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Fuzzy Transportation Model Technique to Determine the Minimum Total Cost Using a Novel Ranking Function

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Abstract

The transportation problem (TP) is employed in many different situations, such as scheduling, performance, spending, plant placement, inventory control, and employee scheduling. When all variables, including supply, demand, and unit transportation costs (TC), are precisely known, effective solutions to the transportation problem can be provided. However, understanding how to investigate the transportation problem in an uncertain environment is essential. Additionally, businesses and organizations should seek the most economical and environmentally friendly forms of transportation, considering the significance of environmental issues and strict environmental legislation. This research employs a novel ranking function to solve the transportation problem (TP), where fuzzy triangular numbers represent the fuzzy demand and supply (DAS). The fuzzy model is transformed and compressed to a crisp model (CM), and the results are compared using the northwest corner method and the least cost method. In addition, a numerical example of the fuzzy transportation model (FTM) is shown.

Keywords: Fuzzy transportation model, Fuzzy set, Ranking function, Triangular fuzzy number, Membership function.

1. Introduction

In today's economy, where there is intense competition, businesses must find new ways to produce and offer value to customers. Finding the best ways and times to deliver goods to clients in the quantities they want efficiently is getting more and more challenging. Around the world, some practical problems are typically solved by focusing on the transportation problem. In the manufacturing sector, the transportation issue is significant, as are other issues. Fuzzy transportation (FT) tries to find the lowest transportation costs (TC) of particular commodities via a capacitated network when the supply and demand of nodes and the capacity and cost of edges are represented as fuzzy integers.

Many researchers have studied the fuzzy transportation problem (FTP). In a fuzzy environment, Maheswari and Ganesan provided a simple technique for solving the FTP in which TC, sources of

supply, and demand at destinations were all represented by fuzzy pentagonal numbers (PFI). The problem of fuzzy transportation (FT) was solved without transforming to its corresponding crisp form, using a ranking methodology and a new pentagonal fuzzy arithmetic [1]. Christi and Kumari applied a robust ranking technique to discover the best fuzzy solution to the transportation problem [2]. To obtain the best solution, Mary and Sivasankari employed the direct method of FTP utilizing fuzzy hexagonal numbers (HFN) with alpha cut (AC) [3]. Purushothkumar et al. used the diagonal optimum technique approach to solve fully fuzzy transportation problems [4].
Several articles have introduced a new ranking function procedure by which the fuzzy number is converted into crisp [5-9].
The primary goal of this article is to propose a comparative method for solving FTP. The remainder of the paper is organized as follows: some fundamental definitions are provided in Section 2 and will be used throughout the article. The ranking function is presented for fuzzy numbers (FN) in Section 3. In Section 4, an algorithm is used to solve FTP. A numerical example is given in Section 5 to demonstrate the proposed strategy. Finally, in Section 6, the conclusions are provided.

2. Preliminaries notions

Simple definitions relevant to this work are presented in this section:

Definition1 [10]: Let $\widetilde{ß}$ be the universal set. A fuzzy set in $\widetilde{ß}$ is a set of ordered pairs, A= $\{\widetilde{ß}, \mu_{\widetilde{ß}}(\mathcal{G}); \mathcal{G} \in \mathring{U} \}$, where $\mu_{\widetilde{ß}}$: $\mathring{U} \to [0,1]$ is called the membership set.

Definition 2 [11]:
A triangular fuzzy number (TFN) $\tilde{\wp}$ is denoted as ($\wp$, d, ę), where $\wp$, d, and ę are real numbers, and its membership function $\mu_{\widetilde{ß}}$ is as follows:

$$\mu_{\widetilde{ß}} = \begin{cases} \frac{\mathcal{G}-\wp}{d-\wp} & \wp \le \mathcal{G} \le d \\ 1 & \mathcal{G} = d \\ \frac{ę-\mathcal{G}}{ę-d} & d \le \mathcal{G} \le ę \end{cases}$$

Definition 3 [12-13]:
A ranking function of a FN *R: F(*$\mathring{U}$*)* $\to$ $\mathring{U}$ is the set of all FN defined on $\mathring{U}$*:*, which maps each FN into a real number. The properties of the RF are the following:

(i) $\widetilde{ß_1} \le \widetilde{ß_2}$ iff $\Re(\widetilde{ß_1}) \le \Re(\widetilde{ß_2})$
(ii) $\widetilde{ß_1} > \widetilde{ß_2}$ iff $\Re(\widetilde{ß_1}) > \Re(\widetilde{ß_2})$

(iii) $\widetilde{ß_1} = \widetilde{ß_2}$iff $\Re(\widetilde{ß_1}) = \Re(\widetilde{ß_2})$

A fuzzy RF meets $\Re\left(k\,\widetilde{ß_1} + l\,\widetilde{ß_2}\right) = k\,\Re(\widetilde{ß_1}) + l\,\Re(\widetilde{ß_2})$ for every $\widetilde{ß_1}, \widetilde{ß_2} \in F(\mathring{U})$.

