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Solving Nonlinear COVID-19 Mathematical Model Using a Reliable Numerical Method

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Abstract

This research aims to numerically solve a nonlinear initial value problem presented as a system of ordinary differential equations. Our focus is on epidemiological systems in particular. The accurate numerical method that is the Runge-Kutta method of order four has been used to solve this problem that is represented in the epidemic model. The COVID-19 mathematical epidemic model in Iraq from 2020 to the next years is the application under study. Finally, the results obtained for the COVID-19 model have been discussed tabular and graphically. The spread of the COVID-19 pandemic can be observed via the behavior of the different stages of the model that approximates the behavior of actual the COVID-19 epidemic in Iraq. In our study, the COVID-19 pandemic will disappear during the next few years within about five years, through the behavior of all stages of the epidemic presented in our research.

Keywords: System of Ordinary Differential Equations, Infectious Diseases, Epidemic Model, COVID-19 Mathematical Model, Runge-kutta Methods.

1. Introduction

The COVID-19 virus was discovered in Wuhan, Hubei in China. In December 2019, the World Health Organization (WHO) declared coronavirus a pandemic on March 11 since it was quickly outbreaking in all of China, [1]. COVID-19 varies from infectious diseases since it includes strongly infectivity during the disease incubation period of 3-7 days or a maximum of 14 days which differs basically across patients, in addition, the time delay between for getting the number of confirmed cases each day and true dynamics,[1]. COVID-19, unlike its close sibling SARS, does not appear any symptoms for COVID-19 on the patients during the incubation phase [2]. One of the most difficult problems in the study of epidemics is predicting future trends, such as how many people will be infected daily, [3,4,5], what policies will be needed to control this epidemic and how the policies will affect the epidemics, and so on [6-8]. Following the epidemic outbreak, all necessary measures have been adopted to understand and control the behavior of this epidemic by the government and individuals together. Several well-known cases in 2003 were SARS [9], 2009 H1N1 flu [10], and the current COVID-19. Epidemic modeling was constructed in 1927 by Kermack and McKendrick's work, Their models were used during cholera and plague outbreaks [11,12]. The most important contributions to epidemiology modeling are to understand virus dynamics best, give good predictions for transmission of the virus, and provide the public health policymakers with beneficial information to control the virus [13-15]. Coronaviruses are three subgroups: alpha, beta, and gamma; the new group is called delta coronaviruses (SARS-CoV). The first appearance of the Coronavirus was in the mid-1960s, and this virus was among humans. In 2020, this virus reappeared and was spread all over the world quickly,[11,12].

Many researchers analyzed different types of epidemics, some of the social epidemics like the obesity mathematical model that was solved numerically using RK4, RK4, RK45 and RK78 methods,[16]. The alcohol consumption model and smoking habit model analyzed the epidemic's behavior analytically and numerically,[17,18]. On the other hand, some disease epidemics analyzed the epidemic's behavior numerically, like the influenza epidemic model discussed and solved using the Runge-Kutta numerical method of orders 4 and 45,[19]. Furthermore, the other analyzed the behavior analytically solved SIR epidemic model using the Tamimi and Ansari iterative method with Laplace transform,[20]. The interest in studying epidemiological mathematical models increases to know whether the epidemic will increase or decrease in the future, especially when coronavirus appears in 2020. The global stability of COVID-19 model was discussed by [21]. The COVID-19 model was analyzed using Euler's and Runge-Kutta methods of orders 2 and 4, applying to Turkey and Iraq [22]. The SIR COVID-19 mathematical model in Kurdistan Region in Iraq, estimates the reproductive number and solve the model numerically by RK4 was analyzed by [23]. A new SEIVR COVID-19 model with Vaccination Campaign was created by [24], and solve numerically using Runge-Kutta method of order 4.

Multi parameters and multivariate in one system are difficult to get the exact or analytic solutions for some real complex modeling. These models are our systems under study. This research is concerned with solving these models efficiently, fast, accuracy, with a common numerical method which is the Runge-Kutta method. Computer programming for these numerical methods is established to quickly and easily get results, [25].

