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Some Results via Gril Semi –p-Open Set

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Abstract

The significance of the work is to introduce the new class of open sets, which is said G-*sp*-open set with some of properties. Then clarify how to calculate the boundary area for these sets using the upper and lower approximation and obtain the best accuracy.

Keywords. G-semi-P open set, G- semi-P closed set , *accuracy seasure* $\mathfrak{M}$(M).

1.Introduction

A nonempty family G *of a topological space* Ẋ is named *a Grill* whenever

i. M ∈ G *and* M ⊆ S ⊆ Ẋ then S ∈ G.

ii. M, S ⊆ Ẋ ∧ M ∪ S ∈ G then M ∈ G ∨ S ∈ G. [1] Suppose that Ẋ is a nonempty set, Then the following families are grills on Ẋ. [1-3]

∅ and p(Ẋ) \ {∅} are trivial examples of a grill on Ẋ

G_{∞} which is the collection of all infinite subsets of Ẋ.

G_{co} which is the collection of all uncountable subsets of Ẋ.

G_p= {Λ: Λ∈ p(Ẋ), p ⊆ Λ} is a specific point grill on Ẋ.

G_A= {S: S∈p(Ẋ), S∩M ≠ ∅ }, and If (Ẋ,$\mathcal{T}$) is a topological space, then the family of all non-nowhere dense subsets called G= {M:$int_{\mathcal{T}}\, cl_{\mathcal{T}}(M) \neq \emptyset$ } is the one of kinds of a grill on Ẋ. Suppose that G is a grill on (Ẋ,$\mathcal{T}$) The operator Ǿ: p(Ẋ)→p(Ẋ) is defined by Ǿ (M)={x ∈ Ẋ\ ů ∩ M ∈ G, *for all* ů ∈ $\mathcal{T}$(Ẋ)},$\mathcal{T}$(Ẋ) indicate the neighborhood of x. A mapping Ψ: p(Ẋ)→p(Ẋ) is defined as Ψ (M) = M ∪ Ǿ (M) for all M ∈ p(Ẋ).[4,5]

The sap Ψ satisfies Kuratowski closure axioms: [3,4]

1. Ψ(∅) = ∅

2. If M ⊆S, then Ψ (M) ⊆ Ψ(S),
3. If M ⊆ Ẋ, then Ψ (Ψ (M)) =Ψ (M),
4. If M,S ⊆ Ẋ, then Ψ (M ∪ S) =Ψ (M) ∪ Ψ (S).

A subset M of (Ẋ,$\mathcal{T}$) is a preopen set if $M \subseteq$ intcl M The complement of a preopen set is named preclosed set. The collection of all preopen sets of Ẋ is indicate by po(Ẋ). The collection of all preclosed sets of Ẋ is indicate by pc(Ẋ).[7]

Now, PCL=∩ {M ⊆ Ẋ ; ů ⊆ M whenever $M^c \in PO$(Ẋ)}. [7]

A subset M of (Ẋ,$\mathcal{T}$) is named semi-p-open set, if and only if there exists a preopen set in Ẋ say Ụ such that Ụ ⊆ M ⊆ PCL Ụ. The collection of all semi-p-open sets of Ẋ is indicated by S-PO(Ẋ). The complement of a semi-pclosed set. The family of all semi-p-closed sets of Ẋ is indicate by S-PC(Ẋ). [7]

It is clear that every preopen set is a S-PO set [7].

2.Preliminaries.

Definition 2.1: [8]

Let Ẋ be a nonempty set and Ř be an equivalence relation on Ẋ , Ḿ ⊆ Ẋ;

The upper approximation of Ḿ for Ř is denoted by Ữ(Ḿ) , which is,

Ữ(Ḿ) $= \bigcup_{x\in Ẋ}$ { Ř(x): Ř(x) ∩ Ḿ ≠ ∅ } such that Ř(x) $is\ the\ equivalence\ class\ of\ x$ and the lower approximation of M for Ř is denoted by ₤ (Ḿ) , which is,

₤(Ḿ) $= \bigcup_{x\in Ẋ}$ { Ř(x): Ř(x) ⊆ Ḿ} .

The boundary region of M for $\mathcal{R}$ is denoted by $\mathcal{B}$(M), which is,

$\mathcal{B}$ (Ḿ) = Ữ(Ḿ) − ₤(Ḿ).

