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Some Results on the Divisible Hyperrings

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Abstract.

A class of hyperrings known as divisible hyperrings will be studied in this paper. It will be presented as each element in this hyperring is a divisible element. Also shows the relationship between the Jacobsen Radical, and the set of invertible elements and gets some results, and linked these results with the divisible hyperring. After going through the concept of divisible hypermodule that presented 2017, later in 2022, the concept of the divisible hyperring will be related to the concept of division hyperring, where each division hyperring is divisible and the converse is achieved under conditions that will be explained in the theorem 3.14. At the end of this paper, it will be clear that the goal of this paper is to study the concept of divisible hyperring by giving some examples, remarks, and results that are related to the concept of divisible hyperrings.

Keywords: divisible hyperring, divisible hypermodule, division hyperring, Jacobson radical.

1.Introduction.

Based on the concept of divisible hypermodule that Sopon Boriboon and Sajee Pianskool introduced in their paper "Baer hypermodule over Krasner hyperring" [1] in 2017, and later by Hashem Bordbar and Irina Cristea in 2022 in the paper "Divisible hypermodule" [2]. This paper presents a study on the concept of divisible hyperring. Before that, would like to re-examine the concept of hyperstructure that was first introduced by the French mathematician Marty in 1937 in the following form. The function $\circledcirc: \mathcal{G} \times \mathcal{G} \longrightarrow P^*(\mathcal{G})$, which is defined as $\circledcirc(\wp, d) = \wp \circledcirc d$, called "hyperoperation" [3], where $P^*(\mathcal{G})$ is the set of all not empty subsets of $\mathcal{G}$. An algebraic hyperstructure $(\mathcal{G},\circledcirc)$ is referred to as a "hypergroupoid" [3]. This hypergroupaid is said to be "semihypergroup" if; $\wp \circledcirc (d \circledcirc q) = (\wp \circledcirc d) \circledcirc q$, for $all\ \wp, d, q \in \mathcal{G}$, i,e: $U_{u \in d \circledcirc q}\, \wp \circledcirc u = U_{v \in \wp \circledcirc d}\, v \circledcirc q$. Also, for any $\mathcal{E} \neq \emptyset\ and\ \mathcal{C} \neq \emptyset$ subsets of $\mathcal{G}$ and $\wp \in \mathcal{G}$, defined $\mathcal{E} \circledcirc \mathcal{C} = U_{e \in \mathcal{E}. c \in \mathcal{C}}\, e \circledcirc c$, $\mathcal{E} \circledcirc \wp = \mathcal{E} \circledcirc \{\wp\}$ and $\wp \circledcirc \mathcal{C} = \{\wp\} \circledcirc \mathcal{C}$. [3], And $(\mathcal{G},\circledcirc)$ is called "quasihypergroup", if $\mathcal{G} \circledcirc x = x \circledcirc \mathcal{G} = \mathcal{G}, for\ all\ x \in \mathcal{G}$, [1]. If the pair $(\mathcal{G},\circledcirc)$ satisfied the conditions of the semihypergroup and the quasihypergroup, then called "hypergroup" [3]. A set

$\emptyset \neq Q$ that contained in $(\mathcal{G},\circledcirc)$ is called "subhypergroup" if it was hypergroup it-self [3]. In 1956 Krasner introduced the concept of hyperring and hypermodule which are known nowadays as Krasner hyperring and Krasner hypermodule respectively. After that many authors introduced many types of hyperring like general hyperring and multiplicative hyperring.

In this paper, the hyperring $\mathcal{R}$ is a Krasner hyperring with unit element 1. R^o is the set of all non-zero divisor elements of a hyperring $\mathcal{R}$, and a left $\mathcal{R}$-hypermodule will be denoted by $\mathcal{M}$.

