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Connectedness via Generalizations of Semi-Open Sets

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Abstract

We use the idea of the grill. This study generalized a new sort of linked space like $\mathcal{G}^*so$-connected and $\mathcal{G}^*so$-hyperconnected and investigated its features, as well as the relationship between it and previously described notions. It also developed new sorts of functions, such as hyperconnected space, and identified their relationship by offering numerous instances and attributes that belong to this set. This set will serve as a starting point for further research into the set many future possibilities. We also use some theorems and observations previously studied and related to the grill and the semi-open to obtain results in this research. We applied the concept of connected to them and obtained results related to connected. The sources related to the connected and semi-open where considered as starting points and an important basis in this research.

Keywords: Grill topological space, $\mathcal{G}^*$s-connected, $\mathcal{G}^*$s-*dis*connected, $\mathcal{G}^*$-semi-open sets.

1. Introduction

In [1,2] found a topological area where the concept of the grill, and the grill has shown to be an effective tool for learning a variety of topological concerns. Subsets of a topological space $(\mathcal{X}, \tau)$ which is a non-empty collection $\mathcal{G}$ and is referred to be a grill whenever (a) $\mathbb{A} \in \mathcal{G}$ and $\mathbb{A} \subseteq \mathbb{B}$ implying $\mathbb{B} \in \mathcal{G}$. (b) $\mathcal{X}$ has a subset $\mathbb{A}$ and $\mathbb{B}$ also $\mathbb{A} \cup \mathbb{B} \in \mathcal{G}$ lead to $\mathbb{A} \in \mathcal{G}$ or $\mathbb{B} \in \mathcal{G}$. A triple $(\mathcal{X}, \tau, \mathcal{G})$ topological space with grills is one type of the topological space. [3] created a distinctive topology with a grill and investigated topological notions. For every topological space $(\mathcal{X}, \tau)$ point x, Neighborhoods are open of $\mathcal{X}$, and embodied by $\tau(\mathcal{X})$. A mapping $\Psi{:}\mathbb{P}(\mathcal{X}) \rightarrow \mathbb{P}(\mathcal{X})$ is referred to as $\oint(\mathbb{A}) = \{ x \in \mathcal{X}: \mathbb{A} \cap \tilde{U} \in \mathcal{G}, \forall \tilde{U} \in \tau(\mathcal{X})$ and $\mathbb{A} \in \mathbb{P}(\mathcal{X})\}$. A mapping $\Psi{:}\ \mathbb{P}(\mathcal{X}) \rightarrow \mathbb{P}(\mathcal{X})$ is referred to as $\Psi(\mathbb{A}) = \mathbb{A} \cup \oint(\mathbb{A})$ for every $\mathbb{A} \in \mathbb{P}(\mathcal{X})$. The map Ψ kuratowski closure axioms are met:

(a) $\Psi(\emptyset)=\emptyset$,

(b) as soon as $\mathbb{A} \subseteq \mathbb{B}$, so $\Psi(\mathbb{A}) \subseteq \Psi(\mathbb{B})$,
(c) as soon as $\mathbb{A} \subseteq \mathfrak{X}$, so $\Psi(\Psi(\mathbb{A})) = \Psi(\mathbb{A})$,
(d) as soon as $\mathbb{A}, \mathbb{B} \subseteq \mathfrak{X}$, so $\Psi(\mathbb{A} \cup \mathbb{B}) = \Psi(\mathbb{A}) \cup \Psi(\mathbb{B})$.