The present study is arranged as follows: In Section 2, the mathematical model of SEIVR COVID-19 model with Vaccination Campaign, Section 3 is establishing the COVID-19 model numerically using Runge-Kutta of 4 order (RK4) method. Section 4, provides some results that are presented tabular and graphically with their discussion. The end of our research in Section 5 supplies the conclusion of this study.

2. COVID-19 Mathematical Model

Mathematical modeling depends on the mathematical language that describes the behavior of a natural phenomenon. The COVID-19 pandemic model including the Vaccination Campaign is of natural phenomenon which can be represented as a system of differential equations for the first order, the model is formed (1),[24].

The population of COVID-19 pandemic model is divided into five individuals named compartments, $S(t), E(t), I(t), V(t)$ and $R(t)$, so this model is made $(SEIVR)$,[24]. The first stage: Susceptible population is the healthy individuals with the symbol (S), the second stage:

Exposed population; the infected individuals in the incubation period who do not show symptoms of disease and are denoted by (E). The third stage: the Infected population is the infected individuals and the given rise to infection after incubation interval and symbolized by (I), the fourth stage: the Vaccinated population is the individuals who take the vaccination and symbolized by (V), the fifth stage: Recovered population are the individuals who die or recover which are coded by (R). The derivatives of all compartments of the coronavirus model are continuous at $t \geq 0$. The solutions of the model are non-negativity, the existence and uniqueness can be viewed in [21]. The system of COVID-19 understudy is offered in (1), [24]:

$$
\begin{aligned}
\frac{dS}{dt} &= \Lambda - (\alpha E + m + \mu)S, \\
\frac{dE}{dt} &= \alpha SE + pVE - (fI + c + \mu)E, \\
\frac{dI}{dt} &= fEI - (z + \mu + \sigma)I, \\
\frac{dV}{dt} &= mS - (pE + \mu)V, \\
\frac{dR}{dt} &= zI + cE - \mu R,
\end{aligned} \tag{1}
$$

where **Table 1.** has a description of the parameters of the COVID-19 model.

Table 1. Description parameters of the COVID-19 model with their values [24]

Parameters	Description	Value of Parameters	Source
Λ	Mobilization rate of Coronavirus	50	[26]
α	Rate of transition from susceptible persons to exposed persons	0.002	estimated
m	The proportion of vaccinated susceptible persons	0.5	estimated
f	The rate at which exposed people become infected	0.008	estimated
p	The rate for vaccinated persons who are exposed to disease	0.08	estimated
z	The recovery rate of infected persons	0.012	estimated
μ	Rate of natural death	0.009	estimated
c	The recovery rate of exposed persons	0.05	estimated
σ	Mortality related to the disease	0.25	estimated

3. Solving COVID-19 Model by Numerical Method

A numerical method is a process to find the solutions approximately at specified points, most initial value problems can be solved numerically like the COVID-19 epidemic system in the present work. In this section, a numerical method RK of order 4 can be utilized to solve the COVID-19 epidemic problem in Iraq in 2020.

Different forms of the RK method concerning the order of the method, in the current work, RK has the form of order 4, so, this work needs four stages to get the final formula [27]. RK_4 the method in a general form can be shown in [27].

First, the form of RK_4 is presented below, the general formula of $S_{i+1}, E_{i+1}, I_{i+1}, V_{i+1}$ and R_{i+1} for the RK_4 method is:

$$S_{i+1} = S_i + \frac{1}{6}\,(g_{S1} + 2g_{S2} + 2g_{S3} + g_{S4}),$$

$$E_{i+1} = E_i + \frac{1}{6}\,(g_{E1} + 2g_{E2} + 2g_{E3} + g_{E4}),$$

$$I_{i+1} = I_i + \frac{1}{6}\,(g_{I1} + 2g_{I2} + 2g_{I3} + g_{I4}),$$

$$V_{i+1} = V_i + \frac{1}{6}\,(g_{V1} + 2g_{V2} + 2g_{V3} + g_{V4}),$$

$$R_{i+1} = R_i + \frac{1}{6}\,(g_{R1} + 2g_{R2} + 2g_{R3} + g_{R4}).$$

where $i = 0,1, \dots, m-1$. (2)