2. Preliminaries.

Definition 2.1 [4]. The hypergroup $(\mathcal{G},\circledcirc)$ is called canonical if;

1. $\circledcirc$ is an associative hyperoperation, i,e $p \circledcirc (d \circledcirc q) = (p \circledcirc d) \circledcirc q$ for every $p, d, q \in \mathcal{G}$;
2. There exist an element "0" $\in \mathcal{G}$, such that $0 \circledcirc p = p \circledcirc 0 = \{p\}. \forall p \in \mathcal{G}$;
3. There exist a unique $-p \in \mathcal{G}, \forall p \in \mathcal{G}$, such that $0 \in p \circledcirc (-p)$;
4. $p \in d \circledcirc q$ implies $d \in p \circledcirc (-q)$.
5. For all $p, d \in \mathcal{G}$, $p \circledcirc d = d \circledcirc p$.

Definition 2.2 [4] . The hyperstructure $(\mathcal{R} \circledcirc,\odot)$ is said to be Krasner hyperring, if:

1. $(\mathcal{R},\circledcirc)$ is a canonical hypergroup;
2. $(\mathcal{R},\odot)$ is a semigroup, have $\alpha\odot 0 = 0\odot\alpha = 0$, for all $\alpha \in \mathcal{R}$
3. $\alpha\odot(\beta \circledcirc \gamma) = \alpha\odot\beta \circledcirc \alpha\odot\gamma$ and $(\beta \circledcirc \gamma)\odot\alpha_1 = \beta\odot\alpha \circledcirc \gamma\odot\alpha$. For all α, β and $\gamma \in \mathcal{R}$.

A Krasner hyperring is commutative if $(\mathcal{R},\odot)$ is a commutative semigroup.

Definition 2.3 [5]. A subset $\mathcal{A}$ of $\mathcal{R}$ is said to be subhyperring if it satisfy the conditions of a hyperring.

Definition 2.4 [5]. Let $(\mathcal{R},\dotplus,\cdot)$ be a commutative hyperring, I be a hyperideal of $\mathcal{R}$, then the set $\mathcal{R}/I = \{x \dotplus I: x \in \mathcal{R}\}$ is a commutative hyperring under hyperaddition $(x \dotplus I) \dotplus (y \dotplus I) = (x \dotplus y) \dotplus I$ and multiplication $(s \dotplus I)(t \dotplus I) = st \dotplus I$. And it is called a quotient hyperring.

Definition 2.5 [1] **.** Let I be a nonempty subset of a Krasner hyperring $\mathcal{R}$, then I is called a " right (resp. left) hyperideal" if for every p and $d \in$ I, and $\alpha \in \mathcal{R}$:

(1) $p - d \subseteq$ I;
(2) $\alpha\odot p \in$ I (resp. $p\odot\alpha \in$ I)

And called hyperideal if it is right and left hyperideal.

Definition 2.6 [4]. The hyperideal I of $\mathcal{R}$ is maximal hyperideal if every hyperideal J in $\mathcal{R}$ with I $\subsetneq$ J $\subseteq \mathcal{R}$ then J = $\mathcal{R}$.

Definition 2.7 [1]. A canonical hypergroup $(\mathcal{M},\oplus)$ is said to be left hypermodule over a hyperring $(\mathcal{R},\circledcirc,\odot)$ with the unit element "1", if the map $\cdot : \mathcal{R} \times \mathcal{M} \longrightarrow \mathcal{M}$ which is defined as:

$$\cdot\,(s, m) \;\mapsto\; s.m = sm \in \mathcal{M}, \text{ for } s \in \mathcal{R}, m \in \mathcal{M}$$

Satisfies the following conditions, for $k, b \in \mathcal{R}$, and $m` \in \mathcal{M}$:

1. $(k \circledcirc b)m = km \oplus bm$

2. $\hbar(m \oplus m`) = \hbar m \oplus \hbar m`$
3. $(\hbar \odot b)m = \hbar(bm)$
4. $0_{\mathcal{R}} . m = 0_{\mathcal{M}}$, where $0_{\mathcal{R}}$ is a zero of $\mathcal{R}$, $0_{\mathcal{M}}$ the secular identity of $\mathcal{M}$.

