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Some Topological and Polynomial Indices (Hosoya and Schultz) for the Intersection Graph of the Subgroup of Z_{r^n}

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Abstract

Let Z_{r^n} be any group with identity element (e) . A subgroup intersection graph of a subset Z_{r^n} is the Graph with V $(\Gamma_{SI}(Z_{r^n})) = Z_{r^n}$ - e and two separate peaks c and d contiguous for c and d if and only if $|\langle c\rangle \cap \langle d\rangle| > 1$, Where $\langle c\rangle$ is a Periodic subset of Z_{r^n} resulting from $c \in Z_{r^n}$. We find some topological indicators in this paper and Multi-border (Hosoya and Schultz) of $\Gamma_{SI}(Z_{r^n})$, where $r \geq 2$, $n > 1$,r is aprime number.

Keyword: Hosoya Polynomial, Schultz Polynomial. , connectivity index, Sum connectivity index, Forgotten index, first Zagreb index, Harmonic index.

1.Introduction

A topological index is a real number associated with the graph, which must be structurally constant. Topological index sometimes called molecular structure descriptor[1]. Many topological indicators have been identified and many applications have been found as a means of nemuls chemical, pharmaceutical and other molecular properties. The Weiner Index is the first topological indicator used in chemistry. More precisely, in 1947, Harold Weiner presented and developed this interesting indicator to determine the physical properties of the hens known as paraffins.

In this paper we examine some topological indicators that depend on the degree of examples of Eccentric connectivity index[2], connectivity index[3], sum connectivity index[4], Zagreb index[5], forgotten index[6], The index of geometric- arithmetic [3], Index of Atom-Bond Connectivity [3] and Harmonic index[7].

In (2019),Abdussakir [8] introduced topological indices about symmetric group graph, also in (2020) G. R. Roshini [9] studied topological indices of transformation graphs , and in (2021) Alaa .J and Akram.S [10] studied topological indices and (hosoya and Schultz) polynomial about subgroup intersection graph of a group Z_r .

One of the graphic concepts obtained from the group is the concept of a subset cross chart of a group introduced by [11]. In refere to the subgroup intersection graph definition by [11], let Graph group be the intersection (G) where G is a graph with V ($r_{SI}(G)$)= G-e and two distinct peaks A and B are adjacent in the r_{SI} (G) if

| 〈a〉 ∩ 〈b〉 |> 1, where 〈a〉 is a periodic subset of G resulting from a ∈G.

2. Method and Materials

In an existing article, all graphs are simple, limited, connected and :directed. For $G = (V(G), E(G))$ graph, the order of G is $p(G) = V(G)$ and the scale of G is $q(G) = E(G)$. Let $deg(u)$ denote the degree of vertex u in G. If $deg(u) = 0$, then u is an isolated vertices. Let $d(u, v)$Indicatex the distance between the peaks u and v in G. The eccentricity $ecc(u)$ of the vertex u is $ecc(u) = sup\{d(u, v): v \in V(G)\}$.

The following definition refers to a graph $G = (V(G), E(G))$.

Eccentric connectivity index of G is [2]

$$\xi^C(G))) = \sum_{u\in V(G)} deg(u).e(u)$$

The index of connectivity of G is[3]

$$X(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{deg(u).deg(v)}}$$

Sum the index of connectivity of G is [4]

$$S(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{deg(u)+deg(v)}}$$

A first zagreb index of G is [5]

$$M_1(G) = \sum_{u\in V(G)} (deg(u))^2$$

A second zagreb index of G is [5]

$$M_2(G) = \sum_{uv\ EG)} deg(u).deg(v)$$

The forgotten index of G is [6]

$$F(G)\ ) = \sum_{u\in V(G)} (deg(u))^3$$

Atom Bond connectivity index of G is [3]

$ABC(G)) = \sum_{uv\in E(G)} \sqrt{\frac{deg(u)+deg(v)-2}{deg(u).deg(v)}}$

Geometric-Arithmetic index of G is [3]

