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Outcome for The Group SL(2,5^7)

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Abstract

The set of all (n×n) non-singular matrices over the field F this set forms a group under the operation of matrix multiplication. This group is called the general linear group of dimension n over the field F, denoted by GL(n, F). The determinant of these matrices is a homomorphism from GL(n, F) into F* and the kernel of this homomorphism was the special linear group and denoted by $\mathcal{SL}$(n, F). Thus $\mathcal{SL}$(n, F) is the subgroup of GL(n, F) which contains all matrices of determinant one. The rational valued characters of the rational representations written as a linear combination of the induced characters for the groups $\mathcal{SL}(2, 5^7)$ discuss in this paper and find the Artin indicator for this group after study the rational valued characters of the rational representations and the induced characters.

Keywords: Artin indicator, induced characters table, rational character table.

1. Introduction

The group of all matrices of determinant 1 is symbolize by $\mathcal{SL}(n,F)$, [1,2], searchers in [3] define the representation of the group.

Authors in [4,5] study the character table of rational representations for the group $\mathcal{SL}(2, p)$, while the authors in [6] study and find the periodical split for the groups PSL(2,31) and PSL(2,37) and the authors in [7] survey and get the calculation for the groups SL(2,U), U = 31 and 37. While the authors in [8] obtain the same results for the $\mathcal{SUT}(2, p)$, where p = 3, 5, 7. Searchers in [10,11] study the New algorithm for number recognition and Symmetric generalized respectively. We apply the same idea in [4-8] to compute the character table of rational representations for the group $\mathcal{SL}(2, 5^7)$. Also we apply the same idea in [9-13] to compute the

Artin indicator for this group. We count all cyclic subgroup, Artin indicator, the rational valued characters of the rational representations and the induced characters for the group $\mathcal{SL}(2,5^7)$ in this work.

2. Basic Concepts

Theorem 2.1: [14-16]

$|\ \mathcal{SL}(2,q^n)\ | = q^n\ (q^{2n} - 1)$.

Definition 2.2: [17-21]

Let H be a cyclic subgroup of a group G, and ϕ be a class function of H. Then

$$\phi\uparrow^G(g) = \frac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\phi(x_i)$$

Definition 2.3: [22-25]

The character induced from the principal character of a cyclic subgroups of G is called Artin character.

Definition 2.4: [26-30]

Let G be a finite group and let χ be any rational valued character on G. The smallest positive number n such that, $n\chi = \sum_c a_c\phi_c$, where $a_c \in Z$ and ϕ_c is Artin character, is called the Artin exponent of G and denoted by A(G).

3- The Outcome

Authors in [4-8] study the character table of rational representations for the group $\mathcal{SL}(2,p)$ we apply that idea and compute the character table of rational representations for the group $\mathcal{SL}(2,5^7)$. Also we apply the same idea in [4,9] to compute the Artin indicator for this group.

The character table of rational representations for the group $\mathcal{SL}(2,5^7)$ is

Table 1. The character table of rational representations for the group $\mathcal{SL}(2,5^7)$

C_g	1	*z*	*c = d*	*zc = zd*	*a*	a^2	a^4	a^{19531}
$\|C_g\|$	1	1	3051757812	3051757812	6103593750	6103593750	6103593750	6103593750
$\|C_G(g)\|$	476837158125000	476837158125000	156250	156250	78124	78124	78124	78124
1_G	1	1	1	1	1	1	1	1
ψ	78125	78125	0	0	1	1	1	1
χ_1	3051601560	-3051601560	39060	-39060	0	4	-4	0
χ_2	762900390	-762900390	9765	9765	1	-1	0	-19530
χ_4	762900390	762900390	9765	9765	-1	0	0	19530
χ_{19531}	156252	-156252	2	-2	0	-4	4	4
θ_1	1959974912	-1959974912	-25088	25088	0	0	0	0
θ_2	979987456	979987456	-12544	-12544	0	0	0	0
θ_3	979987456	-979987456	-12544	12544	0	0	0	0
θ_6	489993728	489993728	-6272	-6272	0	0	0	0
θ_{29}	69999104	-69999104	-896	896	0	0	0	0
θ_{58}	34999552	34999552	-448	-448	0	0	0	0
θ_{87}	34999552	-34999552	-448	448	0	0	0	0
θ_{174}	17499776	17499776	-224	-224	0	0	0	0
θ_{449}	4374944	-4374944	-56	56	0	0	0	0
θ_{898}	2187472	2187472	-28	-28	0	0	0	0
θ_{1347}	2187472	-2187472	-28	28	0	0	0	0
θ_{2694}	1093736	1093736	-14	-14	0	0	0	0
θ_{13021}	156248	-156248	-2	2	0	0	0	0

