

# Magnetic Lens Design Using Analytical Target Field Function

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## Abstract

Analytical field target function has been considered to represent the axial magnetic field distribution of double polepiece symmetric magnetic lens. In this article, with aid of the proposed target function, the syntheses procedure is dependent. The effect of the main two coefficient optimization parameters on the lens field distribution, polepieces shape, and the objective focal properties for lenses operated under zero magnification mode has been studied. The results have shown that the objective properties evaluated in sense of the inverse design procedure are in an excellent correspondence with that of analysis approach. Where the optical properties enhance as the field distribution of the electron lens distributed along a narrow axial interval with high field peak and virsa.

**Keywords:** magnetic lens , polepiece , aberrations , target function

## Introduction

The basic goal of the optimization procedure for any system of a charged particle beam optics is to produce electrodes or pole pieces systems that provide giving focusing properties with minimum aberrations, i.e., the approach that aims to improve the quality of the electron device, in general, and any desirable procedure leading to optimum design of an electron optical system [1]. However, the effect of aberrations leads to a degradation for any system of charged particle beam optics. Indeed, generating aberrations is inevitable when the charged particle beams are extracted, accelerated, transmitted, and focused with electrostatic and magnetic fields. Strictly speaking, aberrations degrade the focused beam spot, limiting the spatial resolution of these instruments [2]. However, the most effective aberrations that deteriorate the efficiency of any electron optical instrument are the spherical and chromatic aberrations. Improvement of the resolution of electron microscope is limited by the aberrations of the objective lens. Spherical and chromatic aberrations are the most effective ones. Unfortunately, both spherical and chromatic aberrations cannot eliminate completely. But they can be minimized to improve the resolution of the electron microscope. Traditionally, effort in improving electron lenses has concentrated on minimizing these aberrations and searching for field distributions with minimum or zero spherical aberration [3].

As it is well known that, in the field of electron and ion optics, there are essentially two different optimization procedures to be followed in the design of charged particle lenses. The first is called analysis in which the process of trial and error is followed; while the second approach is the synthesis as in some times it is called the inverse design procedure. The main feature that characterizes the later approach is that there is a target function which has to be optimized with certain conditions. The axial electrostatic or magnetic field, potential, or the beam trajectory of the electron optical device can be presented by this target function.

## Mathematical formation

### Target Function

Synthesis procedure of magnetic lenses followed in the present paper starts with suggesting the following target function to approximate the axial magnetic field distribution  $B_z(z)$ .

$$(B_z(z)) = \frac{1}{\sqrt{1+a^2}} e^{-\left[\frac{z}{z_0}\right]^2 / \sqrt{1+a^2}} \quad (1)$$

Where  $z$  is the axial coordinate along the optical axis,  $z_0$  may be any axial coordinate taking different values, and an optimization parameter may represent the radius of the magnetic lens pole piece. The first derivative of the magnetic field given by equation (1) can be easily deduced to be as;

$$(B'_z(z)) = \frac{-2z}{z^2(1+a^2)} e^{-\left(\frac{z}{z_0}\right)^2 / \sqrt{1+a^2}} \quad (2)$$

### Magnetic Scalar Potential

As it is well-known that in the current-free region, the axial magnetic field distribution along the optical axis can be determined with aid of the magnetic scalar potential  $V(z)$  as

$$(B_z(z)) = -\mu_0 \frac{dV(z)}{dz} \quad (3)$$

Where  $\mu_0$  is the free-space permeability ( $4\pi \times 10^{-7}$  H/m).



However, to determine the axial magnetic scalar potential distribution  $V(z)$  when the magnetic field is known along the optical axis, equation (3) can be integrated along the axial extension of the lens as follows[1]

$$(V(z) = - \frac{1}{\mu_o} \int_{z_s}^{z_f} B_z(z) dz) \quad (4)$$

Where  $Z_s$  and  $Z_f$  are the terminally of the field distribution.

