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Some Results on Nano Perfect Mappings

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Abstract

The structure of this paper includes introduction the definition of the nano topological space, which was defined by M. L. Thivagar, who defined the lower approximation of G and the upper approximation of G, as well as defined the boundary region of G and some other important definitions that were mentioned in this paper with giving some theories on this subject. Some examples of defining nano perfect mappings are presented along with some basic theories. Also, some basic definitions were presented that form the focus of this paper, including the definition of nano pseudometrizable space, the definition of nano compactly generated space, and the definition of completely nano para-compact. In this paper, we presented images of nano perfect mappings with some definitions and important evidence related to them, then we presented inverse images of nano perfect mappings with related theories.

Keywords: Nano topological space,nano continuous mappings,nano compact space,nano perfect mappings.

1. Introduction

[1] introduced the concept of nano topological spaces with respect to a subset G of a universe U, which known as terms of approximations and boundary region of a subset of an universe using an equivalence relation on it and also known as nano-closed-sets, nano-interior and nano-closure. [2] introduced fibrewise IJ-Perfect bitopological spaces and [3] introduced weak and strong forms of ω- perfect mappings. [4] introduced Rα-compactness on bitopological spaces. [5] introduced on cohomology groups of four-dimensional nilpotent associative algebras. [6,7,8,9,10] introduce some basic concepts that helped us build this work. [11,12 ,13,14] introduce some types of mappings in bitopological spaces and Soft Simply Compact Spaces and other topics related to the

mentioned sources. In this paper, we introduce a nano perfect mappings and several related theorems.

Definition 1.1 : [15] Let U be a non empty finite set of objects is said to be the universe and R be an equivalence relation on U is called indiscernibility relation. Then U is divided into disjoint equivalence classes. Components belonging to the same equivalence class are called the indiscernible with one another. The spouse (U, R) is called the approximation-space. Let $G \subseteq U$. Then,

a) The lower approximation of G with reference to R is the set of all objects which can be categorized as G with reference to R and is symboly by $Lo_R(G)$. That is, $Lo_R(G) = \cup\{R(G): R(G) \subseteq G, g \in U\}$ where R(G) indicate the equivalence class determined by $g \in U$.

b) The upper approximation of G with reference to R is the set of all objects which can be possibly classified as G with reference to R and is symboly by $Up_R(G)$. That is, $Up_R(G) = \cup\{R(G): R(G) \cap G \neq \emptyset, g \in U\}$.

c) The boundary region of G with reference to R is the set of all objects which can be classified neither as G nor as not /G with reference to R and is symboly by $Bo_R(G)$. $Bo_R(G) = Up_R(G) / Lo_R(G)$.

Property 1.2 : [16] If (U, R) is an approximation space and $G,H \subseteq U$, then

a) $Lo_R(G) \subseteq G \subseteq Up_R(G)$.
b) $Lo_R(\emptyset) = Up_R(\emptyset) = \emptyset$.
c) $Lo_R(U) = Up_R(U) = U$.
d) $Up_R(G \cup H) = Up_R(G) \cup Up_R(H)$.
e) $Up_R(G \cap H) \subseteq Up_R(G) \cap Up_R(H)$.
f) $Lo_R(G \cup H) \supseteq Lo_R(G) \cup Lo_R(H)$.
g) $Lo_R(G \cap H) = Lo_R(G) \cap Lo_R(H)$.
h) $Lo_R(G) \subseteq Lo_R(H)$ and $Up_R(G) \subseteq Up_R(H)$ whenever $G \subseteq H$.
i) $Up_R(G^c) = [Lo_R(G)]^C$ and $Lo_R(G^c) = [Up_R(G)]^c$.
j) $Up_R[Up_R(G)] = Lo_R[Up_R(G)] = Up_R(G)$.
k) $Lo_R[Lo_R(G)] = Up_R[Lo_R(G)] = Lo_R(G)$.

Definition 1.3 : [16] If $\tau_R(G)$ is the nano-top-sp.on U with respect to G, then the set B = { U , $Lo_R(G)$, $Bo_R(G)$ } is the nano-basis for $\tau_R(G)$.

Definition 1.4 : [17] Let U be the universe, R be an equivalence relation on U and $\tau_R(G) = \{U, \emptyset, Lo_R(G), Up_R(G), Bo_R(G)\}$, where $G \subseteq U$. Then $\tau_R(G)$ it achieves the following axioms:

a) U and $\emptyset \in \tau_R(G)$.
b) The union of the components of any subset of $\tau_R(G)$ is in $\tau_R(G)$.
c) The intersection of the components of any finite subset of $\tau_R(G)$ is in $\tau_R(G)$.

Therefore, $\tau_R(G)$ is a topology on U called the nano topology (denoted by nano-top.) on U with reference to G. We call (U, $\tau_R(G)$) as the nano topological space (denoted by nano-top-sp.) The components of $\tau_R(G)$ are said to be nano open sets (denoted by nano-ope-sets). The complement of a nano-ope-set is called a nano closed set(denoted by nano-clos-set).

Definition 1.5 : [18] Let (U, $\tau_R(G)$) and (V, $\tau_{R`}(H)$) be nano-top-sp. Then a mapping (dented by map.) f : (U, $\tau_R(G)$) ⟶ (V, $\tau_{R`}(H)$) is nano continuous (denoted by nano-cont.) on U if the inverse image of each nano-ope-set in V is nano-ope. in U.