θ_{26042}	78124	78124	-1	-1	0	0	0	0
ξ	78126	78126	1	1	-2	2	2	-2
η	156248	-156248	-1	1	0	0	0	0

Complete Table 1. The character table of rational representations for the group $\mathcal{SL}(2,5^7)$

C_g	b	b^2	b^3	b^6	b^{29}	b^{58}	b^{87}
$\lvert C_g \rvert$	6103437500	6103437500	6103437500	6103437500	6103437500	6103437500	6103437500
$\lvert C_G(g) \rvert$	78126	78126	78126	78126	78126	78126	78126
1_G	1	1	1	1	1	1	1
ψ	-1	-1	-1	-1	-1	-1	-1
χ_1	0	0	0	0	0	0	0
χ_2	0	0	0	0	0	0	0
χ_3	0	0	0	0	0	0	0
χ_4	0	0	0	0	0	0	0
θ_1	-2	2	4	-4	56	-56	-112
θ_2	1	0	-2	0	-28	0	56
θ_3	2	2	0	0	-56	56	0
θ_4	-1	0	0	0	28	0	0
θ_5	2	-2	-4	4	0	0	224
θ_6	-1	0	2	0	0	0	-112
θ_7	-2	2	0	0	112	-112	-224
θ_8	1	0	0	0	0	0	112
θ	2	-2	-4	4	-56	56	112
θ	-1	0	2	-7	28	56	-56
θ	-2	2	0	-14	56	-56	-56
θ	1	0	-7	7	-28	-28	-28
θ	-2	2	4	-4	-4	-4	-4
θ	1	2	-2	-2	-2	-2	-2
ξ	0	0	0	0	0	0	0
η	4	-4	4	-4	4	-4	4

Complete Table 1. The character table of rational representations for the group $\mathcal{SL}(2,5^7)$

C_g	b^{174}	b^{449}	b^{898}	b^{1347}	b^{2694}	b^{13021}	b^{26042}
$\lvert C_g \rvert$	6103437500	6103437500	6103437500	6103437500	6103437500	6103437500	6103437500
$\lvert C_G(g) \rvert$	78126	78126	78126	78126	78126	78126	78126
1_G	1	1	1	1	1	1	1
ψ	-1	-1	-1	-1	-1	-1	-1
χ_1	0	0	0	0	0	0	0
χ_2	0	0	0	0	0	0	0
χ_3	0	0	0	0	0	0	0
χ_4	0	0	0	0	0	0	0
θ_1	112	896	-896	-1792	1792	-25088	25088
θ_2	0	-448	0	896	-3136	12544	25088
θ_3	0	-896	896	0	-6272	25088	-25088
θ_4	0	448	-1568	-3136	3136	-12544	-12544
θ_5	0	-896	896	1792	-1792	-1792	-1792
θ_6	0	448	896	-896	-896	-896	-896
θ_7	224	896	-896	-896	-896	-896	-896
θ_8	224	-448	-448	-448	-448	-448	-448
θ	-112	-112	-112	-112	-112	-112	-112
θ	-56	-56	-56	-56	-56	-56	-56
θ	-56	-56	-56	-56	-56	-56	-56
θ	-28	-28	-28	-28	-28	-28	-28
θ	-4	-4	-4	-4	-4	-4	-4
θ	-2	-2	-2	-2	-2	-2	-2
ξ	0	0	0	0	0	0	0
η	-4	4	-4	4	-4	4	-4

This group has 22 cyclic subgroups generated by the conjugacy classes of the group. The induced character table for this group is:

Table 2. The induced character table for the group $\mathcal{SL}(2,5^7)$

C_g	**1**	***z***	***c* = *d***	***zc* = *zd***	***a***	a^2	a^4	a^{19531}
$\|C_g\|$	**1**	**1**	**3051757812**	**3051757812**	**6103593750**	**6103593750**	**6103593750**	**6103593750**
$\|C_G(g)\|$	**476837158125000**	**476837158125000**	**156250**	**156250**	**78124**	**78124**	**78124**	**78124**
Φ_1	476837158125000	0	0	0	0	0	0	0
Φ_2	95367431625000	95367431625000	0	0	0	0	0	0
Φ_3	6103515624	0	2	0	0	0	0	0
Φ_4	6103515624	3051757812	3	3	0	0	0	0
Φ_5	6103593750	12207187500	0	0	2	0	0	0
Φ_6	12207187500	24414375000	0	0	0	4	0	0
Φ_7	24414375000	0	0	0	0	0	8	0
Φ_8	119209289531250	238418579062500	0	0	0	0	0	39062
Φ_9	12206875000	24413750000	0	0	0	0	0	0
Φ_{10}	24413750000	0	0	0	0	0	0	0
Φ_{11}	36620625000	73241250000	0	0	0	0	0	0
Φ_{12}	73241250000	0	0	0	0	0	0	0
Φ_{13}	353999375000	707998750000	0	0	0	0	0	0
Φ_{14}	707998750000	0	0	0	0	0	0	0
Φ_{15}	1061998125000	2123996250000	0	0	0	0	0	0
Φ_{16}	2123996250000	0	0	0	0	0	0	0
Φ_{17}	5480886875000	10961773750000	0	0	0	0	0	0
Φ_{18}	10961773750000	0	0	0	0	0	0	0
Φ_{19}	16442660625000	32885321250000	0	0	0	0	0	0
Φ_{20}	32885321250000	0	0	0	0	0	0	0
Φ_{21}	158945719375000	317891438750000	0	0	0	0	0	0
Φ_{22}	317891438750000	0	0	0	0	0	0	0

Complete Table 2. The induced character table for the group $\mathcal{SL}(2,5^7)$

C_g	b	b^2	b^3	b^6	b^{29}	b^{58}	b^{87}
$\|C_g\|$	6103437500	6103437500	6103437500	6103437500	6103437500	6103437500	6103437500
$\|C_G(g)\|$	78126	78126	78126	78126	78126	78126	78126
Φ_1	0	0	0	0	0	0	0
Φ_2	0	0	0	0	0	0	0
Φ_3	0	0	0	0	0	0	0
Φ_4	0	0	0	0	0	0	0
Φ_5	0	0	0	0	0	0	0
Φ_6	0	0	0	0	0	0	0
Φ_7	0	0	0	0	0	0	0
Φ_8	0	0	0	0	0	0	0
Φ_9	2	0	0	0	0	0	0
Φ_{10}	0	2	0	0	0	0	0
Φ_{11}	0	0	6	0	0	0	0
Φ_{12}	0	0	0	12	0	0	0
Φ_{13}	0	0	0	0	58	0	0
Φ_{14}	0	0	0	0	0	116	0
Φ_{15}	0	0	0	0	0	0	174
Φ_{16}	0	0	0	0	0	0	0
Φ_{17}	0	0	0	0	0	0	0
Φ_{18}	0	0	0	0	0	0	0
Φ_{19}	0	0	0	0	0	0	0
Φ_{20}	0	0	0	0	0	0	0
Φ_{21}	0	0	0	0	0	0	0
Φ_{22}	0	0	0	0	0	0	0

Complete Table 2. The induced character table for the group $\mathcal{SL}(2,5^7)$

C_g	b^{174}	b^{449}	b^{898}	b^{1347}	b^{2694}	b^{13021}	b^{26042}
$\lvert C_g \rvert$	6103437500	6103437500	6103437500	6103437500	6103437500	6103437500	6103437500
$\lvert C_G(g) \rvert$	78126	78126	78126	78126	78126	78126	78126
Φ_1	0	0	0	0	0	0	0
Φ_2	0	0	0	0	0	0	0
Φ_3	0	0	0	0	0	0	0
Φ_4	0	0	0	0	0	0	0
Φ_5	0	0	0	0	0	0	0
Φ_6	0	0	0	0	0	0	0
Φ_7	0	0	0	0	0	0	0
Φ_8	0	0	0	0	0	0	0
Φ_9	0	0	0	0	0	0	0
Φ_{10}	0	0	0	0	0	0	0
Φ_{11}	0	0	0	0	0	0	0
Φ_{12}	0	0	0	0	0	0	0
Φ_{13}	0	0	0	0	0	0	0
Φ_{14}	0	0	0	0	0	0	0
Φ_{15}	0	0	0	0	0	0	0
Φ_{16}	348	0	0	0	0	0	0
Φ_{17}	0	898	0	0	0	0	0
Φ_{18}	0	0	1796	0	0	0	0
Φ_{19}	0	0	0	2694	0	0	0
Φ_{20}	0	0	0	0	5388	0	0
Φ_{21}	0	0	0	0	0	26042	0
Φ_{22}	0	0	0	0	0	0	5208