In the same way, one can define the right $\mathcal{R}$-hypermodule. The $\mathcal{R}$- hypermodule $\mathcal{M}$ is said to be unitary if $1.m = m$, where 1 is the unit element of $\mathcal{R}$ and $m \in \mathcal{M}$.

Definition 2.8 [6]. If $\mathcal{M}$ is an $\mathcal{R}$-hypermodule, then $\emptyset \neq \mathcal{N} \subseteq \mathcal{M}$ is called a "Subhypermodule" if and only if $\sigma - a \subseteq \mathcal{N}$ and $\sigma r \in \mathcal{N}$, for each $\sigma, a \in \mathcal{N}, r \in \mathcal{R}$.

Definition 2.9 [2] Let ($\mathcal{R}$,⊚, ⊙) be a hyperring. The element $r_1 \in \mathcal{R}$ is named "right (resp. left)-zero divisor" if there is $0 \neq r_2 \in \mathcal{R}$ such that $r_1 . r_2$ (resp. $r_2 . r_1$) = {0}. And called zero-divisor if it was right and left zero-divisor.

Definition 2.10 [2] Let $\mathcal{M}$ be an $\mathcal{R}$-hypermodule.The element $\mu \in \mathcal{M}$ is called a "divisible element" if for each non-zero divisor $\varsigma \in \mathcal{R}$ there is $\in \mathcal{M}$, such that. $\mu = \varsigma\eta$. If every element in an $\mathcal{R}$-hypermodule $\mathcal{M}$ is a divisible, then $\mathcal{M}$ is named a "divisible hypermodule"

Definition 2.11 [4]. The Jacobson radical of a hyperring $\mathcal{R}$ is the intersection of all maximal-hyperideals and it is denoted by J($\mathcal{R}$).

Remark 2.12 [7]. J($\mathcal{R}$) is a hyperideal in $\mathcal{R}$

Notations:

- $U_{left}(\mathcal{R}) = \{r \in \mathcal{R} \mid \exists \acute{r} \in \mathcal{R}\ such\ that\ \acute{r}.r = 1\}$
- $U_{right}(\mathcal{R}) = \{r \in \mathcal{R} \mid \exists \acute{r} \in \mathcal{R}\ such\ that\ r.\acute{r} = 1\}$
- $U(\mathcal{R}) = \{r \in \mathcal{R} \mid \exists \acute{r} \in \mathcal{R}\ such\ that\ r.\acute{r} = \acute{r}.r = 1\}$

In the following proposition, Davvaz B and Salasi A in [4] proved the part (1 ⟶ 2). Here we add another condition and give the following proposition.

Proposition 2.13. For an element $p \in \mathcal{R}$ the following statements are equivalent

(1) $p \in$ J($\mathcal{R}$)

(2) for any $d \in \mathcal{R}$, 1-$pd \subseteq U(\mathcal{R})$

(3) for any $d, q \in \mathcal{R}$, 1-$pdq \subseteq U(\mathcal{R})$.

Proof. (1⟷ 2 proved in (4, Prop. 2.14)).

Now, will prove (1⟷3). To prove that, for $p \in$ J($\mathcal{R}$), and for all $d, q \in \mathcal{R}$, the set 1- $pdq \subseteq U(\mathcal{R})$. Suppose that $\exists d_o, q_o \in \mathcal{R}$ such that $s \notin U(\mathcal{R})$, for some $s \in$ 1 - $(d_o q_o)\ p$. But $s \notin U(\mathcal{R})$ this lead to $\mathcal{R}s \neq \mathcal{R}$ then there is a maximal hyperideal I such that $\mathcal{R}s \subseteq I$ (4, Prop.2.12), since $s \in$ 1 - $(d_o q_o)\ p$, one obtain that 1∈ $s + (d_o q_o)\ p \subseteq I +$ J($\mathcal{R}$) $\subseteq I$ (by Definition 2.1) that is 1∈ I and this contradiction.