Definition 1.6 : [18] A function f : (U, $\tau_R(G)$) ⟶ (V, $\tau_{R`}(H)$) is a nano-clos. mapping if the image of each nano-clos-set in U is nano-clos. in V.

Definition 1.7 : [13] Let (G, $\tau_R(G)$) be a nano-top-sp.and A ⊆ G.The nano-closure of a set A is symboly by NCI(A) is the intersection of all nano-clos-sets containing A or all clos-super-sets of A; i.e.,the smallest clos-set containing A.

Definition 1.8 : [18] A function f:(U,$\tau_R(G)$)⟶(V,$\tau_{R`}(H)$) is called a nano homeomorphism if :

a) f is 1-1 and onto.
b) f is nano-cont.
c) f is nano-ope.

Definition 1.9 : [19] A set {A_i: i ∈ I } of nano-ope-sets in a nano-top-sp. ((U, $\tau_R(G)$) is said to be nano-ope-cover of a subset B of U if B ⊂ {A_i: i ∈ I } holds.

Definition 1.10 : [19] A space (U, $\tau_{R'}(G)$) is called nano Hausdroff space (denoted by nano-Hausd-sp.) if whenever g and h are distinct points of (U,$\tau_R(G)$) , find disjoint nano-ope-sets A and B such as g ∈ A and h ∈ B.

Definition 1.11 : [19] Let (U,$\tau_R(G)$) be a nano-top-sp. and W be a subspace of G. We said to be space W is nano-comp-sp. iff each open-cover from G cover W has a finite-sub-cover.

Definition 1.12 : [18] Let (G, $\tau_R(G)$) be anano-top-sp., and let $\{U_i : U_i \subset G\}_{i\in\Lambda}$ be a open-cover of G is called locally finite-cover if for all point g ∈ G, find a nbd U_g of g ⊂ U_g such as it intersects only limited many components of the cover, hence such as $U_g \cap U_i \neq \emptyset$, for only a finite number of i ∈ Λ.

Definition 1.13 : [20] A top-sp.(G, τ) and the open-cover $\{U_i: U_i \subset G\}_{i\in I}$ is called refinement if a set of open subsets $\{V_j: V_j \subset G\}_{i\in I}$ which is still an open-cover in itself and such as for each j ∈ J, find an i ∈ I with $V_j \subset U_i$.

Definition 1.14 : [21] A nano-top-sp.(G, $\tau_R(G)$) is called nano regular space (denoted by nano-regu-sp.) iff for all nano closed set (denoted by nano-clos-set) F ⊂ G, and all point g ∉ F find nano open sets(denoted by nano-ope-sets) U and V such as g ∈ U, F ⊂ V and U⋂V = ∅.

Definition 1.15 : [21] A anao-top-sp.(U,$\tau_R(G)$) is called nano-normal-space (denoted by nano-norm-sp.) if for any pair of disjoint nano-clos-sets A and B of U find nano-ope-sets V and W of U such as A ⊆ V and B ⊆ W.

Definition 1.16 : A nano-top-sp.(G, $\tau_R(G)$) is said to be nano paracompact space (denoted by nano-para-comp-sp.) if it satisfies for each nano-ope-cover has a locally finite-nano-open refinement.

Definition 1.17 : A nano-top-sp.(G, $\tau_R(G)$) is said to be nano metrizable-space if there is a metric d : G×G ⟶ [0,∞], such as the nano topology induced by d is τ, and is nano homeomorphic to a metric-space.

2. Images of Nano Perfect Mappings.

We study the Images of nano perfect mappings and some theories related to the topic.

Definition 2.1 : A map f : G ⟶ H is said to be nano perfect (dented by nano-perf.) if it is nano continuous (denoted by nano-cont.), nano closed (denoted by nano-clos.), and for each H ∈ h, f^{-1}(h) is nano compact (denoted by nano-comp.).

We show by the next examples:

Example 2.2 : Let f : U ⟶ V be a map such as U= {a, b, c, d} with U/R = {{a},{b},{c, d}}. Let $G = \{a, c\} \subset U$. Then $\tau_R(G)$ ={U, ∅ ,{a},{a,c,d},{c,d}}. Let V= {x, y, z, w} with V/R' = {{x},{y}, {z,w}} and H= {x,z} ⊂ V . Then $\tau_{R'}(H)$ = {V, ∅ ,{x},{x,z,w},{z,w}}. Such as f(a) = x , f(b) = y , f(c) = z , f(d) = w . Then f is nano-perf-map.

Example 2.3 : Let U ={a,b,c,d} with U/R = {{b},{c},{a,d}}. Let G = {a,b} ⊂ U . Then $\tau_R(G)$ ={ U, ∅ ,{b},{a,b,d},{a,d}}. Let V = {1, 2, 3, 4} with V/R' = {{1},{2},{3},{4}} and H = {1, 3, 4} ⊂ V . Then $\tau_{R'}(H)$ = {V, ∅ ,{1, 3, 4,}}. Define f : U ⟶ V such as f(a) = 1 , f(b) = 2, f(c) = 3, f(4) = 4 then f is not nano-perf-map.

Theorem 2.4 : Let f : G ⟶ H be a nano-perf-map. of G onto H . If the weight of G is infinite, then w(H) ≤ w(G).