In concerning with the inverse design procedure, the problem of determining the magnetic scalar potential distribution  $V(z)$  is an important consequence for determining magnetic lens pole pieces. Therefore, with aid of equation (4),  $V(z)$  may be determined analytically when the field target function is easily to be integrated, or one can use the numerical integration techniques. Fortunately, the cubic spline integration technique that had been introduced by [4] for symmetric fields has been used in this paper. Accordingly, the magnetic field distribution  $B_z(z)$  for each spline interval  $k$  can be represented by the following cubic expression [4].

$$B_{zk}(z) = B_k + B'_k(z - z_k) + 0.5B''_k(z - z_k)^2 + \frac{B''_{k+1} - B''_k}{6(z_{k+1} - z_k)}(z - z_k)^3 \quad (5)$$

Where  $z$  is the axial coordinate along the spline interval such that  $Z_k \leq z \leq Z_{k+1}$ .

In the case of symmetrical magnetic field distribution, the potential values at the terminals  $V_{zs}$  and  $V_{zf}$  have the following property

$$(V_{zf} = -V_{zs} = 0.5NI) \quad (6)$$

Where  $NI$  is the lens excitation.

However, using equations (4) and (5), the magnetic scalar potential distribution  $V(z)$  along the solution domain  $[z_s, z_f]$  can be determined as follows (see, for more details in [4,5,6].

$$(V_{z(k+1)} = -\mu_o^{-1} \sum_{k=1}^{\frac{1}{2}n+1} B_{ak} d_k - \sum_{j=1}^k E_k) \quad (7)$$

Where

$$(E_k = \left\{ V'_{zj} d_k + V''_{zk} \left( \frac{d_k}{2} \right)^2 + \left( V'''_{zk} + \frac{V''_{z(k+1)}}{3} \right) \left( \frac{d_k}{2} \right)^3 \right\}) \quad (8)$$

and  $d_k = z_{k+1} - z_k$

### Polepiece Profile

The final task of any synthesis procedure in the field of electron and ion optical systems is finding the electrodes or /and the pole piece profile which produces the suggested tragic function distribution (i.e, the distribution of the magnetic field in the present investigation). The solution of Laplace's equation in cylindrical coordinates system for rotationally symmetric magnetic fields can be expressed by the following power series expansion [1].

$$V(r, z) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(ni)^2} \left( \frac{r}{2} \right)^{2n} V^{(2n)}(z) \quad (9)$$

However, by using the first two terms of equation (9), the pole piece shape can be obtained from the following equation.

$$R_p(z) = 2 \sqrt{\frac{V(z) - V_p}{V''(z)}} \quad (10)$$

Where  $R_p(z)$  is the radial height of the reconstructed polepiece shape,  $V''(z)$  is the second derivative of  $V(z)$  and  $V_p$  represents the potential at the lens terminal which is equal to (0.5 x total excitation of the lens).

### Electron Lens Aberrations

In high-resolution transmission electron microscopy, lens aberrations play a key role in the imaging and the interpretation of object structures on an atomic scale. Aberrations are beneficial and detrimental to high-resolution imaging at the same time. On the one hand, they introduce unwanted blurring in any imaging plane, hence obscuring the finest object details. On the other hand, they are urgently needed to produce the desired phase contrast of the very thin objects required for high-resolution structure investigations [7]. Actually, any task in the field of the electron and ion optics aims to determine the aberrations of an optical instrument. To evaluate these aberrations, several axial functions must be determined.

In this paper two relating figures of merit are considered to evaluate the objective focal properties, the spherical  $C_s$  and chromatic  $C_c$  aberration coefficients. The coefficients of these two important aberrations are represented by the following integral forms [8].

$$C_s = \frac{\eta}{128V_r} \int_{z_0}^{z_i} \left[ \frac{3\eta}{V_r} B_z^4(z) r^2(z) + 8B_z'^2(z) r^2(z) - 8B_z^2(z) r^2(z) r'(z) \right] dz \quad (11)$$

$$C_c = \frac{\eta}{8V_r} \int_{z_0}^{z_i} B_z^2(z) r_a^2(z) dz \quad (12)$$

where the limits of integrations,  $z_0$  and  $z_i$  are the object and image positions respectively,  $\eta$  is the charge to mass quotient of the electron and  $V_r$  is the relativistically corrected accelerating voltage. The axial functions  $r(z)$  and  $r'(z)$  are the electron beam trajectory and its first derivative. The integrals of aberrations  $C_s$  and  $C_c$  can be totally determined when  $r(z)$  is known along the interval of the lens. Therefore, the beam trajectory  $r(z)$  can be evaluated by solving the following second-order differential equation [1].