Now, to find $g_{S1}, g_{E1}, g_{I1}, g_{V1}$ and g_{R1}, the next steps are done by substituting the form of $g_{S1}, g_{E1}, g_{I1}, g_{V1}$ and g_{R1} in (1) as below; where $Eq_1, Eq_2, Eq_3, Eq_4, Eq_5$ are the equations of (1), h is a step size and t is a time :

$$g_{S1} = hEq_1(t_i, S_i, E_i, I_i, V_i, R_i)$$

$$= h(\Lambda - (\alpha E_i + m + \mu)S_i),$$

$$g_{E1} = hEq_2(t_i, S_i, E_i, I_i, V_i, R_i)$$

$$= h(\alpha S_i E_i + pV_i E_i - (fI_i + c + \mu)E_i),$$

$$g_{I1} = hEq_3(t_i, S_i, E_i, I_i, V_i, R_i)$$

$$= h\,(fE_i I_i - (z + \mu + \sigma)I_i),$$

$$g_{V1} = hEq_4(t_i, S_i, E_i, I_i, V_i, R_i)$$

$$= h\,(mS_i - (pE_i + \mu)V_i),$$

$$g_{R1} = hEq_5(t_i, S_i, E_i, I_i, V_i, R_i)$$

$$= h(zI_i + cE_i - \mu R_i).$$

In the same process, $g_{S2}, g_{E2}, g_{I2}, g_{V2}$ and g_{R2} can be found by substituting the form of $g_{S2}, g_{E2}, g_{I2}, g_{V2}$ and g_{R2} in (1):

$$g_{S2} = hEq_1(t_i + \frac{h}{2}, S_i + \frac{1}{2}g_{S1}, E_i + \frac{1}{2}g_{E1}, I_i + \frac{1}{2}g_{I1}, V_i + \frac{1}{2}g_{V1}, R_i + \frac{1}{2}g_{R1})$$

$$= h\,(\Lambda - (\alpha(E_i + \tfrac{1}{2}g_{E1}) + m + \mu)(S_i + \tfrac{1}{2}g_{S1})),$$

$$g_{E2} = hEq_2\,(t_i + \frac{h}{2}, S_i + \frac{1}{2}g_{S1}, E_i + \frac{1}{2}g_{E1}, I_i + \frac{1}{2}g_{I1}, V_i + \frac{1}{2}g_{V1}, R_i + \frac{1}{2}g_{R1})$$

$= h\,(\alpha\left(S_i + \frac{1}{2}g_{S1}\right)\left(E_i + \frac{1}{2}g_{E1}\right) + p(V_i + \frac{1}{2}g_{V1})(E_i + \frac{1}{2}g_{E1}) - (f(I_i + \frac{1}{2}g_{I1}\,) + c + \mu)(E_i + \frac{1}{2}g_{E1})),$

$$g_{I2} = hEq_3\,(t_i + \frac{h}{2}, S_i + \frac{1}{2}g_{S1}, E_i + \frac{1}{2}g_{E1}, I_i + \frac{1}{2}g_{I1}\,, V_i + \frac{1}{2}g_{V1}, R_i + \frac{1}{2}g_{R1})$$
$$= h\,(f(E_i + \tfrac{1}{2}g_{E1})(I_i + \tfrac{1}{2}g_{I1}) - (z + \mu + \sigma)(I_i + \tfrac{1}{2}g_{I1}\,)),$$

$$g_{V2} = hEq_4\,(t_i + \tfrac{h}{2}, S_i + \tfrac{1}{2}g_{S1}, E_i + \tfrac{1}{2}g_{E1}, I_i + \tfrac{1}{2}g_{I1}\,, V_i + \tfrac{1}{2}g_{V1}, R_i + \tfrac{1}{2}g_{R1})$$