Proof: Let $\underset{\smile}{V}$ be a nano-base for the nano-topology of G such as $|\underset{\smile}{V}|$ = w(G) and let $\underset{\smile}{W}$ consist of every sets of the form $\bigcup_{B\in\underset{\smile}{B}} B$, where $\underset{\smile}{B}$ is a finite subset of $\underset{\smile}{V}$. If W ∈ $\underset{\sim}{W}$, let T_W = H\f(G\W) and let T be the set of nano-ope-sets of H of the form T_W for some W in $\underset{\sim}{W}$. So w(G) is infinite, $|\underset{\sim}{W}|$ = w(G) , so that the cardinality of the set $\underset{\smile}{T}$ of nano-ope-sets of H does not exceed w(G). Furthermore $\underset{\smile}{T}$ is a nano-base for the nano-topology of H. For let J be an nano-ope-set of H and let h be a point of J. Since f^{-1} (h) ⊂ f^{-1}(J) and f^{-1}(h) is nano-comp., find W of $\underset{\smile}{W}$ such as f^{-1}(h) ⊂ W ⊂ f^{-1}(J). Then h ∈ T_W ⊂ J. It follows that w(H) ≤ w(G).

Theorem 2.5 : The class of nano-T_i-spaces (denoted by nano-T_i-sp.) is invariant under nano-perf-maps for i=1,2,3,4.

Proof: Let f : G ⟶ H be a nano-perf. onto map:

(a) If G is a nano-T_1-sp. then all point {g} is a nano-clos-set. So let h ∈ H, then find g ∈ G such as f(g) = h. But f is nano-clos. and {g} is nano-clos.; hence {h} is nano-clos. in H.

(b) let G be a nano-T_2-sp. Let h_1, h_2 be a pair of distinct points of H. The inverse images $f^{-1}(h_1)$ and $f^{-1}(h_2)$ are nano-comp.and disjoint subsets of G, so find nano-ope-sets U, V ⊂ G such as $f^{-1}(h_1)$ ⊂ U, $f^{-1}(h_2)$ ⊂ V and U⋂V=∅. So f is nano-clos., the sets A=H\f(G\U) and B=H\f(G\V) are nano-ope. in H such as $f^{-1}(h_1)$ ⊂ f^{-1}(A) ⊂ U and $f^{-1}(h_2)$ ⊂ f^{-1}(B) ⊂ V, A and B are nano-nbds of h_1 and h_2 respectively. Moreover A⋂ B =[H\f(G\U)] ⋂ [H\f(G\V)]

=H\[f(G\U) ⋃ f(G\V)]

=H\f[G\(U⋂V)]

=H\f(G)

=∅

So H is a nano-T_2-sp.

(c) Let G be a nano-T_3-sp. Let h ∈ H and F be a nano-clos-set in H such as h ∈ H so f^{-1}(h) is nano-comp-sub-set of G and f^{-1}(F) is a nano-clos-sub-set of G and f^{-1}(h) ⋂f^{-1}(F) = ∅. So find U, V nano-ope-sub-sets of G such as f^{-1}(h) ⊂ U and f^{-1}(F) ⊂ V with U⋂V = ∅. So f is nano-clos., find A, B nano-ope-sub-sets of H such as f^{-1}(h) ⊂ f^{-1}(A) ⊂ U and f^{-1}(F) ⊂ f^{-1}(B) ⊂ V. Clearly, h ∈ A, F ⊂ B and A⋂B = ∅ so H is a nano-T_3-sp.

(d) Let G be a nano-T_4-sp. Let F_1, F_2 be distinct nano-clos. subsets of H. So $f^{-1}(F_1)$ and f^{-1} (F_2) are nano-clos-sub-sets of G, find U, V nano-ope-sub-sets of G such as$f^{-1}(F_1)$ ⊂ U, $f^{-1}(F_2)$ ⊂ V. Since f is nano-clos., find nano-ope-sub-sets A, B of H such as$f^{-1}(F_1)$ ⊂ f^{-1}(A) ⊂ U and $f^{-1}(F_2)$ ⊂ f^{-1}(B) ⊂ V. Clearly, F_1 ⊂ A, F_2 ⊂ B and A, B are disjoint, hence H is a nano-T_4-sp.

Theorem 2.6 : Nano-compactness is invariant under nano-perf-maps.

Proof: Let f : G ⟶ H be a nano-perf-map from a nano-comp-sp. G onto a space H. Let $\underset{\smile}{U}$ = { U_α : α ∈ Λ } be any nano-ope-cover of H. then $f^{-1}(\underset{\smile}{U})$ is an nano-ope-cover of G. So find a finite-sub-cover $\{f^{-1}(U_{\alpha_i})\}_{i=1}^{n}$ for G. Then $\{U_{\alpha_i}\}_{i=1}^{n}$ is a finite-sub-cover of H, hence H is nano-comp.

Lemma 2.7 : For each nano-comp-sub-sp. A of a locally nano-comp-sp. G and each nano-ope-set V that contains A find an nano-ope-set U such as A ⊂ U ⊂ NCl(U) ⊂ V and NCl(U) is nano-comp.

