}$ whenever, ɖ , ɖ$_{\mathcal{R}}$ $\tilde{\in}$ $\tilde{Ӿ}$ s.t ɖ$_{\mathcal{M}}$ = {ȩ} and ɖ$_{\mathcal{R}}$ = {ɱ,ɼ}.
In the second step,
P ll chooses (Ƌ,Ð), (F,Ð) s.t Ƌ(ɖ) = {ȩ}∀ ɖ, F(ɖ) = {ɱ,ɼ} ∀ ɖ which are $sʇpg$-$open$ sets.
In the fifth inning:
The first step, P l chooses ɖ$_{\mathcal{N}}$ ≠ ɖ$_{\mathcal{S}}$ whenever, ɖ , ɖ$_{\mathcal{S}}$ $\tilde{\in}$ $\tilde{Ӿ}$ s.t ɖ$_{\mathcal{N}}$ = {ɱ} and ɖ$_{\mathcal{S}}$ = {ȩ,ɼ}.
In the second step,
P ll chooses (Ư,Ð), (Ƞ,Ð) s.t Ư(ɖ) = {ɱ}∀ ɖ, Ƞ(ɖ) = {ȩ,ɼ} ∀ ɖ which are $sʇpg$-$open$ sets.
In the sixth inning:
The first step, P l chooses ɖ$_{\mathcal{O}}$ ≠ ɖ$_{\mathcal{L}}$ whenever, ɖ , ɖ$_{\mathcal{L}}$ $\tilde{\in}$ $\tilde{Ӿ}$ s.t ɖ$_{\mathcal{O}}$ = {ɼ} and ɖ$_{\mathcal{L}}$ = {ȩ,ɱ}.
In the second step,
P ll chooses (Ƈ,Ð), (ƥ,Ð) s.t Ƈ(ɖ) = {ɼ}, ƥ(ɖ) = {ȩ,ɱ} ∀ ɖ which are $sʇpg$-$open$ sets.
Then Ƀ = {{(Ƌ, Ð), (Ư, Ð)}, {(Ư, Ð), (Ƈ, Ð)}, {(Ƌ, Ð), (Ƈ, Ð)},{(Ƌ,Ð),(F,Ð)}, {(Ư, Ð), (Ƞ, Ð)}, {(Ƈ, Ð), (ƥ, Ð)}}. Is the winning strategy for P ll in Ȿg(Ʈ$_1$, Ӿ, ʇ). Hence Player ll ↑ Ȿg(Ʈ$_1$,Ӿ, ʇ). By the same way in Example 5.3, P / ↑ Ȿg(Ʈ$_1$, Ӿ, ʇ).

Remark 5.12. For a space (Ӿ , Ʈ, Ð, ʇ) :
i- If P ll ↑ Ȿg(Ʈ$_1$, Ӿ) then P ll ↑ Ȿg(Ʈ$_1$,Ӿ , ʇ).
ii- If P l ↑ Ȿg(Ʈ$_1$ Ӿ,, ʇ) then P l ↑ Ȿg(Ʈ$_1$, Ӿ).

Remark 5.13. For a space (Ӿ , Ʈ, Ð, ʇ), if P ll ↓ Ȿg(Ʈ$_1$, Ӿ) then P ll ↓ Ȿg(Ʈ$_1$,Ӿ , ʇ).

Theorem 5.14. A space (Ӽ , Ţ, Đ, ỉ) is a $s\text{ỉ}pg$-Ţ1-space) if and only if P // ↑ Ş𝔤(Ţ1 , Ӽ, ỉ).
Proof: (⟹) in the z-th inning P Ⅰ in Ş𝔤(Ţ1 , Ӽ, ỉ) choose$(\text{ḑ}_{\mathcal{M}})_z \neq (\text{ḑ}_{\mathcal{N}})_z$ whenever, $(\text{ḑ}_{\mathcal{M}})_z$, $(\text{ḑ}_{\mathcal{N}})_z$ $\widetilde{\in}$ $\tilde{Ӽ}$, P Ⅱ in Ş𝔤(Ţ0 , Ӽ, ỉ) Choose (Ƌ$_z$, Đ) , (Ƞ$_z$, Đ) are two $s\text{ỉ}pg$-open sets s.t $(\text{ḑ}_{\mathcal{M}})_z$ $\widetilde{\in}$ (Ƌ$_z$, Đ) ∧ $(\text{ḑ}_{\mathcal{N}})_z$ $\widetilde{\notin}$ (Ƌ$_z$, Đ) and $(\text{ḑ}_{\mathcal{N}})_z$ $\widetilde{\in}$, (Ƞ$_z$, Đ) ∧ $(\text{ḑ}_{\mathcal{M}})_z$ $\widetilde{\notin}$ (Ƞ$_z$, Đ) Since(Ӽ , Ţ, Đ, ỉ)is a $s\text{ỉ}pg$-Ţ1-space. Then Ƀ =
{{(Ƌ$_1$, Đ), (Ƞ$_1$, Đ)}, {(Ƌ$_2$, Đ) , (Ƞ$_2$, Đ)}, ... , {(Ƌ$_z$, Đ), (Ƞ$_z$, Đ)}, ... } is the winning strategy for P Ⅱ in Ş𝔤(Ţ1 , Ӽ, ỉ). Hence P Ⅱ ↑ Ş𝔤(Ţ1 , Ӽ, ỉ).
(⟸) Clear.