$$\frac{d^2 r}{dz^2} + \frac{\eta}{8V_r} B_z^2 r = 0 \quad (13)$$

## Results and Discussion

### Effect of the parameter a

In the present work five various values of the parameter  $a$  have been taken under consideration (i.e.,  $a=1,2,3,4,$  and  $5\text{mm}$ ) when the parameter  $z_0$  is kept constant at  $0.5\text{mm}$ . The effect of this parameter on the axial magnetic field distribution and their related axial functions such as  $V(z)$ ,  $R_p(z)$  as well as the objective properties of a symmetric double polepiece magnetic electron lens has been investigated. Figure 1 shows different axial magnetic field distributions corresponding to the five various values of the parameter  $a$ . One can see that, as the parameter  $a$  is increased the physical and the geometrical parameters  $B_{\max}$ , the halfwidth  $W$ , and  $NI$  are varied as shown in figure 2. However, as  $a$  is increased the axial

field would be distributed along a large axial interval, i.e., the halfwidth  $W$  is increased with increasing  $a$ , while the peak axial flux density value  $B_{\max}$  and the excitation of the lens  $NI$  are decreased. The axial magnetic scalar potential distribution  $V(z)$  corresponding to the field distributions is plotted in figure 3. It is noted that the outer region of  $V(z)$  curves about the symmetry plane (i.e.,  $z=0$ ) represents a field-free region, since the potential is approximately constant in these regions. This behavior can be explained according to the equation

$(B_z(z) = -\mu_0 \, dV(z)/dz)$ . This means that the electron beams travel in straight lines in these regions. Therefore, the main effect of the parameter  $a$  is confined in the region between the polepieces, i.e., the effective (air gap) region in which the charged partial beams suffer from refraction effect. The reconstructed polepieces for each value of the parameter  $a$  are shown in figure 4. Actually, one may note that the increasing of  $a$  will significantly change the polepiece profile. The coincidence between this synthesis result and the well known conventional counterparts is that both of them announce the increasing of the bore diameter will redacting the ability of the polepiece in localizing and confining the magnetic flux lines.

The objective properties the focal length  $f_0$ ,  $c_s$ , and  $c_c$  are affected by varying the parameter  $a$  as shown in figure 5. It is noted that as the parameter  $a$  decreases, the lens will be of good performance, hence, the aberrations  $c_s$  and  $c_c$  as well as the focal length  $f_0$  have small values for fields (lenses) of small values of the parameter  $a$ .

It should be mentioned that figure 5 illustrates the values of  $c_s$ ,  $c_c$ , and  $f_0$  at  $NI/V_r^{1/2}=20$  for a lens operated under zero magnification mode. The values  $B_{\max}$ ,  $W$ , and  $NI$  as well as the objective properties  $f_0$ ,  $c_s$ , and  $c_c$  at  $NI/V_r^{1/2}=20$  for different values of the parameter  $a$  are listed in table I.

### Effect of the parameter $z_0$

Five values of the parameter  $z_0$  (0.5, 0.7, 0.9, 1.1, 1.3 mm) have been chosen to investigate the effect of this parameter on the axial functions distributions, polepiece profiles, and the objective focal properties. The parameter  $a$  has been kept constant at 1mm, while the length of the lens which has no effect on the design of the magnetic lens is kept constant at 20mm. Figure 6 shows different field distributions at different  $z_0$  having different half width,  $B_{\max}$ , and  $NI$ . However, variation of  $B_{\max}$ ,  $W$  and  $NI$  with the parameter  $z_0$  is shown in figure 7. The magnetic scalar potential distributions and the polepiece of the magnetic lens for different values of  $z_0$  are shown in figures 8 and 9 respectively. The values of the objective focal length  $f_0$ , the spherical  $c_s$  and chromatic  $c_c$  aberration coefficients for various values of  $z_0$  are shown in figure 10.