Grill topological spaces come in various shapes, sizes, as a discrete topology, and a coffinite topology.
On a space $(\mathfrak{X}, \tau)$; this agrees to a grill $\mathbb{G}$ A topology exists $\tau_{\mathbb{G}}$ on $\mathfrak{X}$ that is given by no one else by $\tau_{\mathbb{G}} = \{\tilde{\mathbb{U}} \subseteq \mathfrak{X}: \Psi(\mathfrak{X} - \tilde{\mathbb{U}}) = \mathfrak{X} - \tilde{\mathbb{U}}\}$, Consequently, $\mathbb{A} \subseteq \mathfrak{X}$, $\Psi(\mathbb{A}) = \mathbb{A} \cup \oint(\mathbb{A})$ [4,5]. $\tau \subseteq \tau_{\mathbb{G}}$ and $\Psi(\mathbb{A}) = c\iota(\mathbb{A})$. Using the following as a basis, we can locate $\tau_{\mathbb{G}}$ on $\mathfrak{X}$ by supplying $\tau_{\mathbb{G}}$ through $\beta(\tau_{\mathbb{G}}, \mathfrak{X}) = \{\Upsilon - \mathbb{A}; \Upsilon \in \tau, \mathbb{A} \notin \mathbb{G}\}$.
On a space $(\mathfrak{X}, \tau)$, there is a grill $\mathbb{G}$, $\tau \subseteq \beta(\mathbb{G}, \tau) \subseteq \tau_{\mathbb{G}}$, where $\beta(\mathbb{G}, \tau)$ basis for $\tau_{\mathbb{G}}$[6]. As an example, to exist in a space $(\mathfrak{X}, \tau)$, $\tau_{\mathbb{G}} = \tau$ whenever $\mathbb{G} = \mathbb{P}(\mathfrak{X}) \setminus \{\emptyset\}$ implying $\tau_{\mathbb{G}} = \tau$ [7].The family of all semi-open set is showed by τ_s. Semi-open is a subset $\mathbb{A}$ of a space $(\mathfrak{X}, \tau)$, if $\mathbb{A} \subseteq c\iota(\underset{\sim}{I}n\mathfrak{t}(\mathbb{A}))$[8]. Let $(\mathfrak{X}, \tau, \mathbb{G})$ be topological space. The subset $\mathbb{A}$ in $\mathfrak{X}$ is known as $\mathbb{G}$-semi-open if $\mathbb{A} \subseteq \Psi(\underset{\sim}{I}n\mathfrak{t}(\mathbb{A}))$, and every Ψ-semi-open is a semi-open.
Various academics have made generalizations using these combinations [9, 10]. The symbol $\underset{\sim}{I}n\mathfrak{t}(\mathbb{A})$ to the set interior $\mathbb{A}$, as well as the sign $c\iota(\mathbb{A})$ is the closure of $\mathbb{A}$ is utilized in this paper.
The space $(\mathfrak{X}, \tau)$ is disconnected if and only if there exists two open disjoint nonempty sets $\mathbb{A}$ and $\mathbb{B}$, such that $\mathbb{A} \cup \mathbb{B} = \mathfrak{X}$. i.e.,
$\mathfrak{X}$ is disconnected if and only if $\mathfrak{X} = \mathbb{A} \cup \mathbb{B}$, $\mathbb{A}, \mathbb{B} \in \tau$, and $\mathbb{A} \cap \mathbb{B} = \phi$ *where* $\mathbb{A}, \mathbb{B} \neq \phi$.
The sets $\mathbb{A}$ and $\mathbb{B}$ form a separation of $\mathfrak{X}$. The space (X, τ) is connected if and only if it is not disconnected. $\mathfrak{X}$ is connected if and only if $\mathfrak{X} \neq \mathbb{A} \cup \mathbb{B}$, $\mathbb{A}, \mathbb{B} \in \tau$, $\mathbb{A} \cap \mathbb{B} = \phi$, *and* $\mathbb{A}, \mathbb{B} \neq \phi$.

2. Grill Semi-Open Sets

Definition 2.1. [2]:
$\mathbb{A}$ is the set that will be Grill semi-open when $\exists\, \tilde{\mathbb{U}} \in \tau$; $\tilde{\mathbb{U}} - \mathbb{A} \notin \mathbb{G}$ and $\mathbb{A} - c\iota_{\mathbb{G}}(\tilde{\mathbb{U}}) \notin \mathbb{G}$. And it is described by $\mathbb{G}^*$-semi-open. $\mathfrak{X} - \mathbb{G}^*$-semi-open is a $\mathbb{G}^*$-semi-closed , as well as the set of all $\mathbb{G}^*$-semi-open at the moment by $\mathbb{G}^*so(\mathfrak{X})$ and the collection of every $\mathbb{G}^*$-semi-closed at the moment by $\mathbb{G}^*sc(\mathfrak{X})$ . .

