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Truncated Inverse Generalized Rayleigh Distribution and Some Properties

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Abstract

Truncated distributions arise naturally in many practical situations. It is a conditional distribution that develops when the parent distribution's domain is constrained to a smaller area. The distribution of a right truncated is one of the types of single truncated that is restricted within a specific field and usually occurs when the specified period for the study is complete. Hence, this paper introduces the Right Truncated Inverse Generalized Rayleigh Distribution (RTIGRD) with two parameters. Then, provided some properties such as probability density function, cumulative distribution function (CDF), survival function, hazard function, rth moment, mean, variance, Moment Generating Function, Skewness, kurtosis, Median, and Mode for Right Truncated Inverse Generalized Rayleigh Distribution on [0, 1].

Keywords: Hazard Function**,** Inverse Generalized Rayleigh Distribution, Right Truncated, Survival Function.

1. Introduction

A Truncated Distribution is a conditional distribution on a specific range that restricts the full range. The purpose of truncated distributions is to get better results. When a distribution is truncated, the domain of the truncated random variable is restricted based on the truncation points of interest, and thus the shape of the distribution changes. In addition, it happens when we are unable to detect or record events that take place inside or outside of a predetermined range or below or above a given threshold. The truncation can be from the left side, the right side, or both sides [1-3].

Galton et al. introduced truncated distribution in 1898 [4] . Then, [5] provided forms for the probability density function, cumulative distribution function, hazard function, characteristic function, mean, mode, median, variance, skewness, and kurtosis of doubly truncated Fréchet distributions. [6] introduced [0, 1]; Truncated Fréchet Gamma and truncated Fréchet inverted Gamma distributions are discussed as special cases. [7] Proposed a new truncated Weibull-G (TW-G). [8] introduced [0, 1] Truncated Fréchet distributions and [0, 1] Truncated Fréchet Weibull as special cases. The cumulative distribution function, rth moment, mean, variance, skewness, kurtosis, mode, median, characteristic function, reliability function, and hazard function. [9] Introduced Truncated Weibull power Lomax distribution with four parameters. [10] introduced [0,1] Truncated Gompertz Exponential distribution and [0,1] truncated Gompertz-G family distribution and then discussed as cases: probability density function (PDF), Cumulative distribution function (CDF), Hazard rate function (HF), Survival function (SF), moments, the mean μ, variance σ2, Moment Generating Function (M.G.F.), Median M, kurtosis KR, and Skewness SK. [11] Introduced the Zero Truncated Discrete Transmuted Generalized Inverse Weibull Distribution (ZT-DTGIW). Jumana introduce [0,1] Truncated Lomax – Lomax ([0,1]TLD) distribution. Some properties of the ([0,1] TLLD) distribution were derived [12].

Therefore, in this study, the Right Truncated Inverse Generalized Raleigh Distribution was derived and some of its statistical and mathematical properties were studied on [0,1] (Probability density function, cumulative distribution function, Survival function, hazard rate function, rth moment, variance, Moment Generating Functions, kurtosis, skewness, median, and mode).

2.Truncated Inverse Generalized Rayleigh Distribution

The Rayleigh distribution derives from the Weibull distribution with two parameters and is a suitable model for life-testing studies [13];

$$f(x) = \frac{2}{\lambda} x e^{-\frac{x^2}{\lambda}} \quad (1)$$

$$F(x) = 1 - e^{-\frac{x^2}{\lambda}} \quad (2)$$

Due to the practical significance of the Rayleigh distribution, numerous extended forms of the Rayleigh distribution have been proposed. For example, [14-23] However, the generalized inverted scale family distributions were introduced by [24]. These newly developed models were formulated by introducing a new shape parameter to the scale family of distributions. These models give major flexibility in modelling complex data and the results drawn from them seem genuine and quite sound [25-27] . Mudholkar and Srivastava [28] suggested a new method for generalization different distributions dependent on c.d.f, which we will be used to generalize the Rayleigh distribution in this paper as follows:

$$G(x) = [F(x)]^{\theta} = [1 - e^{-\frac{x^2}{\lambda}}]^{\theta} \tag{3}$$

$$g(x) = \theta \left[1 - e^{-\frac{x^2}{\lambda}}\right]^{\theta-1} \frac{2x}{\lambda}\, e^{-\frac{x^2}{\lambda}} \tag{4}$$