Proposition 2.2: [9,10]

If Ḿ, ẏ ⊆ Ẋ $then\ the\ following\ properties\ are\ realized$

1. ₤(Ḿ) ⊆ M ⊆ Ữ (Ḿ).
2. Ḿ ⊆ ẏ , $then$ ₤(Ḿ) ⊆ ₤(S) and (ỮḾ) ⊆ Ữ(ẏ).
3. ₤(∅) = Ữ(∅) = ∅ and₤ (Ẋ) = Ữ(Ẋ) = Ẋ .
4. Ữ(Ḿ ∪ ẏ) = Ữ(Ḿ) ∪ Ữ(ẏ).
5. Ữ(Ḿ ∩ ẏ) ⊆ Ữ(Ḿ) ∩ Ữ(ẏ).
6. ₤(Ḿ ∪ ẏ) ⊇ ₤(Ḿ) ∪ ₤(ẏ)
7. ₤(Ḿ ∩ ẏ) ⊆ ₤(Ḿ) ∩ ₤(ẏ).
8. Ữ(Ữ(Ḿ)) = ₤(Ữ(Ḿ)) = Ữ(Ḿ).
9. ₤(₤(Ḿ)) = Ữ(₤(Ḿ)) = ₤(Ḿ).

Example 2.3: let Ẋ= {$\rho_1, \rho_2, \rho_3, \rho_4$} and Ǥ= p(Ẋ) \ {∅} ,

Ř = {$(\rho_1,\rho_1),(\rho_2,\rho_2),(\rho_3,\rho_3),(\rho_4,\rho_4),(\rho_1,\rho_2),(\rho_2,\rho_1)$} ,Ř ($\rho_1$)={ ρ_1,ρ_2} = Ř (ρ_2)

Ř (ρ_3) = {ρ_3} , Ř (ρ_4)= {ρ_4}

Table 2.1. The boundary region

P(Ẋ)	Ữ (Ḿ)	₤ (Ḿ)	$\mathcal{B}$ (Ḿ)
∅	∅	∅	∅
$\{ \rho_1\}$	$\{ \rho_1, \rho_2\}$	∅	$\{ \rho_1, \rho_2\}$
$\{ \rho_2\}$	$\{ \rho_1, \rho_2\}$	∅	$\{ \rho_1, \rho_2\}$
$\{\rho_3\}$	$\{\rho_3\}$	$\{\rho_3\}$	∅
$\{\rho_4\}$	$\{\rho_4\}$	$\{\rho_4\}$	∅
$\{ \rho_1, \rho_2\}$	$\{ \rho_1, \rho_2\}$	$\{ \rho_1, \rho_2\}$	∅
$\{ \rho_1, \rho_3\}$	$\{ \rho_1, \rho_2, \rho_3\}$	$\{\rho_3\}$	$\{ \rho_1, \rho_2\}$
$\{ \rho_1, \rho_4\}$	$\{ s\rho_1, \rho_2, \rho_4\}$	$\{\rho_4\}$	$\{\rho_1, \rho_2\}$
$\{ \rho_2, \rho_3\}$	$\{ \rho_1, \rho_2, \rho_3\}$	$\{\rho_3\}$	$\{\rho_1, \rho_2\}$
$\{ \rho_2, \rho_4\}$	$\{\rho_1, \rho_2, \rho_4\}$	$\{\rho_4\}$	$\{\rho_1, \rho_2\}$
$\{ \rho_3, \rho_4\}$	$\{ \rho_3, \rho_4\}$	$\{ \rho_3, \rho_4\}$	∅
$\{ \rho_1, \rho_2, \rho_3\}$	$\{ \rho_1, \rho_2, \rho_3\}$	$\{ \rho_1, \rho_2, \rho_3\}$	∅
$\{\rho_1, \rho_2, \rho_4\}$	$\{\rho_1, \rho_2, \rho_4\}$	$\{\rho_1, \rho_2, \rho_4\}$	∅
$\{ \rho_2, \rho_3, \rho_4\}$	$\{\rho_1, \rho_2, \rho_3, \rho_4\}$	$\{\rho_3, \rho_4\}$	$\{\rho_1, \rho_2\}$
$\{ \rho_1, \rho_3, \rho_4\}$	$\{\rho_1, \rho_2, \rho_3, \rho_4\}$	$\{\rho_3, \rho_4\}$	$\{\rho_1, \rho_2\}$
$\{\rho_1, \rho_2, \rho_3, \rho_4\}$	$\{\rho_1, \rho_2, \rho_3, \rho_4\}$	$\{\rho_1, \rho_2, \rho_3, \rho_4\}$	∅