Example 2.2.
Let $(\mathfrak{X}, \tau, \mathbb{G})$ topological space to be a grill and $\mathfrak{X} = \{\mathfrak{x}_1, \mathfrak{x}_2, \mathfrak{x}_3\}, \tau = \{\mathfrak{X}, \emptyset, \{\mathfrak{x}_1\}, \{\mathfrak{x}_1, \mathfrak{x}_2\}\}, \mathcal{F} = \{\mathfrak{X}, \emptyset, \{\mathfrak{x}_3\}, \{\mathfrak{x}_3, \mathfrak{x}_2\}\}, \mathbb{G} = \{\tilde{\mathbb{U}} \subseteq \mathfrak{X}; \mathfrak{x}_2 \in \tilde{\mathbb{U}}\}, \oint : \mathbb{P}(\mathfrak{X}) \to \mathbb{P}(\mathfrak{X}), \oint(\mathbb{A}) = \{\mathfrak{x} \in \mathfrak{X}; \forall \tilde{\mathbb{U}} \in \tau_{\mathfrak{x}} ; \tilde{\mathbb{U}} \cap \mathbb{A} \in \mathbb{G}\}, \Psi(\mathbb{A}) = \mathbb{A} \cup \emptyset, \tau_{\mathbb{G}} = \{\mathfrak{X}, \emptyset, \{\mathfrak{x}_3, \mathfrak{x}_2\}, \{\mathfrak{x}_2\}, \{\mathfrak{x}_1, \mathfrak{x}_2\}\}, \mathcal{F}_{\mathbb{G}} = \{\mathfrak{X}, \emptyset, \{\mathfrak{x}_3, \mathfrak{x}_2\}, \{\mathfrak{x}_3\}, \{\mathfrak{x}_1\}\}$, then $\mathbb{G}^*so(\mathfrak{X}) = \{\mathfrak{X}, \emptyset, \{\mathfrak{x}_3\}, \{\mathfrak{x}_1\}, \{\mathfrak{x}_2\}, \{\mathfrak{x}_1, \mathfrak{x}_3\}, \{\mathfrak{x}_1, \mathfrak{x}_2\}, \{\mathfrak{x}_3, \mathfrak{x}_2\}\}$.

Theorem 2.3. [10]:
Every family's union $\mathbb{G}^*$-semi-open set is a $\mathbb{G}^*$-semi-open set.

Remark 2.4. [10]:
Every two $\mathbb{G}^*$-Semi-open sets intersecting need not to be a $\mathbb{G}^*$-semi-open set.

Remark 2.5. [10]:
The family of all $\mathbb{G}^*$-Semi-open sets is characterized as supra topology.

Remark 2.6 [10]:
i. Every open is a $\mathbb{G}^*$-semi-open sets.
ii. Every closed is a $\mathbb{G}^*$-semi-closed sets.

Theorem 2.7. [10]:
Every $\mathbb{G}$-semi-open is a $\mathbb{G}^*$-semi-open.

Proposition 2.8. [10]:
Every Grill topology $(\mathfrak{X},\tau,\mathbb{G})$, $\mathbb{A}$ is an $\mathbb{G}$-semi-open set if and only if $\mathbb{A}$ is a $\mathbb{G}^*$-semi-open sets whenever $\mathbb{G} = \mathbb{P}(\mathfrak{X}) \setminus \{\emptyset\}$.

Remark 2.9. [10]:
The concepts $\mathbb{G}^*$-semi-open sets and the semi-open set are independent.

Definition 2.10. [10]:
The function $\dot{f}: (\mathfrak{X},\tau,\mathbb{G}) \to (Y,\tau',\mathbb{G})$ is known as:

1. $\mathbb{G}^*$-semi-open function, currently "$\mathbb{G}^*$-s-o function" if $\dot{f}(\tilde{U}) \in \mathbb{G}^*so(Y)$, whenever $\tilde{U} \in \mathbb{G}^*so(\mathfrak{X})$.
2. $\mathbb{G}^{**}$-semi-open function, currently "$\mathbb{G}^{**}$-s-o function" if $\dot{f}(\tilde{U}) \in \mathbb{G}^*so(Y)$ whenever $\tilde{U} \in \tau$.
3. $\mathbb{G}^{***}$-semi-open function, currently "$\mathbb{G}^{***}$-s-o function" if $\dot{f}(\tilde{U}) \in \tau'$ whenever $\tilde{U} \in \mathbb{G}^*so(\mathfrak{X})$.