The Generalized Raleigh Distribution can be shown by the transformation of a random variable. If the random variable T has Generalized Raleigh Distribution, then the r. v. $X = \left(\frac{1}{T}\right)$ has an Inverse Generalized Raleigh Distribution (IGRD). Suppose T is a random variable following Inverse Generalized Rayleigh distribution with two parameters θ and λ. Then p.d.f and c.d.f functions of inverse Generalized Rayleigh distribution are given for equations (3) and (4), respectively, by [29];

$$g(t) = \left[1 - e^{-\frac{1}{\lambda t^2}}\right]^{\theta-1} \frac{2\theta}{\lambda t^3}\; e^{-\frac{1}{\lambda t^2}} \tag{5}$$

$$G(t) = 1 - \left[1 - e^{-\frac{1}{\lambda t^2}}\right]^{\theta} \tag{6}$$

when $0 < t < \infty$ and $g\,(t) = 0$ o.w

Hance for truncation for the Inverse Generalized Rayleigh Distribution, Right-Side Truncation for the Inverse Generalized Rayleigh Distribution to called Right Truncated Inverse Generalized Rayleigh distribution (RTIGRD) on [0, 1] by using $f_{RTIGRD}(t) = \frac{g(t)}{G(1)}$ [30], When t=1 in equation (6)

$$G(1) = 1 - \left[1 - e^{-\frac{1}{\lambda}}\right]^{\theta}$$

$$f_{RTIGRD}(t) = \frac{g(t)}{G(1)}$$

The p.d.f of RTIGRD is

$$f_{RTIGRD}(t) = \frac{\left[1-e^{-\frac{1}{\lambda t^2}}\right]^{\theta-1} \frac{2\theta}{\lambda t^3}\, e^{-\frac{1}{\lambda t^2}}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}} \qquad , 0 \le t \le 1$$

The c.d.f of RTIGRD is

$$F_{RTIGRD}(t) = \int_0^t \frac{\left[1-e^{-\frac{1}{\lambda t^2}}\right]^{\theta-1} \frac{2\theta}{\lambda t^3}\, e^{-\frac{1}{\lambda t^2}}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}\, dt$$

Therefore,

$$F_{RTIGRD}(t) = \frac{1-\left[1-e^{-\frac{1}{\lambda t^2}}\right]^{\theta}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}$$

The Survival Function of RTIGRD is

$$S_{RTIGRD}(t) = 1 - F_{RTIGRD}(t)$$

$$= 1 - \frac{1-\left[1-e^{-\frac{1}{\lambda t^2}}\right]^{\theta}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}} = \frac{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}-1+\left[1-e^{-\frac{1}{\lambda t^2}}\right]^{\theta}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}$$

$$S_{RTIGRD}(t) = \frac{\left[1-e^{-\frac{1}{\lambda t^2}}\right]^{\theta}-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}$$

The Hazard Function of RTIGRD is

$$H_{RTIGRD}(t) = \frac{f_{RTIGRD}(t)}{S_{RTIGRD}(t)}$$

$$H_{RTIGRD}(t) = \frac{\dfrac{\left[1-e^{-\frac{1}{\lambda t^2}}\right]^{\theta-1}\frac{2\theta}{\lambda t^3}\, e^{-\frac{1}{\lambda t^2}}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}}{\dfrac{\left[1-e^{-\frac{1}{\lambda t^2}}\right]^{\theta}-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}}$$

$$H_{RTIGRD}(t) = \frac{\left[1-e^{-\frac{1}{\lambda t^2}}\right]^{\theta-1}\frac{2\theta}{\lambda t^3}\, e^{-\frac{1}{\lambda t^2}}}{\left[1-e^{-\frac{1}{\lambda t^2}}\right]^{\theta}-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}$$

Where,

t: is a value of random variable and $0 < t < 1$.