Corollary 5.15.
A space (Ӽ , Ţ, Đ, ỉ) is a $s\text{ỉ}pg$-Ţ1-space if and only if P Ⅰ ↟ Ş𝔤(Ţ1, Ӽ ,ỉ).
Proof: By Theorem 5.14, the proof is over.

Theorem 5.16. For a space (Ӽ , Ţ, Đ, ỉ):
A space (Ӽ , Ţ, Đ, ỉ) is not $s\text{ỉ}pg$-Ţ1-space if and only if P Ⅰ ↑ Ş𝔤(Ţ1, Ӽ,ỉ).
Proof:(⟹) in the z-th inning P Ⅰ in Ş𝔤(Ţ1 , Ӽ, ỉ) choose$(\text{ḑ}_{\mathcal{M}})_z \neq (\text{ḑ}_{\mathcal{N}})_z$ whenever, $(\text{ḑ}_{\mathcal{M}})_z$, $(\text{ḑ}_{\mathcal{N}})_z$ $\widetilde{\in}$ $\tilde{Ӽ}$, P Ⅱ in Ş𝔤(Ţ1 , Ӽ, ỉ) cannot find (Ƌ$_z$, Đ) , (Ƞ$_z$, Đ) are two $s\text{ỉ}pg$-open sets s.t $(\text{ḑ}_{\mathcal{M}})_z$ $\widetilde{\in}$ (Ƌ$_z$, Đ) ∧ $(\text{ḑ}_{\mathcal{N}})_z$ $\widetilde{\notin}$ (Ƌ$_z$, Đ) and $(\text{ḑ}_{\mathcal{N}})_z$ $\widetilde{\in}$, (Ƞ$_z$, Đ) ∧ $(\text{ḑ}_{\mathcal{M}})_z$ $\widetilde{\notin}$ (Ƞ$_z$, Đ) because (Ӽ , Ţ, Đ, ỉ) is not $s\text{ỉ}pg$-Ţ1-space. Hence P Ⅰ ↑ Ş𝔤(Ţ1 , Ӽ, ỉ).
(⟸) Clear.

Corollary 5.17.
If a space (Ӽ , Ţ, Đ, ỉ) is not $s\text{ỉ}pg$-Ţ1-space if and only if P Ⅱ ↟ Ş𝔤(Ţ1 Ӽ, , ỉ).
Proof: Similar way of proof Theorem 4.16.

Definition 5.18.
In the space (Ӽ, Ţ, Đ, ỉ), define a game Ş𝔤(Ţ2 , Ӽ, ỉ) as follows:

P Ⅰ and P Ⅱ are playing an inning for every natural number in the $z\text{-}th$ inning:

The first step, P Ⅰ Choose $(\text{ḑ}_{\mathcal{M}})_z \neq (\text{ḑ}_{\mathcal{N}})_z$ whenever, $(\text{ḑ}_{\mathcal{M}})_z$, $(\text{ḑ}_{\mathcal{N}})_z$ $\widetilde{\in}$ $\tilde{Ӽ}$.