It is clear that these optical properties enhanced as the parameter  $z_0$  decreases, i.e., the small  $z_0$  values makes the field distribution more localized in the air gap region. Table II shows the computed parameters  $B_{\max}$ ,  $W$ , and  $NI$  and values of the lens objective properties  $f_0$ ,  $c_s$ , and  $c_c$  at  $NI/V_r^{1/2}=20$ .

### Conclusion

Results in present article have shown that the objective focal properties  $c_s$ ,  $c_c$ , and  $f_0$  can be minimized to a suitable values under the effect of the bore polepiece radius and a specific axial extension value. Also, the results shown that the behavior of the aberration in sense of synthesis procedure is in excellent with that of analysis one.

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**Table No.(1): Some lens parameters and the objective properties for various values of the parameter a.**

a(mm)	$B_{max}(T)$	NI(A-t)	W(mm)	$C_c(mm)$	$C_s(mm)$	$f_o(mm)$
1	0.4472	471	1.246	0.3181	0.2447	0.4455
2	0.2425	347	1.69	0.4324	0.3328	0.6050
3	0.1644	285	2.054	0.525	0.4044	0.7348
4	0.1240	248	2.3644	0.5890	0.4416	0.8348
5	0.0995	222	2.6398	0.6580	0.4939	0.9323

**Table No. (2): Some lens parameters and the objective properties for various values of the parameter  $Z_o$ .**

$Z_o(mm)$	$B_{max}(T)$	NI(A-t)	W(mm)	$C_c(mm)$	$C_s(mm)$	$F_o(mm)$
0.5	0.4472	471	1.246	0.3181	0.2447	0.4455
0.7	0.5735	747	1.540	0.3936	0.3029	0.5508
0.9	0.6690	1038	1.832	0.4687	0.3607	0.6557
1.1	0.7399	1334	2.130	0.5449	0.4194	0.7620
1.3	0.7926	1632	2.431	0.6058	0.4544	0.8586

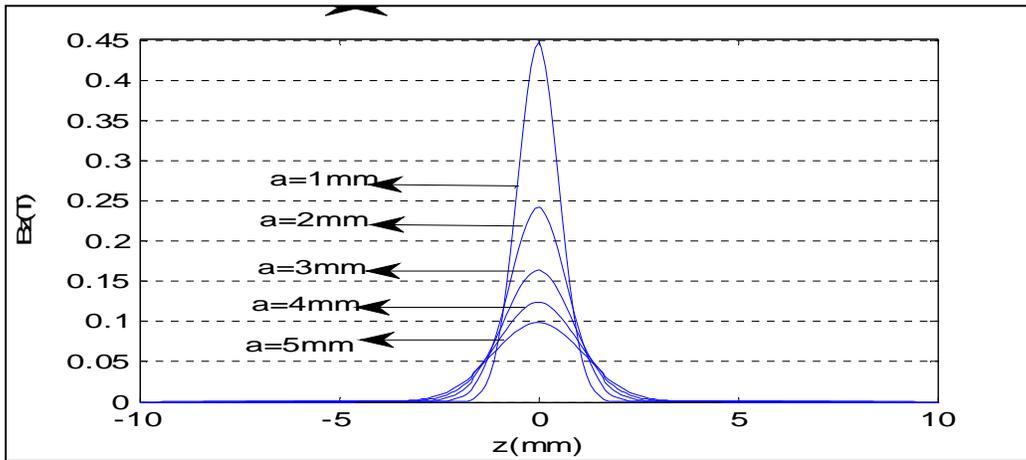


Figure No.(1): The axial magnetic field distribution  $B_z(z)$  for various values of the parameter  $a$  at  $z_0 = 0.5\text{mm}$