Hence, the rational valued characters in the first tables is written as a linear combination of induced characters in the second table

$$1 = \frac{1}{5208}\Phi_{22} + \frac{1}{26042}\Phi_{21} + \frac{1}{5388}\Phi_{20} + \frac{1}{2694}\Phi_{19} + \frac{1}{1769}\Phi_{18} + \frac{1}{898}\Phi_{17} + \frac{1}{348}\Phi_{16} + \frac{1}{174}\Phi_{15} + \frac{1}{116}\Phi_{14} + \frac{1}{58}\Phi_{13} + \frac{1}{12}\Phi_{12} + \frac{1}{6}\Phi_{11} + \frac{1}{2}\Phi_{10} + \frac{1}{2}\Phi_9 + \frac{1}{39062}\Phi_8 + \frac{1}{8}\Phi_7 + \frac{1}{4}\Phi_6 + \frac{1}{2}\Phi_5 + \frac{1}{3}\Phi_4 - 7.4545390170038\Phi_2 - 2.5350887732113\Phi_1,$$

$$\Psi = -\frac{1}{5208}\Phi_{22} - \frac{1}{26042}\Phi_{21} - \frac{1}{5388}\Phi_{20} - \frac{1}{2694}\Phi_{19} - \frac{1}{1796}\Phi_{18} - \frac{1}{898}\Phi_{17} - \frac{1}{348}\Phi_{16} - \frac{1}{174}\Phi_{15} - \frac{1}{116}\Phi_{14} - \frac{1}{58}\Phi_{13} - \frac{1}{12}\Phi_{12} - \frac{1}{6}\Phi_{11} - \frac{1}{2}\Phi_{10} - \frac{1}{2}\Phi_9 + \frac{1}{39062}\Phi_8 + \frac{1}{8}\Phi_7 + \frac{1}{4}\Phi_6 + \frac{1}{2}\Phi_5 + 8.1974976391737\Phi_2 + 3.2782431357639\Phi_1,$$

$$\chi_1 = -\frac{1}{2}\Phi_7 + \Phi_6 - 13020\Phi_4 + 39060\,\Phi_3 + \frac{39706420735680}{95376431625000}\,\Phi_2 + \frac{238031060800780}{476837158125000}\,\Phi_1,$$

$$\chi_2 = -0.49997\,\Phi_8 - \frac{1}{4}\Phi_6 + \frac{1}{2}\Phi_5 + 3255\Phi_4 + \frac{109269428196208}{95376431625000}\Phi_2 + \frac{149031782399292}{476837158125000}\Phi_1,$$

$$\chi_4 = 0.49997\Phi_8 - \frac{1}{2}\Phi_5 + 3255\,\Phi_4 + \frac{189275531789958}{95376431625000}\,\Phi_2 + \frac{109811334644164}{476837158125000}\,\Phi_1,$$

$\chi_{19531} = 0.00001\Phi_8 + \frac{1}{2}\Phi_7 - \Phi_6 - 0.66667\Phi_4 + 2\Phi_3 + 0.0002523350784\,\Phi_2 + 0.0002613036685\,\Phi_1,$

$\theta_1 = 4.81720\Phi_{22} - 0.96337\Phi_{21} + 0.33259\Phi_{20} - 0.66518\Phi_{19} - 0.49889\Phi_{18} + 0.99777\Phi_{17} + 0.32184\Phi_{16} - 0.64368\Phi_{15} - 0.48276\Phi_{14} + 0.96552\Phi_{13} - \frac{1}{3}\Phi_{12} + 0.66667\Phi_{11} + \Phi_{10} - \Phi_9 + 8362.66667\Phi_4 - 25088\Phi_3 + 3.0562055844106\,\Phi_2 - 2.11532393244476\Phi_1,$

$\theta_2 = 4.81720\Phi_{22} + 0.48168\Phi_{21} - 0.58203\Phi_{20} + 0.33259\Phi_{19} - 0.49889\Phi_{17} + 0.32184\Phi_{15} - 0.48276\Phi_{13} - \frac{1}{3}\Phi_{11} + \frac{1}{2}\Phi_9 - 0.00024\Phi_4 - 1.6663882111459\,\Phi_2 - 3.6712421001151\Phi_1,$