Conversely, for any $d, q \in \mathcal{R}$, let 1-$pdq \subseteq U(\mathcal{R})$. To prove that $p \in$ J($\mathcal{R}$), assume $p \notin$ J($\mathcal{R}$), so ∃ I is a maximal hyperideal of $\mathcal{R}$ such that $p \notin I$. Thus $<I, p> \neq I$, and so $<I, p> = \mathcal{R}$. Since 1

$\in \mathcal{R}$, then there is $a \in I$, and $d,q \in \mathcal{R}$ such that $1 \in a + p(dq)$. Hence $a \in 1 - p(dq) \subseteq U(\mathcal{R})$, this implies that $1 \in I$, and this contradiction, therefore $p \in J(\mathcal{R})$.■

3. Main results

The concept of a divisible hyperring will be discussed in this section.

Definition 3.1. The family of all divisible elements of an $\mathcal{R}$-hypermodule $\mathcal{M}$ are defined as

$$d(\mathcal{M}) = \{ y \in \mathcal{M} \mid \text{for each } r \in R^O, \text{ there is } x \in \mathcal{M}, \text{ such that } y = rx\}.$$

Remark 3.2. $d(\mathcal{M})$ is a divisible hypergroup of $(\mathcal{M},+)$.

Proof. Let $m_1, m_2, m_3 \in d(\mathcal{M})$. The associative of "+" is an obvious. Now, to prove $m \circ d(\mathcal{M}) = d(\mathcal{M}) \circ m = d(\mathcal{M})$, let $x \in m \circ d(\mathcal{M})$. It is mean $x \in m \circ y$; $y \in d(\mathcal{M})$. For each $r \in R^O$ there is $z \in \mathcal{M}$ such that $y = rz$; $m \in d(\mathcal{M})$ *mean that* $m = rw, w \in \mathcal{M}$. Follows $x \in (rw) \circ (rz)$, lead to $x \in r(w \circ z) \in d(\mathcal{M})$. In the same way $d(\mathcal{M}) \circ m = d(\mathcal{M})$ can be proved.

Proposition 3.3. If the hyperstructure $(\mathcal{R},\circledcirc,\odot)$ is commutative hyperring then;

1. $d(\mathcal{M})$ is divisible $\mathcal{R}$-Subhypermodule of an $\mathcal{R}$-hypermodule $\mathcal{M}$
2. $d(\mathcal{M}/d(\mathcal{M})) = 0$.

Proof. 1. Let $m, n \in d(\mathcal{M})$, For any $r \in R^O$, $m = rm'$ and $n = rn'$. So $m - n = rm' - rn' = r(m' - n') \in d(\mathcal{M})$. Now, if $y \in d(\mathcal{M})$, $0 \neq a \in \mathcal{R}$ and for any $r \in R^O$, $\exists x \in \mathcal{M}$ such that, $y = rx$. Therefore, $ay = a(rx) = (ar)x = (ra)x = r(ax)$. This implies that $ay \in d(\mathcal{M})$.

2. If $y + d(\mathcal{M}) \in d(\mathcal{M}/d(\mathcal{M}))$ for $y \in d(\mathcal{M})$. Then $\forall r \in R^O, \exists$ $x+d(\mathcal{M}) \in \mathcal{M}/d(\mathcal{M})$ such that, $y+d(\mathcal{M}) = r(x+d(\mathcal{M}))$. Thus $y-rx \in d(\mathcal{M})$. This implies that $\exists x' \in \mathcal{M}$ such that, $(y-rx) = rx'$. It follows that $y=r(x+x')$ and $y \in d(\mathcal{M})$ or equivalently $y + d(\mathcal{M}) = d(\mathcal{M})$, thus $d(\mathcal{M}/d(\mathcal{M})) = 0$.