3. Suggested a Novel Ranking Function of the Fuzzy Transportation Model:

The following is the membership function that is utilized:

$$\tilde{\mathcal{C}}(\widetilde{ß}) = \begin{cases} \frac{\cancel{f}(\mathcal{s}-ŗ)}{\ell-ŗ} & ŗ \le \mathcal{s} \le \ell \\ \cancel{f} & \mathcal{s} = \ell \\ \frac{\cancel{f}(t-\mathcal{s})}{t-\ell} & \ell \le \mathcal{s} \le t \end{cases}$$

Now, utilizing the triangular membership function (TMF) is to find a novel ranking function as follows:

$$\frac{f(s-r)}{\ell-r} = \upsilon \rightarrow s = r + \frac{\alpha}{f}\,(\ell - r) = inf\ \tilde{C}\,(\upsilon)$$

$$\frac{f(t-s)}{t-\ell} = \upsilon \rightarrow s = t - \frac{\alpha}{f}\,(t - \ell) = sup\ \tilde{C}\,(\upsilon).$$

Hence, the following ranking function $\mathcal{J}(\tilde{C})$ can be calculated:

$$\mathcal{J}(\tilde{C}) = \frac{[\frac{1}{2}\int_0^{\delta}\frac{\upsilon^{10}}{2}\left[inf\left(\tilde{C}(\upsilon)\right) + sup\left(\tilde{C}(\upsilon)\right)\right]d\upsilon\,]}{\int_0^{\delta}\frac{\upsilon^{10}}{2}\ d\upsilon}$$

$$\mathcal{J}(\tilde{C}) = \frac{[\frac{1}{2}\int_0^{\delta}\frac{\upsilon^{10}}{2}\left[r + \frac{\upsilon}{\delta}\,(\ell - r) + t - \frac{\upsilon}{\delta}\,(t-\ell)\right]d\upsilon\,]}{\int_0^{\delta}\frac{\upsilon^{10}}{2}\ d\upsilon}$$

$$\mathcal{J}(\tilde{C}) = \frac{[\frac{1}{2}\int_0^{\delta}\frac{\upsilon^{10}}{2}\,r + \frac{\upsilon^{11}}{2\delta}\,(\ell - r) + \frac{\upsilon^{10}}{2}t - \frac{\upsilon^{11}}{2\delta}\,(t-\ell)]d\upsilon\,]}{\int_0^{\delta}\frac{\upsilon^{10}}{2}\ d\upsilon}$$

$$\mathcal{J}(\tilde{C}) = \frac{[\frac{1}{2}[\,\frac{\upsilon^{11}}{22}\,r + \frac{\upsilon^{12}}{24\delta}\,(\ell - r) + \frac{\upsilon^{11}}{22}t - \frac{\upsilon^{12}}{24\delta}\,(t-\ell)]\Big|_0^{\delta}\,]}{\frac{\upsilon^{11}}{22}\Big|_0^{\delta}}$$

$$\mathcal{J}(\tilde{C}) = \frac{[\frac{1}{2}[\,\frac{\delta^{11}}{22}\,r + \frac{\delta^{12}}{24\delta}\,(\ell - r) + \frac{\delta^{11}}{22}t - \frac{\delta^{12}}{24\delta}\,(t-\ell)\,]}{\frac{\delta^{11}}{22}}$$

$$\mathcal{J}(\tilde{A}) = \frac{[\frac{1}{2}\left[\,\frac{\delta^{11}}{22}\,r + \frac{\delta^{12}}{24}\,(\ell - r) + \frac{\delta^{11}}{22}t - \frac{\delta^{12}}{24}\,(t-\ell)\,\right]]}{\frac{\delta^{11}}{22}}$$

$$\mathcal{J}(\tilde{C}) = \frac{[\frac{\delta^{11}}{2}\left[\frac{r}{22} + \frac{(\ell - r)}{24} + \frac{t}{22} - \frac{(t-\ell)}{24}\right]]}{\frac{\delta^{11}}{22}}$$

$$\mathcal{J}(\tilde{C}) = \frac{[\frac{\delta^{11}}{2}\left[\frac{24\,r + 22\,\ell - 22\,r}{528} + \frac{24\,t - 22\,t + 22\,\ell}{528}\right]]}{\frac{\delta^{11}}{22}}$$

$$\mathcal{J}(\tilde{C}) = \frac{[\frac{\delta^{11}}{2}\left[\frac{24\,r + 22\,\ell - 22\,r + 24\,t - 22\,t + 22\,\ell}{528}\right]]}{\frac{\delta^{11}}{22}}$$

$$\mathcal{J}(\tilde{c}) = \frac{[\frac{\lambda^{11}}{2}\left[\frac{2\,\text{ŗ} + 44\,\ell + 2\,t}{528}\right]]}{\frac{\delta^{11}}{22}}$$

$$\mathcal{J}(\tilde{c}) = \frac{[\left[\frac{2\,\text{ŗ} + 44\,\ell + 2\,t}{528}\right]]}{\frac{\delta^{11}}{22}}$$