$GA(G) = \sum_{uv\in E(G)} \frac{2\sqrt{deg(u).deg(v)}}{deg(u)+deg(v)}$

Harmonic index of G is [7]

$$H(G) = \sum_{uv\in E(G)} \frac{2}{deg(u)+deg(v)}$$

3. The Main Result

To get a good look ,: $r \geq 2, n > 1$, a subgroup intersection graph of a group Z_{r^n} is $(\Gamma_{SI}(Z_{r^n}))$ a graph with V $(\Gamma_{SI}(Z_{r^n})) = Z_{r^n} - e$, any two different vertices a and b are adjacent:$|\langle a\rangle \cap \langle b\rangle| > 1$, where $\langle a\rangle$ is the subset created by a $\in Z_{r^n}$.

Theorem 3.1

Let Z_{r^n} be a group with $r \geq 2$, $n > 1$,:then the Eccentric connectivity index of $\Gamma_{SI}(Z_{r^n})$ is $\xi^c(\Gamma_{SI}(Z_{r^n})) = (r^n - 1)(r^n - 2)$

Proof:

$(v) = r^n - 2$, $\forall v \in V(\Gamma_{SI}(Z_{r^n}))$, $v = 1,2,\dots,r^n - 1$ deg

e $(v) = 1$, $\forall v \in V(\Gamma_{SI}(Z_{r^n}))$, $v = 1,2,\dots,r^n - 1$

$$\xi^C(\Gamma_{SI}(Z_{r^n}))) = \sum_{u\in V(\Gamma_{SI}(Z_{r^n}))} deg(u).e(u)$$

$$= \underbrace{deg(1).e(1) + \cdots + deg(r^n - 1).e(r^n - 1)}_{(r^n-1)times}$$

$$=(r^n - 1)(r^n - 2)$$

Theorem 3.2

Let Z_{r^n} be a group with $r \geq 2$, $n > 1$ then the connectivity index of $\Gamma_{SI}(Z_{r^n})$ is $X(\Gamma_{SI}((Z_{r^n})) = 1 + \frac{\sum_{i=3}^{r-1}(r^n-i)}{(r^n-2)}$

Proof:

$(v) = r^n - 2$, $\forall v \in V(\Gamma_{SI}(Z_{r^n}))$, $v = 1,2,\dots,r^n - 1$ deg

$$X(\Gamma_{SI}(Z_{r^n})) = \sum_{uv\in E(\Gamma_{SI}(Z_{r^n}))} \frac{1}{\sqrt{deg(u).deg(v)}}$$

$$= \underbrace{\frac{1}{\sqrt{deg(1).deg(2)}} + \cdots + \frac{1}{\sqrt{deg(1).deg(r^n-1)}}}_{(r^n-2)times}$$

$$+ \underbrace{\frac{1}{\sqrt{deg(2).deg(3)}} + \cdots + \frac{1}{\sqrt{deg(3).deg(r^n-1)}}}_{(r^n-3)times}$$

$$+ \underbrace{\frac{1}{\sqrt{deg(3).deg(4)}} + \cdots + \frac{1}{\sqrt{deg(3).deg(r^n-1)}}}_{(r^n-4)times}$$

$$+ \cdots .. + \frac{1}{\sqrt{deg(r^n-2).deg(r^n-1)}}$$

$$= \frac{(r^n-2)}{(r^n-2)} + \frac{(r^n-3)}{(r^n-2)} + \frac{(r^n-4)}{(r^n-2)} + \cdots + \frac{1}{(r^n-2)}$$

$$= 1 + \frac{\sum_{i=3}^{r^n-1}(r^n-i)}{(r^n-2)}$$

Theorem 3.3

Let Z_{r^n} be a group with $r \geq 2, n > 1$,:then the Sum connectivity index of $\Gamma_{SI}(Z_{r^n})$ is $S(\Gamma_{SI}(Z_{r^n})) = \frac{\sum_{i=2}^{r^n-1}(r^n-i)}{\sqrt{2r^n-4}}$