Remark 3.4. The set $d(\mathcal{R}) = \{a \in \mathcal{R} \mid \text{for all } r \in R^O, \text{ there is } b \in \mathcal{R} \text{ such that. } a=rb\}$.

Remark 3.5. The set $d(\mathcal{R})$ is a right hyperideal of $\mathcal{R}$. Indeed, for any y and x in $d(\mathcal{R})$, y =rs and x=ra, for all s and a belong to $\mathcal{R}$, $y-x \subseteq d(\mathcal{R})$. Also for any $r \in R^o$ there is $s \in \mathcal{R}$ such that, y=rs, thus yt = (rs)t = r(st), this implies $yt \in d(\mathcal{R})$, $t \in \mathcal{R}$.

Definition 3.6 [3] **.** A hyperring $(\mathcal{R},\circledcirc,\odot)$ is a division hyperring if $(\mathcal{R}\backslash\{0\},\odot)$ is a hypergroup with unit element 1.

Proposition 3.7. If $\mathcal{R}$ is a division hyperring, then $d(\mathcal{R}) = \mathcal{R}$.

Proof. It is clear that $d(\mathcal{R}) \subseteq \mathcal{R}$. Now, for any $a \in \mathcal{R}$ and any $r \in R^O = \mathcal{R}^*$, $\exists$ $b = r^{-1}a \in \mathcal{R}$ such that a = rb, therefore $a \in d(\mathcal{R})$.

Proposition 3.8. Let $\mathcal{R}$ be a hyperring that satisfies the property $d(\mathcal{R}) \cap R^O \neq \emptyset$, then:

$d(\mathcal{R}) = \cap \{\hat{R} \mid \hat{R}$ is a left hyperideal of hyperring $\mathcal{R}$ such that. $\hat{R} \cap R^O \neq \emptyset\}$.

Proof. It's clear that $\cap\{\hat{R} \mid \hat{R}$ is a left hyperideal in a hyperring $\mathcal{R}$, such that $\hat{R} \cap R^O \neq \emptyset\} \subseteq d(\mathcal{R})$. Now to prove the converse inclusion, let $y \in d(\mathcal{R})$ and $\hat{R}$ be the left hyperideal of a hyperring $\mathcal{R}$ such that, $\hat{R} \cap R^O \neq \emptyset$, let $y_o \in \hat{R} \cap R^O$, then for any $y \in d(\mathcal{R})$, there is $s \in \mathcal{R}$ such that, $y = y_o s \in \hat{R}$ this gives $d(\mathcal{R}) \subseteq \hat{R}$. Since $\hat{R}$ is an arbitrary hyperideal, so $y \in \cap\{\hat{R}\}$.

Proposition 3.9. If $\mathcal{R}$ is commutative hyperring and $R^O \cap J(\mathcal{R}) \neq \emptyset$, then 1+d($\mathcal{R}$) $\subseteq U(\mathcal{R})$.

Proof. Let $r_o \in R^O \cap J(\mathcal{R})$ and for all $r \in$ d($\mathcal{R}$), $\exists a \in \mathcal{R}$ such that, $r = r_o a$. By (Prop.2.13-3), 1+$r = 1 + r_o a = 1 - 1 \cdot r_o(-a) \in U(\mathcal{R})$ implies $1 + d(\mathcal{R}) \subseteq U(\mathcal{R})$.

Proposition 3.10. If $\mathcal{R}$ is a hyperring satisfy $R^O = R*$, then either d($\mathcal{R}$)=0 or d($\mathcal{R}$)=$\mathcal{R}$.