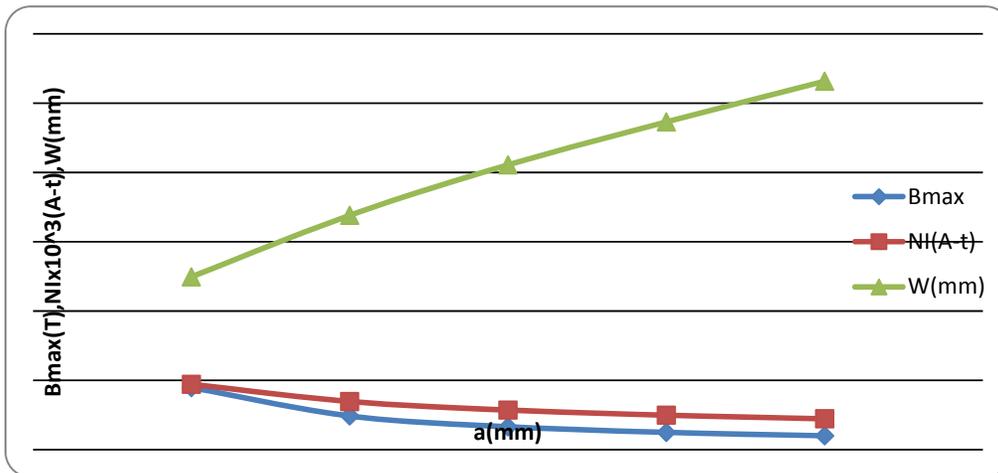


Figure No.(2): Variation of  $B_{\max}$ ,  $W$ , and  $NI$  with  $a$

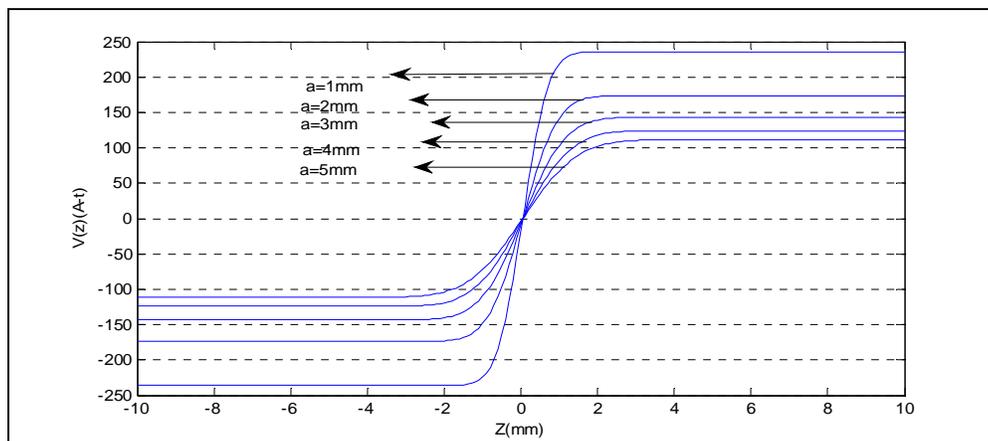


Figure No.(3): The axial magnetic scalar potential distribution  $V(z)$  for various values of  $a$  at  $z_0 = 0.5\text{mm}$

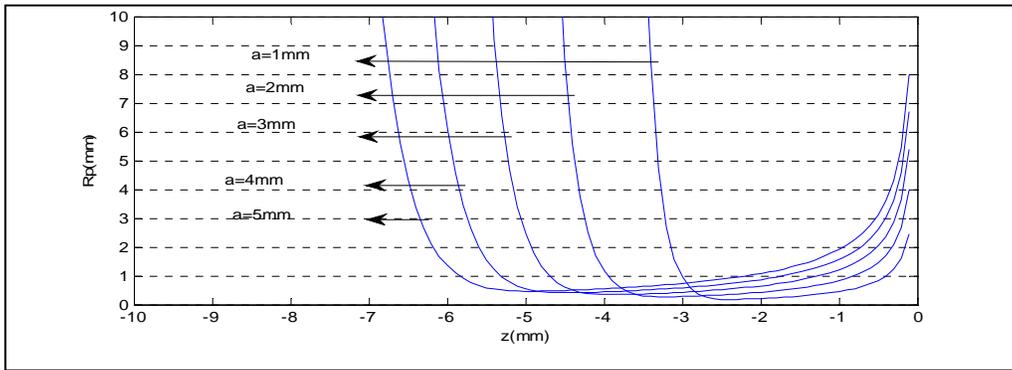


Figure No.(4): Different polepiece shapes for different values of the parameter a

