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Result for the group $\mathcal{SL}(2,17^2)$

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Abstract
The set of all (n×n) non-singular matrices over the field F this set forms a group under the operation of matrix multiplication. This group is called the general linear group of dimension n over the field F, denoted by $\mathrm{GL}(n, F)$. The determinant of these matrices is a homomorphism from $\mathrm{GL}(n, F)$ into F* and the kernel of this homomorphism was the special linear group and denoted by $\mathcal{SL}(n, F)$. Thus $\mathcal{SL}(n, F)$ is the subgroup of $\mathrm{GL}(n, F)$ which contains all matrices of determinant one.
The rational valued characters of the rational representations written as a linear combination of the induced characters for the groups $\mathcal{SL}(2, 17^2)$ discuss in this work and find the Artin indicator for this group after study the rational valued characters of the rational representations and the induced characters.
Keywords: Induced characters table, Artin indicator, rational character table of special linear group $\mathcal{SL}(n, F)$.

1. Introduction
The representation of the group study by [1], the group of all matrices of determinant 1 is $\mathcal{SL}(n,F)$, [2,3]. Authors in [4,5] study the character table of rational representations for the group $\mathcal{SL}(2, p)$, and find the periodical split for the groups PSL(2,31) and PSL(2,37) , SL(2,U), U = 31 and 37, we apply the same idea in [6-13] to compute the character table of rational representations for the group $\mathcal{SL}(2, 17^2)$. Also we apply the same idea in [4,7] to compute the Artin indicator for this group. We count all cyclic subgroup, Artin indicator, the rational valued characters of the rational representations and the induced characters for the group $\mathcal{SL}(2, 17^2)$ in this work.

2. The Fundamentals
Theorem 2.1: [14-16]
$| \mathcal{SL}(2, q^n) | = q^n \ (q^{2n} - 1)$.

Definition 2.4: [17-21]

Let H be a cyclic subgroup of a group G, and ϕ be a class function of H. Then

$$\phi\uparrow^{G}(g)=\frac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\phi(x_i)$$

Definition 2.5: [22-25]

The character induced from the principal character of a cyclic subgroups of G is called Artin character.

Definition 2.6: [26-30]

Let G be a finite group and let χ be any rational valued character on G. The smallest positive number n such that, $n\chi=\sum_{c}a_c\phi_c$, where $a_c\in Z$ and ϕ_c are Artin characters, is called the Artin exponent of G and denoted by A(G).

3. The Results

Authors in [4-7] study the character table of rational representations for the group $\mathcal{SL}(2,p)$ we apply that idea and compute the character table of rational representations for the group $\mathcal{SL}(2,17^2)$. Also we apply the same idea in [4,9] to compute the Artin indicator for this group. The character table of rational representations for the group $\mathcal{SL}(2,17^2)$ is

Table 1. The character table of rational representations for the group $\mathcal{SL}(2,17^2)$

C_g	1	z	c = d	zc = zd	a	a^2	a^3	a^4	a^6	a^8	a^9	a^{12}	a^{16}
$\|C_g\|$	1	1	41760	41760	83810	83810	83810	83810	83810	83810	83810	83810	83810
$\|C_G(g)\|$	24137280	24137280	578	578	288	288	288	288	288	288	288	288	288
1_G	1	1	1	1	1	1	1	1	1	1	1	1	1
ψ	289	289	0	0	1	1	1	1	1	1	1	1	1
χ_1	27840	-27840	96	-96	0	0	0	0	0	0	0	0	0
χ_2	6960	6960	24	24	0	0	0	0	0	0	0	0	0
χ_3	9280	-9280	32	-32	0	0	0	0	0	0	256	0	512
χ_4	3480	3480	12	12	0	0	0	0	0	0	0	144	- 72
χ_6	2320	2320	8	8	0	0	0	0	0	64	-32	0	- 64
χ_8	1740	1740	6	6	0	0	0	0	36	-48	0	- 72	- 192
χ_9	4640	-4640	16	-16	0	0	64	0	-64	0	0	- 128	- 256
χ_{12}	1160	1160	4	4	0	0	0	16	0	-32	-32	- 32	- 32
χ_{16}	870	870	3	3	0	0	9	0	9	-18	-18	- 18	18
χ_{18}	1160	1160	4	4	0	0	-8	0	-16	-32	-32	32	32
χ_{24}	580	580	2	2	0	4	0	-4	-8	-8	8	8	8
χ_{32}	870	870	3	3	0	0	-9	-18	-18	-18	18	18	18
χ_{36}	580	580	2	2	0	0	-4	-8	8	8	8	8	8
χ_{48}	290	290	1	1	1	-1	-2	-2	2	2	2	2	2
χ_{72}	290	290	1	1	0	-2	2	2	2	2	2	2	2
χ_{96}	290	290	1	1	0	-2	2	2	2	2	2	2	2
θ_1	32256	-32256	-112	112	0	0	0	0	0	0	0	0	0
θ_2	16128	16128	-56	-56	0	0	0	0	0	0	0	0	0
θ_5	8064	-8064	-28	28	0	0	0	0	0	0	0	0	0
θ_{10}	4032	4032	-14	-14	0	0	0	0	0	0	0	0	0
θ_{29}	1152	-1152	-4	4	0	0	0	0	0	0	0	0	0
θ_{52}	576	576	-2	-2	0	0	0	0	0	0	0	0	0
ξ	290	290	1	1	-2	2	-2	2	2	2	-2	2	2
η	576	-576	-1	1	0	0	0	0	0	0	0	0	0