$\theta_3 = -4.81720\Phi_{22} + 0.96337\Phi_{21} - 1.16407\Phi_{20} + 0.49889\Phi_{18} - 0.99777\Phi_{17} + 0.48276\Phi_{14} - 0.96552\Phi_{13} + \Phi_{10} + \Phi_9 + 0.00024\Phi_4 - 12544\Phi_3 - \frac{294651553870827}{95376431625000}\Phi_2 + 2.5131832655351\Phi_1,$

$\theta_6 = -2.40860\Phi_{22} - 0.48168\Phi_{21} + 0.58203\Phi_{20} - 1.16407\Phi_{19} - 0.87305\Phi_{18} + 0.49889\Phi_{17} + 0.48276\Phi_{13} - \frac{1}{2}\Phi_9 - 2090.66667\Phi_4 + \frac{1919847155587088}{95376431625000}\Phi_2 + \frac{2016598610418427}{476837158125000}\Phi_1,$

$\theta_{29} = -0.34409\Phi_{22} - 0.06881\Phi_{21} - 0.33259\Phi_{20} + 0.66518\Phi_{19} + 0.49889\Phi_{18} - 0.99777\Phi_{17} + 1.28736\Phi_{15} + \frac{1}{3}\Phi_{12} - 0.66667\Phi_{11} - \Phi_{10} + \Phi_9 + 298.66667\,\Phi_4 - 896\Phi_3 + \frac{7315381744325}{95376431625000}\Phi_2 + 0.2214571432381\Phi_1,$

$\theta_{58} = -0.17204\Phi_{22} - 0.34406\Phi_{21} - 0.16629\Phi_{20} - 0.33259\Phi_{19} + 0.49889\Phi_{18} + 0.49889\Phi_{17} - 0.64368\Phi_{15} + \frac{1}{3}\Phi_{11} - \frac{1}{2}\Phi_9 - 149.33334\Phi_4 + 1.2231960883785\,\Phi_2 + 0.4830853368404\Phi_1,$

$\theta_{87} = -0.17204\Phi_{22} - 0.34406\Phi_{21} - 0.33259\Phi_{19} - 0.49889\Phi_{18} + 0.99777\Phi_{17} + 0.64368\Phi_{16} - 1.28736\Phi_{15} - 0.96552\Phi_{14} + 1.93103\Phi_{13} + \Phi_{10} - \Phi_9 + 149.33334\Phi_4 - 448\Phi_3 + 1.156679561213\Phi_2 + 0.2558004465064\Phi_1,$

$\theta_{174} = -0.08602\Phi_{22} - 0.01720\Phi_{21} - 0.08315\Phi_{20} - 0.16639\Phi_{19} - 0.24944\Phi_{18} - 0.49889\Phi_{17} + 0.64368\Phi_{16} + 0.64368\Phi_{15} + \frac{1}{2}\Phi_9 - 74.66667\Phi_4 + \frac{15256741967469}{9537641625000}\,\Phi_2 + \frac{54673491989306}{476387158125000}\Phi_1,$

$\theta_{449} = -0.21505\Phi_{22} - 0.00430\Phi_{21} - 0.02079\Phi_{20} - 0.04157\Phi_{19} - 0.06236\Phi_{18} - 0.12472\Phi_{17} - 0.32184\Phi_{16} + 0.64368\Phi_{15} + 2.07143\Phi_{14} - 0.96552\Phi_{13} + \frac{1}{3}\Phi_{12} - 0.66667\Phi_{11} - \Phi_{10} + \Phi_9 + 18.66667\Phi_4 - 56\Phi_3 + \frac{3884984923184}{95376431625000}\Phi_2 + \frac{80933664633553}{476387158125000}\Phi_1,$

$\theta_{898} = -\frac{1}{93}\Phi_{22} - 0.00215\Phi_{21} - 0.01039\Phi_{20} - 0.02079\Phi_{19} - 0.03118\Phi_{18} - 0.06236\Phi_{17} - 0.16092\Phi_{16} - 0.32184\Phi_{15} + 0.48276\Phi_{14} - \frac{1}{2}\Phi_{13} - 0.58333\Phi_{12} + \frac{1}{3}\Phi_{11} - \frac{1}{2}\Phi_9 - 9.33333\Phi_4 + \frac{9514432370548}{45376431625000}\,\Phi_2 + \frac{15253832651739}{476387158125000}\,\Phi_1,$