Proof:

$(v) = r^n - 2 \ , \forall v \in V(\Gamma_{SI}(Z_{r^n})),\ v = 1,2,\ldots, r^n - 1\ \ deg$

$$S(\Gamma_{SI}(Z_{r^n})) = \sum_{uv\in E(\Gamma_{SI}(Z_{r^n}))} \frac{1}{\sqrt{deg(u)+deg(v)}}$$

$$= \underbrace{\frac{1}{\sqrt{deg(1)+deg(2)}} + \cdots + \frac{1}{\sqrt{deg(1)+deg(r^n-1)}}}_{(r^n-2)times}$$

$$+ \underbrace{\frac{1}{\sqrt{deg(2)+deg(3)}} + \cdots + \frac{1}{\sqrt{deg(3)+deg(r^n-1)}}}_{(r^n-3)times}$$

$$+ \underbrace{\frac{1}{\sqrt{deg(3)+deg(4)}} + \cdots + \frac{1}{\sqrt{deg(3)+deg(r^n-1)}}}_{(r^n-4)times}$$

$$+ \cdots .. + \frac{1}{\sqrt{deg(r^n-2)+deg(r^n-1)}}$$

$$= \frac{(r^n-2)}{\sqrt{2r^n-4}} + \frac{(r^n-3)}{\sqrt{2r^n-4}} + \frac{(r^n-4)}{\sqrt{2r^n-4}} + \cdots + \frac{1}{\sqrt{2r^n-4}}$$
$$= \frac{\sum_{i=2}^{r^n-1}(r^n-i)}{\sqrt{2r^n-4}}$$

Theorem 3.4

Let Z_{r^n} be a group with $r \geq 2, n > 1$ then the first zegrab index of $\Gamma_{SI}(Z_{r^n})$ is $M_1(\Gamma_{SI}(Z_{r^n})) = (r^n-1)(r^n-2)^2$

Proof:

$(v) = r^n - 2 \ , \forall v \in V(\Gamma_{SI}(Z_{r^n})),\ v = 1,2,\ldots, r^n-1 \ deg$

$M_1(\Gamma_{SI}(Z_{r^n})) = \sum_{u\in V(\Gamma_{SI}(Z_{r^n}))}(deg(u))^2$

$= \underbrace{(deg(1))^2 + \cdots + (deg(r^n-1))^2}_{(r^n-1) times}$

$= (r^n-1)(r^n-2)^2$

Theorem 3.5

Let Z_{r^n} be a group with $r \geq 2, n > 1$ then the second zegrab index of $\Gamma_{SI}(Z_{r^n})$ is $M_2(\Gamma_{SI}(Z_{r^n})) = (r^n-2)^3 + (r^n-2)^2 \sum_{i=3}^{r^n-1}(r^n-i)$

Proof:

$(v) = r^n - 2 \ , \forall v \in V(\Gamma_{SI}(Z_{r^n})),\ v = 1,2,\ldots, r^n-1 \ deg$

$M_2(\Gamma_{SI}(Z_{r^n})) = \sum_{uv\ E(\Gamma_{SI}(Z_{r^n}))} deg(u).deg(v)$

$= \underbrace{deg(1).deg(2) + \cdots + deg(1).deg(r^n-1)}_{(r^n-2) times} +$
$\underbrace{deg(2).deg(3) + \cdots + deg(2).deg(r^n-1)}_{(r^n-3)\text{times}} +$
$\underbrace{deg(3).deg(4) + \cdots + deg(3).deg(r^n-1)}_{(r^n-4) times} + \cdots + deg(r^n-2).deg(r^n-1)$

$$= (r^n-2)^3 + (r^n-3)(r^n-2)^2 + (r^n-4)(r^n-2)^2 + \cdots + (r^n-2)^2$$

$$= (r^n-2)^3 + (r^n-2)^2 \sum_{i=3}^{r^n-1}(r^n-i)$$

Theorem 3.6

Let Z_{r^n} be a group with r ≥ $2, n > 1$ then the forgotten index of $\Gamma_{SI}(Z_{r^n})$ is $F(\Gamma_{SI}(Z_{r^n})) = (r^n - 1)(r^n - 2)^3$

