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Theoretical Analysis Of The Photon Production Rate in the Quark-Gluon Interaction According To The Quantum Cromodynamic QCD Theory

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Abstract

In this work, we have used the QCD dynamic scenario of the quark gluon interaction to investigate and study photon emission theoretically based on quantum theory. The QCD theory is implemented by deriving the photon emission rate equation of the state of ideal QGP at a chemical potential. The photon rate of the quark-gluon interaction has to be calculated for the anti up-gluon interaction in the $\bar{u}g \rightarrow \bar{d}\ \gamma$ system at the temperature of system $(180 \leq T \leq 360)\ MeV$ with critical temperature (T_c =132.38, 158.86, 178.72 and 198.57) MeV and photon energy $(1 \leq E \leq 10)$ GeV. We investigated a significant effect of critical temperature, strength coupling, and photon energy on the photon rate contribution. Here, the increased photon emission rate and decreased strength coupling of the quark-gluon reaction due to the decrease in temperature of the system from 360 MeV to 180 MeV are predicted. Photon energy in the range (1 to 10) GeV and the rate spectrum of four varieties of critical temperatures are presented.

The interesting point in our results is the minimum value of photon rate, especially in the photon energy E=10 GeV of $n_f = 3$ flavor which reflects the poor coupling between quark and gluon in the the $\bar{u}g \rightarrow \bar{d}\ \gamma$ system which was already expected. The features of QCD results are achieved in the case of $n_f = 3$ flavors for the photon energy E=1 to 10 GeV, the strength coupling and the photo meason rate are calculated theoretically. We can notice that the asymptotic behavior, which was characterized by a hadronic phase limit, will be satisfied.

Keywords: Photon Production Rate, Quark-Gluon Interaction, QCD Theory.

1.Introduction

Photon production is an important tool for probing the strong interaction matter in the Large Hadronic Collision (LHC) at CERN and relativistic heavy-ion collisions (RHIC) at BNL [1]. All the photons are divided into decay photons, which come from hadron decays, and direct photons. Furthermore, the direct photons are divided into prompt photons and thermal photons [2]. Mostly the quark-gluon has been created at relativistic heavy-ion collision experiments at the Relativistic Heavy Ion Collider (BNL) and the LHC at CERN [3]. High temperatures are not an extreme feature of the quark-gluon interaction that's produced in heavy ion collisions [4]. The strong interaction of quark-gluon undergoes a transition from one phase to another at an approximated temperature of 150 MeV [5]. The medium below the transition temperature is characterized by hadrons as having primary degrees of freedom. On the other hand, the medium above the transition temperature is characterized by quarks and gluons, the element degrees of freedom of QCD [6].

The standard model is an important theory that establishes the study of the dynamics and characteristics of quarks in nuclear matter [7]. The QCD is known to undergo the transition temperature at which partial deconfinement is restored. The nature of the QCD transition is discussed because of its relevance to heavy ion collisions [8]. The standard model of elementary particles in physics is the mathematical framework that describes the interactions of elementary particles: electromagnetism, weak interactions, and strong interactions [9]. Hadi, J.M., Al-agealy et al. (2016) evaluated the rate of the photon for quark-gluon collisions produced at Compton scattering using quantum consideration with different photon energy spectra [10]. In 2018, Hadi J. M. Al-Agealy et al. discussed the behavior of photons emitted from quark-gluon systems with different fugacity coefficients at high energy collisions using color quantum theory [11]. In 2020, Ahmed M. Ashwiekh et al. studied the flow rate of hard photon emission from quark-antiquark interaction at high temperatures using the lowest-order approximation of QCD theory. Results show an increase in flow rate with an increase in temperature of the media, which indicates a logarithmically divergent thermal effect on the photon product [12]. Elaf Mohammed et al. in 2022 calculated and analyzed the photon rate produced from the interaction of the quark with the anti-quark during the annihilation process depending on the phenomenology of QCD [13]. This paper aims to analyze and study the photon production rate from the interaction of quark and gluon via QCD theory simulations and investigate the effect of critical temperature, strength coupling, and photon distribution energy on its rate.

2.Theory

The hard photon rate produced by the quark-gluon interaction with momentum P and energy E is given via QCD theory [14].

$$\mathrm{R}_{qg}^{H}(E,P) = -\frac{1}{(2\pi)^3}\, \mathrm{f}_B(\mathrm{E})\mathrm{Im}\prod{}_{qg}^{H}(E,P) \qquad (1)$$

Where $\mathrm{f}_B(\mathrm{E})$ is the Bose-Einstein distribution function and the imaginary part $\mathrm{Im}\prod_{qg}^{H}(E,P)$ of the retarded self-energy polarization of photons at finite temperature is given by [15].

$$\mathrm{Im}\prod_{qg}^{H}(E,P) = (-1)\frac{N_c C_a}{\pi^4} {g_E}^2 {g_H}^2 \frac{T}{E^2} \left|\mathrm{I}_{\mathrm{T},L}\right| \quad \int_0^\infty \left(f_q(P) - f_q(E-P)\right)(P^2 + (E-P)^2)\, dP \quad (2)$$

Where N_c is color number, C_a is the Casimir operator, g_E is the quantum electrodynamics coupling,, g_H is the quantum chromodynamic coupling, $f_q(P)$ is the Fermi distribution function of a quark, $\left|\mathrm{I}_{\mathrm{T},L}\right|$ is the self-integral of the system equal $\mathrm{I}_{\mathrm{T},L} = \mathrm{I}_\mathrm{T} - \mathrm{I}_\mathrm{L}$ where I_T and I_L are dimensionless constants [16].

The Eq. (3-6) together with Eq.(3-3) and for all electric charges $\sum e_q^2$ for quarks leads to

$$\mathrm{Im}\prod\ {}_{qg}^{H}(E,P) = (-1)\frac{N_c C_a}{\pi^4} {g_E}^2 {g_H}^2 \frac{T}{E^2} \left|\mathrm{I}_\mathrm{T} - \mathrm{I}_\mathrm{L}\right| \sum e_q^2 \int_0^\infty \left(f_q(P) - f_q(E-P)\right)(P^2 + (E-P)^2)\, dP \quad (3)$$

The Juttner distribution function $f_q(P)$ and $f_q(p-E)$ as function of the fugacity coefficient λ_q for quarks and writs [17]

$$f_q(P) = \frac{\lambda_q}{e^{\frac{(p+\mu_q)}{T}} + \lambda_q} \quad (4)$$

$$f_q(p-E) = \frac{\lambda_q}{e^{\frac{(E-p-\mu_q)}{T}} + \lambda_q} \quad (5)$$

where μ_q is the chemical potential coefficient. Obviously, the chemical potential with fugacity of quarks and gluons is$\lambda_{q,g} = e^{\frac{\mu_{q,g}}{T}}$ [18].

Substituting both Eq.(4) and Eq.(5) in Eq.(3) and expanding $(P^2 + (E-P)^2) = P^2 + E^2 - 2EP + P^2 = 2P^2 + E^2 - 2EP$ to results .

$$\mathrm{Im}\prod_{qg}^{H}(E,P) = (-1)\frac{N_c C_a}{\pi^4} {g_E}^2 {g_H}^2 \frac{T}{E^2} \left|\mathrm{I}_\mathrm{T} - \mathrm{I}_\mathrm{L}\right| \sum e_q^2 \left[\int_0^\infty \frac{(2P^2+E^2-2EP)}{\left(\frac{e^{\frac{(p+\mu_q)}{T}}}{\lambda_q}+1\right)} dP - \int_0^\infty \frac{(2P^2+E^2-2EP)}{\left(\frac{e^{\frac{(E-p-\mu_q)}{T}}}{\lambda_q}+1\right)} dP\right] \quad (6)$$

We assume that

$$A_1 = \int_0^\infty \frac{(2P^2+E^2-2EP)}{\left(\frac{e^{\frac{(p+\mu_q)}{T}}}{\lambda_q}+1\right)} dP = \int_0^\infty [\frac{e^{\frac{-(p+\mu_q)}{T}}}{{\lambda_q}^{-1}} - \frac{e^{\frac{-2(p+\mu_q)}{T}}}{{\lambda_q}^{-2}} + \frac{e^{\frac{-3(p+\mu_q)}{T}}}{{\lambda_q}^{-3}} - \frac{e^{\frac{-4(p+\mu_q)}{T}}}{{\lambda_q}^{-4}} - \cdots .. \frac{e^{\frac{-n(p+\mu_q)}{T}}}{{\lambda_q}^{-n}}](2P^2 - 2EP + E^2)dP) \quad (7)$$

$$A_2 = \int_0^\infty \frac{(2P^2+E^2-2EP)}{\left(\frac{e^{\frac{(E-p-\mu_q)}{T}}}{\lambda_q}+1\right)} dP = \quad (8)$$