$$= h\,(m(S_i + \tfrac{1}{2}g_{S1}) - \left(p(E_i + \tfrac{1}{2}g_{E1}) + \mu\right)(V_i + \tfrac{1}{2}g_{V1})\,),$$
$$g_{R2} = hf_5\,(t_i + \frac{h}{2}, S_i + \frac{1}{2}g_{S1}, E_i + \frac{1}{2}g_{E1}, I_i + \frac{1}{2}g_{I1}\,, V_i + \frac{1}{2}g_{V1}, R_i + \frac{1}{2}g_{R1})$$
$$= h\left(z(I_i + \tfrac{1}{2}g_{I1}\,) + c(E_i + \tfrac{1}{2}g_{E1}) - \mu(\,R_i + \tfrac{1}{2}g_{R1})\right).$$

In the third stage, try to find $g_{S3}, g_{E3}, g_{I3}, g_{V3}$ and g_{R3}, by substituting the form of $g_{S3}, g_{E3}, g_{I3}, g_{V3}$ and g_{R3} in (1) as below:

$$g_{S3} = hEq_1\,(t_i + \frac{h}{2}, S_i + \frac{1}{2}g_{S2}, E_i + \frac{1}{2}g_{E2}, I_i + \frac{1}{2}g_{I2}, V_i + \frac{1}{2}g_{V2}, R_i + \frac{1}{2}g_{R2})$$

$$= h\,(\Lambda - (\alpha(E_i + \tfrac{1}{2}g_{E2}) + m + \mu)(S_i + \tfrac{1}{2}g_{S2})),$$
$$g_{E3} = hEq_2\,(t_i + \frac{h}{2}, S_i + \frac{1}{2}g_{S2}, E_i + \frac{1}{2}g_{E2}, I_i + \frac{1}{2}g_{I2}, R_i + \frac{1}{2}g_{R2})$$
$= h\,(\alpha(S_i + \frac{1}{2}g_{S2})(E_i + \frac{1}{2}g_{E2}) + p(V_i + \frac{1}{2}g_{V2})(E_i + \frac{1}{2}g_{E2}) - (f(I_i + \frac{1}{2}g_{I2}\,) + c + \mu)(E_i + \frac{1}{2}g_{E2})),$

$$g_{I3} = hEq_3\,(t_i + \frac{h}{2}, S_i + \frac{1}{2}g_{S2}, E_i + \frac{1}{2}g_{E2}, I_i + \frac{1}{2}g_{I2}\,, V_i + \frac{1}{2}g_{V2}, R_i + \frac{1}{2}g_{R2})$$
$$= h\,(f(E_i + \tfrac{1}{2}g_{E2})(I_i + \tfrac{1}{2}g_{I2}) - (z + \mu + \sigma)(I_i + \tfrac{1}{2}g_{I2}\,)),$$

$$g_{V3} = hEq_4\,(t_i + \tfrac{h}{2}, S_i + \tfrac{1}{2}g_{S2}, E_i + \tfrac{1}{2}g_{E2}, I_i + \tfrac{1}{2}g_{I2}\,, V_i + \tfrac{1}{2}g_{V2}, R_i + \tfrac{1}{2}g_{R2})$$

$$= h\,(m(S_i + \tfrac{1}{2}g_{S2}) - \left(p(E_i + \tfrac{1}{2}g_{E2}) + \mu\right)(V_i + \tfrac{1}{2}g_{V2})\,),$$

$$g_{R3} = hEq_5\,(t_i + \frac{h}{2}, S_i + \frac{1}{2}g_{S2}, E_i + \frac{1}{2}g_{E2}, I_i + \frac{1}{2}g_{I2}\,, V_i + \frac{1}{2}g_{V2}, R_i + \frac{1}{2}g_{R2})$$

$$= h\left(z(I_i + \tfrac{1}{2}g_{I2}\,) + c(E_i + \tfrac{1}{2}g_{E2}) - \mu(\,R_i + \tfrac{1}{2}g_{R2})\right).$$

