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# A Proposed Wavelet and Forecasting Wind Speed with Application

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## Abstract

Time series analysis is the statistical approach used to analyze a series of data. Time series is the most popular statistical method for forecasting, which is widely used in several statistical and economic applications. The wavelet transform is a powerful mathematical technique that converts an analyzed signal into a time-frequency representation. The wavelet transform method provides signal information in both the time domain and frequency domain. The aims of this study are to propose a wavelet function by derivation of a quotient from two different Fibonacci coefficient polynomials, as well as a comparison between ARIMA and wavelet-ARIMA. The time series data for daily wind speed is used for this study. From the obtained results, the proposed wavelet-ARIMA is the most appropriate wavelet for wind speed. As compared to wavelets the proposed wavelet is the most appropriate wavelet for wind speed forecasting, it gives us less value of MAE and RMSE.

**Keywords:** ARIMA, Fibonacci Coefficient Polynomials, Proposed Wavelet**,** Time Series, Wavelet Transform.

## 1. Introduction

Wavelet analysis is an approach for resolving difficult issues in mathematics, physics, and engineering. Wavelet transform is the improved form of Fourier transform since the Fourier transform is a helpful tool for studding the component of a stationary data. However, it is incapable to analyse non-stationary signals, whereas wavelet transform allows for the analysis of non-stationary signal components [1]. Morlet, Arens, Fourgeau, Giard, and Grossman [2] were the first to use the name wavelet in their work in the early 1980s. Jean Morlet and Alex Grossman introduced the concept of wavelets in 1982. The mother wavelet is a family of functions formed by translating and dilation of a single function. Wavelets are mathematical functions that divide data into distinct frequency components and analyse each component with a resolution that matches to its scale [3, 4]. Wavelets are commonly used in time series analysis [5, 6]. In (2013) Ramesh and Pachiyappan [7] proposed a hybrid predicts approach consisting of wavelet transforms and ANN to predict the wind speed. Study shows that the proposed method improves the predict accuracy of wind speed and justifies the application's ability to predict short-term wind speed. In (2013) Ramana et al. [8] introduced a wavelet neural networks, that is the mixture of wavelets analysis and neural networks for rainfall forecast Darjeeling station, India. Used discrete wavelet transforms. In (2013) Al Wadi et al. [9] used a maximal overlaps discrete wavelet transform (MODWT) to improve the accuracy of time series data forecasting. The findings demonstrate that combining MODWT with the ARIMA model improves predicting accuracy. In (2014) Chandra et al. [10] used Morlet and Mexican hat wavelets for wind speed predicting based on adaptive wavelet neural networks. The results of Morlet wavelet wind forecasting were the most accurate of all of these methods. In (2015) Kumar et al. [11] proposed a new technique for forecasting time series data based on ARIMA model and wavelet transform. As a result, the results demonstrated that combining ARIMA with wavelet is effective and efficient. In (2015) Ji, Cai, and Zhang [12] identified a wavelet transform in combination with a neuron fuzzy network to prediction the wind power interval. The efficacy of the neuron fuzzy network structure is demonstrated by the creation of prediction intervals based on wind power data. In (2016) Lamben et al. [13] proposed a new wavelet function named golden wavelet generated by fourth derivation of a quotient from two different Fibonacci coefficient polynomials distinct. The golden wavelet was applied the cardiac arrhythmia classification in ECG signals. The obtained results using the golden wavelet are better than these using other wavelet functions. In (2016) Sang et al. [14] discussed four main problems in wavelet transform: inconsistent usage of continuous and discrete wavelet techniques, mother wavelet selection, temporal scale selection, and uncertain evaluation in wavelet-aided predicting. Finally, wavelet models have the potential to improve hydrological data set forecasting. In (2017) Saini and Ahja [15] used propagation trained artificial neural network and wavelet transform to Predict wind speed. The results of this study show the low value of root mean square of error and mean absolute of error, suggest that the proposed scheme can be used effectively to predict wind speed for a short period, i.e. one hour ahead of the forecast. In (2018), Bunrit et al. [16] we applied multiresolution analysis of wavelet transform for commodities prices time series forecasting. The variances of errors from the proposed method of data sets are much less than the direct use of the actual series data for forecasting. Gossler et al. (2018) [17] published a comparison of Golden and Mexican hat wavelets, finding that Golden hat wavelets have double that much vanishing moment than Mexican hat wavelets. In (2021) Gossler et al. [18] comparison of Gaussian and golden wavelets were performed. The derivative of specified basis functions generates these wavelets.