Definition 2.4:[11]

let Ẋ be a nonempty set and Ḿ ⊆ Ẋ *such that* Ř is any relation on Ẋ so, by using the concepts of lower and upper approximation.

$$\mathfrak{M}(\acute{M}) = \frac{|₤(\acute{M})|}{|Ữ(\acute{M})|} \quad , |₤(\acute{M})| \neq \emptyset$$

we can define the second accuracy measure of Ḿ which is called a semi-accuracy measure of approximation.

$$\mathfrak{M}_{\xi}(\acute{M}) = \frac{|Ữ(₤(\acute{M}))|}{|Ữ(\acute{M})|} \quad , |₤(\acute{M})| \neq \emptyset$$

The third measure is called pre –accuracy measure of approximation.

$$\mathfrak{M}_{p}(\acute{M}) = \frac{|₤(Ữ\acute{M})|}{|Ữ(\acute{M})|} \quad , |₤(\acute{M})| \neq \emptyset$$

Example 2. 5:

Let $Ẋ = \{\rho_1, \rho_2, \rho_3, \rho_4\}$ and, G= p(Ẋ) \ {∅},

Ř= $\{(\rho_1, \rho_1), (\rho_2, \rho_2), (\rho_3, \rho_3), (\rho_4, \rho_4), (\rho_2, \rho_3), (\rho_3, \rho_2)\}$

Ř (ρ_1)=$\{ \rho_1\}$,Ř $(\rho_2) = \{\rho_2, \rho_3\}$ = Ř(ρ_3) ,Ř $(\rho_4) = \{\rho_4\}$.

Table 2. 2. Accuracy measure of approximation

P(Ẋ)	Ữ (Ḿ)	₤ (Ḿ)	$\mathcal{B}$ (Ḿ)	₤ (Ữ (Ḿ))	Ữ (₤ Ḿ))
Ẋ	Ẋ	Ẋ	∅	Ẋ	Ẋ
∅	∅	∅	∅	∅	∅
$\{ \rho_1\}$	$\{ \rho_1\}$	$\{ \rho_1\}$	∅	$\{ \rho_1\}$	$\{ \rho_1\}$
$\{ \rho_2\}$	$\{ \rho_2, \rho_3\}$	∅	$\{ \rho_2, \rho_3\}$	$\{ \rho_2, \rho_3\}$	∅
$\{\rho_3\}$	$\{\rho_2, \rho_3\}$	∅	$\{ \rho_2, \rho_3\}$	$\{\rho_2, \rho_3\}$	∅