θ: Shape parameter and $\theta > 0$.

λ: Scale parameter and $\lambda > 0$.

Figures (1),(2),(3) and (4) plot the p.d.f , c.d.f , SF and HF for the RTIGRD for some cases of θ and λ

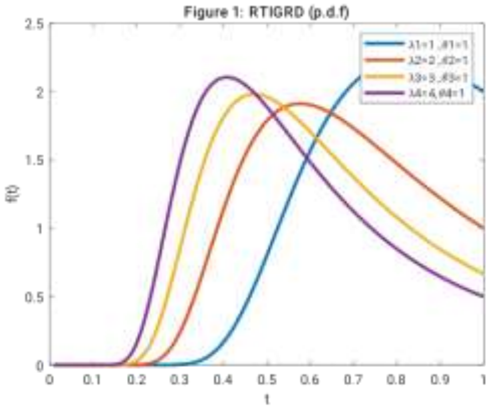

Figure 1. probability density function

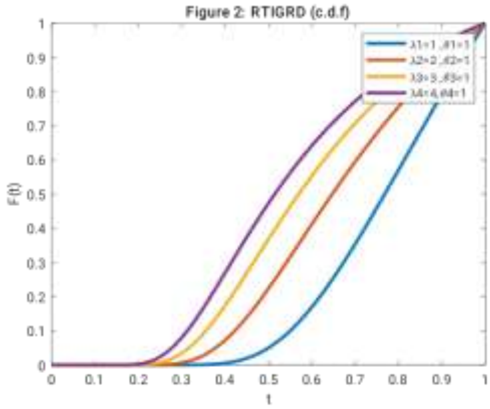

Figure 2. cumulative distribution function

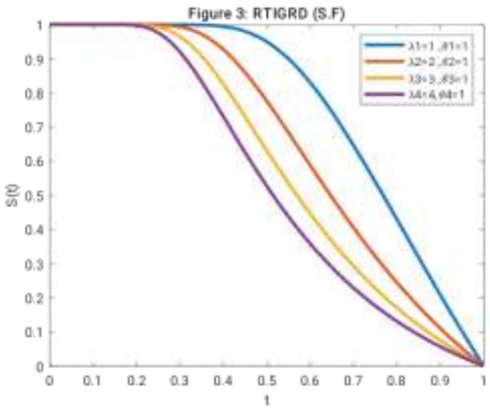

Figure 3. Survival function

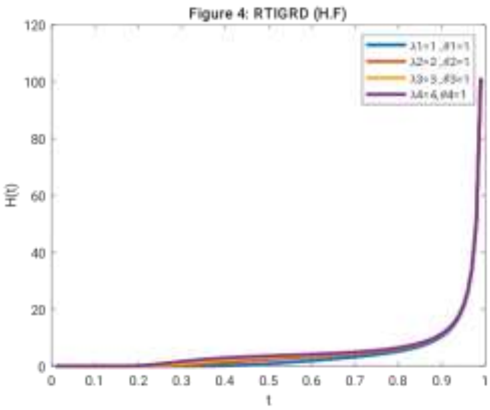

Figure 4. Hazard function

3.Some properties of Right Truncated Inverse Generalized Rayleigh Distribution

In this section, some properties are given for RTGRD. However, some properties are complicated to solve. For this reason, use numerical analysis to find it. We made some simplifications for the p.d.f. by using the Binomial theorem and Tyler series

$$(a \mp x)^n = \sum_{j=0}^{n} \binom{n}{j} (\mp x)^j a^{n-j}$$

$$\left[1 - e^{-\frac{1}{\lambda t^2}}\right]^{\theta-1} = \sum_{j=0}^{\theta-1} \binom{\theta-1}{j} (-e^{-\frac{1}{\lambda t^2}})^j$$