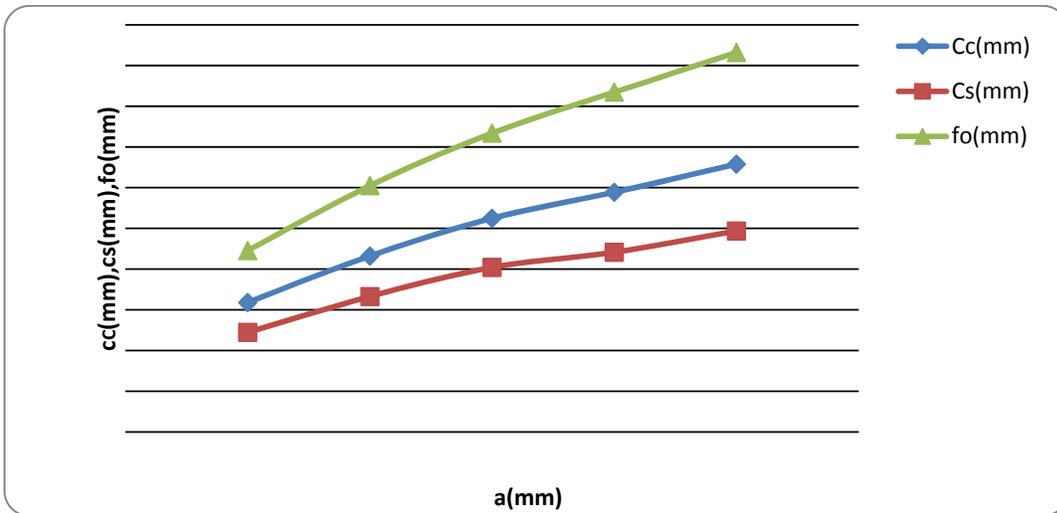


Figure No.(5): Variation of  $f_o$ ,  $c_s$ , and  $c_c$  with the parameter a at  $NI/V_r^{1/2}=20$

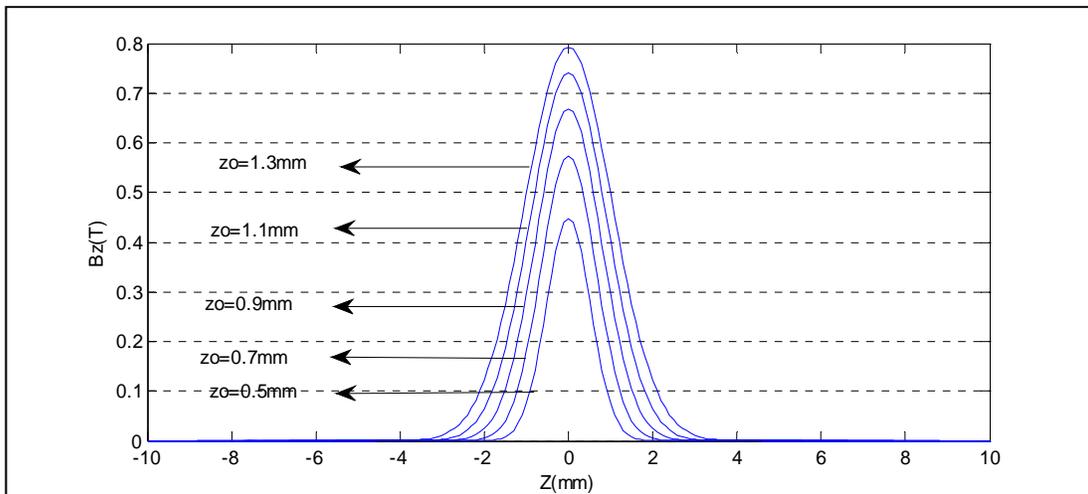


Figure No.(6): Different field distributions for different  $z_0$  values at  $a=1mm$