In the second step, P Ⅱ Choose (Ƌ$_z$, Đ), (Ʊ$_z$, Đ) are two $s\text{ỉ}pg$-Open soft sets s.t (ḑ$_{\mathcal{M}}$)$_z$ $\widetilde{\in}$ (Ƌ$_z$, Đ) , (ḑ$_{\mathcal{N}}$)$_z$ $\widetilde{\in}$ (Ʊ$_z$, Đ) and (Ƌ$_z$, Đ) $\widetilde{\cap}$ (Ʊ$_z$, Đ)={ $\widetilde{\emptyset}$ }.

Then P Ⅱ wins in the game Ş𝔤(Ţ0 , Ӽ, ỉ) if

Ƀ = {{(Ƌ$_1$, Đ), (Ʊ$_1$, Đ)},{(Ƌ$_2$, Đ), (Ʊ$_2$, Đ)},…**{** (Ƌ$_z$, Đ), (Ʊ$_z$, Đ)}…...} be a collection of a soft-ỉ- *pre* open set in Ӽ s.t ∀ $(\text{ḑ}_{\mathcal{M}})_z$, $(\text{ḑ}_{\mathcal{N}})_z$ $\widetilde{\in}$ Ӽ, ∃ (Ƌ$_z$, Đ), (Ʊ$_z$, Đ) ∈ Ƀ s.t(ḑ$_{\mathcal{M}}$)$_z$ $\widetilde{\in}$ (Ƌ$_z$, Đ) , (ḑ$_{\mathcal{N}}$)$_z$ $\widetilde{\in}$ (Ʊ$_z$, Đ) and (Ƌ$_z$, Đ) $\widetilde{\cap}$ (Ʊ$_z$, Đ)={ $\widetilde{\emptyset}$ }. Otherwise, P / wins in the game Ş𝔤(Ţ2 , Ӽ, ỉ).

For Example, 5.11. ∀$(\text{ḑ}_{\mathcal{M}})_z \neq (\text{ḑ}_{\mathcal{N}})_z$ whenever $(\text{ḑ}_{\mathcal{M}})_z$, $(\text{ḑ}_{\mathcal{N}})_z$ $\widetilde{\in}$ Ӽ, ∃ (Ƌ$_z$, Đ), (Ʊ$_z$, Đ) ∈ Ƀ s.t(ḑ$_{\mathcal{M}}$)$_z$ $\widetilde{\in}$ (Ƌ$_z$, Đ) , (ḑ$_{\mathcal{N}}$)$_z$ $\widetilde{\in}$ (Ʊ$_z$, Đ) and (Ƌ$_z$, Đ) $\widetilde{\cap}$ (Ʊ$_z$, Đ)={ $\widetilde{\emptyset}$ }.
So Ƀ = {{(Ƌ, Đ), (Ʊ, Đ)},{(Ʊ,Đ),(Ƈ,Đ)},{(Ƌ,Đ),(Ƈ,Đ)},{(Ƌ,Đ),(F,Đ)},
{(Ʊ, Đ),(Ƞ,Đ)}, {(Ƈ, Đ), (ƥ, Đ)}}. Is the winning startegy for P Ⅱ in Ş𝔤(Ţ2 , Ӽ, ỉ). Hence P Ⅱ ↑ Ş𝔤(Ţ2 , Ӽ, ỉ). By the same way in Example 5.3, P / ↑ Ş𝔤(Ţ2 , Ӽ, ỉ).

Remark 5.19. For a space (Ӽ, Ţ, Đ, ʠ):

i- If P II ↑ Şģ(Ţ$_2$, Ӽ) then P II ↑ Şģ(Ţ$_2$, Ӽ , ʠ).

ii- If P I ↑ Şģ(Ţ$_2$, Ӽ , ʠ) then P I ↑ Şģ(Ţ$_2$, Ӽ).

Remark 5.20. For a space (Ӽ, Ţ, Đ, ʠ), if Player II ↓ Şģ(Ţ$_2$, Ӽ) then P II ↓ Şģ(Ţ$_2$ Ӽ, , ʠ).

Theorem 5.21. A space (Ӽ, Ţ, Đ, ʠ) is *sʠpg*-Ţ$_2$-*space* if and only if P // ↑ Şģ(Ţ$_2$, Ӽ, ʠ).