The aims of this study are to propose a wavelet function by sixth derivation of a quotient from two different Fibonacci coefficient polynomials, as well as to compare ARIMA and wavelet-ARIMA to determine the best-fitted model.

In this study, introduction, autoregressive integrated moving average model, wavelet transform and some of mother wavelets are introduced. Next, the proposed wavelet is introduced and then the results of these models are compared. Finally, conclusions are given.

## 2. Methodology

### 2.1 Autoregressive integrated Moving Average Model (ARIMA)

The (ARIMA) is an appropriate model for the stationary time-series data. It is denoted by $ARIMA(p,d,q)$, $p$ is the autoregressive order, $q$ is the moving average order, and $d$ is the differencing order. Box and Jenkins generalized this model in 1970 [19].

The general mathematical ARIMA model for non-stationary time series can be defined as [4]:

$$\phi(B)(1-B)^d x_t = \mu + \theta(B)a_t \qquad (1)$$

Where:

$t$: Indexes time.

$B$ : The backshift operator.

$\phi(B) = 1 - \phi_1 B - \phi_2 B^2 \dots . -\phi_p B^p$ is the p-order autoregressive operator.

$\theta(B) = 1 - \theta_1 B - \theta_2 B^2 \dots . -\theta_q B^q$ is the q-order moving average operator.

$a_t$ : Error term at time t.

$$a_t = NID(0, \sigma_a^2) \qquad (2)$$

Identification, parameter estimate, diagnostic checking, and forecasting are the four stages of the model building process.

### 2.2 Wavelet Transform

A wave is typically characterized as a time-varying oscillating function, such as a sinusoid. The term wavelet refers to a small oscillation that decays quickly. The wavelets transform is first introduced for transient continuous signal time - frequency domain analysis, and subsequently expanded to the concept for multi-resolution wavelet transform utilizing filtering approximations [20].

A signal is represented by a wavelet transform in the form of specific short time intervals [21- 23]:

$$\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}}\psi\left(\frac{t-b}{a}\right) \qquad (3)$$

Where: $\psi\left(\frac{t-b}{a}\right)$ function parent, $a$ scale factor, $b$ time shift.

Continuous and discrete wavelet transforms are the two types of wavelet transforms:

Continuous wavelet transform is defined as follows:

$$W_{a,b}(t) = \frac{1}{\sqrt{|a|}}\int_{-\infty}^{\infty} x(t)\psi\left(\frac{t-b}{a}\right)dt \qquad (4)$$

The transmitted signal evaluated by the signal analysis that is scaled in the time domain coefficients. This signal is "compressed" for $(a < 1)$ and "stretched" for $(a > 1)$.

Discrete wavelet transforms is defined as follows [16]

$$W_{a,b}(t) = \frac{1}{\sqrt{|a|}}\sum_{k=1}^{N}\psi\left(\frac{t-b}{a}\right)x(t) \qquad (5)$$

Where, $N$ is normalization constant.

The wavelet transform is based on the $\psi \in L^2(R)$ function, often known as the mother wavelet or wavelet.

The following conditions are met by this function [1, 5, 11]:

i. $$C_\psi = \int_0^\infty \frac{|\hat{\psi}(\omega)|^2}{|\omega|}d\omega < \infty \qquad (6)$$

Where, $\hat{\psi}(\omega)$ is the Fourier transform of $\psi(t)$. The admissibility condition assures that $\hat{\psi}(\omega)$ decrease to zero quickly as $(\omega \to 0)$. It is necessary $\hat{\psi}(0) = 0$ to verify that $C_\psi < \infty$ .

$$\int_{-\infty}^{\infty} \psi(t)\, dt = 0 \quad (7)$$

ii. Wavelet function is that have unit energy [13, 24]. That is

$$\int_{-\infty}^{\infty} |\psi(t)|^2\, dt < \infty \quad (8)$$

## 2.3 Haar wavelet

The Haar wavelet was the first mother wavelet introduced by Hungarian mathematician Alfred Haar in 1909 [3, 9, 25]. It is an orthogonal wavelet and only has one vanishing moment, making it inappropriate for smooth function reconstruction.