To solve both integrals Eq.(7) and Eq.(8), we can simply write both integrals in six terms;

$$J_1 = \int_0^\infty \left[\frac{e^{\frac{-(p+\mu_q)}{T}}}{{\lambda_q}^{-1}} - \frac{e^{\frac{-2(p+\mu_q)}{T}}}{{\lambda_q}^{-2}} + \frac{e^{\frac{-3(p+\mu_q)}{T}}}{{\lambda_q}^{-3}} - \frac{e^{\frac{-4(p+\mu_q)}{T}}}{{\lambda_q}^{-4}} + \frac{e^{\frac{-5(p+\mu_q)}{T}}}{{\lambda_q}^{-5}} - \cdots .. \frac{e^{\frac{-n(p+\mu_q)}{T}}}{{\lambda_q}^{-n}}\right](2P^2)dP \quad (9)$$

The second integral is

$$J_2 = \int_0^\infty [\frac{e^{\frac{-(p+\mu_q)}{T}}}{{\lambda_q}^{-1}} - \frac{e^{\frac{-2(p+\mu_q)}{T}}}{{\lambda_q}^{-2}} + \frac{e^{\frac{-3(p+\mu_q)}{T}}}{{\lambda_q}^{-3}} - \frac{e^{\frac{-4(p+\mu_q)}{T}}}{{\lambda_q}^{-4}} + \frac{e^{\frac{-5(p+\mu_q)}{T}}}{{\lambda_q}^{-5}} - \cdots .. \frac{e^{\frac{-n(p+\mu_q)}{T}}}{{\lambda_q}^{-n}}](2PE)dP \quad (10)$$

The third integral is

$$J_3 = \int_0^\infty [\frac{e^{\frac{-(p+\mu_q)}{T}}}{{\lambda_q}^{-1}} - \frac{e^{\frac{-2(p+\mu_q)}{T}}}{{\lambda_q}^{-2}} + \frac{e^{\frac{-3(p+\mu_q)}{T}}}{{\lambda_q}^{-3}} - \frac{e^{\frac{-4(p+\mu_q)}{T}}}{{\lambda_q}^{-4}} + \frac{e^{\frac{-5(p+\mu_q)}{T}}}{{\lambda_q}^{-5}} - \cdots .. \frac{e^{\frac{-n(p+\mu_q)}{T}}}{{\lambda_q}^{-n}}](E^2)dP \quad (11)$$

The fourth term is

$$J_4 = \int_0^\infty [\frac{e^{\frac{-(E-p-\mu_q)}{T}}}{\lambda_q{}^{-1}} - \frac{e^{\frac{-2(E-p-\mu_q)}{T}}}{\lambda_q{}^{-2}} + \frac{e^{e^{\frac{-3(E-p-\mu_q)}{T}}}}{\lambda_q{}^{-3}} - \frac{e^{\frac{-4(E-p-\mu_q)}{T}}}{\lambda_q{}^{-4}} + \frac{e^{\frac{-5(E-p-\mu_q)}{T}}}{\lambda_q{}^{-5}} - \cdots \ldots \ldots \frac{e^{\frac{-n(E-p-\mu_q)}{T}}}{\lambda_q{}^{-n}}](2P^2)dP \qquad (12\ )$$

The fifth integral term is

$$J_5 = \int_0^\infty [\frac{e^{\frac{-(E-p-\mu_q)}{T}}}{\lambda_q{}^{-1}} - \frac{e^{\frac{-2(E-p-\mu_q)}{T}}}{\lambda_q{}^{-2}} + \frac{e^{e^{\frac{-3(E-p-\mu_q)}{T}}}}{\lambda_q{}^{-3}} - \frac{e^{\frac{-4(E-p-\mu_q)}{T}}}{\lambda_q{}^{-4}} + \frac{e^{\frac{-5(E-p-\mu_q)}{T}}}{\lambda_q{}^{-5}} - \cdots \ldots \ldots \frac{e^{\frac{-n(E-p-\mu_q)}{T}}}{\lambda_q{}^{-n}}](2PE)dP \qquad (3)$$

The final integral term is

$$J_6 = \int_0^\infty [\frac{e^{\frac{-(E-p-\mu_q)}{T}}}{\lambda_q{}^{-1}} - \frac{e^{\frac{-2(E-p-\mu_q)}{T}}}{\lambda_q{}^{-2}} + \frac{e^{e^{\frac{-3(E-p-\mu_q)}{T}}}}{\lambda_q{}^{-3}} - \frac{e^{\frac{-4(E-p-\mu_q)}{T}}}{\lambda_q{}^{-4}} + \frac{e^{\frac{-5(E-p-\mu_q)}{T}}}{\lambda_q{}^{-5}} - \cdots \ldots \ldots \frac{e^{\frac{-n(E-p-\mu_q)}{T}}}{\lambda_q{}^{-n}}](E^2)dP \qquad (14)$$

The solutions of the six term are given by .

$$J_1 = 2T^3 \left[\frac{\lambda_q{}^1 e^{\frac{-\mu_q}{T}}}{1^3} - \frac{\lambda_q{}^2 e^{\frac{-2\mu_q}{T}}}{2^3} + \frac{\lambda_q{}^3 e^{\frac{-3\mu_q}{T}}}{3^3} - \frac{\lambda_q{}^4 e^{\frac{-4\mu_q}{T}}}{4^3} + \frac{\lambda_q{}^5 e^{\frac{-5\mu_q}{T}}}{5^3} - \cdots \frac{\lambda_q{}^n}{n^3} e^{\frac{-n\mu_q}{T}}\right] \Gamma\,(\,3) \qquad (15)$$

The second term in term J_2 is.

$$J_2 = 2ET^2\Gamma(\,2) \left[\frac{\lambda_q{}^1 e^{\frac{-\mu_q}{T}}}{1^2} - \frac{\lambda_q{}^2 e^{\frac{-2\mu_q}{T}}}{2^2} + \frac{\lambda_q{}^3 e^{\frac{-3\mu_q}{T}}}{3^2} - \frac{\lambda_q{}^{4e^{\frac{-4\mu_q}{T}}}}{4^2} + \frac{\lambda_q{}^5 e^{\frac{-5\mu_q}{T}}}{5^2} - \cdots \frac{\lambda_q{}^n e^{\frac{-n\mu_q}{T}}}{n^2}\right] \qquad (16\ )$$

The third integral term in J_3 is

$$J_3 = E^2T[\,\frac{\lambda_q{}^1 e^{\frac{-\mu_q}{T}}}{1} - \frac{\lambda_q{}^2 e^{\frac{-2\mu_q}{T}}}{2} + \frac{\lambda_q{}^3 e^{\frac{-3\mu_q}{T}}}{3} - \frac{\lambda_q{}^{4e^{\frac{-4\mu_q}{T}}}}{4} + \frac{\lambda_q{}^5 e^{\frac{-5\mu_q}{T}}}{5} - \cdots \frac{\lambda_q{}^{ne^{\frac{-n\mu_q}{T}}}}{n})\Gamma(\,1\,) \qquad (\,17)$$

On the other hand, the fourth term is

$$J_4 == 2T^3\Gamma(3)\,[\frac{\lambda_q{}^1 e^{\frac{-(E-\mu_q)}{T}}}{1^3} - \frac{\lambda_q{}^2 e^{\frac{-2(E-\mu_q)}{T}}}{2^3} + \frac{\lambda_q{}^3 e^{\frac{-3(E-\mu_q)}{T}}}{3^3} - \frac{\lambda_q{}^4 e^{\frac{-4(E-\mu_q)}{T}}}{4^3} + \frac{\lambda_q{}^5 e^{\frac{-5(E-\mu_q)}{T}}}{5^3} + \cdots \frac{\lambda_q{}^n}{n^3} e^{\frac{-n(E-\mu_q)}{T}}\ ] \qquad (18)$$

The fifth integral term is

$$J_5 = 2ET^2\Gamma(2)\,[\,\frac{\lambda_q{}^1 e^{\frac{-(E-\mu_q)}{T}}}{1^2} - \frac{\lambda_q{}^2 e^{\frac{-2(E-\mu_q)}{T}}}{2^2} + \frac{\lambda_q{}^3 e^{\frac{-3(E-\mu_q)}{T}}}{3^2} + \frac{\lambda_q{}^4 e^{\frac{-4(E-\mu_q)}{T}}}{4^2} - \frac{\lambda_q{}^5 e^{\frac{-5(E-\mu_q)}{T}}}{5^2} + \cdots \frac{\lambda_q{}^n}{n^2} e^{\frac{-n(E-\mu_q)}{T}}\ ] \qquad (19)$$

The sixth integral term gives

$$J_6 = E^2T\Gamma(1)\left(\frac{\lambda_q{}^1 e^{\frac{-(E-\mu_q)}{T}}}{1} - \frac{\lambda_q{}^2 e^{\frac{-2(E-\mu_q)}{T}}}{2} + \frac{\lambda_q{}^3 e^{\frac{-3(E-\mu_q)}{T}}}{3} - \frac{\lambda_q{}^4 e^{\frac{-4(E-\mu_q)}{T}}}{4} + \frac{\lambda_q{}^5 e^{\frac{-5(E-\mu_q)}{T}}}{5} \ldots\ldots\ldots + \frac{\lambda_q{}^n}{n} e^{\frac{-n(E-\mu_q)}{T}}\right) \qquad .(\,20)$$

Inserting the results of J_1 , J_2 ,and J_3 in Eq.(7) gives A_1 .