Proof: (⟹) in the z-th inning , P I in Şģ(Ţ$_2$, Ӽ, ʠ) Choose $(d_{\mathcal{M}})_z \neq (d_{\mathcal{N}})_z$ whenever, $(d_{\mathcal{M}})_z$, $(d_{\mathcal{N}})_z$ ∈̃ X̃, P II in Şģ(Ţ$_2$, Ӽ, ʠ) Choose (Ꭿ$_z$, Đ), (Ư$_z$, Đ) are two *sʠpg*-Open sets s.t $(d_{\mathcal{M}})_z$ ∈̃ (Ꭿ$_z$, Đ) , $(d_{\mathcal{N}})_z$ ∈̃ (Ư$_z$, Đ) and (Ꭿ$_z$, Đ) ∩̃ (Ư$_z$, Đ)={ ∅̃ }. Since (Ӽ, Ţ, Đ, ʠ) is *sʠpg*-Ţ$_2$-space. Then Ƀ = {{(Ꭿ$_1$, Đ), (Ư$_1$, Đ)},{(Ꭿ$_2$, Đ), (Ư$_2$, Đ)},…{ (Ꭿ$_z$, Đ), (Ư$_z$, Đ)}…...} is the winning strategy for P II in Şģ(Ţ$_2$, Ӽ, ʠ). Hence P II ↑ Şģ(Ţ$_2$, Ӽ, ʠ).

(⟸) Clear.

Corollary 5.22.

A space (Ӽ, Ţ, Đ, ʠ) is a *sʠpg*-Ţ$_2$-space if and only if P I ⇞ Şģ(Ţ$_2$, Ӽ, , ʠ).

Proof: By Theorem 5.21, the proof is over.

Theorem 5.23. For a space (Ӽ, Ţ, Đ, ʠ):

A space (Ӽ, Ţ, Đ, ʠ) is not a *sʠpg*-Ţ$_2$-space if and only if P I ↑ Şģ(Ţ$_2$,Ӽ , ʠ).

Proof: (⟹) in the z-th inning, P I in Şģ(Ţ$_2$, Ӽ, ʠ Choose $(d_{\mathcal{M}})_z \neq (d_{\mathcal{N}})_z$ whenever, $(d_{\mathcal{M}})_z$, $(d_{\mathcal{N}})_z$ ∈̃ X̃, P II in Şģ(Ţ$_2$, Ӽ, ʠ) cannot find (Ꭿ$_z$, Đ),(Ư$_z$, Đ) are two *sʠpg*-Open sets s.t$(d_{\mathcal{M}})_z$ ∈̃ (Ꭿ$_z$, Đ), $(d_{\mathcal{N}})_z$ ∈̃ (Ư$_z$, Đ) and (Ꭿ$_z$, Đ) ∩̃ (Ư$_z$, Đ)={ ∅̃ }, because(Ӽ, Ţ, Đ, ʠ) is not *sʠpg*-Ţ$_2$-space. Hence P I ↑ Şģ(Ţ$_2$, Ӽ, ʠ).

(⟸) Clear.

Corollary 5.24.

A space (Ӽ, Ţ, Đ, ʠ) is not a *sʠpg*-Ţ$_2$-space if and only if P II ⇞ Şģ(Ţ$_2$, Ӽ , ʠ).

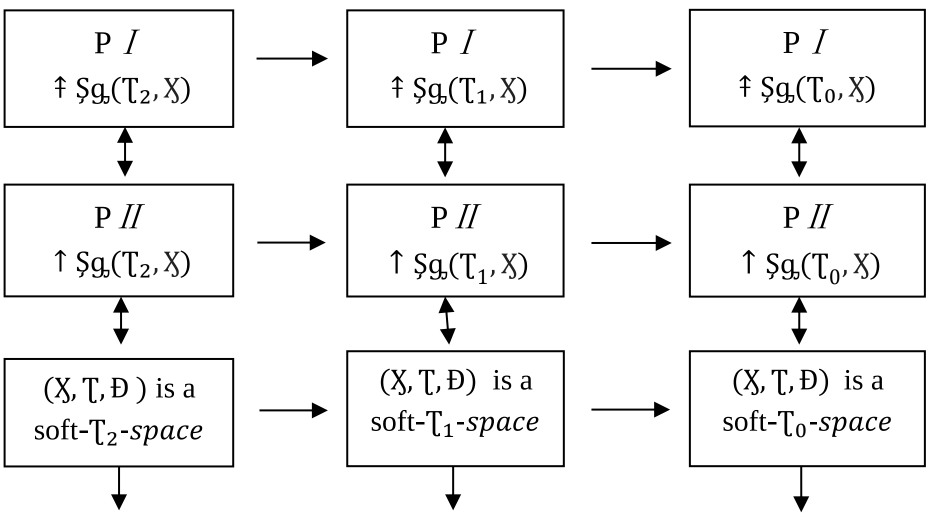
Proof: By Theorem 5.23, the proof is over.