The Haar wavelet function $\psi(t)$ can be described as [1, 26]:

$$\psi(t) = \begin{cases} 1 & 0 \le t < \frac{1}{2} \\ -1 & \frac{1}{2} \le t \le 1 \\ 0 & Otherwise \end{cases} \quad (9)$$

So, we get only two conditions and two equations

$$h_0 + h_1 = \sqrt{2} \quad (10)$$

$$h_0{}^2 + h_1{}^2 = 1 \quad (11)$$

The solution to these equations is

$$h_0 = h_1 = \frac{1}{\sqrt{2}} \quad (12)$$

## 2.4 Daubechies wavelet

The Daubechies wavelet is a discrete wavelet named by Belgian physicist Ingrid Daubechies [3, 27]. The most commonly used Daubechies in practical applications are db2-db20 (even index only) a db2 is also called  Haar wavelet. It is an orthogonal wavelet family and the number of moments is equal to half the length of the support [28]. The conditions for Daubechies db4 wavelet lead for the following set of equations:

$$h_0 + h_1 + h_2 + h_3 = \sqrt{2} \quad (13)$$

$$h_1 + 2h_2 + 3h_3 = 0 \quad (14)$$

$$h_0{}^2 + h_1{}^2 + h_2{}^2 + h_3{}^2 = 1 \quad (15)$$

$$h_0 h_2 + h_1 h_3 = 0 \quad (16)$$

## 2.5 Coiflet Wavelets

The Coiflet wavelet is a discrete wavelet proposed by Ingrid Daubechies at Ronald Coifman's request to have vanishing moment scaling functions. It is an orthogonal wavelet. This wavelet is not symmetric but near symmetric [3]. Its nature is more symmetric than the Daubechies wavelet [27].This wavelet function has $(2N)$ vanishing moments and its scaling function has $(2N-1)$ vanishing moments.

The set of equations for coefficients for Coifet2 is:

$$h_{-2} + h_{-1} + h_0 + h_1 + h_2 + h_3 = \sqrt{2} \quad (17)$$

$$-2h_{-2} - h_{-1} + h_1 + 2h_2 + 3h_3 = 0 \quad (18)$$

$$h_{-2}{}^2 + h_{-1}{}^2 + h_0{}^2 + h_1{}^2 + h_2{}^2 + h_3{}^2 = 1 \quad (19)$$

$$h_{-2}h_0 + h_{-1}h_1 + h_0 h_2 + h_1 h_3 = 0 \quad (20)$$

$$h_{-2}h_2 + h_{-1}h_3 = 0 \quad (21)$$

### 2.6 Mexican Hat Wavelet

The Mexican wavelet is obtained after the second derivative of a Gaussian function. This wavelet is non-orthogonal and infinite support. This wavelet is symmetric and explicit expression of $\psi(t)$.

The analytic formula of $\psi(t)$ for Mexican hat wavelet [3, 10] as follows:

$$\psi(t) = (1 - t^2)e^{-0.5t^2} \quad (22)$$

### 2.7 Golden Hat Wavelet

Gossler et al (2018) [17], proposed a Golden Hat function generated by FCPs as the fourth derivative of the quotient between $p_0(t) = 1$ and $p_2(t) = t^2 + t + 2$, expressed by the equation:

$$\psi(t) = \frac{24(5t^4+10t^3-10t^2-15t-1)}{(t^2+t+2)^5} \quad (23)$$

The researcher named the new wavelet function $\psi(t)$ as golden wavelet, because of the relation between Fibonacci sequences and the golden ratio.

### 2.8 Proposed A New Wavelet Function

The researcher proposed a new wavelet function generated by Fibonacci coefficient polynomials (FCPs).

#### 2.8.1 Fibonacci Coefficient Polynomials

Fibonacci coefficient polynomials (FCPs) introduced by Garth Mills and Mitchell (2007), and they are building by Fibonacci sequences. The Fibonacci sequence one of the most well-known mathematical formulas. The sum of the two numbers that precede it determines the next number in the sequence. The sequence is $0, 1, 1, 2, 3, 5, 8, 13, 21, 34, \textit{and so on}$ [29].

We define polynomial sequence $\{p_n(t)\}_{n=0}^{\infty}$ by setting $p_0(t) = 1$ and

$$p_n(t) = \sum_{k=0}^{n} F_{k+1} t^{n-k} \quad , n \geq 1 \quad (24)$$

$p_n(t)$ is called Fibonacci coefficient polynomial (FCP) of order $n$ [30, 31].

The Fibonacci numbers $F_k$ are the terms of the sequence

$$F_k = F_{k-1} + F_{k-2} \quad , k \geq 2 \quad (25)$$

With initial terms are $F_0 = 0$ and $F_1 = 1$.