$$A_1=\int_0^\infty \frac{\lambda_q(2P^2+E^2-2EP)}{e^{\frac{(p+\mu_q)}{T}}+\lambda_q}dP = 2T^3\left[\frac{\lambda_q{}^1 e^{\frac{-\mu_q}{T}}}{1^3}-\frac{\lambda_q{}^2 e^{\frac{-2\mu_q}{T}}}{2^3}+\frac{\lambda_q{}^3 e^{\frac{-3\mu_q}{T}}}{3^3}-\frac{\lambda_q{}^4 e^{\frac{-4\mu_q}{T}}}{4^3}+\frac{\lambda_q{}^5 e^{\frac{-5\mu_q}{T}}}{5^3}-\cdots\frac{\lambda_q{}^n}{n^3}e^{\frac{-n\mu_q}{T}}\right]\Gamma(3)+2\mathrm{E}T^2\Gamma(2)\left[\left[\frac{\lambda_q{}^1 e^{\frac{-\mu_q}{T}}}{1^2}-\frac{\lambda_q{}^2 e^{\frac{-2\mu_q}{T}}}{2^2}+\frac{\lambda_q{}^3 e^{\frac{-3\mu_q}{T}}}{3^2}-\frac{\lambda_q{}^4 e^{\frac{-4\mu_q}{T}}}{4^2}+\frac{\lambda_q{}^5 e^{\frac{-5\mu_q}{T}}}{5^2}-\cdots\frac{\lambda_q{}^n}{n^2}e^{\frac{-n\mu_q}{T}}\right]\right]+E^2T\left[\left[\frac{\lambda_q{}^1 e^{\frac{-\mu_q}{T}}}{1}-\frac{\lambda_q{}^2 e^{\frac{-2\mu_q}{T}}}{2}+\frac{\lambda_q{}^3 e^{\frac{-3\mu_q}{T}}}{3}-\frac{\lambda_q{}^4 e^{\frac{-4\mu_q}{T}}}{4}+\frac{\lambda_q{}^5 e^{\frac{-5\mu_q}{T}}}{5}-\cdots\frac{\lambda_q{}^n}{n}e^{\frac{-n\mu_q}{T}}\right]\right)\Gamma(1) \quad \textbf{(21)}$$

On the other hand, we insert the J_4 , J_5, and J_6 in Eq.(8) to give the integral term A_2 .

$$A_2=\int_0^\infty \frac{\lambda_q(2P^2+E^2-2EP)}{e^{\frac{(E-p-\mu_q)}{T}}+\lambda_q}dP$$

$$= 2T^3\Gamma(3)\left[\frac{\lambda_q{}^1 e^{\frac{-(E-\mu_q)}{T}}}{1^3}-\frac{\lambda_q{}^2 e^{\frac{-2(E-\mu_q)}{T}}}{2^3}+\frac{\lambda_q{}^3 e^{\frac{-3(E-\mu_q)}{T}}}{3^3}-\frac{\lambda_q{}^4 e^{\frac{-4(E-\mu_q)}{T}}}{4^3}+\frac{\lambda_q{}^5 e^{\frac{-5(E-\mu_q)}{T}}}{5^3}+\cdots\frac{\lambda_q{}^n}{n^3}e^{\frac{-n(E-\mu_q)}{T}}\right]$$
$$+2\mathrm{E}T^2\Gamma(2)\left[\frac{\lambda_q{}^1 e^{\frac{-(E-\mu_q)}{T}}}{1^2}-\frac{\lambda_q{}^2 e^{\frac{-2(E-\mu_q)}{T}}}{2^2}+\frac{\lambda_q{}^3 e^{\frac{-3(E-\mu_q)}{T}}}{3^2}-\frac{\lambda_q{}^4 e^{\frac{-4(E-\mu_q)}{T}}}{4^2}+\frac{\lambda_q{}^5 e^{\frac{-5(E-\mu_q)}{T}}}{5^2}+\cdots\frac{\lambda_q{}^n}{n^2}e^{\frac{-n(E-\mu_q)}{T}}\right]$$
$$+E^2T\Gamma(1)\left[\frac{\lambda_q{}^1 e^{\frac{-(E-\mu_q)}{T}}}{1}-\frac{\lambda_q{}^2 e^{\frac{-2(E-\mu_q)}{T}}}{2}+\frac{\lambda_q{}^3 e^{\frac{-3(E-\mu_q)}{T}}}{3}-\frac{\lambda_q{}^4 e^{\frac{-4(E-\mu_q)}{T}}}{4}+\frac{\lambda_q{}^5 e^{\frac{-5(E-\mu_q)}{T}}}{5}+\cdots\frac{\lambda_q{}^n}{n}e^{\frac{-n(E-\mu_q)}{T}}\right] \quad (22)$$

We assume $J=A_1+A_2$, then with using Eq.(21) and Eq.(22), we get

$$J=2T^3\left[\frac{\lambda_q{}^1 e^{\frac{-\mu_q}{T}}}{1^3}-\frac{\lambda_q{}^2 e^{\frac{-2\mu_q}{T}}}{2^3}+\frac{\lambda_q{}^3 e^{\frac{-3\mu_q}{T}}}{3^3}-\frac{\lambda_q{}^4 e^{\frac{-4\mu_q}{T}}}{4^3}+\frac{\lambda_q{}^5 e^{\frac{-5\mu_q}{T}}}{5^3}-\cdots\frac{\lambda_q{}^n}{n^3}e^{\frac{-n\mu_q}{T}}\right]\Gamma(3)$$
$$+2\mathrm{E}T^2\Gamma(2)\left[\frac{\lambda_q{}^1 e^{\frac{-\mu_q}{T}}}{1^2}-\frac{\lambda_q{}^2 e^{\frac{-2\mu_q}{T}}}{2^2}+\frac{\lambda_q{}^3 e^{\frac{-3\mu_q}{T}}}{3^2}-\frac{\lambda_q{}^4 e^{\frac{-4\mu_q}{T}}}{4^2}+\frac{\lambda_q{}^5 e^{\frac{-5\mu_q}{T}}}{5^2}-\cdots\frac{\lambda_q{}^n}{n^2}e^{\frac{-n\mu_q}{T}}\right]$$
$$+E^2T\left[\frac{\lambda_q{}^1 e^{\frac{-\mu_q}{T}}}{1}-\frac{\lambda_q{}^2 e^{\frac{-2\mu_q}{T}}}{2}+\frac{\lambda_q{}^3 e^{\frac{-3\mu_q}{T}}}{3}-\frac{\lambda_q{}^4 e^{\frac{-4\mu_q}{T}}}{4}+\frac{\lambda_q{}^5 e^{\frac{-5\mu_q}{T}}}{5}-\cdots\frac{\lambda_q{}^n}{n}e^{\frac{-n\mu_q}{T}}\right)\Gamma(1)+2T^3\Gamma(3)\left[\frac{\lambda_q{}^1 e^{\frac{-(E-\mu_q)}{T}}}{1^3}\right.$$
$$\left.-\frac{\lambda_q{}^2 e^{\frac{-2(E-\mu_q)}{T}}}{2^3}+\frac{\lambda_q{}^3 e^{\frac{-3(E-\mu_q)}{T}}}{3^3}-\frac{\lambda_q{}^4 e^{\frac{-4(E-\mu_q)}{T}}}{4^3}+\frac{\lambda_q{}^5 e^{\frac{-5(E-\mu_q)}{T}}}{5^3}+\cdots\frac{\lambda_q{}^n}{n^3}e^{\frac{-n(E-\mu_q)}{T}}\right]$$
$$+2\mathrm{E}T^2\Gamma(2)\left[\frac{\lambda_q{}^1 e^{\frac{-(E-\mu_q)}{T}}}{1^2}-\frac{\lambda_q{}^2 e^{\frac{-2(E-\mu_q)}{T}}}{2^2}+\frac{\lambda_q{}^3 e^{\frac{-3(E-\mu_q)}{T}}}{3^2}-\frac{\lambda_q{}^4 e^{\frac{-4(E-\mu_q)}{T}}}{4^2}+\frac{\lambda_q{}^5 e^{\frac{-5(E-\mu_q)}{T}}}{5^2}+\cdots\frac{\lambda_q{}^n}{n^2}e^{\frac{-n(E-\mu_q)}{T}}\right]$$
$$+E^2T\Gamma(1)\left[\frac{\lambda_q{}^1 e^{\frac{-(E-\mu_q)}{T}}}{1}-\frac{\lambda_q{}^2 e^{\frac{-2(E-\mu_q)}{T}}}{2}+\frac{\lambda_q{}^3 e^{\frac{-3(E-\mu_q)}{T}}}{3}-\frac{\lambda_q{}^4 e^{\frac{-4(E-\mu_q)}{T}}}{4}+\frac{\lambda_q{}^5 e^{\frac{-5(E-\mu_q)}{T}}}{5}+\cdots\frac{\lambda_q{}^n}{n}e^{\frac{-n(E-\mu_q)}{T}}\right] \quad (23)$$