Complete Table 1. The character table of rational representations for the group $\mathcal{SL}(2,17^2)$

C_g	b	b^2	b^3	b^6	b^{29}	b^{58}	b^{87}
$\lvert C_g \rvert$	6103437500	6103437500	6103437500	6103437500	6103437500	6103437500	6103437500
$\lvert C_G(g) \rvert$	78126	78126	78126	78126	78126	78126	78126
1_G	1	1	1	1	1	1	1
ψ	-1	-1	-1	-1	-1	-1	-1
χ_1	0	0	0	0	0	0	0
χ_2	0	0	0	0	0	0	0
χ_3	0	0	0	0	0	0	0
χ_4	0	0	0	0	0	0	0
θ_1	-2	2	4	-4	56	-56	-112
θ_2	1	0	-2	0	-28	0	56
θ_3	2	2	0	0	-56	56	0
θ_4	-1	0	0	0	28	0	0
θ_5	2	-2	-4	4	0	0	224
θ_6	-1	0	2	0	0	0	-112
θ_7	-2	2	0	0	112	-112	-224
θ_8	1	0	0	0	0	0	112
θ	2	-2	-4	4	-56	56	112
θ	-1	0	2	-7	28	56	-56
θ	-2	2	0	-14	56	-56	-56
θ	1	0	-7	7	-28	-28	-28
θ	-2	2	4	-4	-4	-4	-4
θ	1	2	-2	-2	-2	-2	-2
ξ	0	0	0	0	0	0	0
η	4	-4	4	-4	4	-4	4

This group has 26 cyclic subgroups generated by the conjugacy classes of the group. The induced character table for this group is:

Table 2. The induced character table for the group $\mathcal{SL}(2,17^2)$

C_g	1	z	$c = d$	$zc = zd$	a	a^2	a^3	a^4	a^6	a^8	a^9	a^{12}	a^{16}
$\lvert C_g \rvert$	1	1	41760	41760	83810	83810	83810	83810	83810	83810	83810	83810	83810
$\lvert C_G(g) \rvert$	24137280	24137280	578	578	288	288	288	288	288	288	288	288	288
Φ_1	24137280	0	0	0	0	0	0	0	0	0	0	0	0
Φ_2	12068640	12068640	0	0	0	0	0	0	0	0	0	0	0
Φ_3	83520	0	2	0	0	0	0	0	0	0	0	0	0
Φ_4	83520	41760	2	2	0	0	0	0	0	0	0	0	0
Φ_5	83810	167620	0	0	2	0	0	0	0	0	0	0	0
Φ_6	41760	0	0	0	0	4	0	0	0	0	0	0	0
Φ_7	27840	55680	0	0	0	0	6	0	0	0	0	0	0
Φ_8	20880	0	0	0	0	0	0	8	0	0	0	0	0
Φ_9	13920	0	0	0	0	0	0	0	12	0	0	0	0
Φ_{10}	10440	0	0	0	0	0	0	0	0	16	0	0	0
Φ_{11}	9280	18560	0	0	0	0	0	0	0	0	18	0	0
Φ_{12}	6960	0	0	0	0	0	0	0	0	0	0	24	0
Φ_{13}	5220	0	0	0	0	0	0	0	0	0	0	0	32
Φ_{14}	4640	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{15}	3480	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{16}	2610	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{17}	2320	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{18}	1740	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{19}	1160	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{20}	870	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{21}	83232	166464	0	0	0	0	0	0	0	0	0	0	0
Φ_{22}	41760	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{23}	16704	33408	0	0	0	0	0	0	0	0	0	0	0
Φ_{24}	8352	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{25}	2880	5760	0	0	0	0	0	0	0	0	0	0	0
Φ_{26}	1440	0	0	0	0	0	0	0	0	0	0	0	0