The following is the proposed formula for a class of valuation ranking functions:

$$\mathcal{J}(\tilde{c}) = \frac{2\,\text{ŗ} + 44\,\ell + 2\,t}{48}$$

4. Algorithm Fuzzy Transportation Model Technique to Determine the Minimum Total Cost using a Novel Ranking Function

In this section, a novel RF is used to discover the best solution for the FTP. The RF of each FN in the fuzzy problem under consideration is used and resulted in an equivalent crisp linear programming problem (CLPP), which is solved using two methods performed on each mathematical operation on FN. In contrast, in our proposed method, crisp numbers perform all arithmetic operations. The following are the steps in the suggested method:

Step1**.** Triangular Fuzzy Transportation Problems (TFTP) with fuzzy demand, supply, and cost were considered. The parameters are in triangular fuzzy number expressions.

Step2. The FT problems were converted to crisp value programming problems (CVPP) by the new RF

$$\mathcal{J}(\tilde{c}) = \frac{2\,\text{ŗ} + 44\,\ell + 2\,t}{48}$$

Step3. Transportation problem is tested to be balanced or unbalanced.

Step4. The **c**risp transportation model (CTM) is solved using NWC and LCM methods.

Step5. The two methods are compared to find the optimal solution (OS).
Step6. An example is given to test the optimality.
Step7. The chosen FTP example1 is solved utilizing all the provided approaches, indicating that it is possible to choose the optimal method among the NWC and LCM.

Example1:

Consider a transportation problem (TP) with parameters as triangular fuzzy numbers.

Table 1. Fuzzy Transportation Problem

	FB_1	FB_2	FB_3	**Fuzzy Supply**
FA_1	[14,16,18]	[56,58,60]	[44,46,48]	[66,68,70]
FA_2	[82,84,86]	[30,32,34]	[14,16,18]	[38,40,42]
FA_3	[86,88,90]	[26,28,30]	[68,70,72]	[86,88,90]
Fuzzy Demand	[38,40,42]	[66,68,70]	[86,88,90]	

The given fuzzy problem (FP) can be converted into a crisp value problem (CVP) applying the new ranking

$$\mathcal{J}(\tilde{C}) = \frac{2\,r + 44\,\ell + 2\,t}{48}$$

Table 2. Crisp Transportation Problem (new ranking function)

	B_1	B_2	B_3	Supply
A_1	16	58	46	68
A_2	84	32	16	40
A_3	88	28	70	88
Demand	40	68	88	

Next, the transportation problem (TP) is checked proving to be balanced, as the sum of supply and demand is equal to 196.

The problem is solved using

1) is shown in **Table 3**:

Table 3. The North west corner method

	$\boldsymbol{B_1}$	$\boldsymbol{B_2}$	$\boldsymbol{B_3}$	**Supply**
$\boldsymbol{A_1}$	16 40	58 28	46	68
$\boldsymbol{A_2}$	84	32 40	16 0	40
$\boldsymbol{A_3}$	88	28	70 88	88
Demand	40	68	88	

The total cost is found to be equal to 9704 (= 16*40+58*28+32*40+16*0+70*88)

2) Least cost method procedure is shown in **Table 4**:

Table 4. The procedure of Least cost method

	$\boldsymbol{B_1}$	$\boldsymbol{B_2}$	$\boldsymbol{B_3}$	**Supply**
$\boldsymbol{A_1}$	16 40	58	46 28	68
$\boldsymbol{A_2}$	84	32	16 40	40
$\boldsymbol{A_3}$	88	28 68	70 20	88
Demand	40	68	88	

The total cost is found to be equal to = 16*40+46*28+16*40+28*68+70*20= 5872

The two methods are compared as shown in **Table 5**.

Table 5. Total cost for North West Corner and Least Cost Methods

No	Method	Transportation Cost
1	North West Corner Method	9704
2	Least Cost Method	5872

The total cost was calculated for each method of transportation model according to **Table 5**. The value describes the total cost of decision-makers in the transportation model. Generally, it is chosen to be a monotonically decreasing function concerning the transportation model, which means the lower the total cost value, the stronger the relationship between the compared results. The minimum determined the fuzzy probabilistic preference relation between the two methods. Obtain the comparison result obtained with the total cost relation, which is in the form of ranking the numbers. According to the values, the lower the deal, the better the transportation model's ranking will be.

5. Conclusion

This research proposes a novel RF to determine the lowest total FTP cost. The cost of transportation was considered, as well as supply and demand. In nature, fuzzy triangular numbers are more realistic and general. A new classification process was used to change the FTP of the triple number to CTP. As shown in the comparison table (**Table 5**), the proposed least-cost method produced better results than the northwest corner method. This procedure is easy to understand and implement.

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