Then, we have

$$J = 2T^3\left[\frac{\lambda_q^{\ 1}}{1^3}\left(e^{\frac{-\mu_q}{T}}+e^{\frac{-(E-\mu_q)}{T}}\right)-\frac{\lambda_q^{\ 2}}{2^3}\left(e^{\frac{-2\mu_q}{T}}+e^{\frac{-2(E-\mu_q)}{T}}\right)+\frac{\lambda_q^{\ 3}}{3^3}\left(e^{\frac{-3\mu_q}{T}}+e^{\frac{-3(E-\mu_q)}{T}}\right)-\frac{\lambda_q^{\ 4}}{4^3}\left(e^{\frac{-4\mu_q}{T}}+e^{\frac{-4(E-\mu_q)}{T}}\right)+\frac{\lambda_q^{\ 5}}{5^3}\left(e^{\frac{-5\mu_q}{T}}+e^{\frac{-5(E-\mu_q)}{T}}\right)-\cdots\frac{\lambda_q^{\ n}}{n^3}\left(e^{\frac{-n\mu_q}{T}}+e^{\frac{-n(E-\mu_q)}{T}}\right)\right]\Gamma(3)+2ET^2\Gamma(2)\left[\frac{\lambda_q^{\ 1}}{1^2}\left(e^{\frac{-\mu_q}{T}}+e^{\frac{-(E-\mu_q)}{T}}\right)-\frac{\lambda_q^{\ 2}}{2^2}\left(e^{\frac{-2\mu_q}{T}}+e^{\frac{-2(E-\mu_q)}{T}}\right)+\frac{\lambda_q^{\ 3}}{3^2}\left(e^{\frac{-3\mu_q}{T}}+e^{\frac{-3(E-\mu_q)}{T}}\right)-\frac{\lambda_q^{\ 4}}{4^2}\left(e^{\frac{-4\mu_q}{T}}+e^{\frac{-4(E-\mu_q)}{T}}\right)+\frac{\lambda_q^{\ 5}}{5^2}\left(e^{\frac{-5\mu_q}{T}}+e^{\frac{-5(E-\mu_q)}{T}}\right)-\cdots\frac{\lambda_q^{\ n}}{n^2}\left(e^{\frac{-n\mu_q}{T}}+e^{\frac{-n(E-\mu_q)}{T}}\right)\right]+E^2T\left[\frac{\lambda_q^{\ 1}}{1}\left(e^{\frac{-\mu_q}{T}}+e^{\frac{-(E-\mu_q)}{T}}\right)-\frac{\lambda_q^{\ 2}}{2}\left(e^{\frac{-2\mu_q}{T}}+e^{\frac{-2(E-\mu_q)}{T}}\right)+\frac{\lambda_q^{\ 3}}{3}\left(e^{\frac{-3\mu_q}{T}}+e^{\frac{-3(E-\mu_q)}{T}}\right)-\frac{\lambda_q^{\ 4}}{4}\left(e^{\frac{-4\mu_q}{T}}+e^{\frac{-4(E-\mu_q)}{T}}\right)+\frac{\lambda_q^{\ 5}}{5}\left(e^{\frac{-5\mu_q}{T}}+e^{\frac{-5(E-\mu_q)}{T}}\right)\ldots\frac{\lambda_q^{\ n}}{n}\left(e^{\frac{-n\mu_q}{T}}+e^{\frac{-n(E-\mu_q)}{T}}\right)\right)\Gamma(1) \quad (24)$$

Inserting Eq.(24) in Eq.(6) to give

$$\mathrm{Im}\prod_{qg}^{H}(E,P) = (-1)\frac{N_c C_a}{\pi^4}g_E^{\ 2}g_H^{\ 2}\frac{T}{E^2}\left|\mathrm{I_T}-\mathrm{I_L}\right| \sum e_q^2 \times 2T^3\left[\frac{\lambda_q^{\ 1}}{1^3}\left(e^{\frac{-\mu_q}{T}}+e^{\frac{-(E-\mu_q)}{T}}\right)-\frac{\lambda_q^{\ 2}}{2^3}\left(e^{\frac{-2\mu_q}{T}}+e^{\frac{-2(E-\mu_q)}{T}}\right)+\frac{\lambda_q^{\ 3}}{3^3}\left(e^{\frac{-3\mu_q}{T}}+e^{\frac{-3(E-\mu_q)}{T}}\right)-\frac{\lambda_q^{\ 4}}{4^3}\left(e^{\frac{-4\mu_q}{T}}+e^{\frac{-4(E-\mu_q)}{T}}\right)+\frac{\lambda_q^{\ 5}}{5^3}\left(e^{\frac{-5\mu_q}{T}}+e^{\frac{-5(E-\mu_q)}{T}}\right)-\cdots\frac{\lambda_q^{\ n}}{n^3}\left(e^{\frac{-n\mu_q}{T}}+e^{\frac{-n(E-\mu_q)}{T}}\right)\right]\Gamma(3)+2ET^2\Gamma(2)\left[\frac{\lambda_q^{\ 1}}{1^2}\left(e^{\frac{-\mu_q}{T}}+e^{\frac{-(E-\mu_q)}{T}}\right)-\frac{\lambda_q^{\ 2}}{2^2}\left(e^{\frac{-2\mu_q}{T}}+e^{\frac{-2(E-\mu_q)}{T}}\right)+\frac{\lambda_q^{\ 3}}{3^2}\left(e^{\frac{-3\mu_q}{T}}+e^{\frac{-3(E-\mu_q)}{T}}\right)-\frac{\lambda_q^{\ 4}}{4^2}\left(e^{\frac{-4\mu_q}{T}}+e^{\frac{-4(E-\mu_q)}{T}}\right)+\frac{\lambda_q^{\ 5}}{5^2}\left(e^{\frac{-5\mu_q}{T}}+e^{\frac{-5(E-\mu_q)}{T}}\right)-\cdots\frac{\lambda_q^{\ n}}{n^2}\left(e^{\frac{-n\mu_q}{T}}+e^{\frac{-n(E-\mu_q)}{T}}\right)\right]+E^2T\left[\frac{\lambda_q^{\ 1}}{1}\left(e^{\frac{-\mu_q}{T}}+e^{\frac{-(E-\mu_q)}{T}}\right)-\frac{\lambda_q^{\ 2}}{2}\left(e^{\frac{-2\mu_q}{T}}+e^{\frac{-2(E-\mu_q)}{T}}\right)+\frac{\lambda_q^{\ 3}}{3}\left(e^{\frac{-3\mu_q}{T}}+e^{\frac{-3(E-\mu_q)}{T}}\right)-\frac{\lambda_q^{\ 4}}{4}\left(e^{\frac{-4\mu_q}{T}}+e^{\frac{-4(E-\mu_q)}{T}}\right)+\frac{\lambda_q^{\ 5}}{5}\left(e^{\frac{-5\mu_q}{T}}+e^{\frac{-5(E-\mu_q)}{T}}\right)\ldots\frac{\lambda_q^{\ n}}{n}\left(e^{\frac{-n\mu_q}{T}}+e^{\frac{-n(E-\mu_q)}{T}}\right)\right)\Gamma(1) \quad (25)$$

We have

$$\mathrm{Im}\prod_{qg}^{H}(E,P) = (-1)\frac{N_c C_a}{\pi^4}g_E^{\ 2}g_H^{\ 2}\frac{T}{E^2}\left|\mathrm{I_T}-\mathrm{I_L}\right| \sum e_q^2 \times \left[2T^3\Gamma(3)\left(\frac{\lambda_q^{\ 1}}{1^3}-\frac{\lambda_q^{\ 2}}{2^3}+\frac{\lambda_q^{\ 3}}{3^3}-\frac{\lambda_q^{\ 4}}{4^3}+\frac{\lambda_q^{\ 5}}{5^3}-\cdots\frac{\lambda_q^{\ n}}{n^3}\right)+2ET^2\Gamma(2)\left(\frac{\lambda_q^{\ 1}}{1^2}-\frac{\lambda_q^{\ 2}}{2^2}+\frac{\lambda_q^{\ 3}}{3^2}-\frac{\lambda_q^{\ 4}}{4^2}+\frac{\lambda_q^{\ 5}}{5^2}-\cdots\frac{\lambda_q^{\ n}}{n^2}\right)+E^2T\Gamma(1)\left(\frac{\lambda_q^{\ 1}}{1}-\frac{\lambda_q^{\ 2}}{2}+\frac{\lambda_q^{\ 3}}{3}-\frac{\lambda_q^{\ 4}}{4}+\frac{\lambda_q^{\ 5}}{5}-\cdots\frac{\lambda_q^{\ n}}{n}\right)\right]\left[\left(e^{\frac{-\mu_q}{T}}+e^{\frac{-(E-\mu_q)}{T}}\right)+\left(e^{\frac{-2\mu_q}{T}}+e^{\frac{-2(E-\mu_q)}{T}}\right)+\left(e^{\frac{-3\mu_q}{T}}+e^{\frac{-3(E-\mu_q)}{T}}\right)+\left(e^{\frac{-4\mu_q}{T}}+e^{\frac{-4(E-\mu_q)}{T}}\right)+\left(e^{\frac{-5\mu_q}{T}}+e^{\frac{-5(E-\mu_q)}{T}}\right)\ldots+\left(e^{\frac{-n\mu_q}{T}}+e^{\frac{-n(E-\mu_q)}{T}}\right)\right) \quad (26)$$