Complete Table 2. The induced character table for the group $\mathcal{SL}(2,17^2)$

C_g	a^{18}	a^{24}	a^{32}	a^{36}	a^{48}	a^{72}	a^{96}	b	b^2	b^5	b^{10}	b^{29}	b^{58}
$\|C_g\|$	83810	83810	83810	83810	83810	83810	83810	82943	82943	82943	82943	82943	82943
$\|C_G(g)\|$	288	288	288	288	288	288	288	290	290	290	290	290	290
Φ_1	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_2	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_3	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_4	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_5	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_6	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_7	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_8	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_9	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{10}	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{11}	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{12}	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{13}	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{14}	36	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{15}	0	48	0	0	0	0	0	0	0	0	0	0	0
Φ_{16}	0	0	64	0	0	0	0	0	0	0	0	0	0
Φ_{17}	0	0	0	72	0	0	0	0	0	0	0	0	0
Φ_{18}	0	0	0	0	96	0	0	0	0	0	0	0	0
Φ_{19}	0	0	0	0	0	144	0	0	0	0	0	0	0
Φ_{20}	0	0	0	0	0	0	192	0	0	0	0	0	0
Φ_{21}	0	0	0	0	0	0	0	2	0	0	0	0	0
Φ_{22}	0	0	0	0	0	0	0	0	2	0	0	0	0
Φ_{23}	0	0	0	0	0	0	0	0	0	10	0	0	0
Φ_{24}	0	0	0	0	0	0	0	0	0	0	20	0	0
Φ_{25}	0	0	0	0	0	0	0	0	0	0	0	58	0
Φ_{26}	0	0	0	0	0	0	0	0	0	0	0	0	116

Hence, the rational valued characters in the first tables is written as a linear combination of induced characters in the second table

$$1= 0.008621\Phi_{26} + 0.01724\Phi_{25} + 0.05\ \Phi_{24} - 0.1\,\Phi_{23} - 0.5\ \Phi_{22} - .5\ \Phi_{21} + 0.00521\ \Phi_{20} + 0.00694\ \Phi_{19} + 0.1042\ \Phi_{18} + 0.01389\ \Phi_{17} + 0.01563\ \Phi_{16} + 0.02083\,\Phi_{15} + 0.2778\ \Phi_{14} + 0.03125\ \Phi_{13} + 0.04167\,\Phi_{12} + 0.5556\,\Phi_{11} + 0{,}0625\,\Phi_{10} + 0.08333\,\Phi_9 + 0.125\,\Phi_8 + 0.16667\,\Phi_7 + 0.25\,\Phi_6 + 0.5\,\Phi_5 + 0.5\,\Phi_4 - 0.01671\,\Phi_2 - 0.00701\,\Phi_1,$$

$$\Psi = -0.008621\Phi_{26} - 0.01724\Phi_{25} - 0.05\ \Phi_{24} + 0.1\,\Phi_{23} + 0.5\ \Phi_{22} - 0.5\ \Phi_{21} + 0.00521\ \Phi_{20} + 0.00694\ \Phi_{19} + 0.1042\ \Phi_{18} + 0.01389\ \Phi_{17} + 0.01563\ \Phi_{16} + 0.02083\,\Phi_{15} + 0.2778\ \Phi_{14} + 0.03125\ \Phi_{13} + 0.04167\,\Phi_{12} + 0.5556\,\Phi_{11} + 0{,}0625\,\Phi_{10} + 0.08333\,\Phi_9 + 0.125\,\Phi_8 + 0.16667\,\Phi_7 + 0.25\,\Phi_6 + 0.5\,\Phi_5 + 0.5\,\Phi_4 - 0.01669\,\Phi_2 + 0.00701\,\Phi_1,$$