Definition 2.11. [2]:
A function $\dot{f} : (\mathfrak{X},\tau,\mathbb{G}) \to (Y,\tau',\mathbb{G})$ is said to be;

1. $\mathbb{G}^*s$- continuous function, currently "$\mathbb{G}^*$s-continuous function "$if\ \dot{f}^{-1}(\tilde{U}) \in \mathbb{G}^*so(\mathfrak{X})$for all $\tilde{U} \in \tau$.
2. Strongly $\mathbb{G}^*s$-continuous function, currently “strongly $\mathbb{G}^*$s-continuous function” $if\ \dot{f}^{-1}(\tilde{U}) \in \tau$, fore ever $\tilde{U} \in \mathbb{G}^*so(Y)$.
3. $\mathbb{G}^*s$-irresolute function, presently "$\mathbb{G}^*$s-irresolute function " if $\dot{f}^{-1}(\tilde{U}) \in \mathbb{G}^*so(\mathfrak{X})$, fore ever $\tilde{U} \in \mathbb{G}^*so(Y)$.

3. Grill Semi-Open Sets in Grill Connected Space

Definition 3.1:
The space $(\mathfrak{X}, \tau, \mathbb{G})$ is a $\mathbb{G}^*so$- disconnected if and only if there exists two $\mathbb{G}^*s$-open disjoint nonempty sets $\mathbb{A}$ and $\mathbb{B}$ such that $\mathbb{A}\cup\mathbb{B} = \mathfrak{X}$. i.e.,
$\mathfrak{X}$ is a $\mathbb{G}^*so$- disconnected if and only if $\mathfrak{X} = \mathbb{A}\cup\mathbb{B}$, $\mathbb{A},\mathbb{B} \in \mathbb{G}^*so(\mathfrak{X})$, and $\mathbb{A} \cap \mathbb{B} = \phi$.
The sets $\mathbb{A}$ and $\mathbb{B}$ form a $\mathbb{G}^*so$-separation of $\mathfrak{X}$. The space $(\mathfrak{X}, \tau, \mathbb{G})$ is a $\mathbb{G}^*so$- connected if and only if it is not $\mathbb{G}^*so$- disconnected. $\mathfrak{X}$ is a $\mathbb{G}^*so$- connected if and only if $\mathfrak{X} \neq \mathbb{A}\cup\mathbb{B}$, $\mathbb{A}, \mathbb{B} \in \mathbb{G}^*so(\mathfrak{X})$, $\mathbb{A} \cap \mathbb{B} = \phi, and\ \mathbb{A},\mathbb{B} \neq \phi$.

Example 3.2.
Let $(\mathfrak{X}, \tau, \mathbb{G})$ topològical *s*pace to be a grill a*nd* $\mathfrak{X} = \{x_1, x_2, x_3\}, \tau = \tau_{\mathbb{G}} = \{\mathfrak{X}, \emptyset, \{x_2\}, \{x_3\}\}$, $\mathbb{G} = \mathbb{P}(\mathfrak{X}) \setminus \{\emptyset\}$ is a $\mathbb{G}^*so$- connected space.

Remark 3.3.
Every disconnected set is a $\mathbb{G}^*so$-disconnected.

Proof.
Let $(\mathfrak{X}, \mathbb{G})$ be a disconnected space, then there exists $\neq \emptyset, \mathcal{Z} \neq \emptyset$, $\mathcal{W}, \mathcal{Z} \in \tau$, $\mathcal{W} \cap \mathcal{Z} = \emptyset$, $and\ \mathcal{W} \cup \mathcal{Z} = \mathfrak{X}$,since every open set is $\mathbb{G}^*s$-open set. Therefore, $\mathfrak{X}$ is $\mathbb{G}^*so$-disconnected.

Remark 3.4.

The space (Ҳ, τ, $\mathbb{G}$) is a $\mathbb{G}^* so$- disconnected with any grill and $\mathbb{G}^* so(Ҳ) = \mathbb{P}(Ҳ)$ if Ҳ contains more than one element since there exists $\mathbb{A}$ and $\mathbb{A}^c \in \mathbb{G}^* so(Ҳ)$, $\mathbb{A} \cup Ƀ = Ҳ$, $\mathbb{A} \cap Ƀ = \phi, and\ \mathbb{A}, Ƀ \neq \phi$.

Remark 3.5.
The space (Ҳ, τ, $\mathbb{G}$) is a $\mathbb{G}^* so$- disconnected , $\mathbb{G} = \emptyset$, and $\tau_{\mathbb{G}} = \{Ҳ, \emptyset\}$, so $\mathbb{G}^* so$- connected space.