We assume that

$$\gamma(E,T,\lambda_q,\mu_q) = \left[2T^3\Gamma(3)\left(\frac{\lambda_q^{\ 1}}{1^3}-\frac{\lambda_q^{\ 2}}{2^3}+\frac{\lambda_q^{\ 3}}{3^3}-\frac{\lambda_q^{\ 4}}{4^3}+\frac{\lambda_q^{\ 5}}{5^3}-\cdots\frac{\lambda_q^{\ n}}{n^3}\right)+2ET^2\Gamma(2)\left(\frac{\lambda_q^{\ 1}}{1^2}-\frac{\lambda_q^{\ 2}}{2^2}+\frac{\lambda_q^{\ 3}}{3^2}-\frac{\lambda_q^{\ 4}}{4^2}+\frac{\lambda_q^{\ 5}}{5^2}-\cdots\frac{\lambda_q^{\ n}}{n^2}\right)+E^2T\Gamma(1)\left(\frac{\lambda_q^{\ 1}}{1}-\frac{\lambda_q^{\ 2}}{2}+\frac{\lambda_q^{\ 3}}{3}-\frac{\lambda_q^{\ 4}}{4}+\frac{\lambda_q^{\ 5}}{5}-\cdots\frac{\lambda_q^{\ n}}{n}\right)\right]\left[\left(e^{\frac{-\mu_q}{T}}+e^{\frac{-(E-\mu_q)}{T}}\right)+\left(e^{\frac{-2\mu_q}{T}}+e^{\frac{-2(E-\mu_q)}{T}}\right)+\left(e^{\frac{-3\mu_q}{T}}+e^{\frac{-3(E-\mu_q)}{T}}\right)+\left(e^{\frac{-4\mu_q}{T}}+e^{\frac{-4(E-\mu_q)}{T}}\right)+\left(e^{\frac{-5\mu_q}{T}}+e^{\frac{-5(E-\mu_q)}{T}}\right)\ldots+\left(e^{\frac{-n\mu_q}{T}}+e^{\frac{-n(E-\mu_q)}{T}}\right)\right) \quad (27)$$

The Eq.(26) together Eq.(27) give

$$\mathrm{Im}\prod_{qg}^{H}(E,P) = (-1)\frac{N_c C_a}{\pi^4}g_E^{\ 2}g_H^{\ 2}\frac{T}{E^2}\left|\mathrm{I_T}-\mathrm{I_L}\right| \sum e_q^2\ \gamma(E,T,\lambda_q,\mu_q) \quad (28)$$

Inserting the Casimiro operator $C_a = \frac{(N_C^2-1)}{2N_c} = \frac{4}{3}$ relative to the color number $N_c = 3$ [19] and the effective strength coupling parameter for QCD theory is $\alpha_{QCD}(\mu^2) = \frac{g_H^{\ 2}}{4\pi}$ [20] and quantum electrodynamics QED coupling constant is $\alpha_{\mathrm{QED}} = \frac{g_E^{\ 2}}{4\pi}$[21] in Eq.(28) to become

$$\mathrm{Im}\prod_{qg}^{H}(E,P) = (-1)4\frac{N}{\pi^2}\frac{16}{3}\alpha_{\mathrm{QED}}\alpha_{QCD}(\mu^2)\sum e_q^2\ \left|\mathrm{J_T}-\mathrm{J_L}\right|\ \frac{T}{E^2}\gamma(E,T,\lambda_q,\mu_q) \quad (29)$$

Inserting Eq.(29) in Eq.(1) to result

$$\mathrm{R}_{qg}^{H}(E,P) = \frac{8N}{3\pi^5}\alpha_{\mathrm{QED}}\alpha_{QCD}(\mu^2)\sum e_q^2\ \left|\mathrm{J_T}-\mathrm{J_L}\right|\ \frac{T}{E^2}\mathrm{F}_B(\mathrm{E})\ \frac{T}{E^2}\gamma(E,T,\lambda_q,\mu_q) \quad (30)$$

The Bosonic function distribution for gluon $\mathrm{f}_B(\mathrm{E}) = \frac{\lambda_g}{e^{\frac{E}{T}}-\lambda_g}$ [22],then Eq.(30) becomes

$$\mathrm{R}_{qg}^{H}(E,P) = \left(\frac{8N}{3\pi^5}\right)\frac{\lambda_g}{e^{E/T}-\lambda_g}\alpha_{\mathrm{QED}}\alpha_{QCD}(\mu^2)\frac{T}{E^2}\left|\mathrm{I_T}-\mathrm{I_L}\right|\ \sum e_q^2\ \gamma(E,T,\lambda_q,\mu_q) \quad (31)$$

the strong coupling constant is [23].

$$\alpha_{QCD}(\mu^2) = \frac{6\pi}{(33-2\,N_F)\,ln\frac{8T}{T_c}} \quad (32)$$

Where N_F is the flavor number of quarks, T is the temperature of system and T_c is the critical temperature for the quark – gluon interaction; it is given by [24].

$$T_c = (\frac{90B}{\pi^2 d_{gq}})^{\frac{1}{4}} \tag{33}$$

where B is the bag coefficient and d_{gq} is the degeneracy factor for gluons and quarks. It can be given by expression [25].

$$d_{gq} = d_g + \frac{7}{8}\,(d_q + d_{\bar{q}}) \tag{34}$$

where d_g is the number of gluons degrees of freedom as a function of the gluons spin state n_s and gluons color states n_c and d_q is the number of quark degrees of freedom as function of the number of quark colour n_c, spin n_s and flavour degrees of freedom n_f. Inserting Eq.(34) in Eq.(33) to result

$$T_c = (\frac{90B}{\pi^2[(n_s \times n_c) + \frac{7}{4}\,(n_c \times n_s \times n_f)]})^{\frac{1}{4}} \tag{35}$$

3.Results

An essential estimation of the critical temperature is predicted near the phase transition scale, which is called the hadronic phase. One of the most technical predictions of the calculation of the critical temperature depends on the bag coefficient B in Eq.(33) and the degeneracy factors for gluons and quarks d_{gq} in Eq.(34) relative to the spin state n_s, color states n_c and flavour degrees n_f. Inserting $n_s = 2$ together with $n_c = 8$ for gluons and $n_c = 3$, $n_s = 2$ and $n_f = 3$ for quarks system and the Bag constant (200,240,270 and 300)MeV in Eq.(35) to give the results listed in **Table (1)**

Table 1 . Result of critical temperature uses the Bag mode of the quark –gluon system for $\bar{u}$g →$\bar{d}$ γ System.

Bag constant $B^{1/4} MeV$	Critical temperature T_c MeV
200	132.38
240	158.86
270	178.72
300	198.57

To evaluate the strength coupling from Eq.(32), we inert the critical temperature of the $\bar{u}$g →$\bar{d}$ γ system from **Table (1),** taking the temperature of $\bar{u}$g →$\bar{d}$ γ system in range (T=180, 210, 240, 270, 300, 330 and 360) MeV and $n_f = 3$, the results of strength coupling are shown in **Table 2**.

Table 2 . Strength coupling for $\bar{u}$g →$\bar{d}$ γ system at different critical temperature with variety temperature of system

T_C	α_{QCD}						
	T=180MeV	T=210MeV	T=240MeV	T=270MeV	T=300MeV	T=330MeV	T=360MeV
132.386068274	0.2925	0.2748	0.2610	0.2500	0.2409	0.2333	0.2267
158.8632819288	0.3167	0.2960	0.2801	0.2675	0.2571	0.2484	0.2409
178.7211921699	0.3346	0.3116	0.2940	0.2801	0.2688	0.2593	0.2512
198.579102411	0.3524	0.3269	0.3077	0.2925	0.2801	0.2698	0.2610

..

For simplicity in calculating the rate of the photon produced from the interaction of the quark-gluon system, we estimate the total quark charge and total flavour number in the quark system. The quark charge is the summation charge of $\sum e_q{}^2 = {}^5/_9$ of $\bar{u}$g →$\bar{d}$ γ system where the charge of the up quark is $+{}^2/_3$ and the anti-down is $-{}^1/_3$,. while the net favor is $n_f = 3$ for the interaction system. The photon rate produced from the interaction of the anti-up with the anti-down system is calculated using Eq. (31) by inserting the photon energy from experimental data in range $E = 1\ to\ 10GeV$ [26], critical temperature from **Table 1**, strength coupling $\alpha_{QCD}(\mu^2)$ from **Table 2**, and fugacity of λ_q=0.02 for quark, λ_g=0.06 for gluon [23], taking the self-integral constants $\mathrm{I_T} = 4.45$ and $\mathrm{I_L} = -4.26$ [15] and using the chemical potential $\mu_q = 500\ MeV$ [27] and $\alpha_{\mathrm{QED}} = 1/137$ and $N = 3$.

Results are shown in tables (3), (4), (5), (6) and figures (1), (2), (3) and (4) taking $\mathrm{I_T} = 4.45$ and $\mathrm{I_L} = -4.26$ with λ_g=0.06 for gluon , $\lambda_{\bar{q}}$=0.02 for quark in $\bar{u}$g →$\bar{d}$ γ system .