$\{\rho_4\}$	$\{\rho_4\}$	$\{\rho_4\}$	$\emptyset$	$\{\rho_4\}$	$\{\rho_4\}$
$\{\rho_1, s\rho_2\}$	$\{\rho_1,\rho_2,\rho_3\}$	$\{\rho_1\}$	$\{\rho_2,\rho_3\}$	$\{\rho_1,\rho_2,\rho_3\}$	$\{\rho_1\}$
$\{\rho_1,\rho_3\}$	$\{\rho_1,\rho_2,\rho_3\}$	$\{\rho_1\}$	$\{\rho_2,\rho_3\}$	$\{\rho_1,\rho_2,\rho_3\}$	$\{\rho_1\}$
$\{\rho_1,\rho_4\}$	$\{\rho_1,\rho_4\}$	$\{\rho_1,\rho_4\}$	$\emptyset$	$\{\rho_1,\rho_4\}$	$\{\rho_1,\rho_4\}$
$\{\rho_2,\rho_3\}$	$\{\rho_2,\rho_3\}$	$\{\rho_2,\rho_3\}$	$\emptyset$	$\{\rho_2,\rho_3\}$	$\{\rho_2,\rho_3\}$
$\{\rho_2,\rho_4\}$	$\{\rho_2,\rho_3,\rho_4\}$	$\{\rho_4\}$	$\{\rho_2,\rho_3\}$	$\{\rho_2,\rho_3,\rho_4\}$	$\{\rho_4\}$
$\{\rho_3,\rho_4\}$	$\{\rho_2,\rho_3,\rho_4\}$	$\{\rho_4\}$	$\{\rho_2,\rho_3\}$	$\{\rho_2,\rho_3,\rho_4\}$	$\{\rho_4\}$
$\{\rho_1,\rho_2,\rho_3\}$	$\{\rho_1,\rho_2,\rho_3\}$	$\{\rho_1,\rho_2,\rho_3\}$	$\emptyset$	$\{\rho_1,\rho_2,\rho_3\}$	$\{\rho_1,\rho_2,\rho_3\}$
$\{\rho_1,\rho_2,\rho_4\}$	$\dot{X}$	$\{\rho_1,\rho_4\}$	$\{\rho_2,\rho_3\}$	$\dot{X}$	$\{\rho_1,\rho_4\}$
$\{\rho_1,\rho_3,\rho_4\}$	$\dot{X}$	$\{\rho_1,\rho_4\}$	$\{\rho_2,\rho_3\}$	$\dot{X}$	$\{\rho_1,\rho_4\}$
$\{\rho_2,\rho_3,\rho_4\}$	$\{\rho_2,\rho_3,\rho_4\}$	$\{\rho_2,\rho_3,\rho_4\}$	$\emptyset$	$\{\rho_2,\rho_3,\rho_4\}$	$\{\rho_2,\rho_3,\rho_4\}$

Table 2. 3 Accuracy measure of approximation

.

$P(\dot{X})$	$\mathfrak{M}(\acute{M})$	$\mathfrak{M}_{\xi}(\acute{M})$	$\mathfrak{M}_{p}(\acute{M})$
$\dot{X}$	1	1	1
$\emptyset$	1	1	1
$\{\rho_1\}$	0	0	1
$\{\rho_2\}$	0	0	1
$\{\rho_3\}$	1	1	1
$\{\rho_4\}$	1/3	1/3	1
$\{\rho_1,\rho_2\}$	1/3	1/3	1
$\{\rho_1,\rho_3\}$	1	1	1
$\{\rho_1,\rho_4\}$	1	1	1
$\{\rho_2,\rho_3\}$	1/3	1/3	1
$\{\rho_2,\rho_4\}$	1/3	1/3	1
$\{\rho_3,\rho_4\}$	1	1	1
$\{\rho_1,\rho_2,\rho_3\}$	½	½	1
$\{\rho_1,\rho_2,\rho_4\}$	½	½	1
$\{\rho_1,\rho_3,\rho_4\}$	1	1	1
$\{\rho_2,\rho_3,\rho_4\}$	1	1	1

3. Grill semi-p-open sets

Definition 3.1 *Let* $(\dot{X},\mathcal{T},G)$ *be a grill topological space and* $M \subseteq \dot{X}$, then M is called Grill semi-p-open set denoted by "G-SPO set " if $\exists$ v $\in PO(\dot{X})$ such that v-M G $\wedge$ M-PCL(v) $\notin$G. The set of all G-SPO sets is denoted by G-SPO($\dot{X}$).

Example3.2 Let $\dot{X}=\{\rho_1,\rho_2,\rho_3\}$, $\mathcal{T}=\{\dot{X},\emptyset,\{\rho_1\}\}$

$PO(\dot{X})=\{\mathring{u} \subseteq \dot{X};\ \rho_1 \in \mathring{u}\} \cup \emptyset$, $PC(\dot{X})=\{\mathcal{F} \subseteq \dot{X}\ ;\ \rho_1 \notin \mathcal{F}\} \cup \dot{X}$.

Then G-SPO $(\dot{X}) = p(\dot{X})$.