$$\chi_1 = -24\,\Phi_{20} + 48\Phi_{18} - 8\,\Phi_{15} + 24\,\Phi_4 + 96\,\Phi_3 + 0.16378\Phi_2 - 0.16753\Phi_1,$$

$$\chi_2 = -6\,\Phi_{20} - 8\,\Phi_{19} - 6\,\Phi_{18} - 4.5\,\Phi_{16} + 12\ \Phi_{15} + 12\,\Phi_4 - 0.04095\,\Phi_2 - 0.00004\,\Phi_1,$$

$$\chi_3 = 5.33333\,\Phi_{20} + 7.11111\,\Phi_{19} + 10.66667\,\Phi_{18} - 7.11111\,\Phi_{17} - 8\,\Phi_{16} - 7.11111\,\Phi_{14} + 13\,\Phi_{13} + 14.22222\,\Phi_{11} - 16\,\Phi_4 + 32\,\Phi_3 + 0.03272\Phi_2 - 0.06129\,\Phi_1,$$

$$\chi_4 = 1.5\,\Phi_{20} + 2\ \Phi_{19} - 3\ \Phi_{18} - 4\,\Phi_{17} - 4.5\,\Phi_{16} - 3\,\Phi_{15} - 2.25\,\Phi_{13} + 6\,\Phi_{12} + 6\,\Phi_4 + 32\,\Phi_3 - 0.02047\ \Phi_2 - 0.02049\,\Phi_1,$$

$\chi_6 = 0.66667\ \Phi_{20} + 0.88889\ \ \Phi_{19} + 1.33333\ \ \Phi_{18} + 1.7778\ \Phi_{17} - 2\ \Phi_{16} - 2.66667\ \Phi_{15} - 1.77778\ \Phi_{14} - 2\ \Phi_{13} - 1.77778\ \Phi_{11} + 4\ \Phi_{10} + 4\ \Phi_4 + 0.01092\ \ \Phi_2 - 0.01452\ \Phi_1$,

$\chi_8 = \Phi_{20} + 1.33333\ \ \Phi_{19} + 2\ \Phi_{18} + 2.66667\ \Phi_{17} + 3\ \Phi_{16} - 4\ \Phi_{15} - 5.33333\ \Phi_{14} - 6\ \Phi_{13} - 3\ \Phi_{12} + 3\ \Phi_9 + 3\ \Phi_4 - 0.01024\ \ \Phi_2 - 0.0078\ \Phi_1$,

$\chi_9 = 1.33333\ \Phi_{20} + 1.77778\ \ \Phi_{19} + 2.66667\ \ \Phi_{18} + 3.55556\ \Phi_{17} + 4\ \Phi_{16} - 5.33333\ \Phi_{15} - 7.11111\ \Phi_{14} - 8\ \Phi_{13} - 5.33333\ \Phi_{12} - 5.33333\ \Phi_9 + 10.66667\ \Phi_7 - 8\ \Phi_4 + 16\ \Phi_3 - 0.02192\ \ \Phi_2 - 0.03395\ \Phi_1$,

$\chi_{12} = 0.6667\ \ \Phi_{20} + 0.22222\ \ \Phi_{19} + 0.33333\ \ \Phi_{18} + 0.4444\ \Phi_{17} + 0.5\ \Phi_{16} + 0.66667\ \Phi_{15} + 0.88889\ \Phi_{14} - \ \Phi_{13} - 1.33333\ \Phi_{12} - 1.77778\ \ \Phi_{11} - 2\ \Phi_{10} + 2\ \Phi_8 + 2\ \Phi_4 - 0.00409\ \Phi_2 - 0.00686\ \Phi_1$,

$\chi_{16} = \ \ 0.09375\ \Phi_{20} + 0.125\ \ \Phi_{19} + 0.1875\ \ \Phi_{18} + 0.25\ \Phi_{17} + 0.28125\ \Phi_{16} + 0.375\ \Phi_{15} + 0.5\ \Phi_{14} + 0.5625\ \Phi_{13} - 0.75\ \Phi_{12} - \ \Phi_{11} - 1.125\ \Phi_{10} + 0.75\ \Phi_9 + 1.5\ \Phi_7 + 1.5\ \Phi_4 - 0.0105\ \ \Phi_2 - 0.00658\ \Phi_1$,