Proof. If d($\mathcal{R}$) $\neq 0$, then for any y in d($\mathcal{R}$)\{0}, have $y^2 \neq 0$ and $\exists\, x \in \mathcal{R}$ such that $y = y^2 x$, thus $y(1 - yx) = 0$, this implies $yx = 1$ so,

$$y \in U_{right}(\mathcal{R}) \quad \dashrightarrow \quad (1)$$

Also, $1 - yx = 0$ leads to $y - yxy = 0$ coming after $y(1 - xy) = 0$. But

$R^O = R*$, then $xy = 1$, thus

$$y \in U_{left}(\mathcal{R}) \quad \dashrightarrow \quad (2)$$

Now by (1) and (2) get $y \in U(\mathcal{R})$. Since $y \in U(\mathcal{R}) \cap d(\mathcal{R})$ *implies* $d(\mathcal{R}) = \mathcal{R}$.

Definition 3.11. The hyperring $\mathcal{R}$ is called "divisible hyperring" if any element belongs to $\mathcal{R}$ is a divisible element.

The following corollary comes straightaway from definition 3.11 and remark 3.4

Corollary 3.12. $\mathcal{R}$ is divisible hyperring if and only if d($\mathcal{R}$)=$\mathcal{R}$.

Corollary 3.13. Every division hyperring is divisible hyperring

Next, will refer to the class $\breve{R}$ of $\mathcal{R}$ having the properties:

- $d(\mathcal{R}) \neq 0$
- If $\{I_j\}_{j\in J}$ is a set of all maximal right hyperideals of a hyperring $\mathcal{R}$, then have $d(\mathcal{R}) \cap I_j = 0$, or $d(R) \cap I_j \cap R^O \neq \emptyset, \forall j \in J$.

Theorem 3.14. A hyperring $\mathcal{R}$ is a division if and only if

a) $\mathcal{R}$ is divisible hyperring
b) $\mathcal{R} \in \breve{R}$
c) $R^0 = U(\mathcal{R})$

Proof. Let I_j be a maximal right hyperideal of a hyperring $\mathcal{R}$, by (a), $\mathcal{R}$ is divisible. This implies that d($\mathcal{R}$)=$\mathcal{R}$. The condition (b) gives I_j=0 or $I_j \cap R^O \neq \emptyset$ for maximal right hyperideal I_j of $\mathcal{R}$. Finally, by (c), either I_j = 0 or $I_j = \mathcal{R}$, but $I_j \neq \mathcal{R}$, so $I_j = 0$.

For the other side, since $\mathcal{R}$ is a division then by Corollary 3.13, $\mathcal{R}$ is divisible hyperring. Now to prove (b), since d($\mathcal{R}$)=$\mathcal{R}$ therefore $d(\mathcal{R}) \neq 0$, and if $\{I_j\}_{j\in J}$ is a set of all maximal right hyperideals of a hyperring $\mathcal{R}$, then $d(R) \cap I_j \cap R^O \neq \emptyset$. Finally, since $\mathcal{R}$ is a division then every element is unite and so $\mathcal{R}$ has no zero divisor hence $R^0 = U(\mathcal{R})$, then (c) holds. ■

Theorem 3.15. A divisible hyperring can be written as $\mathcal{R}$=x$\mathcal{R}$ for any x $\in R^O$.

Proof. It is enough to show that $\mathcal{R} \subseteq$ x$\mathcal{R}$. Since $\mathcal{R}$ is divisible hyperring, then for any $a \in \mathcal{R}$ there is $b \in \mathcal{R}$ such that $a =$ xb and so $a \in$ x$\mathcal{R}$, thus $\mathcal{R} \subseteq$ x$\mathcal{R}$.

As the definition of the direct sum of two hyperideals in [8] and the direct sum of subhypermodules in [1], we will define the direct sum of two subhyperrings as follows;

Definition 3.16. The direct sum of two subhyperrings $\mathcal{S}$ *and* $\mathcal{T}$, is denoted by $\mathcal{S}\oplus\mathcal{T}$ such that for each element $k \in \mathcal{S}\oplus\mathcal{T}$, there is unique elements $s \in \mathcal{S}, t \in \mathcal{T}$, $k = s + t$.