Example 3.3: *Let* $\dot{X}=\{\rho_1,\rho_2,\rho_3,\rho_4\}$, $\mathcal{T}=\{\dot{X},\emptyset,\{\rho_1\},\{\rho_4\},\{\rho_1,\rho_4\}\}$, G= $p(\dot{X}) \setminus \{\emptyset\}$,

$PO(\dot{X})=\{\dot{X},\emptyset,\{\rho_1\},\{\rho_4\},\{\rho_1,\rho_4\},\{\rho_1,\rho_2,\rho_4\},\{\rho_1,\rho_3,\rho_4\}\}$. $PC(\dot{X})=$ $\{\dot{X},\emptyset,\{\rho_2,\rho_3,\rho_4\},\{\rho_1,\rho_2,\rho_3\},\{\rho_2,\rho_3\},\{\rho_3\},\{\rho_2\}\}$, then G-SPO($\dot{X}$)= $\{\dot{X},\emptyset,\{\rho_1\},\{\rho_4\},$ $\{\rho_1,\rho_2\},\{\rho_1,\rho_3\},\{\rho_1,\rho_4\},\{\rho_2,\rho_4\},\{\rho_3,\rho_4\},\{\rho_1,\rho_2,\rho_3\},\{\rho_1,\rho_2,\rho_4\},\{\rho_2,\rho_3,\rho_4\},\{\rho_1,\rho_3,\rho_4\}\}$.

Remark 3.4: [7] $\bigcup_{i\in\wedge} PCL(\mathring{u}_i) \subseteq PCL(\bigcup_{i\in\wedge} \mathring{u}_i)$.

Proposition 3.5: If $M_i \in$ G-SPO($\dot{X}$) $\forall\ i \in \wedge$, *then* $\bigcup_{i\in\wedge} M_i \in$ G-SPO($\dot{X}$).

Proof: Let $M_i \in$ G-SPO($\dot{X}$), $\exists\ \mathring{u} \in$ PO($\dot{X}$), ($\mathring{u}_i - M_i$) $\notin$G $\wedge$(M_i −PCL($\mathring{u}_i$))$\notin$ G$\forall\ i \in\wedge$. this implies, $\bigcup_i$($\mathring{u}_i$- M_i) $\notin$ G, *so* ($\bigcup_i \mathring{u}_i - \bigcup_i M_i$) $\subseteq \bigcup_i$($\mathring{u}_i - M_i$) $\notin$ G, therefore, ($\bigcup_i \mathring{u}_i - \bigcup_i M_i$) $\notin$ G, On the other hands, (M_i − PCL ($\mathring{u}_i$)) $\notin$ G $\forall\ i\ \in \wedge$,$\bigcup_i \left(M_i - PCL(\mathring{u}_i)\right) \notin$ G,
($\bigcup_i M_i - \bigcup_i PCL$ ($\mathring{u}_i$)) $\subseteq\ \bigcup_i\left(M_i - PCL(\mathring{u}_i)\right) \notin$ G so, $\bigcup_i M_i - \bigcup_i(PCL(\mathring{u}_i)) \notin$ G
,since $\bigcup_i PCL(\mathring{u}_i) \subseteq PCL(\bigcup_I \mathring{u}_i)$, there for ($\bigcup_{i\in\wedge} M_i - PCL(\bigcup_i \mathring{u}_i)$) $\subseteq$ ($\bigcup_i M_i - \bigcup_i PCL(\mathring{u}_i)$) $\notin$ G
so,($\bigcup_i M_i - PCL(\bigcup_i \mathring{u}_i)$) $\notin$ G.

Corollary 3.6: If $\mathcal{F}_i\ \in$ G-SPC($\dot{X}$), then $\bigcap_i \mathcal{F}_i \in$ G-SPC($\dot{X}$).

Remark 3.7: *let* $M, S \in$ G -SPO($\dot{X}$) then $M \cap S$ need not to be a G-SPO set.