Proof:

$(v) = r^n - 2\ , \forall v \in V(\Gamma_{SI}(Z_{r^n})),\ v = 1,2,\ldots, r^n - 1\ \ deg$

$$F(\Gamma_{SI}(Z_{r^n})) = \sum_{u\in V(\Gamma_{SI}(Z_{r^n}))}(deg(u))^3$$

$$= \underbrace{(deg(1))^3 + \cdots + (deg(r^n - 1))^3}_{(r^n-1)\ times}$$

$$= (r^n - 1)(r^n - 2)^3$$

Theorem 3.7

Let Z_{r^n} be a group with $r \geq 2, n > 1$, then the Atom Bond connectivity index of $\Gamma_{SI}(Z_{r^n})$ is

$$ABC(\Gamma_{SI}(Z_{r^n}))) = \sqrt{2r^n - 6} + \frac{\sqrt{2r^n - 6}\sum_{i=3}^{r^n-1}(r^n - i)}{(r^n - 2)}$$

Proof:

$(v) = r^n - 2\ , \forall v \in V(\Gamma_{SI}(Z_{r^n})),\ v = 1,2,\ldots, r^n - 1\ \ deg$

$$ABC(\Gamma_{SI}(Z_{r^n})) = \sum_{uv\in E(\Gamma_{SI}(Z_{r^n}))}\sqrt{\frac{deg(u) + deg(v) - 2}{deg(u)\,.\,deg(v)}}$$

$$= \underbrace{\sqrt{\frac{deg(1) + deg(2) - 2}{deg(1)\,.\,deg(2)}} + \cdots + \sqrt{\frac{deg(1) + deg(r^n - 1) - 2}{deg(1)\,.\,deg(r^n - 1)}}}_{(r^n-2)times}$$

$$+ \underbrace{\sqrt{\frac{deg(2) + deg(3) - 2}{deg(2)\,.\,deg(3)}} + \cdots + \sqrt{\frac{deg(2) + deg(r^n - 1) - 2}{deg(2)\,.\,deg(r^n - 1)}}}_{(r^n-3)times}$$

$$+ \underbrace{\sqrt{\frac{deg(3) + deg(4) - 2}{deg(3)\,.\,deg(4)}} + \cdots + \sqrt{\frac{deg(3) + deg(r^n - 1) - 2}{deg(3)\,.\,deg(r^n - 1)}}}_{(r^n-4)times} + \cdots$$

$$+ \sqrt{\frac{deg(r^n - 2) + deg(r^n - 1) - 2}{deg(r^n - 2)\,.\,deg(r^n - 1)}}$$

$$= \sqrt{2r^n-6} + \frac{(r^n-3)\sqrt{2r^n-6}}{(r^n-2)} + \frac{(r^n-4)\sqrt{2r^n-6}}{(r^n-2)} + \cdots + \frac{\sqrt{2r^n-6}}{(r^n-2)}$$

$$= \sqrt{2r^n-6} + \frac{\sqrt{2r^n-6}\sum_{i=3}^{r^n-1}(r^n-i)}{(r^n-2)}$$

Theorem 3.8

Let Z_{r^n} be a group with $r \geq 2\,, n > 1$ then the Geometric –Arithmetic index of $\Gamma_{SI}(Z_{r^n})$ is $GA(\Gamma_{SI}(Z_{r^n})) = (r^n-2) + \sum_{i=3}^{r^n-1}(r^n-i)$

Proof:

$(v) = r^n - 2\ , \forall v \in V(\Gamma_{SI}(Z_{r^n})),\ v = 1,2,\ldots,r^n-1 \quad deg$

$$GA(\Gamma_{SI}(Z_{r^n}))) = \sum_{uv\in E(\Gamma_{SI}(Z_{r^n}))} \frac{2\sqrt{deg(u).deg(v)}}{deg(u)+deg(v)}$$

$$= \underbrace{\frac{2\sqrt{deg(1).deg(2)}}{deg(1)+deg(2)} + \cdots + \frac{2\sqrt{deg(1).deg(r^n-1)}}{deg(1)+deg(r^n-1)}}_{(r^n-2)times}$$

$$+ \underbrace{\frac{2\sqrt{deg(2).deg(3)}}{deg(2)+deg(3)} + \cdots + \frac{2\sqrt{deg(2).deg(r^n-1)}}{deg(2)+deg(r^n-1)}}_{(r^n-3)times}$$

$$+ \underbrace{\frac{2\sqrt{deg(3).deg(4)}}{deg(3)+deg(4)} + \cdots + \frac{2\sqrt{deg(3).deg(r^n-1)}}{deg(3)+deg(r^n-1)}}_{(r^n-4)times}$$

$$+ \cdots . + \frac{2\sqrt{deg(r^n-2).deg(r^n-1)}}{deg(r^n-2)+deg(r^n-1)}$$

$$= \frac{2(r^n-2)^2}{2r^n-4} + \frac{2(r^n-3)(r^n-2)}{2r^n-4} + \frac{2(r^n-4)(r^n-2)}{2r^n-4} + \cdots + \frac{2(r^n-2)}{2r^n-4}$$

$$= (r^n-2) + \sum_{i=3}^{r^n-1}(r^n-i)$$

Theorem 3.9

Let Z_{r^n} be a group with $r \geq 2\,, n > 1$ then the Harmonic index of $\Gamma_{SI}(Z_{r^n})$ is

$H(\Gamma_{SI}(Z_{r^n}))) = 1 + \frac{2}{2r^n-4}\sum_{i=3}^{r^n-1}(r^n-i)$

Proof:

$(v) = r^n - 2\ , \forall v \in V(\Gamma_{SI}(Z_{r^n})),\ v = 1,2,\ldots,r^n-1\ deg$

$$H(\Gamma_{SI}(Z_{r^n}))) = \sum_{uv\in E(\Gamma_{SI}(Z_{r^n}))} \frac{2}{deg(u)+deg(v)}$$

$$= \underbrace{\frac{2}{deg(1)+deg(2)} + \cdots + \frac{2}{deg(1)+deg(r^n-1)}}_{(r^n-2)times}$$

$$+ \underbrace{\frac{2}{deg(2)+deg(3)} + \cdots + \frac{2}{deg(2)+deg(r^n-1)}}_{(r^n-3)times}$$

$$+ \underbrace{\frac{2}{deg(3)+deg(4)} + \cdots + \frac{2}{deg(3)+deg(r^n-1)}}_{(r^n-4)times} + \cdots . + \frac{2}{deg(r^n-2)+deg(r^n-1)}$$

$$= \frac{2(r^n-2)}{2r^n-4} + \frac{2(r^n-3)}{2r^n-4} + \frac{2(r^n-4)}{2r^n-4} + \cdots + \frac{2}{2r^n-4}$$

$$= 1 + \frac{2}{2r^n-4} \sum_{i=3}^{r^n-1} (r^n - i)$$

4.(Hosoya and Schultz) Polynomial of $\Gamma_{SI}(Z_{r^n})$

In this section, we find the (Hosoya and Schultz) Polynomial of $\Gamma_{SI}(Z_{r^n})$.