Thus,

$$f_{RTIGRD}(t) = \frac{\sum_{j=0}^{\theta-1} \binom{\theta-1}{j} (-1)^j \, e^{-\frac{j}{\lambda t^2}} \frac{2\theta}{\lambda t^3} \, e^{-\frac{1}{\lambda t^2}}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}$$

$$f_{RTIGRD}(t) = \frac{\sum_{j=0}^{\theta-1} \binom{\theta-1}{j} (-1)^j \, \frac{2\theta}{\lambda t^3} \, e^{-\frac{j+1}{\lambda t^2}}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}$$

let

$$e^{-\frac{j+1}{\lambda t^2}} = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \frac{(j+1)^k}{(\lambda t^2)^k}$$

$$f_{RTIGRD}(t) = \frac{\sum_{j=0}^{\theta-1} \binom{\theta-1}{j} (-1)^j \, \frac{2\theta}{\lambda t^3} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \frac{(j+1)^k}{(\lambda t^2)^k}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}$$

$$= \frac{\sum_{k=0}^{\infty} \sum_{j=0}^{\theta-1} \frac{(-1)^{k+j}}{k!} \binom{\theta-1}{j} \, \frac{(j+1)^k}{(\lambda)^k \, t^{2k}} \frac{2\theta}{\lambda t^3}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}$$

$$= \frac{\sum_{k=0}^{\infty} \sum_{j=0}^{\theta-1} \frac{(-1)^{k+j}}{k!} \binom{\theta-1}{j} \, \frac{2\theta(j+1)^k}{(\lambda)^{k+1} \, t^{2k+3}}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}$$

$$f_{RTIGRD}(t) = \frac{\sum_{k=0}^{\infty} \sum_{j=0}^{\theta-1} \frac{(-1)^{k+j}}{k!} \binom{\theta-1}{j} \, \frac{2\theta(j+1)^k}{(\lambda)^{k+1}} \, t^{-2k-3}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}} \qquad , 0 \le t \le 1$$

3.1 rth moment:

The rth moment can be derived as follow:

$$E(t^r) = \int_0^1 t^r \; f_{RTIGRD}(t) \; dt$$

$$= \int_0^1 t^r \frac{\sum_{k=0}^{\infty} \sum_{j=0}^{\theta-1} \frac{(-1)^{k+j}}{k!} \binom{\theta-1}{j} \frac{2\theta(j+1)^k}{(\lambda)^{k+1}} t^{-2k-3}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}} \; dt$$

$$= \frac{\sum_{k=0}^{\infty} \sum_{j=0}^{\theta-1} \frac{(-1)^{k+j}}{k!} \binom{\theta-1}{j} \frac{2\theta(j+1)^k}{(\lambda)^{k+1}}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}} \int_0^1 t^{r-2k-3} \, dt$$

$$= \frac{\sum_{k=0}^{\infty} \sum_{j=0}^{\theta-1} \frac{(-1)^{k+j}}{k!} \binom{\theta-1}{j} \frac{2\theta(j+1)^k}{(\lambda)^{k+1}}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}} \; \frac{t^{r-2k-2}}{r-2k-2} \Big|_0^1$$

$$E(t^r) = \frac{\sum_{k=0}^{\infty} \sum_{j=0}^{\theta-1} \frac{(-1)^{k+j}}{k!} \binom{\theta-1}{j} \frac{2\theta(j+1)^k}{(\lambda)^{k+1} \; (r-2k-2)}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}$$