Proof: For each g ∈ A take a nano-nbd V_g of the point g such as NCl(V_g) ⊂ V and a nano-nbd W_g of g such as NCl(W_g) is nano-comp. The set NCl(U_g), where $U_g = V_g \cap W_g$ is nano-comp. because it is a nano-clos-sub-set of the nano-comp-sp. NCl(W_g). So find a finite set $\{g_1, g_2, \dots, g_k\} \subset A$ such as $A \subset U = U_{g_1} \cup U_{g_2} \cup \dots \cup U_{g_k}$. The set $NCl(U) = NCl(U_{g_1}) \cup NCl(U_{g_2}) \cup \dots \cup NCl(U_{g_k})$ is nano-comp. and clearly we have $NCl(U) \subset NCl(V_{g_1}) \cup NCl(V_{g_2}) \cup \dots \cup NCl(V_{g_k}) \subset V$.

Theorem 2.8 : Locally nano-compactness is invariant under nano-perf-maps.

Proof: Let f : G ⟶ H be a nano-perf-map of a locally nano-comp-sp. G onto a space H. By Lemma 2.7, for each h ∈ H find an nano-ope-set V ⊂ G such as f^{-1}(h) ⊂ V and NCl(V) is nano-comp. The set W = H\f(G\V) is a nbd of h and since W = H\f(G\V) ⊂ H\(G\NCl(V)) ⊂ f(NCl(V)), the nano-closure NCl(W) is a nano-comp-sub-sp.of H.

Theorem 2.9 : A nano-cont. map f : G ⟶ H is nano-clos. if and only if for each point h ∈ H and each nano-ope-set U ⊂ G such as f^{-1}(h) ⊂ U, then find an nano-ope-set V of H such as h ∈ V and f^{-1}(V) ⊂ U.

Proof : (⇒) Let f be a nano-cont. map f : G⟶ H is nano-clos. and g ∈ H. Let U be an nano-ope-set in G, such as f^{-1}(h) ⊂ U, so G \ U is nano-clos. in G. This implies f(G \ U) is nano-clos. in H. Let V = H \ f(G \ U), then V is an nano-ope-sub-set of H such as h ∈ V and f^{-1}(V) = G \ f^{-1}(f(G \ U)) ⊂ U.

(⇐) Assume that the assumption holds. Let F be a nano-clos-sub-set of G and h ∈ H \ f(F) and each nano-ope-set U ⊂ G such as f^{-1}(h) ⊂ U find an nano-ope-set V of H such as h ∈ V and f^{-1}(V) ⊂ U. It is easy to show that V ⊂ H \ f(F). Hence H \ f(F) is nano-ope. in H and this implies f(F) is nano-clos. in H.

Lemma 2.10 : If f : G ⟶ H is a nano-perf. onto map and $\{A_\alpha\}_{\alpha\in\Lambda}$ is a locally finite family of subsets of G, then $\{f(A_\alpha)\}_{\alpha\in\Lambda}$ is a locally finite family of subsets of H.

Proof: Let h be a point of H. If g ∈ f^{-1}(h) , then find an nano-ope-set V_g of G such as g ∈ V_g and $\Lambda_g = \{ \alpha \in \Lambda : V_g \cap A_\alpha \neq \emptyset \}$ is finite. Since f^{-1}(h) is nano-comp., find a finite subset B of f^{-1}(h) such as $f^{-1}(h) \subset \bigcup_{g\in B} V_g$. By Theorem 2.9, find an nano-ope-set W of H such as h ∈ W and $f^{-1}(W) \subset \bigcup_{g\in B} V_g$. If $W \cap f(A_\alpha) \neq \emptyset$, then $f^{-1}(W) \cap A_\alpha \neq \emptyset$, so that $\alpha \in M = \bigcup_{g\in B}. \Lambda_g$ Since M is a finite subset of Λ, it follows that the family $\{f(A_\alpha)\}_{\alpha\in\Lambda}$ is locally finite.

Theorem 2.11 : For each point-finite nano-ope-cover $\{U_s\}_{s\in S}$ of a nano-normal space G find an nano-open-cover $\{V_s\}_{s\in S}$ of G such as $NCl(V_s) \subset U_s$ for each s ∈ S.

Theorem 2.12 : If each nano-ope-cover of a top-sp. G has a locally finite nano-clos. refinement, then each nano-ope-cover of G has also a locally finite nano-ope. refinement.

Theorem 2.13 : Nano-para-compactness is invariant under nano-perf-maps.

Proof: Let f : G ⟶ H be a nano-perf-map from a nano-para-comp-sp. G onto a nano-top-sp. H. Let $\underset{\sim}{V}$ be any nano-ope-cover of the space H. So G is nano-para-comp., $\{f^{-1}(V)\}_{V\in\underset{\sim}{V}}$ has a locally finite nano-ope. improvement $\{U_s\}_{s\in S}$. Therefore G is norm-sp., therefore by Theorem 2.11,find a nano-clos-cover $\underset{\sim}{F} = \{F_s\}_{s\in S}$ of G such as $F_s \subset U_s$ for each s ∈ S. Since $\underset{\sim}{F}$ is locally finite and nano-clos., it follows that $\{f(F_s)\}_{s\in S}$ is a locally finite nano-clos.improvement of the cover $\underset{\sim}{V}$. Hence by Theorem 2.12, $\underset{\sim}{V}$ has an nano-ope. locally finite improvement and hence H is nano-para-comp.