Table 3. Rate of photon production at T_c =132.38 MeV , $\mathrm{I_T} = 4.45$ and $\mathrm{I_L} = -4.26$ with λ_g=0.06, $\lambda_{\bar{q}}$=0.02 in $\bar{u}$g →$\bar{d}$ γ system .

$E_\gamma\ GeV$	$\mathrm{R}^H_{qg}(E,P)\,\frac{1}{GeV^2 fm^4}$						
	T=180MeV α_{QCD} =0.2925	T=210MeV α_{QCD} =0.2748	T=240MeV α_{QCD} =0.2610	T=270MeV α_{QCD} =0.2500	T=300MeV α_{QCD} =0.2409	T=330MeV α_{QCD} =0.2333	T=360MeV α_{QCD} =0.2267
1	8.0004E-12	3.7264E-11	1.2547E-10	3.3889E-10	7.8194E-10	1.6049E-09	3.0106E-09
2	1.2629E-14	1.2604E-13	7.4724E-13	3.1194E-12	1.0159E-11	2.7551E-11	6.5013E-11
3	4.5519E-17	9.8877E-16	1.0445E-14	6.7986E-14	3.1444E-13	1.1319E-12	3.3708E-12
4	1.7034E-19	8.1351E-18	1.5491E-16	1.5921E-15	1.0596E-14	5.1299E-14	1.9529E-13
5	6.4597E-22	6.7993E-20	2.3400E-18	3.8075E-17	3.6571E-16	2.3886E-15	1.1663E-14
6	2.4658E-24	5.7269E-22	3.5661E-20	9.1982E-19	1.2766E-17	1.1263E-16	7.0626E-16
7	9.4472E-27	4.8446E-24	5.4620E-22	2.2347E-20	4.4845E-19	5.3484E-18	4.3101E-17
8	3.6278E-29	4.1093E-26	8.3919E-24	5.4486E-22	1.5816E-20	2.5509E-19	2.6430E-18
9	1.3952E-31	3.4919E-28	1.2920E-25	1.3315E-23	5.5925E-22	1.2201E-20	1.6258E-19
10	5.3715E-34	2.9708E-30	1.9919E-27	3.2592E-25	1.9811E-23	5.8476E-22	1.0023E-20

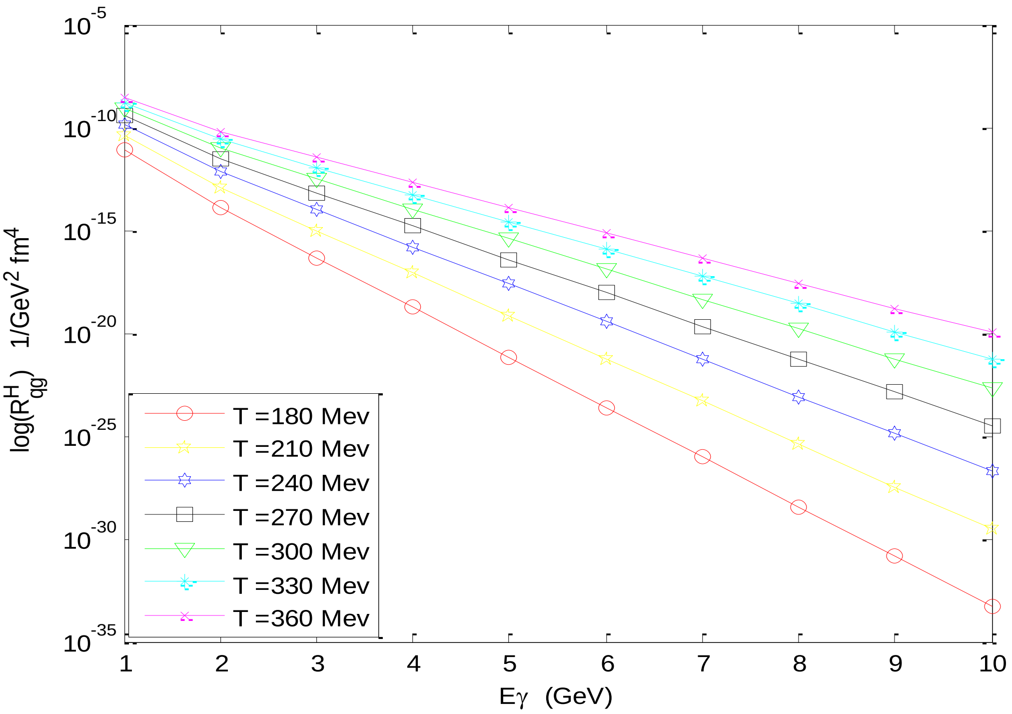

Figure1. The rate of photon produces as afunction to gamma energy produced at T_c =132.38MeV

Table 4. Rate of photon production at $T_c = 158.86\ MeV$, $I_T = 4.45$ and $I_L = -4.26$ with λ_g=0.06, $\lambda_{\bar{q}}$=0.02 in system $\bar{u}$g →$\bar{d}$ γ system .

$E_\gamma\ GeV$	$R^{H}_{qg}(E,P)\frac{1}{GeV^2 fm^4}$						
	T=180MeV α_{QCD} =0.3167	T=210MeV α_{QCD} =0.2960	T=240MeV α_{QCD} =0.2801	T=270MeV α_{QCD} =0.2675	T=300MeV α_{QCD} =0.2571	T=330MeV α_{QCD} =0.2484	T=360MeV α_{QCD} =0.2409
1	8.6621E-12	4.0145E-11	1.3465E-10	3.6256E-10	8.3445E-10	1.7090E-09	3.2001E-09
2	1.3674E-14	1.3579E-13	8.0190E-13	3.3373E-12	1.0841E-11	2.9338E-11	6.9104E-11
3	4.9284E-17	1.0652E-15	1.1209E-14	7.2736E-14	3.3555E-13	1.2053E-12	3.5829E-12
4	1.8443E-19	8.7640E-18	1.6625E-16	1.7033E-15	1.1308E-14	5.4627E-14	2.0757E-13
5	6.9940E-22	7.3250E-20	2.5112E-18	4.0735E-17	3.9026E-16	2.5436E-15	1.2397E-14
6	2.6697E-24	6.1696E-22	3.8270E-20	9.8408E-19	1.3623E-17	1.1994E-16	7.5070E-16
7	1.0229E-26	5.2191E-24	5.8616E-22	2.3909E-20	4.7856E-19	5.6954E-18	4.5814E-17
8	3.9278E-29	4.4270E-26	9.0059E-24	5.8292E-22	1.6878E-20	2.7163E-19	2.8093E-18
9	1.5106E-31	3.7618E-28	1.3865E-25	1.4245E-23	5.9680E-22	1.2993E-20	1.7281E-19
10	5.8157E-34	3.2005E-30	2.1377E-27	3.4869E-25	2.1141E-23	6.2269E-22	1.0654E-20

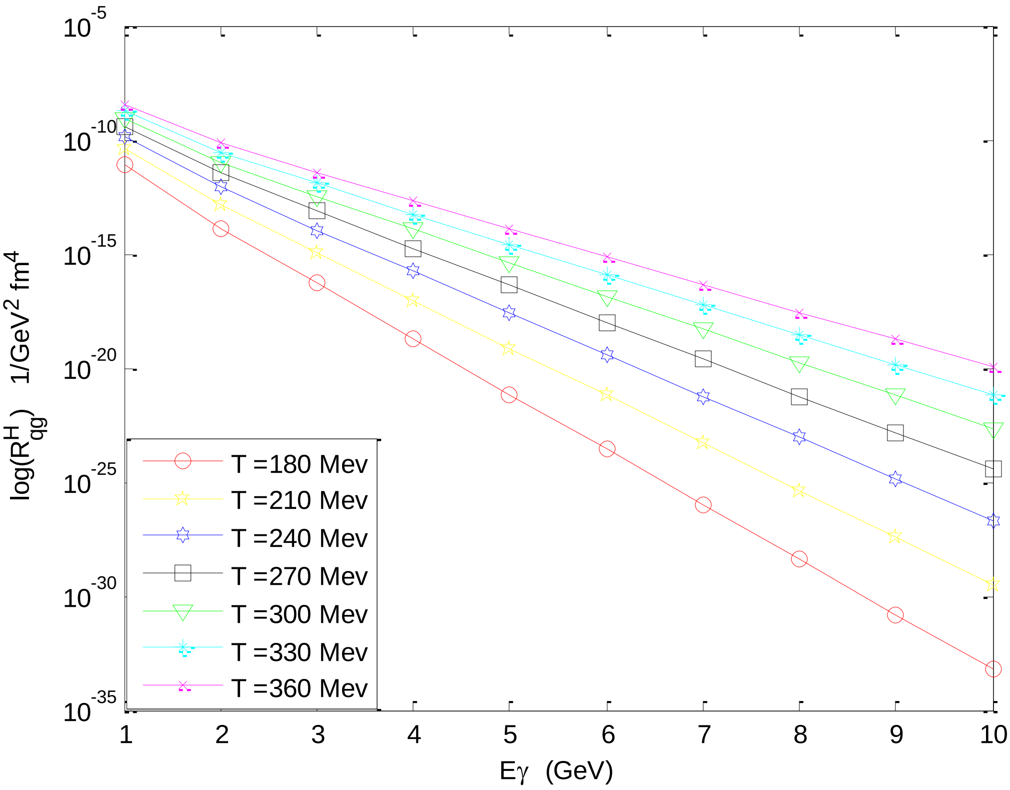

Figure2. The rate of photon produces as afunction to gamma energy produced at $T_c = 158.86MeV$.