#### 2.8.2 Proposed Wavelet

The researcher proposed a new wavelet function generated by (FCPs) by the sixth derivative of the quotient between $p_0(t)$ and $p_2(t)$. The proposed wavelet is:

$$\psi(t) = \frac{5040A+25200B}{C^7} \quad (26)$$

Where

$$A = -t^6 + 3t^5 + 3t^2 - 1 \quad (27)$$

$$B = t^4 + 3t^3 - t \quad (28)$$

$$C = t^2 + t + 2 \quad (29)$$

A wavelet $\psi(t)$ has $N$ vanishing moments with a fast decay if and only if there exists $g(t)$ with a fast decay such that [17]:

$$\psi(t) = (-1)^N \frac{d^N}{dt^N}\, g(t) \qquad (30)$$

Where $g(t)$ is a function of quotient between $p_0(t)$ and $p_2(t)$.

### 2.8.3 Conditions of Proposed Wavelet

In order to show that $\psi(t)$ defied in (26) is a wavelet, it must satisfy the following conditions:

1. Admissibility Condition.

To verify this condition, we use the Fourier transform (FT) time derivatives property:

$$\hat{\psi}(\omega) = (i\omega)^6 G(i\omega) \qquad (31)$$

Where

$G(i\omega)$ is the FT of the $g(t)$, Thus can be given by

$$G(i\omega) = \int_{-\infty}^{\infty} g(t) e^{-i\omega t} dt$$

$$= \frac{2\pi}{\sqrt{7}} e^{0.5(i\omega - \sqrt{7}|\omega|)} \qquad (32)$$

$$\hat{\psi}(\omega) = (i\omega)^6 \frac{2\pi}{\sqrt{7}} e^{0.5(i\omega - \sqrt{7}|\omega|)} \qquad (33)$$

The obtained result was

$$C_\psi = \int_{-\infty}^{\infty} \frac{|\hat{\psi}(\omega)|^2}{|\omega|} d\omega = \frac{2714.2976\pi^2}{117649} < \infty \qquad (34)$$

2. The second step, was verify the condition of (8).

The obtained result was

$$\int_{-\infty}^{\infty} |\psi(t)|^2\, dt = \frac{22809600\,\pi}{117649\sqrt{7}} < \infty \qquad (35)$$

To obtain a wavelet $\psi(t)$ satisfying the unit energy condition in (8), it is necessary to multiply the proposed wavelet function obtained in (26) by the normalizing coefficient ($N_C$) [24].

$$N_C = \frac{1}{\sqrt{\int_{-\infty}^{\infty} |\psi(t)|^2 dt}} \qquad (36)$$

## 3 Data Analysis and Results

### 3.1 Data Description

In order to illustrate an appropriate model, the average of daily wind speed (m/s) data sets are collected from the meteorological directorate of Sulaimani for the period (Jan. 2016- Dec. 2020), have been used Matlab and R programming. The plot of wind speed data is represented in Figure1.

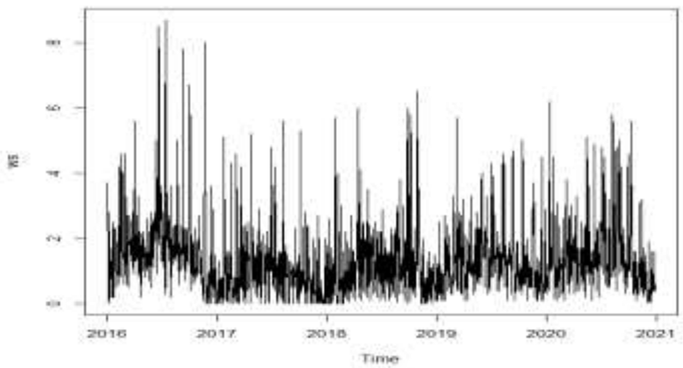


**Figure 1-** Daily Wind Speed Data Series

### 3.2 Results of ARIMA Model

ARIMA models are created automatically using R's auto.arima function. $ARIMA(2,1,1)$ is the determined model, and it has a minimum Akaike Information Criterion (AIC) value (5110.9). This means that the $ARIMA(2,1,1)$ model is the best among all the other models. The parameters have been estimated using R statistical software. The model's parameter estimates are given in Table 1.