The fourth stage needs to find $g_{S4}, g_{E4}, g_{I4}, g_{V4}$ and g_{R4}, substituting the form of $g_{S4}, g_{E4}, g_{I4}, g_{V4}$ and g_{R4} in (1) as below:
as below:

$$g_{S4} = hEq_1\,(t_i + h, S_i + g_{S3}, E_i + g_{E3}, I_i + g_{I3}, V_i + g_{V3}, R_i + g_{R3})$$
$$= h\,(\Lambda - (\alpha(E_i + g_{E3}) + m + \mu)(S_i + g_{S3})),$$
$$g_{E4} = hEq_2\,(t_i + h, S_i + g_{S3}, E_i + g_{E3}, I_i + g_{I3}, V_i + g_{V3}, R_i + g_{R3})$$
$$= h\,(\alpha(S_i + g_{S3})(E_i + g_{E3}) + p(V_i + g_{V3})(E_i + g_{E3}) - (f(I_i + g_{I3}\,) + c + \mu)(E_i + g_{E3})),$$

$$g_{I4} = hEq_3\ (t_i + h, S_i + g_{S3}, E_i + g_{E3}, I_i + g_{I3}, V_i + g_{V3}, R_i + g_{R3})$$
$$= h\ (f(E_i + g_{E3})(I_i + g_{I3}) - (z + \mu + \sigma)(I_i + g_{I3}\ )),$$
$$g_{V4} = hEq_4\ (t_i + h, S_i + g_{S3}, E_i + g_{E3}, I_i + g_{I3}, V_i + g_{V3}, R_i + g_{R3})$$
$$= h\ (m(S_i + g_{S3}) - (p(E_i + g_{E3}) + \mu)(V_i + g_{V3})\ ),$$
$$g_{R4} = hEq_5\ (t_i + h, S_i + g_{S3}, E_i + g_{E3}, I_i + g_{I3}, V_i + g_{V3}, R_i + g_{R3})$$
$$= h\ \ (z(I_i + g_{I3}\ ) + c(E_i + g_{E3}) - \mu(\ R_i + g_{R3})).$$

Finally, by substituting in (2), all of $g_{S1}, g_{E1}, g_{I1}, g_{V1}, g_{R1}, g_{S2}, g_{E2}, g_{I2}, g_{V2}, g_{R2}, g_{S3}, g_{E3}, g_{I3}, g_{V3}, g_{R3}, g_{S4}, g_{E4}, g_{I4}, g_{V4}, g_{R4}$ for four steps, the numerical results for each of $S(t), E(t), I(t), V(t)$ and $R(t)$ can be computed.

4. Results and Discussion

The initial conditions of (1) are taken from the Iraq data from the World Health Organization website [28,29], and it is as follows: $S_0 = 500, E_0 = 46, I_0 = 23, V_0 = 0,$ and $R_0 = 12.$ Numerical results for the COVID-19 epidemic problem in Iraq in 2020 are discussed and analyzed in this section. Table 2 gives the results of each of the epidemic stages $S(t), E(t), I(t), V(t)$ and $R(t)$ for RK_4 method, that RK_4 results converge with step sizes 0.02 in a weak and 0.003 in a day. But the results of step size h = 0.003 are more accurate than the results of step size h = 0.02, this is the principle of numerical methods.

Figures 1, 2, 3, and 4 are to show the behavior of all epidemic stages $S(t)$, $E(t)$, $I(t)$, $V(t)$ and $R(t)$ for RK_4 the method in two cases:

Case1: when the step size h is 0.02 in a weak during 10 and 15 years; the results are presented in **figures 1 and 2,** from 2020 the beginning of the breakout of the virus until 2030 and 2035 respectively.

Case2: when the step size h is 0.003 in a day for 10 and 15 years; the results are presented in figures 3 and 4, from 2020 the beginning of the breakout of the virus until 2030 and 2035 respectively.