$\chi_{18} = 0.6667\ \ \Phi_{20} + 0.22222\ \ \Phi_{19} + 0.33333\ \ \Phi_{18} + 0.4444\ \Phi_{17} + 0.5\ \Phi_{16} + 0.66667\ \Phi_{15} + 0.88889\ \Phi_{14} + \ \Phi_{13} + 1.33333\ \Phi_{12} - 1.77778\ \ \Phi_{11} - 2\ \Phi_{10} - 1.33333\ \Phi_9 - 1.33333\ \Phi_7 + 2\ \Phi_4 - 0.00206\ \ \Phi_2 - 0.00383\ \Phi_1$,

$\chi_{24} = 0.04167\ \Phi_{20} + 0.5556\ \ \ \ \ \Phi_{19} + 0.08333\ \ \ \ \ \Phi_{18} + 0.01111\ \Phi_{17} + 0.125\ \Phi_{16} + 0.66667\ \Phi_{15} + 0.22222\ \Phi_{14} + 0.25\ \Phi_{13} + 0.33333\ \Phi_{12} + 0.44444\ \ \Phi_{11} - 0.5\ \Phi_{10} - 0.66667\ \Phi_9 - 0.5\ \Phi_8 + \Phi_6 + \Phi_4 - 0.00401\Phi_2 - 0.00455\ \Phi_1$,

$\chi_{32} = \ \ 0.09375\ \Phi_{20} + 0.125\ \ \Phi_{19} + 0.1875\ \ \Phi_{18} + 0.25\ \Phi_{17} + 0.28125\ \Phi_{16} + 0.375\ \Phi_{15} + 0.5\ \Phi_{14} + 0.5625\ \Phi_{13} + 0.75\ \Phi_{12} + \Phi_{11} - 1.125\ \Phi_{10} - 1.5\ \Phi_9 - 2.25\ \Phi_8 - 1.5\ \Phi_7 + 1.5\ \Phi_4 + 0.0018\ \Phi_2 - 0.0010801\ \Phi_1$,

$\chi_{36} = \ 0.04167\ \Phi_{20} + 0.5556\ \Phi_{19} + 0.08333\ \Phi_{18} + 0.011111\ \Phi_{17} + 0.125\ \Phi_{16} + 0.66667\ \Phi_{15} + 0.22222\ \Phi_{14} + \ 0.25\ \Phi_{13} + 0.33333\ \Phi_{12} + 0.44444\ \ \Phi_{11} + 0.5\ \Phi_{10} + 0.66667\ \Phi_9 - \Phi_8 - 0.66667\ \Phi_7 + \Phi_4 - 0.00102\ \ \Phi_2 - \ 0.00282\ \Phi_1$,

$\chi_{48} = 0.01042\ \Phi_{20} + 0.01389\ \Phi_{19} + 0.2083\ \Phi_{18} + 0.02778\ \Phi_{17} + 0.03125\ \Phi_{16} + 0.04167\ \Phi_{15} + 0.05556\ \Phi_{14} + 0.625\ \Phi_{13} + 0.08333\Phi_{12} + 0.11111\Phi_{11} + 0.125\ \Phi_{10} + 0.16667\ \Phi_9 - 0.25\Phi_8 - 0.33333\Phi_7 - 0.25\ \Phi_6 + 0.5\ \Phi_5 + 0.5\ \Phi_4 - 0.00728\ \Phi_2 - \ 0.00268\ \Phi_1$,

$\chi_{72} = 0.01042\Phi_{20} + 0.01389\ \Phi_{19} + 0.2083\ \Phi_{18} + 0.02778\ \Phi_{17} + 0.03125\ \Phi_{16} + 0.04167\ \Phi_{15} + 0.05556\ \Phi_{14} + 0.625\ \Phi_{13} + 0.08333\Phi_{12} + 0.11111\Phi_{11} + 0.125\ \Phi_{10} + 0.16667\ \Phi_9 + 0.25\Phi_8 - 0.33333\Phi_7 - 0.5\ \Phi_6 + 0.5\ \Phi_4 - 0.00342\ \Phi_2 - \ 0.00171\ \Phi_1$,