Table 5. Rate of photon production at $T_c = 178.72\ MeV$, $I_T = 4.45$ and $I_L = -4.26$ with λ_g=006, $\lambda_{\bar{q}}$=0.02 in $\bar{u}$g →$\bar{d}$ γ system.

$E_\gamma\ GeV$	$R^H_{qg}(E,P)\frac{1}{GeV^2 fm^4}$						
	T=180MeV α_{QCD} =0.3346	T=210MeV α_{QCD} =0.3116	T=240MeV α_{QCD} =0.2940	T=270MeV α_{QCD} =0.2801	T=300MeV α_{QCD} =0.2688	T=330MeV α_{QCD} =0.2593	T=360MeV α_{QCD} =0.2512
1	9.1511E-12	4.2255E-11	1.4133E-10	3.7970E-10	8.7229E-10	1.7837E-09	3.3357E-09
2	1.4446E-14	1.4292E-13	8.4169E-13	3.4951E-12	1.1332E-11	3.0621E-11	7.2032E-11
3	5.2066E-17	1.1212E-15	1.1765E-14	7.6174E-14	3.5077E-13	1.2580E-12	3.7347E-12
4	1.9484E-19	9.2247E-18	1.7450E-16	1.7838E-15	1.1820E-14	5.7016E-14	2.1637E-13
5	7.3888E-22	7.7100E-20	2.6357E-18	4.2661E-17	4.0796E-16	2.6549E-15	1.2922E-14
6	2.8204E-24	6.4939E-22	4.0168E-20	1.0306E-18	1.4240E-17	1.2518E-16	7.8251E-16
7	1.0806E-26	5.4935E-24	6.1524E-22	2.5039E-20	5.0026E-19	5.9445E-18	4.7755E-17
8	4.1495E-29	4.6597E-26	9.4526E-24	6.1047E-22	1.7643E-20	2.8352E-19	2.9283E-18
9	1.5959E-31	3.9595E-28	1.4553E-25	1.4919E-23	6.2387E-22	1.3561E-20	1.8013E-19
10	6.1440E-34	3.3687E-30	2.2437E-27	3.6517E-25	2.2100E-23	6.4993E-22	1.1105E-20

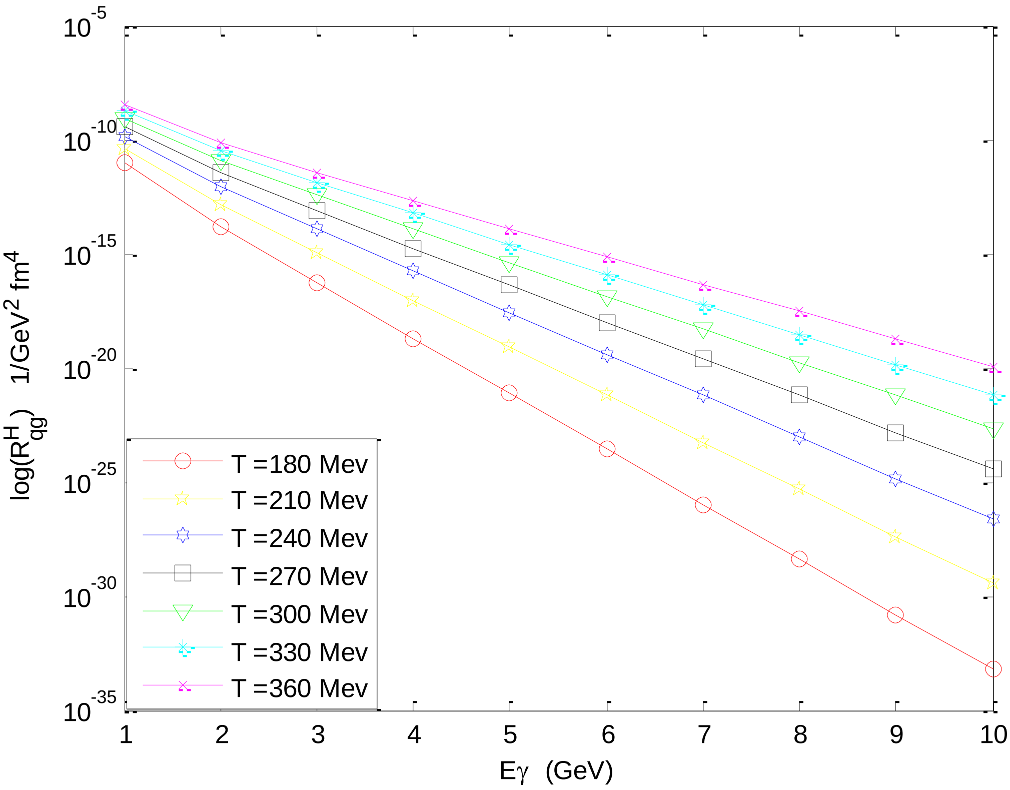

Figure 3. The rate of photon produces as afunction to gamma energy produced at $T_c = 178.72\ MeV$.

Table 6. Rate of photon production at $T_c = 198.57\ MeV$, $I_T = 4.45$ and $I_L = -4.26$ with λ_g=0.06, $\lambda_{\bar{q}}$=0.02 in $\bar{u}$g →$\bar{d}$ γ system .

$E_\gamma\ GeV$	$R^H_{qg}(E,P)\,\frac{1}{GeV^2 fm^4}$						
	T=180MeV α_{QCD} =0.3524	T=210MeV α_{QCD} =0.3269	T=240MeV α_{QCD} =0.3077	T=270MeV α_{QCD} =0.2925	T=300MeV α_{QCD} =0.2801	T=330MeV α_{QCD} =0.2698	T=360MeV α_{QCD} =0.2610
1	9.6378E-12	4.4340E-11	1.4789E-10	3.9646E-10	9.0917E-10	1.8564E-09	3.4671E-09
2	1.5214E-14	1.4998E-13	8.8077E-13	3.6494E-12	1.1811E-11	3.1868E-11	7.4870E-11
3	5.4834E-17	1.1765E-15	1.2312E-14	7.9536E-14	3.6560E-13	1.3093E-12	3.8818E-12
4	2.0520E-19	9.6798E-18	1.8260E-16	1.8626E-15	1.2320E-14	5.9338E-14	2.2489E-13
5	7.7817E-22	8.0904E-20	2.7581E-18	4.4544E-17	4.2521E-16	2.7630E-15	1.3431E-14
6	2.9704E-24	6.8143E-22	4.2034E-20	1.0761E-18	1.4843E-17	1.3028E-16	8.1334E-16
7	1.1381E-26	5.7645E-24	6.4381E-22	2.6144E-20	5.2141E-19	6.1866E-18	4.9636E-17
8	4.3702E-29	4.8896E-26	9.8916E-24	6.3742E-22	1.8389E-20	2.9506E-19	3.0437E-18
9	1.6807E-31	4.1549E-28	1.5229E-25	1.5577E-23	6.5024E-22	1.4113E-20	1.8723E-19
10	6.4707E-34	3.5349E-30	2.3479E-27	3.8129E-25	2.3034E-23	6.7640E-22	1.1543E-20

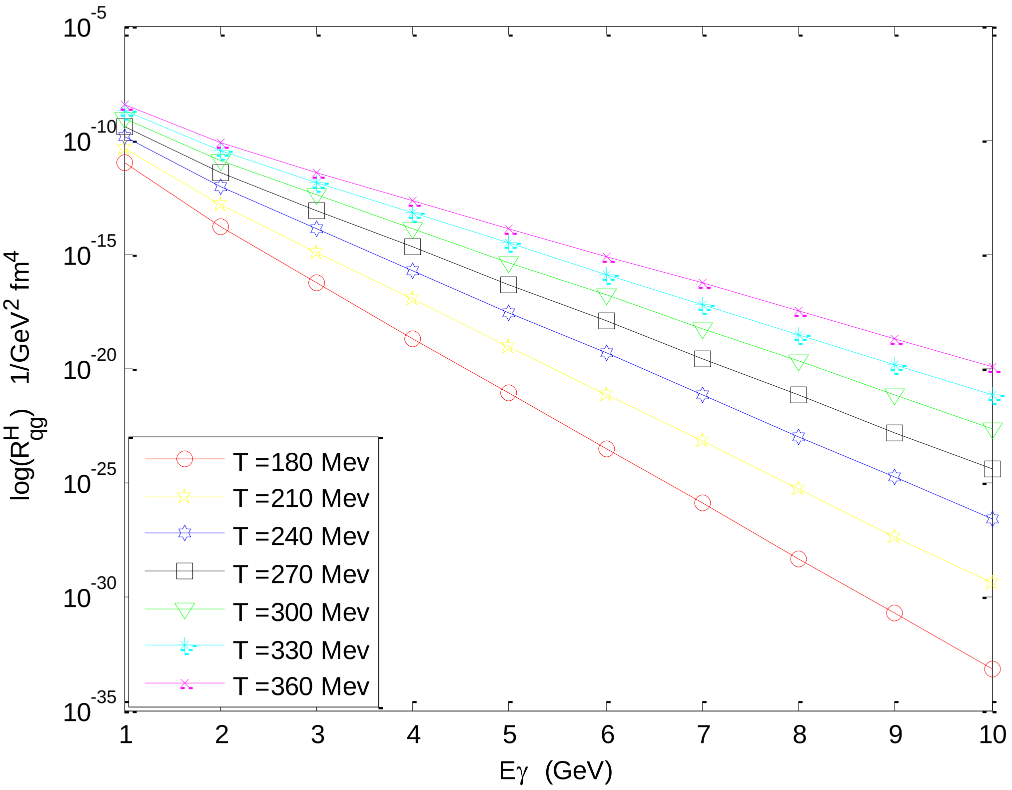

Figure4. The rate of photon produces as afunction to gamma energy produced at $T_c = 198.57MeV$.