Figures 1, 2, 3, and 4 describe the behavior of each phase of the COVID-19 epidemic. The behavior of $S(t)$, the population has decreased gradually since the virus began, then will be stable at the end of 2022 to the following years. On the other hand, it is noticeable that the $E(t)$ has increased since the beginning of the spread of the virus, reaching its highest possible in 2021. Then it starts declining from 2021 to the end of 2022. After that, it settles to the same level. It is clear that the behavior of $I(t)$ the population has been increasing since the beginning of the epidemic and begins to decrease gradually after 2023, it will maintain the same level after 2025. At the beginning of the spread of the virus, the vaccine was not available until the end of 2021, therefore, the behavior of the vaccine curve before the middle of 2021 should be ignored. With the vaccination campaign began, we note that the curve of $V(t)$ gradually raised to 2023 and remained in a noticeable rise until 2025, then began to decline and stabilize at a certain level in 2025 for the coming years. Finally, the curve $R(t)$ began to increase and rise gradually until 2026. The increase would be very little in the coming years for the curve $R(t)$.

All the present study results agree with the previous studies, [24] for all stages of the Coronavirus epidemic in the results. The MATLAB program of version R2017a has helped to supply numerically solving the pandemic model at the present work.

Table 2. Numerical solutions for the COVID-19 model

Population	Step Size (*h*)	10 Years	15 Years
S	0.02	89.3593198158165	87.5900398275115
	0.003	89.3598850965887	87.5900398209401
E	0.02	28.4319382456529	32.09564036298
	0.003	28.4309679558064	32.0956403779949
I	0.02	210.285219546379	183.971491059345
	0.003	210.294376974796	183.971490970528
V	0.02	19.9586546831238	17.1116618577972
	0.003	19.9595517884747	17.1116618474704
R	0.02	68.2557582669928	84.1216912610883
	0.003	68.2524273304788	84.1216910919618

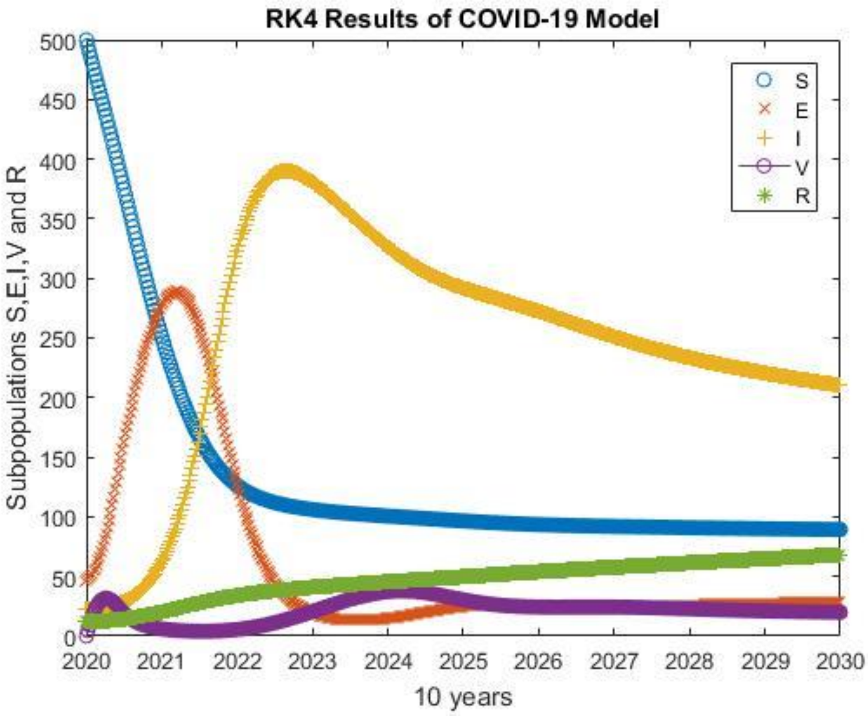

Figure 1. RK_4 numerical results of COVID-19 model of S, E, I, V, R from 2020 with step size $h = 0.02$ during 10 years.