$\chi_{96} = 0.01042\Phi_{20} + 0.01389\ \Phi_{19} + 0.2083\ \Phi_{18} + 0.02778\ \Phi_{17} + 0.03125\ \Phi_{16} + 0.04167\ \Phi_{15} + 0.05556\ \Phi_{14} + 0.625\ \Phi_{13} + 0.08333\Phi_{12} + 0.11111\Phi_{11} + 0.125\ \Phi_{10} + 0.16667\ \Phi_9 + 0.25\Phi_8 - 0.33333\Phi_7 - 0.5\ \Phi_6 + 0.5\ \Phi_4 - 0.005553\ \Phi_2 - \ 0.00171\ \Phi_1$,

$\theta_1 = 27.03448\,\Phi_{26} - 54.06897\,\Phi_{25} + 22.4\,\Phi_{24} - 44.8\,\Phi_{23} - 56\,\Phi_{22} - 56\,\Phi_{21} + 56\,\Phi_4 - 112\,\Phi_3 - 0.81904\,\Phi_2 + 0.12698\,\Phi_1,$

$\theta_2 = 13.51724\,\Phi_{26} + 27.03448\,\Phi_{25} - 39.2\,\Phi_{24} + 22.4\,\Phi_{23} - 28\,\Phi_{21} - 28\,\Phi_4 + 0.40952\,\Phi_2 + 0.19058\,\Phi_1,$

$\theta_5 = -6.75862\,\Phi_{26} + 13.51724\,\Phi_{25} - 19.6\,\Phi_{24} + 14\,\Phi_{22} - 14\,\Phi_{21} + 14\,\Phi_4 - 28\,\Phi_3 + 0.13754\,\Phi_2 + 0.078404\,\Phi_1,$

$\theta_{10} = -3.37931\,\Phi_{26} - 6.75862\,\Phi_{25} + 9.8\,\Phi_{24} - 19.6\,\Phi_{23} + 7\,\Phi_{21} - 7\,\Phi_4 - 0.01451\,\Phi_2 - 0.01143\,\Phi_1,$

$\theta_{29} = -0.13793\,\Phi_{26} - 0.27586\,\Phi_{25} - 0.8\,\Phi_{24} - 1.6\,\Phi_{23} + 2\,\Phi_{22} - 2\,\Phi_{21} + 2\,\Phi_4 - 4\,\Phi_3 + 0.02513\,\Phi_2 + 0.01183\,\Phi_1,$

$\theta_{52} = -0.06897\,\Phi_{26} - 0.13793\,\Phi_{25} - 0.4\,\Phi_{24} - 0.8\,\Phi_{23} + 2\,\Phi_{22} + \Phi_{21} - \Phi_4 - 0.00801\,\Phi_2 - 0.00271\,\Phi_1,$

$\zeta = 0.01042\,\Phi_{20} + 0.01389\,\Phi_{19} + 0.2083\,\Phi_{18} + 0.02778\,\Phi_{17} + 0.03125\,\Phi_{16} + 0.04167\,\Phi_{15} + 0.05556\,\Phi_{14} + 0.625\,\Phi_{13} + 0.08333\Phi_{12} - 0.11111\Phi_{11} + 0.125\,\Phi_{10} + 0.16667\,\Phi_9 + 0.25\Phi_8 - 0.33333\,\Phi_7 + 0.5\,\Phi_6 - \Phi_5 + 0.5\,\Phi_4 + 0.01369\,\Phi_2 + 0.00089\,\Phi_1,$

$\eta = -0.03448\,\Phi_{26} + 0.06897\,\Phi_{25} - 0.2\,\Phi_{24} + 0.4\,\Phi_{23} - 2\,\Phi_{22} + 2\,\Phi_{21} + 0.5\,\Phi_4 - \Phi_3 - 0.0305\,\Phi_2 - 0.00181\Phi_1.$

Therefore $\mathcal{A}(\mathcal{SL}(2,17^2)) = 24137280\chi_1$.

4. Conclusion

In this paper we compute the cyclic subgroup which need that to get the Artin indicator, the rational valued characters of the rational representations and the induced characters for the group $\mathcal{SL}(2,17^2)$.

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Conflict of Interest

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