**Table 1**- The Estimates of $ARIMA(2,1,1)$ Model

| Coefficients | Value | S.E |
|---|---|---|
| AR1 | 0.4581 | 0.0240 |
| AR2 | -0.0975 | 0.0240 |
| MA1 | -0.969 | 0.0065 |

Since the model is:

$$(1-0.4581B+0.0975\,B^2)(1-B)y_t=\delta+(1+0.969\,B)a_t \quad (37)$$

After estimation the parameters the Box-Ljung $Q$ statistic is used to verify the model's overall adequacy. The Q statistic as follows:

$$Q=n(n+2)\sum_{k=1}^{K}\frac{r^2(k)}{n-k} \quad (38)$$

Where, $r(k)$ is the residual autocorrelation at lag $k$, $n$ is the number of residuals and $K$is the number of lags. Because the p-value of the test is $(0.2663)$ and greater than $(0.05)$, and the value of Box-Ljung tests is $(23.47)$, this $ARIMA$ model is appropriate for future forecasting.

For testing the accuracy of the model, we analysed the performance of model is evaluated by using the Mean Absolute Error (MAE), Mean Absolute Scaled Error (MASE), and Root Mean Squares Error (RMSE) [8, 11 ].

$$\mathrm{MAE}=\frac{1}{n}\sum_{t=1}^{n}\left|Y_t-\widehat{Y}_t\right| \quad (39)$$

$$\mathrm{MASE}=\frac{\frac{1}{n}\sum_{t=1}^{n}|Y_t-\widehat{Y}_t|}{\frac{1}{n-2}\sum_{t=2}^{n}|Y_t-Y_{t-2}|} \quad (40)$$

$$\mathrm{RMSE}=\left[\frac{1}{n}\sum_{t=1}^{n}\left(Y_t-\widehat{Y}_t\right)^2\right]^{\frac{1}{2}} \quad (41)$$

Where $\widehat{Y}_t$ is the predicted value, $Y_t$ is the actual value, and $n$ is the number of observations.

### 3.3 Results of Wavelet-ARIMA

We used proposed wavelet, Golden hat, Mexican hat, Daubechies, and Coiflet to transform the wind speed data using the continuous wavelet transform (CWT). The CWT was used up to 64 scales, and the average of all wavelet coefficients was computed. Matlab was used to implement the CWT. An ARIMA model is fitted after the decomposition. Other wavelet functions' results were investigated for comparison with the proposed wavelet-ARIMA. Table (2) shows that the

(MAE), (MASE), and (RMSE), of the proposed wavelet-ARIMA model are fewer than the MAE, RMSE, and MASE of the direct use of ARIMA model, indicating that the suggested wavelet-ARIMA model has better predictive capacity. It signifies that the proposed wavelet-ARIMA approach outperforms the direct usage of the ARIMA model for the provided data set.

**Table 2-** The Estimates of $ARIMA(2,1,1)$ Model

| Model | MAE | MASE | RMSE |
|---|---|---|---|
| ARIMA | 0.68824 | 0.86761 | 0.97999 |
| Proposed Wavelet-ARIMA | 0.03363 | 0.56867 | 0.04553 |

**Table 3-** Comparison of Wavelet-ARIMA

| Mother Wavelet | MAE | MASE | RMSE |
|---|---|---|---|
| Proposed wavelet | 0.03363 | 0.56867 | 0.04553 |
| Golden hat | 0.04373 | 0.63261 | 0.05947 |
| Mexican hat | 0.05917 | 0.57855 | 0.05267 |
| Daubechies 1(Haar) | 0.26526 | 0.82055 | 0.32138 |
| Daubechies 2 | 0.09405 | 0.47276 | 0.13043 |
| Daubechies 3 | 0.08737 | 0.52716 | 0.12207 |
| Daubechies 4 | 0.09419 | 0.52482 | 0.12841 |
| Daubechies 5 | 0.07266 | 0.47772 | 0.10203 |
| Coiflet1 | 0.12240 | 0.58794 | 0.17193 |
| Coiflet2 | 0.07794 | 0.47228 | 0.10910 |
| Coiflet3 | 0.06637 | 0.44251 | 0.09289 |
| Coiflet4 | 0.07659 | 0.50719 | 0.10880 |
| Coiflet 5 | 0.05917 | 0.42861 | 0.08277 |

## 4. Conclusions

The aims of this study are to propose a wavelet function, as well as to compare ARIMA and wavelet- $ARIMA$ to determine the best-fitted model. The CWT was used to decompose the data. From the previous results, it is clear that the model for daily wind speed forecasting is the $ARIMA(2,1,1)$ model, and the wavelet- $ARIMA$ model is better than direct use of $ARIMA$ for daily wind speed forecasting. As compared to wavelets the proposed wavelet is the most appropriate wavelet for wind speed forecasting, it gives us less value of MAE (0.03363) and RMSE (0.04553).

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