Remark 3.6.
If $\tau_{\mathbb{G}} = \mathcal{F}_{\mathbb{G}}$ and $\tau_{\mathbb{G}} \neq$ I indiscrete, when $\mathbb{G} = \mathbb{P}(Ҳ) \setminus \{\emptyset\}$, that is mean (Ҳ, τ, $\mathbb{G}$) is a $\mathbb{G}^* so$-disconnected .

Theorem 3.7.
The space (Ҳ, τ, $\mathbb{G}$) is a $\mathbb{G}^* so$-connected if and only Ҳ cannot be written as union of two non-empty disjoint closed set's.

Proof.
Let Ҳ is a $\mathbb{G}^* so$-connected if $Ҳ = \mathbb{A} \cup Ƀ$ such that $\mathbb{A}$, Ƀ $\in \mathbb{G}^* sc(Ҳ), \mathbb{A} \cap Ƀ = \phi, and\ \mathbb{A}, Ƀ \neq \phi$. So $\mathbb{A} = Ƀ^c\ and\ Ƀ = \mathbb{A}^c$, then $Ҳ = \mathbb{A} \cup Ƀ$, $\mathbb{A}$, Ƀ $\in \mathbb{G}^* so(Ҳ)$, $\mathbb{A} \cap Ƀ = \phi, and\ \mathbb{A}, Ƀ \neq \phi$, that's mean Ҳ is a $\mathbb{G}^* so$- disconnected. That's contradiction. Therefore, Ҳ cannot be written as a union of two non-empty disjoint closed set. Now, if Ҳ is a $\mathbb{G}^* so$- disconnected space, so Ҳ= $\mathbb{A} \cup Ƀ$; $\mathbb{A}$, Ƀ $\in \mathbb{G}^* so(Ҳ)$, $\mathbb{A} \cap Ƀ = \phi, and\ \mathbb{A}, Ƀ \neq \phi$. So $\mathbb{A} = Ƀ^c\ and\ Ƀ = \mathbb{A}^c$,then $\mathbb{A}$ and Ƀ $\in \mathbb{G}^* sc(x)$, but that is contradiction. Therefore, Ҳ is a $\mathbb{G}^* so$-connected space.

Theorem 3.8.
The space (Ҳ, τ, $\mathbb{G}$) is a $\mathbb{G}^* so$-connected if and only if the only subsets of the space Ҳ which are $\mathbb{G}^* s$-open and $\mathbb{G}^* s$-closed are Ҳ and $\emptyset$.

Proof.
Let $\mathbb{A}$,$\mathbb{A}^c \in \mathbb{G}^* so(x)$, $\mathbb{A} \neq Ҳ$, $and\ \mathbb{A} \neq \emptyset$, so $Ҳ = \mathbb{A} \cup \mathbb{A}^c$, $\mathbb{A} \cap \mathbb{A}^c = \phi$, $and\ \mathbb{A}, \mathbb{A}^c \neq \phi$. Then, Ҳ is a $\mathbb{G}^* so$-disconnected and that is a contradiction. So, where $\mathbb{A} \subseteq Ҳ$, $\mathbb{A}$, $\mathbb{A}^c \in \mathbb{G}^* so(x)$, then, $\mathbb{A} = Ҳ$ or $\mathbb{A} = \emptyset$. Conversely Ҳ is a $\mathbb{G}^* so$-disconnected space; This means that $Ҳ = \mathcal{W} \cup \mathcal{Z}$, $\mathcal{W}, \mathcal{Z} \in \mathbb{G}^* so(x)$, $\mathcal{W} \cap \mathcal{Z} = \phi$, and $\mathcal{W}, \mathcal{Z} \neq \emptyset$ implies that $\mathcal{W} = \mathcal{Z}^c$ and $\mathcal{Z} = \mathcal{W}^c$. So $\mathcal{W}, \mathcal{Z} \in \mathbb{G}^* sc(x)$ (that is a contradiction**)**.Therefore, Ҳ is a $\mathbb{G}^* so$-connected space.

Remark 3.9.
If $\dot{f} : (Ҳ, τ, \mathbb{G}) \to (Y, \tau', \mathbb{G})$ is a $\mathbb{G}^* so\text{-}irresolute$ and onto function and Y is a $\mathbb{G}^* so\text{-}connected$ space, then Ҳ is not necessary $\mathbb{G}^* so\text{-}connected$ space.