When r = 1, the mean of RTIGRD equal to

$$\mu = E(t) = \frac{\sum_{k=0}^{\infty} \sum_{j=0}^{\theta-1} \frac{(-1)^{k+j}}{k!} \binom{\theta-1}{j} \frac{2\theta(j+1)^k}{(\lambda)^{k+1} \; (1-2k-2)}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}$$

$$\mu = E(t) = \frac{\sum_{k=0}^{\infty} \sum_{j=0}^{\theta-1} \frac{(-1)^{k+j}}{k!} \binom{\theta-1}{j} \frac{2\theta(j+1)^k}{(\lambda)^{k+1} \; (-2k-1)}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}$$

When r=2, we will get $E(t^2)$

$$\mathrm{E}(t^2) = \frac{\sum_{k=0}^{\infty} \sum_{j=0}^{\theta-1} \frac{(-1)^{k+j}}{k!} \binom{\theta-1}{j} \frac{2\theta(j+1)^k}{(\lambda)^{k+1} \; (-2k)}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}$$

$$E(t^2) = \frac{\sum_{k=0}^{\infty} \sum_{j=0}^{\theta-1} \frac{(-1)^{k+j}}{k!} \binom{\theta-1}{j} \frac{\theta(j+1)^k}{(\lambda)^{k+1}\ (-k)}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}$$

when r=3

$$E(t^3) = \frac{\sum_{k=0}^{\infty} \sum_{j=0}^{\theta-1} \frac{(-1)^{k+j}}{k!} \binom{\theta-1}{j} \frac{\theta(j+1)^k}{(\lambda)^{k+1}\ (1-2k)}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}$$

When r=4

$$E(t^4) = \frac{\sum_{k=0}^{\infty} \sum_{j=0}^{\theta-1} \frac{(-1)^{k+j}}{k!} \binom{\theta-1}{j} \frac{\theta(j+1)^k}{(\lambda)^{k+1}\ (2-2k)}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}$$

3.2 Variance

The Variance (Var) of RTIGRD can be found as follows:

$$\sigma^2 = Var(t) = E(t^2) - [E(t)]^2$$

$$Var(t) = \frac{\sum_{k=0}^{\infty} \sum_{j=0}^{\theta-1} \frac{(-1)^{k+j}}{k!} \binom{\theta-1}{j} \frac{\theta(j+1)^k}{(\lambda)^{k+1}\ (-k)}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}} - \left[\frac{\sum_{k=0}^{\infty} \sum_{j=0}^{\theta-1} \frac{(-1)^{k+j}}{k!} \binom{\theta-1}{j} \frac{2\theta(j+1)^k}{(\lambda)^{k+1}\ (-2k-1)}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}\right]^2$$

$$= \frac{\left[1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}\right] \sum_{k=0}^{\infty} \sum_{j=0}^{\theta-1} \frac{(-1)^{k+j}}{k!} \binom{\theta-1}{j} \frac{\theta(j+1)^k}{(\lambda)^{k+1}\ (-k)} - \left[\sum_{k=0}^{\infty} \sum_{j=0}^{\theta-1} \frac{(-1)^{k+j}}{k!} \binom{\theta-1}{j} \frac{2\theta(j+1)^k}{(\lambda)^{k+1}\ (-2k-1)}\right]^2}{\left[1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}\right]^2}$$

$$= \frac{\sum_{k=0}^{\infty}\sum_{j=0}^{\theta-1}\frac{(-1)^{k+j}}{k!}\binom{\theta-1}{j}\frac{2\theta(j+1)^k}{(\lambda)^{k+1}}\left[\frac{1}{(-2k)}\left[1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}\right]-\sum_{k=0}^{\infty}\sum_{j=0}^{\theta-1}\frac{(-1)^{k+j}}{k!}\binom{\theta-1}{j}\frac{2\theta(j+1)^k}{(\lambda)^{k+1}\,(-2k-1)}\right]}{\left[1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}\right]^2}$$