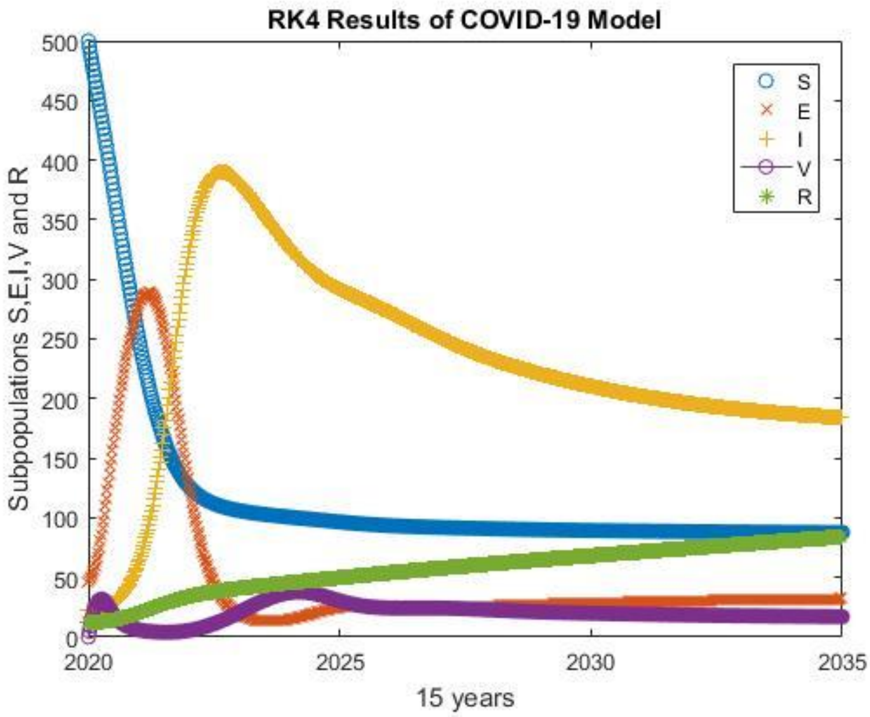

Figure 2. RK_4 numerical results of COVID-19 model of S, E, I, V, R from 2020 with step size $h = 0.02$ during 15 years.