Example 3.10.
Let $\dot{f} : (\mathbb{R}, D, \mathbb{G}) \to (\mathbb{R}, I, \mathbb{G})$, such that $\dot{f}$(ҳ) = ҳ , for all ҳ ∈ $\mathbb{R}$, and $\mathbb{G} = \mathbb{P}(Ҳ) \setminus \{\emptyset\}$, so $\dot{f}$ is a $\mathbb{G}^* so\text{-}\ irresolute$ and onto function, $(\mathbb{R}, I, \mathbb{G})$ is a $\mathbb{G}^* so$-connected space, and $(\mathbb{R}, D, \mathbb{G})$ is a $\mathbb{G}^* so$-disconnected space.

Theorem 3.11.
If $\mathbb{A}$ and Ƀ are a $\mathbb{G}^* so$-connected spaces of (Ҳ, τ, $\mathbb{G}$) and $\mathbb{A} \cap Ƀ \neq \phi$, then is a $\mathbb{G}^* so$-connected space.$\mathbb{A} \cup Ƀ$

Proof.
Let (Ҳ, τ, $\mathbb{G}$) be a grill topological space and $\mathbb{A}$, Ƀ ⊆ Ҳ ; $\mathbb{A}$, Ƀ is a $\mathbb{G}^* so$-connected space. Now, if $\mathbb{A} \cup Ƀ$ is a $\mathbb{G}^* so$-disconnected space, so $\mathbb{A} \cup Ƀ = \mathcal{W} \cup \mathcal{Z}$, $\mathcal{W}, \mathcal{Z} \in \mathbb{G}^* so(x)_{(\mathbb{A} \cup Ƀ)}$, $\mathcal{W} \cap \mathcal{Z} = \phi$, and $\mathcal{W}, \mathcal{Z} \neq \emptyset$, then $\mathbb{A} \subseteq \mathbb{A} \cup Ƀ$, $\mathbb{A} \subseteq \mathcal{W} \cup \mathcal{Z}$, $\mathbb{A} \subseteq \mathcal{W}\ or\ \mathbb{A} \subseteq \mathcal{Z}$.

Similarly, $\mathbb{B}$ leads to either $\mathbb{A} \subseteq \mathcal{W}$ and $\mathbb{B} \subseteq \mathcal{W}$ then $\mathbb{A} \cup \mathbb{B} \subseteq \mathcal{W}$ then $\mathcal{Z} = \emptyset$ C! Or $\mathbb{A} \subseteq \mathcal{Z}$ and $\mathbb{B} \subseteq \mathcal{Z}$ then $\mathbb{A} \cup \mathbb{B} \subseteq \mathcal{Z}$ then $\mathcal{W} = \emptyset$ C! Or $\mathbb{A} \subseteq \mathcal{W}$ and $\mathbb{B} \subseteq \mathcal{Z}$ then $\mathbb{A} \cap \mathbb{B} \subseteq \mathcal{W} \cap \mathcal{Z}$ then $\mathbb{A} \cap \mathbb{B} = \emptyset$ that's contradiction Or $\mathbb{A} \subseteq \mathcal{Z}$ and $\mathbb{B} \subseteq \mathcal{W}$ then $\mathbb{A} \cap \mathbb{B} \subseteq \mathcal{W} \cap \mathcal{Z}$ then $\mathbb{A} \cap \mathbb{B} = \emptyset$ (that's contradiction!). So, $\mathbb{A} \cup \mathbb{B}$ is a $\mathbb{G}^*so$-connected space.

Remark 3.12.

We can generalize **Theorem 3.11.** to a family of a $\mathbb{G}^*so$-connected sets as follows: let $\{\mathbb{A}_\propto\}_{\propto \in \Lambda}$ be a family of a $\mathbb{G}^*so$-connected subsets of a space $(\mathbb{X}, \tau, \mathbb{G})$ and $\bigcap_{\propto \in \Lambda} \mathbb{A}_\propto \neq \emptyset$, then $\bigcup_{\propto \in \Lambda} \mathbb{A}_\propto$ is a $\mathbb{G}^*so$-connected set.

4. Grill Semi-Open Sets in Grill Connected Space Hyperconnected

Definition 4.1.