Example 3.8: *Let* $\dot{X} = \{\rho_1, \rho_2, \rho_3, \rho_4\}, \mathcal{T} = \left\{\dot{X}, \emptyset, \{\rho_1\}, \{\rho_4\}, \{\rho_1, \rho_4\}\right\}$. *Then*
$PO(\dot{X}) = \left\{\dot{X}, \emptyset, \{\ \rho_1\ \}, \{\rho_4\}, \{\rho_1, \rho_4\}, \{\rho_1, \rho_2, \rho_4\}, \{\rho_1, \rho_3, \rho_4\}\right\}$,
$PC(\dot{X}) = \{\dot{X}, \emptyset, \{\rho_2, \rho_3, \rho_4\}, \{\rho_1, \rho_2, \rho_3\},\ \{\rho_2, \rho_3\}, \{\rho_3\}, \{\rho_2\}\}$, when G=p($\dot{X}$)\ {$\emptyset$},
Hence G-$SPO(\dot{X}) = \{\dot{X}, \emptyset\ , \{\rho_1\}, \{\rho_4\}, \{\rho_1, \rho_2\}, \{\rho_1, \rho_3\}, \{\rho_1, \rho_4\}$,
$\{\rho_2, \rho_4\}, \{\rho_3, \rho_4\}, \{\rho_1, \rho_2, \rho_3\}, \{\rho_1, \rho_2, \rho_4\}, \{\rho_2, \rho_3, \rho_4\}, \{\rho_1, \rho_3, \rho_4\}\}$, *let* $M = \{\rho_1, \rho_2, \rho_3\}$ *and*
$S = \{\rho_2, \rho_4\}$, *then* M *and* S are G-SPO($\dot{X}$) But $M \cap S = \{\rho_2\}$,Which is not a G-SPO($\dot{X}$).

Remark 3.9: **let** M ,$S \in$ G -SPC($\dot{X}$) then $M \cup S$ need not be a G-SPC set.

See Example 2.8, *let* $M = \{\rho_1, \rho_2, \rho_3\}, S = \{\rho_2, \rho_4\}, M^c = \{\rho_4\}, S^c = \{\rho_2, \rho_3\}$,M^c ,S^c *are* G-$SPC(\dot{X})$, and $M^c\ \cup\ S^c = \{\rho_1, \rho_3, \rho_4\}$ which is not a G-SPC($\dot{X}$).

Remark 3.10: [7] Each open set is a preopen set.

Proposition 3.11: Each open set is a G -SPO set.

Proof: Let $M \in \mathcal{T}$ by Remark 2.4, so M is a preopen set; $\exists M \in po(\dot{X})$, such that, M-M = {$\emptyset$} $\notin$ G, And M-PCL (M) = {$\emptyset$ } $\notin$ G, *therefor* M is a G-SPO set.

Corollary 3.12: If F is a closed set, then F is a G-SPC set.

Proposition 3.13: Every semi-PO set is a G-SPO set.

Proof: Let $M \in$ S-PO($\dot{X}$) for that $\exists \mathring{u} \in PO(\dot{X})$ *such that* $\mathring{u} \subseteq M \subset PCL(M)$, further more $\mathring{u}$-M= $\{\emptyset\} \notin$ G $\wedge$ M-$PCL(M) = \{\emptyset\} \notin$ G. Hence, M is a G-SPO set.
As for the reverse proposition (2.13), it is not necessarily to be achieved.

Example 3.14: suppose that
$\dot{X} = \{\rho_1, \rho_2, \rho_3, \rho_4\}, \mathcal{T} = \{\dot{X}, \emptyset, \{\rho_1\}, \{\rho_4\}, \{\rho_1, \rho_4\}\}$,
G= $\emptyset$,$PO(\dot{X}) = \{\dot{X}, \emptyset, \{\rho_1\}, \{\ \rho_4\}, \{\ \rho_1, \rho_4\}, \{\rho_1, \rho_2, \rho_4\}, \{\rho_1, \rho_3, \rho_4\}\}$,
$PC(\dot{X}) = \left\{\dot{X}, \emptyset, \{\rho_2, \rho_3, \rho_4\}\ , \{\rho_1, \rho_2, \rho_3\}, \{\rho_2, \rho_3\}, \{\rho_3\}, \{\rho_2\}\right\}$,
G-SPO($\dot{X}$)=p($\dot{X}$). Then $\{\rho_2\} \in$ G-$SPO(\dot{X})$,But $\{\rho_2\} \notin$ G -SPO($\dot{X}$).

Corollary 3.15: The set of all G-SPO is a supra topological space.