Definition 2.14 : [15] Let $\underset{\sim}{A} = \{A_s\}_{s\in S}$ be a cover of a set G; the star of a set M ⊂ G with reference to $\underset{\sim}{A}$ is the set St(M, A) = ⋃ $\{A_s : M\cap A_s \neq \emptyset\}$. The star of one~point set {G} with reference to a cover $\underset{\sim}{A}$ is said to be the star of the point g with reference to $\underset{\sim}{A}$ and is symboly by St(g, $\underset{\sim}{A}$).

Definition 2.15 : A nano-pseudometrizable-space is a pair (G,d) where G is a set and d is a function d:G×G⟶[0,∞], is said to be nano-pseudometrizable, that satisfies the following properties for all g,h,i in G:

a) d(g,h) = 0 if and only if g=h.
b) d(g,h) = d(h,g).
c) d(g,h) ≤ d(g,i) + d(i,h).

Lemma 2.16 : A space G is nano-pseudometrizable if and only if find a sequence $\left\{\underset{\sim n}{F}\right\}_{n\in N}$ of locally finite nano-clos-covering of G such as for all point g of G and all nano-ope-set U such as g ∈ U find an integer n such as St(g, $\underset{\sim n}{F}$) ⊂ U.

Theorem 2.17 : If f : G ⟶ H is a nano-perf. onto map. and G is nano-pseudometizable-space, then H is a nano-pseudometizable-space.

Proof: Choose a nano-pseudometric d on G which induces the nano-topology of G. For all positive integer n , let $\underset{\sim n}{E}$ be a locally finite nano-clos. improvement of the covering $\left\{B_{1/_{2n}}(G)\right\}_{g\in G}$ of G. Then by Lemma 2.10, $\underset{\sim n}{F}$ = { f(E) : E ∈ $\underset{\sim n}{E}$} is a locally finite nano-clos-covering of H for all positive integer n. We shall complete the proof by showing that the sequence $\left\{\underset{\sim n}{F}\right\}_{n\in N}$ satisfies the condition of Lemma 2.16. Let J be an nano-ope-set of H and let h be a point of J. Then f^{-1} (h) is nano-comp. and f^{-1}(h) ⊂ f^{-1}(J). The nano-cont. real value function ∅ know in f^{-1}(h), by putting ∅(g) =d(g, G\\$f^{-1}$(J)) if g ∈ f^{-1}(h), is bounded and attains its bounds. Since ∅(g) >0 if g ∈ f^{-1}(h), it follows that find a point g_o ∈ f^{-1}(h) such as ∅ (g) ≥ $\emptyset(g_0) > 0$ if g ∈ f^{-1}(h). Thus find a positive integer n such as d(g, G\f(J)) ≥ $1/_2{}^n$ if g ∈ f^{-1}(h). suppose that F $\underset{\sim n+1}{F}$ and h ∈ F. Then F = f(E), where E ∈ $\underset{\sim n+1}{E}$ and E⋂f^{-1}(h) ≠ ∅. Let g be a point of E⋂f^{-1}(h) and suppose find g_2 ∈ G such as g_2 ∈ E⋂(G\\$f^{-1}$(J)). Since there g_1 ∈ G such that E ⊂ $B_{1/_2}$n+1(g_1), it follows that d(g, g_2) ≥ d(g, g_1) + d(g_1, g_2) < $1/_2{}^n$.

Which is a inconsistency. Hence E ⊂ f^{-1}(J) so that F ⊂ J. Thus St(h, $\underset{\sim n}{F}$) ⊂ J.

Corollary 2.18 : If f : G ⟶ H is a nano-perf. onto map and G is a nano-metrizable-space, then H is a nano-metrizable-space.

Defintion 2.19 : In a nano-top-sp. G is a nano-compactly-generated-sp. (denoted by nano k-space) in which a subset is nano-clos. if it is intersection with any nano-comp-sub-set is clos.

Lemma 2.20 : A nano-Hausd-sp. G is a nano k-space if and only if for all A ⊂ G, the set A is nano-clos. in G on condition the intersection of A with any nano-comp-sub-sp. F of the space G is nano-clos. in F.

Theorem 2.21 : If there exists a nano-perf-map f : G ⟶ H of a nano k-sp. G onto a top-sp. H, then H is a nano k-sp.

Proof: Let A ⊂ H such as A⋂F is nano-clos. in F for each nano-comp-sub-set F of H. It suffices to show that A is nano-clos. in H. Let K be a nano-comp-sub-set of G, so f(K) is nano-comp. in H, hence A⋂f(K) is nano-clos. in f(K), so f^{-1}(A)⋂K is nano-clos. in K. But G is a nano k-sp., so f^{-1}(A) is nano-clos. in G. Thus ff^{-1}(A) = A is nano-clos. in H, hence H is a nano k-sp.

3. Inverse Images of Nana Perfect Mappings.

We study the Inverse Images of nano perfect mappings and several related theorems.

Theorem 3.1 : Nano-regularity is an inverse of nano-perf-maps.

Proof: Let f : G ⟶ H be a nano-perf-map onto a nano-regu-sp. H. Take a point g ∈ G and a nano-clos-set F ⊂ G such as g ∉ F. The set F⋂f^{-1}(f(g)) is nano-comp. and does not contain g, since G is nano-Hausd-sp. Find disjoint nano-ope-sets U_1, V_1 ⊂ G such as g ∈ U_1 and F⋂f^{-1}(f(g)) ⊂ V_1. The set f(F\\$V_1$) is nano-clos. in H and does not contain f(g), hence by nano-regularity of H, find disjoint nano-ope-sets U_2 , V_2 ⊂ H such as f(g) ∈ U_2 and f(F\\$V_1$) ⊂ V_2. The sets U = U_1⋂f^{-1}(U_2) and V = V_1⋂f^{-1}(V_2) are nano-ope. in G, disjoint, and contain the point g and the set F alternately.