Remark 5.25. For a space (Ӽ, Ţ, Đ, ʠ) :

i. If **P II** ↑ Şģ(Ţ$_{i+1}$, Ӽ, ʠ) then **P II** ↑ Şģ(Ţ$_i$, Ӽ, ʠ)**,** where $i = \{0,1\}$**.**

ii.If **P II** ↑ Şģ(Ţ$_i$, Ӽ)**;** then **P II** ↑ Şģ(Ţ$_i$, ʠ)**,** where $i = \{0,1,2\}$**.**

The following **(Figure**) clarifies relationships in Theorem 5.6, Theorem 5.14, Theorem 5.21 and Remark 5.25.



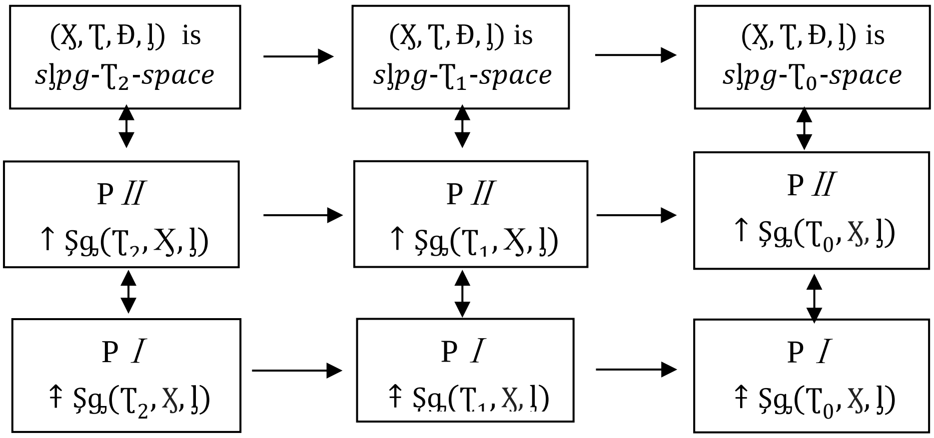


Figure 2. The winning and losing strategy for any player in $Sg(T_i, X)$ and $Sg(T_i, J)$ where i={0,1,2}.

Remark 5.26. For a space (X, T, J):

i- If P I ↑ $Sg(T_i, X, J)$ then P I ↑ $Sg(T_{i+1}, X, J)$, where $i = \{0,1\}$.

ii- If P I ↑ $Sg(T_i, X, J)$ then P I ↑ $Sg(T_i, X)$, where $i = \{0,1,2\}$.

The following (**Figure**) clarifies relationships in Theorem 5.8, Theorem 5.16, Theorem 5.23 and Remark 5.26.