3.3 Moment Generating Function

The Moment Generating Function of RTIGRD can be derived as follow:

$$\mathcal{M}_t(\text{ŧ}) = E(e^{\text{ŧ}t}) = \int_0^1 e^{\text{ŧ}t}\ f_{RTIGRD}(t)\ dt$$

$$\mathcal{M}_t(\text{ŧ}) = \int_0^1 e^{\text{ŧ}t}\frac{\sum_{k=0}^{\infty}\sum_{j=0}^{\theta-1}\frac{(-1)^{k+j}}{k!}\binom{\theta-1}{j}\frac{2\theta(j+1)^k}{(\lambda)^{k+1}}t^{-2k-3}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}\ dt$$

$$= \frac{\sum_{k=0}^{\infty}\sum_{j=0}^{\theta-1}\frac{(-1)^{k+j}}{k!}\binom{\theta-1}{j}\frac{2\theta(j+1)^k}{(\lambda)^{k+1}}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}\int_0^1 e^{\text{ŧ}t}\ t^{-2k-3}\,dt$$

Use,

$$e^{\text{ŧ}t} = \sum_{n=0}^{\infty}\frac{(\text{ŧ}t)^n}{n!}$$

$$= \frac{\sum_{k=0}^{\infty}\sum_{j=0}^{\theta-1}\frac{(-1)^{k+j}}{k!}\binom{\theta-1}{j}\frac{2\theta(j+1)^k}{(\lambda)^{k+1}}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}\int_0^1 \sum_{n=0}^{\infty}\frac{(\text{ŧ}t)^n}{n!}\ \ t^{-2k-3}\,dt$$

$$= \frac{\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\sum_{j=0}^{\theta-1}\frac{(-1)^{k+j}\ (\text{ŧ})^n}{k!\ n!}\binom{\theta-1}{j}\frac{2\theta(j+1)^k}{(\lambda)^{k+1}}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}\int_0^1\ t^{n-2k-3}\ dt$$

$$= \frac{\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\sum_{j=0}^{\theta-1}\frac{(-1)^{k+j}\ (\text{ŧ})^n}{k!\ n!}\binom{\theta-1}{j}\frac{2\theta(j+1)^k}{(\lambda)^{k+1}}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}\frac{t^{n-2k-2}}{n-2k-2}\Big|_0^1$$

$$\mathcal{M}_t(\text{ŧ}) = \frac{\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\sum_{j=0}^{\theta-1}\frac{(-1)^{k+j}\ (\text{ŧ})^n}{k!\ n!}\binom{\theta-1}{j}\frac{2\theta(j+1)^k}{(\lambda)^{k+1}\,(n-2k-2)}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}$$

3.4 Kurtosis

The kurtosis of RTIGRD can be found as follows:
kurtosis of RTIGRD can be found as follows:

$$kr=\frac{\mu_3}{(\mu_2)^2}-3=\frac{E(t^4)-4\mu E(t^3)+6\mu^2E(t^2)-3\mu^4}{(\sigma^2)^2}-3$$