In any grill topological space $(\mathbb{X}, \tau, \mathbb{G})$ is said to be:

1. $*$-hyperconnected if $\mathbb{A}$ is $\tau_{\mathbb{G}}$-dense ($cl_{\mathbb{G}}(\mathbb{A}) = \mathbb{X}$) for every non-empty open subset $\mathbb{A}$ of $\mathbb{X}$.
2. $\mathbb{G}^*$-hyperconnected if $\mathbb{X} - cl_{\mathbb{G}}(\mathbb{A}) \notin \mathbb{G}$ for every non-empty open subset $\mathbb{A}$ of $\mathbb{X}$.
3. $\mathbb{G}^*s$-hyperconnected if $\mathbb{X} - cl_{\mathbb{G}}(\mathbb{A}) \notin \mathbb{G}$ for every non-empty $\mathbb{G}^*s\text{-}open$ subset $\mathbb{A}$ of $\mathbb{X}$.
4. $\mathbb{G}^*so$-hyperconnected if $cl_{\mathbb{G}}(\mathbb{A}) = \mathbb{X}$ for all $\mathbb{A} \in \mathbb{G}^*so(\mathbb{X})$.

Proposition 4.2.

1. Every $\mathbb{G}^*so$-hyperconnected is $*$-hyperconnected.
2. Every $*$-hyperconnected is $\mathbb{G}^*$-hyperconnected.
3. Every $\mathbb{G}^*s$-hyperconnected is $\mathbb{G}^*$-hyperconnected.
4. Every $\mathbb{G}^*so$-hyperconnected is $\mathbb{G}^*s$-hyperconnected.

Proof.

1. Let $\mathbb{A}$ be a $\mathbb{G}^*so$-hyperconnected, then $cl_{\mathbb{G}}(\mathbb{A}) = \mathbb{X}$, then $\mathbb{A}$ is a $*$-hyperconnected (since $\mathbb{A} \in \mathbb{G}^*so(\mathbb{X})$ then $\mathbb{A} \in \tau$).
2. Let $\mathbb{A}$ be a $*$-hyperconnected. This means that $cl_{\mathbb{G}}(\mathbb{A}) = \mathbb{X}$. So, $\mathbb{X} - cl_{\mathbb{G}}(\mathbb{A}) = \emptyset \notin \mathbb{G}$. Therefore, $\mathbb{A}$ is a $\mathbb{G}^*$-hyperconnected.
3. Let $\mathbb{A}$ be an open in $\mathbb{G}^*s$-hyperconnected. This means that $\mathbb{X} - cl_{\mathbb{G}}(\mathbb{A}) \notin \mathbb{G}^*s$-hyperconnected (since every open set in $\mathbb{G}^*s$-hyperconnected is an open set in $\mathbb{G}^*$-hyperconnected. So, $\mathbb{X} - cl_{\mathbb{G}}(\mathbb{A}) \notin \mathbb{G}$ .Therefore, $\mathbb{A}$ is a $\mathbb{G}^*$-hyperconnected.
4. Let $\mathbb{A}$ be an open set in $\mathbb{G}^*so$-hyperconnected. This means that $cl_{\mathbb{G}}(\mathbb{A}) = \mathbb{X}$. So, $\mathbb{X} - cl_{\mathbb{G}}(\mathbb{A}) \notin \mathbb{G}$. Therefore, $\mathbb{A}$ is a $\mathbb{G}^*s$-hyperconnected.

The following diagram shows the relationship between the types of hyperconnected.

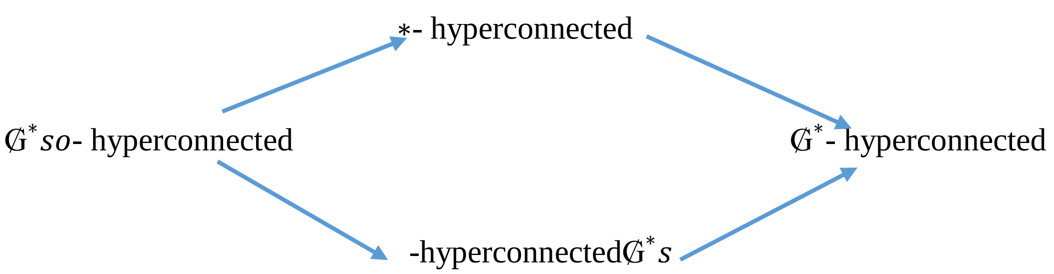

Diagram 1
Hyperconnected space via grill space

4. Conclusion

In this research, we studied a connect space in the grill semi-open topological spaces, some examples are showed, and some theorems are applied for these new sets. We also found some new properties of these sets.

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