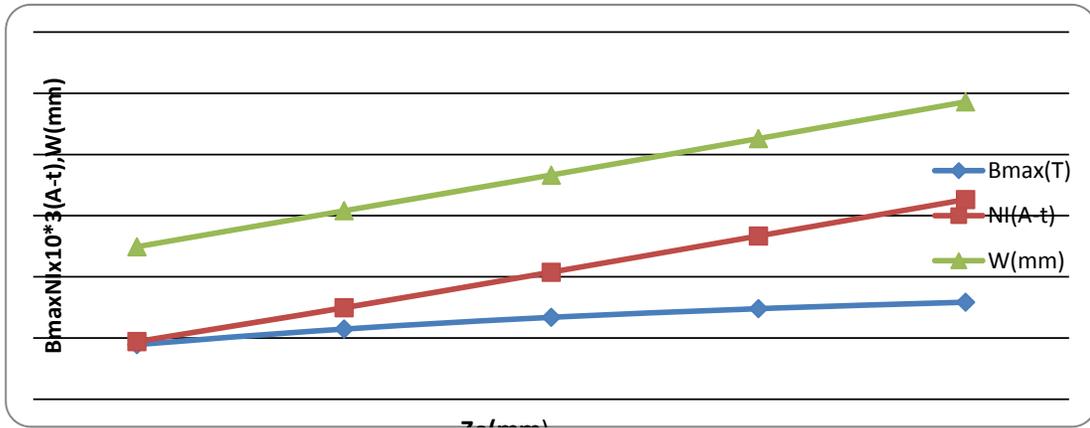
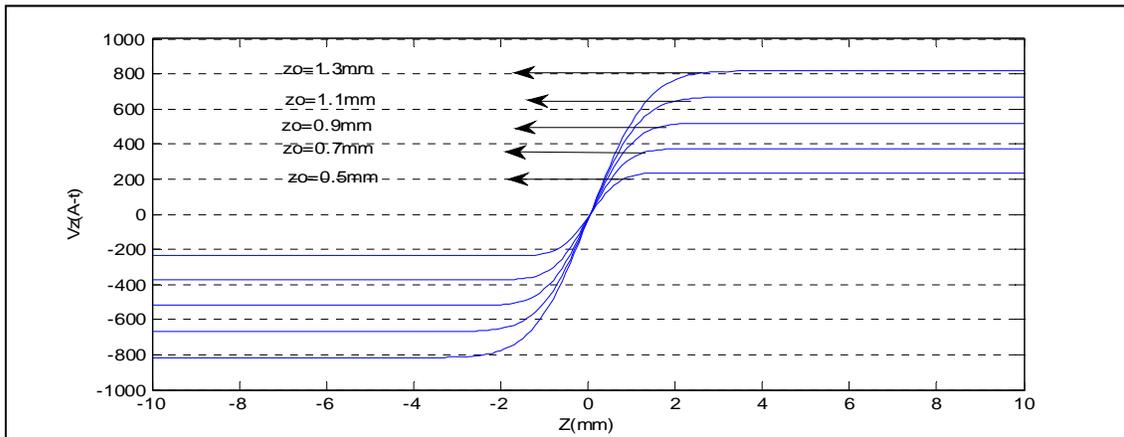


Figure No.(7):  $B_{max}$  W, NI with the parameter  $z_0$  for  $a=1mm$



FigureNo.(8): The magnetic scalar potential distribution for different values of  $z_0$  for  $a=1mm$