$\theta_{1347} = -\frac{1}{93}\Phi_{22} - 0.00215\Phi_{21} - 0.01039\Phi_{20} - 0.02079\Phi_{19} - 0.03118\Phi_1 - 0.06236\Phi_{17} - 0.16092\Phi_{16} - 0.32184\Phi_{15} - 0.48276\Phi_{14} + 0.96552\Phi_{13} - 1.16667\Phi_{12} + \Phi_{10} - \Phi_9 + 9.33333\Phi_4 - 28\Phi_3 + \frac{818622717026}{9537641625000}\Phi_2 + \frac{6596680087056}{476387158125000}\,\Phi_1,$

$\theta_{2694} = -\frac{1}{186}\Phi_{22} - 0.00108\Phi_{21} - 0.00529\Phi_{20} - 0.01039\Phi_{19} - 0.01559\Phi_{18} - 0.03118\Phi_{17} - 0.08046\Phi_{16} - 0.16092\Phi_{15} - 0.24138\Phi_{14} - 0.48276\Phi_{13} + 0.58333\ \Phi_{12} - 1.16667\ \Phi_{11} + \frac{1}{2}\ \Phi_9 - 4.66667\ \Phi_4 + \frac{1797860434764}{95376431625000}\ \Phi_2 + \frac{5071174336039}{476387158125000}\Phi_1,$

$\theta_{13021} = -\frac{1}{1302}\Phi_{22} - 0.00015\Phi_{21} - \frac{1}{1347}\Phi_{20} - 0.00148\Phi_{19} - \frac{1}{449}\Phi_{18} - 0.00445\Phi_{17} - \frac{1}{87}\Phi_{16} - 0.02299\Phi_{15} - \frac{1}{29}\Phi_{14} - 0.06897\Phi_{13} - \frac{1}{3}\Phi_{12} + 0.66667\Phi_{11} + \Phi_{10} - \Phi_9 + 0.66667\Phi_4 - 2\Phi_3 + 0.0002294029092\Phi_2 + 0.0014695113016\ \Phi_1,$

$\theta_{26042} = -\frac{1}{2604}\Phi_{22} - \frac{1}{13021}\Phi_{21} - \frac{1}{2694}\Phi_{20} - \frac{1}{1347}\Phi_{19} - \frac{1}{898}\Phi_{18} - \frac{1}{449}\ \Phi_{17} - \frac{1}{174}\Phi_{16} - \frac{1}{87}\ \Phi_{15} - \frac{1}{58}\Phi_{14} - \frac{1}{29}\Phi_{13} - \frac{1}{6}\Phi_{12} - \frac{1}{3}\ \Phi_{11} + \Phi_{10} + \frac{1}{2}\Phi_9 - \frac{1}{3}\Phi_4 + \frac{135292955728}{95376431625000}\Phi_2 + 0.0007616100819\Phi_1,$

$\zeta = -\frac{1}{19531}\Phi_8 + \frac{1}{4}\Phi_7 + \frac{1}{2}\Phi_6 - \frac{1}{2}\Phi_5 + \frac{1}{3}\Phi_4 + \frac{5086419272}{95376431625000}\Phi_2 + \frac{54932148435}{476387158125000}\Phi_1,$

$\eta = -\frac{1}{1302}\Phi_{22} + 0.00015\Phi_{21} - \frac{1}{1347}\Phi_{20} + 0.00149\Phi_{19} - \frac{1}{449}\Phi_{18} + 0.00445\Phi_{17} - \frac{1}{87}\ \Phi_{16} + 0.02299\ \Phi_{15} - \frac{1}{29}\Phi_{14} + 0.06897\Phi_{13} - \frac{1}{3}\Phi_{12} + 0.66667\Phi_{11} - 2\Phi_{10} + 2\Phi_9 + \frac{1}{3}\Phi_4 - \Phi_3 - 0.0075290764184\ \Phi_2 - 0.0012407775638\Phi_1.$

Therefore $\mathcal{A}(\mathcal{SL}(2,5^7)) = 476837158125000\chi_1$.

4. Conclusion

From our work we deduced that $\mathcal{A}(\mathcal{SL}(2,5^n))$ where n is an odd number and $n \geq 3$ equal to the order of the group and the number of the induced characters Φ_s equal to the number of the cyclic subgroups generated by the conjugacy classes of the group.

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Conflict of Interest

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