Now, let's calculate the following example;

Example 3.16: Let $\dot{X}$= $\{\rho_1, \rho_2, \rho_3\}$, $\mathcal{T}$= {$\dot{X}$,$\emptyset$,$\{\rho_1\}$}
Then G-SPO ($\dot{X}$) = $p(\dot{X})$ and $\check{R}$ = {(ρ_1, ρ_1), (ρ_2,ρ_2), (ρ_3,ρ_3)} $\check{R}$ (ρ_1)={ρ_1 }, $\check{R}$(ρ_2)={ρ_2},
$\check{R}$ (ρ_3) = {ρ_3}.

Table 3. 1 Grill of Accuracy measure of approximation

G-SPO(Ẋ)	**Ữ (G-SPO(Ẋ))**	**£(G-SPO(Ẋ))**	**$\mathcal{B}$(G-SPO(Ẋ))**	**£(Ữ (G-SPO(Ẋ)**	**£ (G-SPO(Ẋ)**
Ẋ	**Ẋ**	**Ẋ**	∅	**Ẋ**	**Ẋ**
∅	∅	∅	∅	∅	∅
$\{\rho_1\}$	$\{\rho_1\}$	$\{\rho_1\}$	∅	$\{\rho_1\}$	$\{\rho_1\}$
$\{\rho_2\}$	$\{\rho_2\}$	$\{\rho_2\}$	∅	$\{\rho_2\}$	$\{\rho_2\}$
$\{\rho_3\}$	$\{\rho_3\}$	$\{\rho_3\}$	∅	$\{\rho_3\}$	$\{\rho_3\}$
$\{\rho_1,\rho_2\}$	$\{\rho_1,\rho_2\}$	$\{\rho_1,\rho_2\}$	∅	$\{\rho_1,\rho_2\}$	$\{\rho_1,\rho_2\}$
$\{\rho_1,\rho_3\}$	$\{\rho_1,\rho_3\}$	$\{\rho_1,\rho_3\}$	∅	$\{\rho_1,\rho_3\}$	$\{\rho_1,\rho_3\}$
$\{\rho_2,\rho_3\}$	$\{\rho_2,\rho_3\}$	$\{\rho_2,\rho_3\}$	∅	$\{\rho_2,\rho_3\}$	$\{\rho_2,\rho_3\}$

Table 3. 2 Grill of Accuracy measure of approximation

G-SPO(Ẋ)	$\mathfrak{M}(\text{G}-\text{SPO}(\dot{X}))$	$\mathfrak{M}_{\xi}(\text{G}-\text{SPO}(\dot{X}))$	$\mathfrak{M}_{p}(\text{G}-\text{SPO}(\dot{X}))$
Ẋ	1	1	1
∅	1	1	1
$\{\rho_1\}$	1	1	1
$\{\rho_2\}$	1	1	1
$\{\rho_3\}$	1	1	1
$\{\rho_1,\rho_2\}$	1	1	1
$\{\rho_1,\rho_3\}$	1	1	1
$\{\rho_2,\rho_3\}$	1	1	1

By Example 3.3

G-SPO(Ẋ)= {Ẋ, ∅, $\{\rho_1\},\{\rho_4\},\{\rho_1,\rho_2\},\{\rho_1,\rho_3\},\{\rho_1,\rho_4\},\{\rho_2,\rho_4\},\{\rho_3,\rho_4\},\{\rho_1,\rho_2,\rho_3\}$, $\{\rho_1,\rho_2,\rho_4\},\{\rho_2,\rho_3,\rho_4\},\{\{\rho_1,\rho_3,\rho_4\}\}$.

Ř = $\{(\rho_1,\rho_1),(\rho_2,\rho_2),(\rho_3,\rho_3),(\rho_4,\rho_4),(\rho_3,\rho_4),(\rho_4,\rho_3)\}$

Ř (ρ_1)=$\{\rho_1\}$, Ř(ρ_2)=$\{\rho_2\}$, Ř(ρ_4), Ř(ρ_3)=$\{\rho_4,\rho_3\}$.