kr =

$$\frac{\left\{\frac{\sum_{k=0}^{\infty}\sum_{j=0}^{\theta-1}\frac{(-1)^{k+j}}{k!}\binom{\theta-1}{j}\frac{\theta(j+1)^k}{(\lambda)^{k+1}\ (2-2k)}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}\right\}-4\left\{\frac{\sum_{k=0}^{\infty}\sum_{j=0}^{\theta-1}\frac{(-1)^{k+j}}{k!}\binom{\theta-1}{j}\frac{2\theta(j+1)^k}{(\lambda)^{k+1}\ (-2k-1)}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}\right\}\left\{\frac{\sum_{k=0}^{\infty}\sum_{j=0}^{\theta-1}\frac{(-1)^{k+j}}{k!}\binom{\theta-1}{j}\frac{\theta(j+1)^k}{(\lambda)^{k+1}\ (1-2k)}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}\right\}+6\left\{\frac{\sum_{k=0}^{\infty}\sum_{j=0}^{\theta-1}\frac{(-1)^{k+j}}{k!}\binom{\theta-1}{j}\frac{2\theta(j+1)^k}{(\lambda)^{k+1}\ (-2k-1)}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}\right\}^2\left\{\frac{\sum_{k=0}^{\infty}\sum_{j=0}^{\theta-1}\frac{(-1)^{k+j}}{k!}\binom{\theta-1}{j}\frac{\theta(j+1)^k}{(\lambda)^{k+1}\ (-k)}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}\right\}-3\left\{\frac{\sum_{k=0}^{\infty}\sum_{j=0}^{\theta-1}\frac{(-1)^{k+j}}{k!}\binom{\theta-1}{j}\frac{2\theta(j+1)^k}{(\lambda)^{k+1}\ (-2k-1)}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}\right\}^4}{\left(\frac{\sum_{k=0}^{\infty}\sum_{j=0}^{\theta-1}\frac{(-1)^{k+j}}{k!}\binom{\theta-1}{j}\frac{2\theta(j+1)^k}{(\lambda)^{k+1}}\left[\frac{1}{(-2k)}\left[1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}\right]-\sum_{k=0}^{\infty}\sum_{j=0}^{\theta-1}\frac{(-1)^{k+j}}{k!}\binom{\theta-1}{j}\frac{2\theta(j+1)^k}{(\lambda)^{k+1}\ (-2k-1)}\right]}{\left[1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}\right]^2}\right)^2}-3$$

3.5 Skewness

The Skewness of RTIGRD can be found as follows:

$$sk=\frac{\mu_3}{(\mu_2)^{\frac{3}{2}}}=\frac{E(t^3)-3\mu E(t^2)+2\mu^3}{(\sigma^2)^{\frac{3}{2}}}$$

sk =

$$\frac{\frac{\sum_{k=0}^{\infty}\sum_{j=0}^{\theta-1}\frac{(-1)^{k+j}}{k!}\binom{\theta-1}{j}\frac{\theta(j+1)^k}{(\lambda)^{k+1}(1-2k)}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}-3\left\{\frac{\sum_{k=0}^{\infty}\sum_{j=0}^{\theta-1}\frac{(-1)^{k+j}}{k!}\binom{\theta-1}{j}\frac{2\theta(j+1)^k}{(\lambda)^{k+1}(-2k-1)}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}\right\}\left\{\frac{\sum_{k=0}^{\infty}\sum_{j=0}^{\theta-1}\frac{(-1)^{k+j}}{k!}\binom{\theta-1}{j}\frac{\theta(j+1)^k}{(\lambda)^{k+1}(-k)}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}\right\}+2\left\{\frac{\sum_{k=0}^{\infty}\sum_{j=0}^{\theta-1}\frac{(-1)^{k+j}}{k!}\binom{\theta-1}{j}\frac{2\theta(j+1)^k}{(\lambda)^{k+1}(-2k-1)}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}\right\}^3}{\left\{\frac{\sum_{k=0}^{\infty}\sum_{j=0}^{\theta-1}\frac{(-1)^{k+j}}{k!}\binom{\theta-1}{j}\frac{2\theta(j+1)^k}{(\lambda)^{k+1}}\left[\frac{1}{(-2k)}\left[1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}\right]-\sum_{k=0}^{\infty}\sum_{j=0}^{\theta-1}\frac{(-1)^{k+j}}{k!}\binom{\theta-1}{j}\frac{2\theta(j+1)^k}{(\lambda)^{k+1}(-2k-1)}\right]}{\left[1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}\right]^2}\right\}^{\frac{3}{2}}}$$