Theorem 3.17. If $\mathcal{S}$ and $\mathcal{T}$ be subhyperrings of a hyperring $\mathcal{R}$, and $\mathcal{S}, \mathcal{T}$ are divisible, then $\mathcal{S}\oplus\mathcal{T}$ is divisible.

Proof. Let $k \in \mathcal{S}\oplus\mathcal{T}$ and $r \in R^O$, $\exists s \in \mathcal{S}$ and $t \in \mathcal{T}$ such that $k = s + t$. Now, $\mathcal{T}$ *and* $\mathcal{S}$ are divisible, so $\mathcal{S} = x\mathcal{S}$ and $\mathcal{T} = x\mathcal{T}$, for $x \in R^O$. Hence $s = xs_1$ and $t = xt_1$ for some $s_1 \in \mathcal{S}$ *and* $t_1 \in \mathcal{T}$. Thus $k = s + t = xs_1 + xt_1 = x(s_1 + t_1)$. If $u \in s_1 + t_1 \subseteq \mathcal{S} + \mathcal{T}$. Then, $k = xu$.

Definition 3.18 [2]. The function ℓ from the hyperring ($\mathcal{R}$ ⊚,⊙) with $1_{\mathcal{R}}$ into the hyperring $(\mathcal{S}, \dot{+}', \cdot')$, with $1_{\mathcal{S}}$, is a hyperring homomorphism if for each $k, b \in \mathcal{R}$,

1. $\ell(k$ ⊚ $b) = \ell(k) \dot{+}' \ell(b)$
2. $\ell(k$ ⊙ $b) = \ell(k) \cdot' \ell(b)$
3. $\ell(1_{\mathcal{R}}) = 1_{\mathcal{S}}$.

Remark 3.19. A function ℓ is named as surjective $\mathcal{R}$-homomorphism if Im(ℓ) = $\mathcal{S}$.

Proposition 3.20. Let $\mathcal{S}$ be a subhyperring of a divisible hyperring $\mathcal{R}$, then the quotient hyperring $\frac{\mathcal{R}}{\mathcal{S}}$ is a divisible hyperring

Proof. Let $a + \mathcal{S} \in d(\frac{\mathcal{R}}{\mathcal{S}})$, so $\exists a' \in \mathcal{R}$ such that $a = ra'$ for $r \in R^O$. Thus

$a + \mathcal{S} = ra' + \mathcal{S} = r(a' + \mathcal{S})$, which implies $\frac{\mathcal{R}}{\mathcal{S}}$ is divisible hyperring.

Corollary 3.21. Let f be a surjective R-homomorphism where $\mathcal{R}$ and $\mathcal{S}$ are hyperrings. If $\mathcal{R}$ is a divisible hyperring, then so is $\mathcal{S}$.

Examples 3.22. [2]. The hyperoperation "⊚" and the multiplication " ⊙" are defined by the following in tables 1 and 2 on $\mathcal{R}$ = {0, 1, p, d}.

Table 1: additive hyperoperayion

⊚	0	1	p	d
0	{0}	1	p	d
1	1	$\mathcal{R}$	$\{1.p.d\}$	$\{1,p,d\}$
p	p	$\{1,p,d\}$	$\mathcal{R}$	$\{1,p,d\}$
d	d	$\{1,p,d\}$	$\{1,p,d\}$	$\mathcal{R}$

Table 2: multiplication

$\odot$	0	1	p	d
0	0	0	0	0
1	0	1	p	d
p	0	p	d	1
d	0	d	1	p

Then $\mathcal{R}$ is a hyperring and every nonzero element is a divisible, thus $\mathcal{R}$ is divisible hyperring

Conclusion.

In this research, we discussed the concept of divisible hyperring. Some of the properties of divisible hyperring were studied. Clarify some concepts related to this concept.

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