Table 3.3 G- SPO of Accuracy measure of approximation

G-SPO(Ẋ)	Ữ(G-SPO(Ẋ))	£(G-SPO(Ẋ))	$\mathcal{B}$(G-SPO(Ẋ))	£ (Ữ (G-SPO(Ẋ)	Ữ(£ (GSPO(Ẋ)
Ẋ	Ẋ	Ẋ	∅	Ẋ	Ẋ
∅	∅	∅	∅	∅	∅
$\{\rho_1\}$	$\{\rho_1\}$	$\{\rho_1\}$	∅	$\{\rho_1\}$	$\{\rho_1\}$
$\{\rho_4\}$	$\{\rho_3,\rho_4\}$	∅	$\{\rho_3,\rho_4\}$	$\{\rho_3,\rho_4\}$	∅
$\{\rho_1,\rho_2\}$	$\{\rho_1,\rho_2\}$	$\{\rho_1,\rho_2\}$	∅	$\{\rho_1,\rho_2\}$	$\{\rho_1,\rho_2\}$
$\{\rho_1,\rho_3\}$	$\{\rho_1,\rho_3,\rho_4\}$	$\{\rho_1\}$	$\{\rho_3,\rho_4\}$	$\{\rho_1,\rho_3,\rho_4\}$	$\{\rho_1\}$
$\{\rho_1,\rho_4\}$	$\{\rho_1,\rho_3,\rho_4\}$	$\{\rho_1\}$	$\{\rho_3,\rho_4\}$	$\{\rho_1,\rho_3,\rho_4\}$	$\{\rho_1\}$
$\{\rho_2,\rho_4\}$	$\{\rho_2,\rho_3,\rho_4\}$	$\{\rho_2\}$	$\{\rho_3,\rho_4\}$	$\{\rho_2,\rho_3,\rho_4\}$	$\{\rho_2\}$
$\{\rho_3,\rho_4\}$	$\{\rho_3,\rho_4\}$	$\{\rho_3,\rho_4\}$	∅	$\{\rho_3,\rho_4\}$	$\{\rho_3,\rho_4\}$
$\{\rho_1,\rho_2,\rho_3\}$	Ẋ	$\{\rho_1,\rho_2\}$	$\{\rho_3,\rho_4\}$	Ẋ	$\{\rho_1,\rho_2\}$
$\{\rho_1,\rho_2,\rho_4\}$	Ẋ	$\{\rho_1,\rho_2\}$	$\{\rho_3,\rho_4\}$	Ẋ	$\{\rho_1,\rho_2\}$
$\{\rho_2,\rho_3,\rho_4\}$	$\{\rho_2,\rho_3,\rho_4\}$	$\{\rho_2,\rho_3,\rho_4\}$	∅	$\{\rho_2,\rho_3,\rho_4\}$	$\{\rho_2,\rho_3,\rho_4\}$
$\{\rho_1,\rho_3,\rho_4\}$	$\{\rho_1,\rho_3,\rho_4\}$	$\{\rho_1,\rho_3,\rho_4\}$	∅	$\{\rho_1,\rho_3,\rho_4\}$	$\{\rho_1,\rho_3,\rho_4\}$

Table 3. 4. G- SPO of Accuracy measure of approximation

Ǥ-SPO(Ẋ)	$\mathfrak{M}(Ǥ - \mathrm{SPO}(Ẋ))$	$\mathfrak{M}_{\xi}(Ǥ - \mathrm{SPO}(Ẋ))$	$\mathfrak{M}_{p}(Ǥ - \mathrm{SPO}(Ẋ))$
Ẋ	1	1	1
∅	1	1	1
$\{ ϱ_1\}$	0	0	1
$\{ ϱ_4\}$	1	1	1
$\{ ϱ_1, ϱ_2\}$	1/3	1/3	1
$\{ ϱs_1, ϱ_3\}$	1/3	1/3	1
$\{ϱ_1, ϱ_4\}$	1/3	1/3	1
$\{ϱ_2, ϱ_4\}$	1	1	1
$\{ϱ_3, ϱ_4\}$	½	½	1
$\{ ϱ_1, ϱ_2, ϱ_3\}$	½	½	1
$\{ ϱ_1, ϱ_2, ϱ_4\}$	1	1	1
$\{ ϱ_2, ϱ_3, ϱ_4\}$	1	1	1
$\{ ϱ_1, ϱ_3, ϱ_4\}$	1	1	1

4. Conclusion

The aim of our study is to define the Ǥ-SPO sets and study some of the properties of these sets, and then find the boundary area for the family of Ǥ-SPO (Ẋ). and try to get the best accuracy for the set when it equals 1 for most of M ∈ Ǥ-SPO (Ẋ).

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