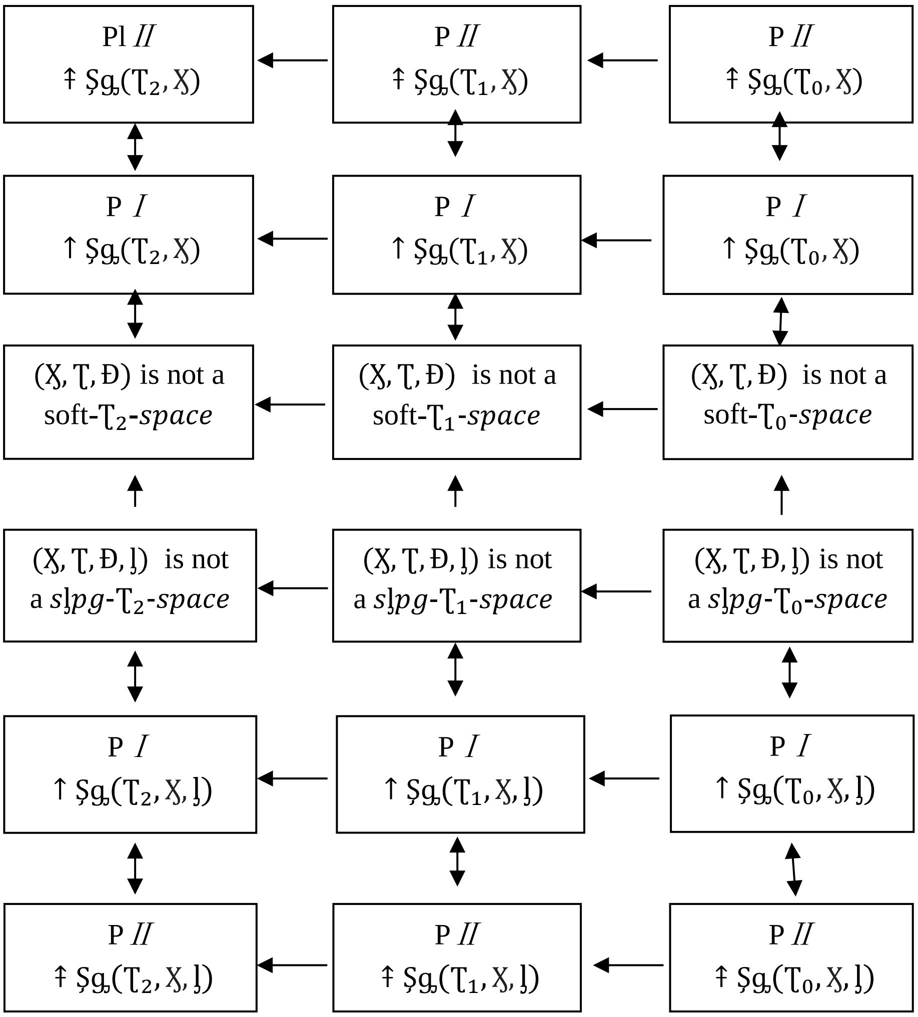


Figure 3. The winning and losing strategy where X is not $sJpg$-T_i-space and not soft T_i-space.

References

1. Shabir, M; Naz,M. On Soft to topological spaces. *Com put Math .Appl*. **2011,** *61*:1786-1799.
2. Hussain,S. ; Ahmad, B. Soft separation axioms in soft topological spaces. *HJMS.* **2015,** *44(3)*: 559-568.
3. Kadil, A.; Tantawy, O. A. E.; El-Sheikn, S. A ; Abd El-latif, A. M. γ- operation and decompositions of some forms of soft continuity in soft topological spaces, *AFMI*. **2014** ***7***:181-196.
4. Abd El-latif, A.M. Supra soft topological spaces *Jo ̈kull journal*. **2014**, *8(4)*: 1731-1740.
5. Kandil, A.; Tantawy, O. A. E.; El-Sheikh, S.A ; Abd El-latif, A .M. (Soft ideal theory, soft local function and generated soft topological spaces ,*Appl .Math . Inf .Sci* . **2014,***8(4):* 1595-1603.
6. Nasaf ,A. A.; Radwan, A. E.; Ibrahem, F. ; Esmaeel, R. B. () Soft α-compactness, via soft ideals, *Jou. of Advances in Math, June*. **2016,** *12(4):* 6178-6184.
7. Esmaeel, R. B.; Naser, A. I. some properties of-Ĩ-semi open soft sets with respect to soft Ideals, *IJpAM*. **2016,***4:* 545-561.
8. Esmaeel, R. B.; Nasir, A. I. ; Bayda Atiya Kalaf. () on α-gĨ-closed soft sets, *Sci. Inter. (Lahore)*, **2018**, *30(5):* 703-705.
9. Maji ,P. K.; Biswas , R .; Roy ,A.R. () soft set theory. *Comput. Math. Appl* . **2003,***45*: 555-562.
10. Ali, M. I.; Feng ,F.; Liu ,X.; Min ,W. K .; Shaber .M. () On Some new operations in soft set theory *.com put. Math. Appl* . **2009,***57(9):*1547-1553.
11. Zorlutuna, I.;Akdag ,M.; Min ,W . K.; Atmaca .S. () Remark On soft topological spaces, *Ann. fuzzy Math. Inform*. **2012**, *3(2)*: 171-185.
12. Subhashini, J. Soft Pre Generalized closed sets in a soft topological space, *International Journal of Engineering Trends and Technology*, **2014** , *12*, Number 7-Jun.
13. Mohammad, R.J. ; Esmaeel, R.B. New Games Via soft- $\mathcal{J}$-Semi-g-Separation axioms, *Ibn Al-Haitham Journal for pure and Applied S*cience. **2020,***33(4)*: 12.