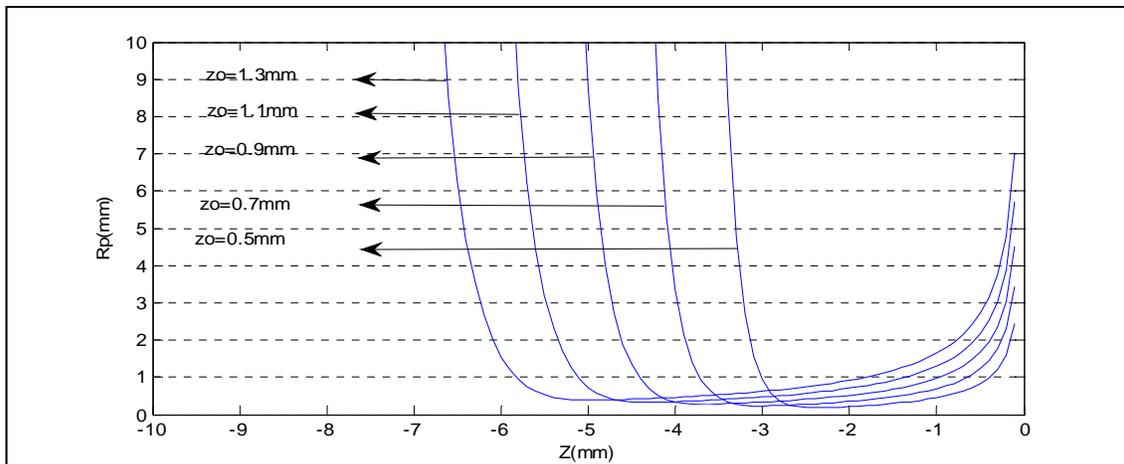
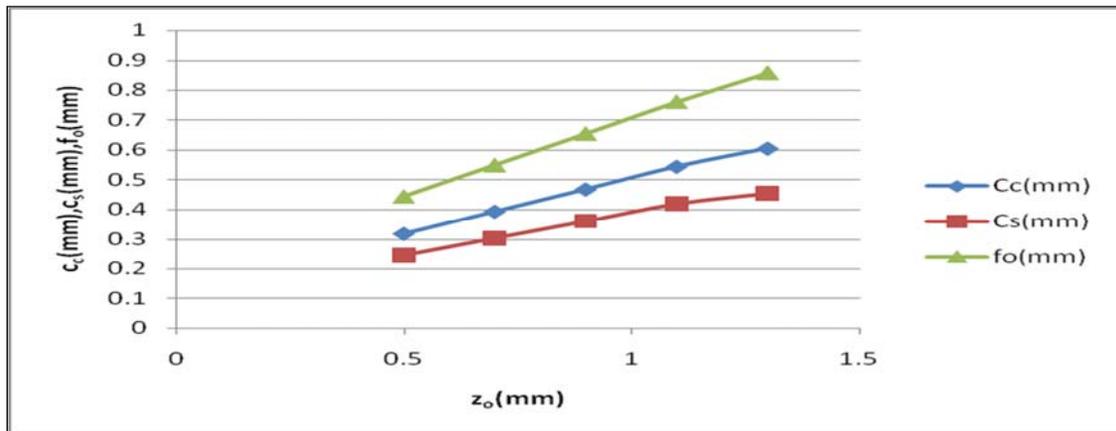


Figure No.(9): The reconstructed polepieces at different values of  $z_0$



FigureNo.(10): Variation of  $c_c, c_s,$  and  $f_0$  with the parameter  $z_0$  at  $NI/V_r^{1/2}=20$

## تصميم عدسة مغناطيسية باستخدام دالة هدف مجالي تحليلي

عماد حميد احمد الداودي

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### الخلاصة

تم في هذا البحث تمثيل المجال المغناطيسي المحوري لعدسة ثنائية القطب المتناظرة بدالة هدف تحليلية وبالاعتماد على أسلوب التوليف تم استخدام تلك الدالة. حيث تم دراسة تأثير اهم عاملي امثلية في دالة الهدف على كل من توزيع مجال العدسة و شكل اقطابها و الخواص البصرية الشبئية باستعمال نمط التشغيل الصفري . والنتائج التي توصلت اليها باستخدام أسلوب التصميم العكسي بينت مدى التطابق الكبير مع التي درست باستخدام أسلوب التحليل. حيث ان الخواص البصرية تتحسن عندما يكون مجال العدسة الالكترونية موزع على طول منطقة محورية ضيقة بقمة مجال عالية وتسوء تلك الخواص اذا كان المجال موزع على منطقة محورية واسعة.

الكلمات المفتاحية: العدسة المغناطيسية, الاقطاب, الزيوغ, دالة الهدف