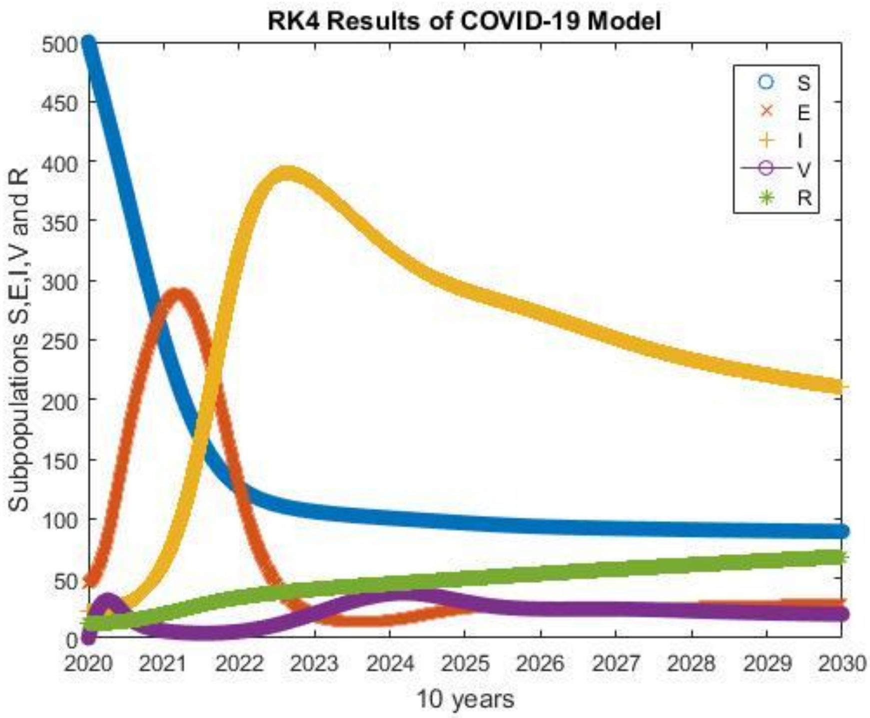

Figure 3. RK_4 numerical results of COVID-19 model of S, E, I, V, R from 2020 with step size $h = 0.003$ during 10 years.

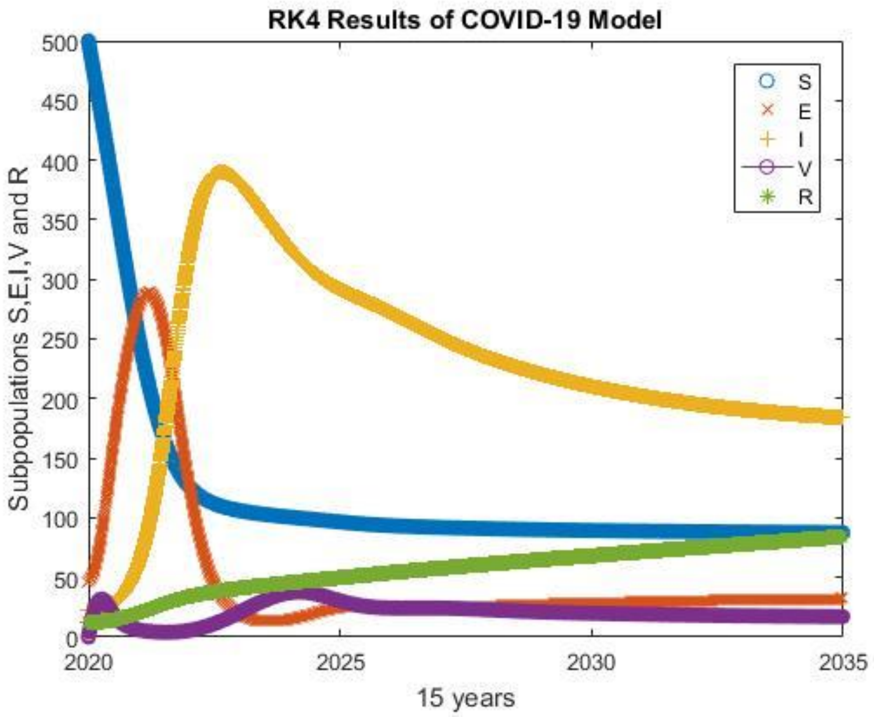

Figure 4 . RK_4 numerical results of COVID-19 model of S, E, I, V, R from 2020 with step size $h = 0.003$ during 15 years.

5. Conclusion

This model is compatible with the Corona Virus epidemic in Iraq in all its stages, and the observation data for Iraq is used as initial values, so the obtained results match the spread of the virus in Iraq.

Using RK_4 the method gives accurate results. As well, it is an effective iterative numerical method that is applied for solving the nonlinear system of differential equations for initial value problems, it has given accurate and reliable results at present work, for the Corona Virus epidemic model.

In the current research, the used numerical method helped us analyze the outbreak of the epidemic in the COVID-19 model in Iraq. The results obtained displayed that the number of susceptible populations gradually decreased in some years, then maintained its level after 3 years of disease beginning. While the number of exposed and infective populations increased in the same years, then it has some stability after (2-5) years from the starting of the disease. When the vaccination campaign began, the number of vaccinated gradually rises for 3 years, then begins to decline and stabilize at a certain level in 2025 for the coming years. On the other hand, note a slight rise from the beginning of the disease to the coming years, and then it stabilizes for the recovered population.

We conclude our research, through the behavior of all epidemic stages, with the expectation that the epidemic will disappear during the next few years within about five years.

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