Remark 3.2 : The remaining axioms of separation are not inverse invariants of nano-perf-maps. Also complete nano-regularity is not inverse invariant of nano-perf-maps.

See [8] Exercis 3.10.c (c) , problem 3.12.20 (e) , Example 5.1.40, and the book.

Theorem 3.3 : If f : G ⟶ H is a nano-perf-map, then for each nano-comp-sub-sp. R ⊂ H the inverse image f^{-1}(R) is nano-comp.

Proof : Let $\underset{\sim}{U}$ = { U_α : α ∈ Λ } be a family of nano-ope-sets of G such as f^{-1}(R) ⊂ $\cup_{\alpha\in\Lambda} U_\alpha$. If h ∈R, then find a finite subset M(h) of Λ such as f^{-1}(h) ⊂ $\cup_{\alpha\in M(h)} U_\alpha$. Since f is a nano-clos.map, by Theorem 2.6 find an nano-ope-set V_h of H such as h ∈ V_h and $f^{-1}(V_h)$ ⊂ $\cup_{\alpha\in M(h)} U_\alpha$. Since R is nano-comp., find a finite subset B of R such as R ⊂ $\cup_{h\in B} V_h$. Hence f^{-1}(R) ⊂ $\cup_{h\in B} f^{-1}(V_h)$ ⊂ $\cup_{h\in B}\cup_{\alpha\in M(h)} U_\alpha$. Thus if M = $\cup_{h\in B}$ M(h), then M is a finite subset of Λ and f^{-1}(R) ⊂ $\cup_{\alpha\in M} U_\alpha$. Thus f^{-1}(R) is nano-comp.

Theorem 3.4 : Nano-compactness and local nano-compactness are inverse invariants of nano-perf-maps.

Proof: The inverse invariance of nano-compactness follows directly from Theorem 3.3 . If f : G ⟶ H is a nano-perf-map and H is a locally nano-comp-sp., then for each g ∈ G, find a nbd U ⊂ G such as f(U) is contained in a nano-comp-sub-sp. I of the space H. Since f(NCl(U)) ⊂ NCl(f(U)) ⊂ R, the set NCl(U) ⊂ f^{-1}(R) is nano-comp.

Theorem 3.5 : Nano-para-compactness is an inverse invariant of nano-perf-maps.

Proof: Let f : G ⟶ H be a nano-perf-map. onto a nano-para-comp-sp.H. Consider an nano-ope-cover $\{U_s\}_{s\in S}$ of the nano-space G and for each h ∈ H choose a finite set S(h) ⊂ S such as f^{-1}(h) ⊂ $\bigcup_{s\in S(h)} U_s$. Since f is nano-clos., by Theorem 2.9, find a nbd V_h of the point h such as f^{-1}(h) ⊂ $f^{-1}(V_h)$ ⊂ $\bigcup_{s\in S(h)} U_s$. The nano-ope-cover $\{V_h\}_{h\in H}$ of H has an nano-ope. locally finite improvement $\{W_t\}_{t\in T}$. The family $\{f^{-1}(W_t)\}_{t\in T}$ is an nano-ope. locally finite-cover of G and for each t ∈ T find a h_t ∈ H satisfying $f^{-1}(W_t)$ ⊂ $f^{-1}(V_{h_t})$ ⊂ $\bigcup_{s\in S(h_t)} U_s$. So the family { $f^{-1}(W_t)$ ⋂ U_s : t ∈ T and s ∈ S(h_t) } is an nano-ope. locally finite improvement of $\{U_s\}_{s\in S}$.

Theorem 3.6 : The Cartesian product G×H of a nano-para-comp-sp. G and a nano-comp-sp. H is nano-para-comp.

Proof: Consider the projection p : G×H ⟶ H. Since H is nano-comp., then p is nano-perf. Since H is nano-para-comp., therefore by Theorem 3.5, G×H is nano-para-comp.

Definition 3.7 : [22] A family $\{B_\alpha: \alpha \in \Gamma\}$ of subsets of a space G is said to be a weak improvement of covering $\underset{\sim}{U}$ of G if find a subset $\Gamma^`$ of Γ such as $\{B_\alpha: \alpha \in \Gamma^`\}$ is a covering of G and a improvement of $\underset{\sim}{U}$.

Definition 3.8 : A nano-top-sp. G is called completely nano-para-comp. if all nano-ope-covering of G has a weak improvement of the form $\{ V_\alpha: \alpha \in \Lambda \}$ where $\Lambda = \bigcup_{n\in N}\Lambda_n$ and $\{ V_\alpha: \alpha \in \Lambda_n \}$ is a star finite nano-ope-covering of G for all n.

Theorem 3.9 : Let f : G ⟶ H be a nano-perf. onto map. If H is a completely nano-para-comp-sp., then G is a completely nano-para-comp-sp.