3.6 Median

The Median of RTIGRD can be found as follows:

$$F_{RTIGRD}(t)=\frac{1}{2}$$

$$\frac{1-\left[1-e^{-\frac{1}{\lambda t^2}}\right]^{\theta}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}=\frac{1}{2}$$

$$1-\left[1-e^{-\frac{1}{\lambda t^2}}\right]^{\theta}=\frac{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}{2}$$

$$2-2\left[1-e^{-\frac{1}{\lambda t^2}}\right]^{\theta}=1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}$$

$$1-2\left[1-e^{-\frac{1}{\lambda t^2}}\right]^{\theta}=-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}$$

$$\left[1-e^{-\frac{1}{\lambda t^2}}\right]^{\theta}=\frac{1+\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}{2}$$

$$1-e^{-\frac{1}{\lambda t^2}}=\left[\frac{1+\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}{2}\right]^{\frac{1}{\theta}}$$

$$e^{-\frac{1}{\lambda t^2}}=1-\left[\frac{1+\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}{2}\right]^{\frac{1}{\theta}}$$

$$\ln e^{-\frac{1}{\lambda t^2}} = \ln\left[1-\left[\frac{1+\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}{2}\right]^{\frac{1}{\theta}}\right]$$

$$-\frac{1}{\lambda t^2} = \ln\left[1-\left[\frac{1+\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}{2}\right]^{\frac{1}{\theta}}\right]$$

$$\frac{1}{\lambda t^2} = \ln\left[1-\left[\frac{1+\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}{2}\right]^{\frac{1}{\theta}}\right]^{-1}$$

$$t^2 = \frac{1}{\lambda \ln\left[1-\left[\frac{1+\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}{2}\right]^{\frac{1}{\theta}}\right]^{-1}}$$

$$t_{Median} = \sqrt{\frac{1}{\lambda \ln\left[1-\left[\frac{1+\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}{2}\right]^{\frac{1}{\theta}}\right]^{-1}}}$$

3.7 Mode

The Mode of RTIGRD can be found as follows:

$$f_{RTIGRD}(t) = \frac{\left[1-e^{-\frac{1}{\lambda t^2}}\right]^{\theta-1} \frac{2\theta}{\lambda t^3}\ e^{-\frac{1}{\lambda t^2}}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}} \qquad , 0 \le t \le 1$$

$$f_{RTIGRD}(t) = \frac{\sum_{j=0}^{\theta-1}\binom{\theta-1}{j}(-1)^j\, e^{-\frac{j}{\lambda t^2}} \frac{2\theta}{\lambda t^3}\ e^{-\frac{1}{\lambda t^2}}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}$$

$$f_{RTIGRD}(t) = \frac{\sum_{j=0}^{\theta-1}\binom{\theta-1}{j}(-1)^j\ \frac{2\theta}{\lambda t^3}\ e^{-\frac{j+1}{\lambda t^2}}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}$$

$$f_{RTIGRD}(t) = \frac{M_j}{t^3}\ e^{-\frac{j+1}{\lambda t^2}}$$

, where

$$M_j = \frac{\sum_{j=0}^{\theta-1}\binom{\theta-1}{j}(-1)^j \ \frac{2\theta}{\lambda}}{1-\left[1-e^{-\frac{1}{\lambda}}\right]^{\theta}}$$

$$\therefore \quad \frac{df_{RTIGRD}(t)}{dt} = \frac{2M_j t^3 \frac{j+1}{\lambda t^3} e^{-\frac{j+1}{\lambda t^2}} - 3M_j e^{-\frac{j+1}{\lambda t^2}} t^2}{t^6} = 0$$

$$2\sum_{j=0}^{\theta-1}\frac{j+1}{\lambda} - 3\ t^2 = 0$$

$$2\sum_{j=0}^{\theta-1}\frac{j+1}{\lambda} = 3\ t^2$$

$$2\sum_{j=0}^{\theta-1}\frac{j+1}{3\lambda} = \ t^2$$

$$t_{Mode} = \sqrt{2\sum_{j=0}^{\theta-1}\frac{j+1}{3\lambda}}$$

4. Conclusions

The Right Truncated Inverse Generalized Rayleigh Distribution was considered in this paper. Numerical methods were used for deriving the properties of RTIGRD as survival function, hazard function, rth moment, mean, variance, Moment Generating Function, skewness, kurtosis, median, and mode.

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