Definition 4.1(12):

Let G be a connected graph , then a Hosoya Polynomial of graph G is defined by $\mathrm{H}(\mathrm{G};\mathrm{x}) = \sum_{\mathrm{k}=0}^{diam(G)} d(G,k)\, \mathrm{x}^{\mathrm{k}}$, where $d(G,k)$ is the number of pairs of vertices of a graph G that are at distance k apart , for $k = 0,1,2,\ldots, diam(G)$,where

$diam(G) = \max_{\mathrm{u,v\in V(G)}} \mathrm{d(u,v)}$.

<u>Note 4.2(13)</u>: 1- $d(G,0) = p(G)$

2- $d(G,1) = q(G)$

Definition 4.3(14):

Let G be a connected graph , then a Schultz Polynomial of a graph G is defined by $\mathrm{Sc}(\mathrm{G};\mathrm{x}) = \sum_{\substack{\mathrm{u,v\in V(G)} \\ \mathrm{u\neq v}}} (deg(u)+deg(v))\mathrm{x}^{\mathrm{d(u,v)}}$, where $deg(u)$ is the degree of the vertices u and $deg(v)$ is the degree of vertices v, $d(u,v)$ is the distance between u and v.

Theorem 4.4:

$\mathrm{H}(\Gamma_{SI}(Z_{r^n}); x) = c_0 + c_1 x$, where $r \geq 2$, $n > 1$, $c_0 = r^n - 1$,

$c_1 = \sum_{i=2}^{r^n-1} (r^n - i)$

Proof:

For every $r \geq 2, n > 1$, we note that every vertices of graph $\Gamma_{SI}(Z_{r^n})$ is adjacent of all vertices of graph $\Gamma_{SI}(Z_{r^n})$, then diam($\Gamma_{SI}(Z_{r^n})) = 1$, its mean is H($\Gamma_{SI}(Z_{r^n}), x) = c_0 + c_1 x$ where

$c_i = d(\Gamma_{SI}(Z_{r^n}), i), \forall\, i = 0,1$

It is clear that $c_0 = d(\Gamma_{SI}(Z_{r^n}), 0) = |\Gamma_{SI}(Z_{r^n})| = r^n - 1$

Now we find the size of $\Gamma_{SI}(Z_{r^n})$, we note that there is m_{r^n-1} of edges s.t

$m_1 = r^n - 2, m_2 = r^n - 3, \ldots., m_{r^n-1} = 1$

Then,: $c_1 = m_1 + m_2 + \cdots + m_{r^n-1}$

We can write:

$$c_1 = \sum_{i=2}^{r^n-1} (r^n - i)$$

Theorem 4.5:

$Sc(\Gamma_{SI}(Z_{r^n}); x) = \sum_{i=2}^{r^n-1} (r^n - i)(2r^n - 4)x$, where $r \geq 2$, $n > 1$.

Proof:

$(v) = r^n - 2, \forall v \in V(\Gamma_{SI}(Z_{r^n})),\ v = 1,2,\ldots, r^n - 1\ deg$

$d(u,v) = 1, \forall u, v \in V(\Gamma_{SI}(Z_{r^n})$

$Sc(\Gamma_{SI}(Z_{r^n}); x) = \sum_{u,v \in V(\Gamma_{SI}(Z_{r^n}))} (deg(u) + deg(v)) x^{d(u,v)}$

$$= \underbrace{(deg(1) + deg(2))\,x + \cdots + (deg(1) + deg(r^n - 1))x}_{(r^n-2)times}$$
$$+ \underbrace{(deg(2) + deg(3))x + \cdots + (deg(2) + deg(r^n - 1))x}_{(r^n-3)times} + \cdots$$
$$+ (deg(r^n - 1) + deg(r^n - 2))x$$
$$= \sum_{i=2}^{r^n-1} (r^n - i)(2r^n - 4)x$$

5. Conclusions

This article has presented the formulae of some degree-based and eccentric-based topological indices of subgroup intersection graph of a group Z_{r^n}, where $r \geq 2, n > 1$. For further research, Revise on subgroup intersection graph of a group Z_{r^n}, where $r \geq 2, n > 1$, r is a prime number.

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