Proof: Let $\underset{\sim}{U} = \{ U_\alpha: \alpha \in \Lambda \}$ be an nano-ope-cover of G. If h ∈ H, find a finite subset Λ(h) of Λ such as $f^{-1}(h) \subset \bigcup_{\alpha\in\Lambda(h)}U_\alpha$. So f is a nano-clos. map, find an nano-ope-set W_h of H such as $h \in W_h$ and $f^{-1}(W_h) \subset \bigcup_{\alpha\in\Lambda(h)}U_\alpha$, and $\underset{\sim}{W} = \{ W_h: h \in H \}$ is an nano-ope-covering of H. So H is completely nano-para-comp., find a family a $\{V_\beta: \beta \in \Gamma\}$ of nano-ope-sets of H such that :

(a) $\Gamma = \bigcup_{n\in N}\Gamma(n)$ and the family $\{ V_\beta: \beta \in \Gamma(n) \}$ is a star finite-covering of H for all n, and

(b) Find a subsets $\Gamma^`$ of Γ such as $\{V_\beta: \beta \in \Gamma^`\}$ is a improvement of $\underset{\smile}{W}$.

If $\beta \in \Gamma^`$, choose h(β) in H such as $V_\beta \subset W_{h(\beta)}$. Let O be the family of nano-ope-sets of G which consists of each sets $F^{-1}(V_\beta)\cap U_\alpha$, where $\beta \in \Gamma^`$ and $\alpha \in \Lambda(h(\beta))$, together with each sets $f^{-1}(V_\beta)$, where $\beta \in \Gamma\backslash\Gamma^`$. Then O consists of countably many star finite nano-ope-coverings of G and O is a weak improvement of $\underset{\smile}{U}$ since the subfamily $\{ f^{-1}(V_\beta)\cap U_\alpha: \alpha \in \Lambda(h(\beta)), \beta \in \Gamma^`\}$ is a covering of G and a improvement of U. Then if H is completely nano-para-comp., then G is completely nano-para-comp.

Definition 3.10 : Let G be a nano-Hausd-sp.and let C be a family consisting of those subsets of G that have nano-clos.interssections with all nano-comp-sub-sps.of G. The set G with the nano-topology generated by the family C of nano-clos-subsets will be symboly by kG is nano-ope. if and only if its intersection with any nano-comp-sub-sp. R of the space G is nano-ope.in R. The nano-topology of kG is finer than the nano-topology of G.

Proposition 3.11 : The nano-spaces kG and G have the same nano-comp-sub-sps.

Proposition 3.12 : Define for each nano-cont. map f: G ⟶ H where G and H nano-Hausd-sps., the map kf assigning to g ∈ kG the point f(g) ∈ kH . And Ɠg: kG ⟶ G by Ɠg(g) = g. Then:

(a) Ɠg: kG ⟶ G is nano-cont.

(b) kf: kG ⟶ kH is nano-cont.

(c) kf satisfies the equality foƓg = ƓHokf.

Theorem 3.13 : For a nano-cont. map f : G ⟶ H of a nano-Hausd-sp. G to a nano k-sp. H the statement are equivalent :

(a) The map. f is nano-perf.

(b) For each nano-comp-sub-sp. R ⊂ H the restriction $f_I : f^{-1}(R) \longrightarrow R$ is nano-perf.

(c) For each nano-comp-sub-sp. R ⊂ H the inverse image $f^{-1}(R)$ is nano-comp.

Proposition 3.14 : If the composition gof of nano-cont. map f : G ⟶ H and j : H ⟶ R, where H is a nano-Hausd-sp., is nano-perf., then the map $j\big|_{f(g)}$ and f are nano-perf .

Proof : (a) For each point r ∈ R the fiber $(j\,|_{f(G)})^{-1}$ (r) =f(G) ∩ j^{-1}(r) = f($(jof)^{-1}$(r)) is nano-comp., because the fiber $(jof)^{-1}$(r) is nano-comp. Any nano-clos. subset of f(G) is of the form A∩f(G), where A is nano-clos. in H. As the inverse image f^{-1}(A) is nano-clos. in G and jof is a nano-clos. map, the set $(j\,|_{f(G)})$ (A∩f(G)) = j(A∩f(G)) = jof (f^{-1}(A)), is nano-clos. in R, we have $j\,|_{f(G)}$ is a nano-clos. map and thus the map $j\,|_{f(G)}$ is a nano-clos. map and thus the map $j\,|_{f(G)}$ is nano-perf.

(b) For each point h ∈ H the fiber f^{-1}(h) = [$(jof)^{-1}$(j(h)]∩ f^{-1}(h) is nano-comp. To conclude the proof it suffices to show that f is nano-clos. For each nano-clos-set F ⊂ G the map $(jof)\,|_F$ is nano-perf., so that by the first section of us proof the restriction $j\,|_{f(F)}$ is nano-perf. ; since the latter map ability be continuously extended through NCl(f(F)), it follows by Lemma 2.11 that f(F) = NCl(f(F)), and thus f is a nano-clos. map.

Theorem 3.15 : If find a nano-perf- map f : G ⟶ H of G onto a nano k-sp. H, then G is a nano k-sp.

Proof: Let us consider spaces kG and map kf : kG ⟶ kH.Since H is a nano k-sp., we have H = kH and kf = foꞡg. From Theorem 3.13, it follows that kf is a nano-perf-map and this along with Proposition 3.14, implies that ꞡg is nano-perf. As ꞡg is a one-to-one map, it is a nano-homeomorphism; therefore G is a nano k-sp.

4. Conclusion

The main purpose of this paper is to present and study images of nano perfect mappings, as well as to present invers images of nano perfect mappings.

References

1. Thivagar, M. L.; Richard, C. On Nano Continuity. *Mathematical Theory and Modelling***. 2013**, *3*, *7*, 32–37.
2. Yousif, Y. Y.; Hussain, L. A. Fibrewise IJ-Perfect Bitopological Spaces. Journal of Physics: Conference Series, IOP Publishing, Ibn Al-Haitham 1^stInternational Scientific Conference. December 2017, **2018**, Volume *1003*, 13-14, , doi :10.1088/1742-6596/1003/1/012063, pp. 1-12.
3. Ashaea, G. S. ;Yousif, Y.Y. Weak and Strong Forms of ω-Perfect Mappings. *Iraqi Journal of Science*. **2020**, *Special Issue*, 45-55.
4. Majeed, R. N. Rα-Compactness on Bitopological Spases. *Journal of Al-Nahrain University Science*. **2010**, Vol.*13 (3)*, September,134-137.
5. Mohammed, N. F.; Gasim, S. G.; Mohammed, A. S. On Cohomology Groups of Four-Dimensional Nilpotent Associative Algebras. Baghdad Science *Journal. University of Baghdad – Collage of Science for Woman.* Published Online First: September, DOI: http://dx.doi.org/10.21123/bsj.2022.19.2.029, **2021**, *Vol. 19, No.2*, PP.329-335, 202.
6. Nasef, A. A.; Aggour, A. I.; Aggour, S. M. On some classes of nearly open sets in nano topological spaces. *Journal of the Egyptian Mathematical Society*. **2016**, *24*,585-589.
7. Vadivel, A. ; Padma, A. ; Saraswathi, M.; Saravanakumar, G. Nano Continuous Mappings via Nano $\mathcal{M}$ Open Sets. Applications and Applied Mathematics : *An International Journal (AAM). December* **2021**, ISSN : 1932-9466, *Issue 2* , *Vol.16*, 1099-1119,.
8. Engelking, R. General Topology. Heldermann. **1989**, Berlin.

9. Mohammed, N. F. On Semi α-connected Subspaces. *Baghdad Science Journal*. University of Baghdad – Collage of Science for Woman. **2010**, *Vol. 7, No.1*, PP.174-179.
10. Majeed, R. N. Compactness on Bitopological Spases. Journal of Al-Qadisiyah for Computer Science and Mathematics. **2011**, *Vol. 3(1),* pp. 1-7.
11. Thivagar, M. L. ; Richard, C. On nano continuity. *International Journal of Mathematics and statistics invention*. **2013**, 1(1), 31–37.
12. Parimala, M. ; Indirani, C. ; Jafari, S. ON NANO b – OPEN SETS IN NANO TOPOLOGICAL SPACES. *Jordan Journal of Mathematics and Statistics (JJMS) 9(3)*. **2016** pp 173-184.
13. Rajasekaran, I. ; Nethaji, O. Simple Forms of Nano Open Sets in an Ideal Nano Topological Spaces. *Journal of New Theory*. **2018**, ISSN: 2149-1402, Number: 24, pages: 35-43.
14. El-Atik, A. A. ; Hassan, H. Z. Some nano topological structures via ideals and graphs. *Journal of the Egyptian Mathematical Society*, **2020**.
15. Krishnaprakash, S. ; Ramesh, R. ; Suresh, R. Nano Compactness and Nano Connectedness in Nano Topological space. *International Journal of Pure and Applied Mathematics*. **2018**, *Vol. 119, no.13*, pp. 107 –115.
16. Bourbaki, N . *General Topology*; Part I. *Addison Wesley. Reading. Mass*, **1996**.
17. Upadhya, A. C. ; Mamata, M. k. ON NANO GP-REGULAR AND NANO GP-NORMAL SPACES. *The International journal of analytical and experimental modal analysis. Karnataka. India.* July, **2020**, ISSN NO:0886-9367, p:3770-3775.
18. Pears, A. R. Dimension Theorem of General Spaces. Cambridge University Press. *London. New York, Melbourne*, **1975**.
19. Ashaea, G. S. ; Yousif, Y. Y. Some Types of Mappings in Bitopological Spaces. *Baghdad Science Journal.* University of Baghdad – Collage of Science for Woman, Published Online First: December 2020, , DOI: http://dx.doi.org/10.21123/bsj.2020.18.1.0149, **2021,** *18,1*, 149-155.
20. Noori, S. ; Yousif, Y. Y. Soft Simply Compact Spaces. Iraqi Journal of Science. University of Baghdad, The 1st Conferences of Mathematics Held at Mustansiriyah University – College of Basic Education in 5-6 Feb. **2020**, DOI:10.24996/ijs.2020.*SI.1.14*, *Special Issue*,108-113.
21. Esmaeel, R.B. ; Saeed, S. Nano ag-open set. Journal of Physics: Conference Seriesthis link is disabled. **2021**, *1897(1)*, 012031.
22. Esmaeel,R. B.; Jassam, A. A. On Nano ḟ-pre-g-Open Set. *Ibn Al-Haitham Journal for Pure and Applied Science.* **2020**, *33(3),*1-12.