4.Discussion

Due to the photon rate expression in Eq. (31), we can find that the rate relates to many parameters such as strength coupling, temperature of the system, photon energy, fugacity of quark and gluon, quark charge, flavour number of the system, critical temperature, and strong coupling $\gamma(E,T,\lambda_q,\mu_q)$. The critical temperature is influenced by the Bag constant and flavour number. The critical temperature T_c in **Table 1** increases with increasing the bag constant.It reaches a minimum of 132.38 at minimum bag constant of 200MeV and a maximum of 198.57 at a maximum bag constant 300MeV. This indicates that we consider the bag constant dependent on the density of quark-gluon matter in the $\bar{u}$g $\rightarrow\bar{d}$ γ system and this agrees with the results of the CERN SPS on the formation of a quark-gluon [28]. On the other hand, one of the main motivations for performing the calculation with different critical temperatures is to know the effect of the critical temperature on the photons that are produced. However, the critical temperature is the main factor affecting the rate of photons through its effect on strength coupling. The strength coupling result in **Table 2** shows that it increases with an increase in the critical temperature. Moreover, the strength of coupling decreased with the increased in temperature of the system from 180 MeV to 360 MeV because the coupling between quarks and gluons decreases with increasing the temperature from 180 MeV to 360 MeV in $\bar{u}$g $\rightarrow\bar{d}$ γ system. However, the strength coupling is related to the critical temperature and the temperature of the $\bar{u}$g $\rightarrow\bar{d}$ γ system due to Eq. (34). It can be seen from **Table 2** that the strength coupling increases with increasing the critical temperature from 132.38 MeV to 198.57 MeV and decreases with increasing the temperature of the system from 180 MeV to 360 MeV. Due to Eq.(31), the calculation of photon rate produced from interaction in $\bar{u}$g $\rightarrow\bar{d}$ γ system at the temperature system in range 180 MeV $\leq$ T and $T \leq$

360 has been done in **Tables (3), (4), (5) and (6)** and is plotted in **Figures (1), (2), (3) and (4),** respectively.

In the theoretical calculation of photon rate in order of $\frac{1}{GeV^2 fm^4}$ for anti-up gluon–anti down photon interaction with different critical temperatures and strength couplings α_{QCD} =0.2925, 0.2748, 0.2610, 0.2500, 0.2409, 0.2333 and 0.2267 in range of temperature of the system from 180 to 360 MeV. The maximum photon rate in all tables (3), (4), (5) and (6) is a maximum of $3.0106E-09\ \frac{1}{GeV^2 fm^4}$ for T_c =132.38 MeV, $\alpha_{QCD} = 3.2001E-09\frac{1}{GeV^2 fm^4}$, for $T_c = 158.86\ MeV$, $3.3357E-09\frac{1}{GeV^2 fm^4}$ for $T_c = 178.72\ MeV$ and $3.4671E-09\frac{1}{GeV^2 fm^4}$ for $T_c = 198.57\ MeV$ at $E = 1\ GeV$ and T=360 MeV with a minimum strength coupling of α_{QCD} =0.2610, while the photon rates are minimum in tables (3),(4),(5) and (6) for a minimum of $5.3715E-34\ \frac{1}{GeV^2 fm^4}$ for T_c =132.38 MeV, $5.8157E-34\frac{1}{GeV^2 fm^4}$, for$T_c = 158.86\ MeV$, $6.1440E-34\frac{1}{GeV^2 fm^4}$ for $T_c = 178.72\ MeV$ and $6.4707E-34\frac{1}{GeV^2 fm^4}$ for $T_c = 198.57\ MeV$ at $E = 10\ GeV$ and T=180 MeV with a maximum strength coupling of $\alpha_{QCD} = 0.2925$. Figures 1, 2, 3 and 4 indicate the behavior of photon emission rate. There is a decrease with increased photons energy E(GeV) at a critical temperature of 132.38, 158.86, 178.72 and 198.57 MeV and variety temperatures of the system with flavors number $n_f = 3$. The photon emission rate increases with the increase in temperature of the system and decreases with the strength of coupling at the critical temperature of $\bar{u}$g →$\bar{d}$ γ system. Moreover, the rate of photon emission in **Table 6** and **Figure 4** with the critical temperature ($T_c = 198.57$)MeV is larger than the photon rate in other **Tables (3)**, **(4), and (5).**

As we can see, the contribution of rate produced from interaction $\bar{u}$g →$\bar{d}$ γ system in all **Tables (3), (4), (5), and (6)** and four **Figures (1), (2), (3), (4), and (5)** reach the maximum at the photons energy $E \leq 2GeV$ comparing the minimum at $E \geq 2GeV$ and reach the minimum at E=10 GeV. In fact, the theoretical calculation of the photon rate increases in the high and is affected by increasing the temperature of the system and decreasing the coupling between quark and gluon for all critical temperatures in **Tables (3), (4), (5) and (6)** for all four critical temperatures in the $\bar{u}$g →$\bar{d}$ γ system with $n_f = 3$.

5.Conclusion

In the present work, the photon rate is calculated. It also studies the effect of the critical temperature and strong coupling of massless quark flavors at chemical potential $\mu_q = 500\ MeV$ at the interaction of $\bar{u}$g →$\bar{d}$ γ system. The equations of photon rate, critical temperatures and strength coupling are derived for the quark-gluon state consisting of an anti-up quark interacting with a gluon to produce an anti-down with photons gamma. Also, the total flavour number of the quark gluon state is $n_f = 3$. We can conclude that there is a significant influence on the strength coupling and critical temperature term in the case of the study of the photon rate properties of the quark-gluon interaction. The strength coupling and critical temperature are the main pure features of the QCD effect on the photon rate of the anti up -gluon interaction at the range of temperature system 180-360 MeV.

The QCD features of the critical temperature, strength coupling, and photon energy distribution for the photon emission state were quantitatively achieved in the system for n_f = 2 + 1. This is a unique feature of the photon rate spectrum. The interesting point in our results is the minimum value of photon rate, especially in the photon energy E=10 GeV of $n_f = 3$ flavor, which reflects the weak correlation between quarks and gluons already expected. We conclude that the photon emission rate production at high energy is a good tool to study nucleon structure.

References

1. Al-agealy, H.J.; Ghadhban, R.Q.; Hassooni, M.A.; Theoretical Study of the Photons Production Kinetic In Hot Quark-Gluon Plasma Matter. *Ibn AL-Haitham Journal For Pure and Applied Sciences*, **2020**, *33*(4),34-41.

2. Sinyukov, Y.; Shapoval, V.; Direct Photon Production in High-Energy Heavy Ion Collisions within the Integrated Hydrokinetic Model. *J*, **2022**, *5*(1), 1-14.

3. Aarts, G.; Allton, C.; Hands, S.; Jäger, B.; Praki, C.; Skullerud, J. I.; Nucleons and parity doubling across the deconfinement transition. *Physical Review D*, **2015,** *92*(1),014503.

4. Wang, X.; Shovkovy, I.A.; Yu, L.; Huang, M.; Ellipticity of photon emission from strongly magnetized hot QCD plasma. *Physical Review D*,**2020**, *102*(7), p.076010.

5. Bhattacharya, T.; Buchoff, M.I.; Christ, N.H.; Ding, H.T.; Gupta, R., Jung, C., Karsch, F., Lin, Z., Mawhinney, R.D., McGlynn, G. and Mukherjee, S. QCD phase transition with chiral quarks and physical quark masses. *Physical review letters*,**2014**, *113*(8),.082001.

6. Cè, M.; Harris, T.; Meyer, H.B.; Steinberg, A.; Toniato, A.; Rate of photon production in the quark-gluon plasma from lattice QCD. *Physical Review D*, **2020**,*102*(9),091501.

7. Al-Agealy, H. J.; Hussein, H. H.; Hussein, S.M.; May. Theoretical estimation of Photons flow rate Production in quark gluon interaction at high energies. In *Journal of Physics: Conference Series*. *IOP Publishing.* **2018,** *1003,* 1, 012119.

8. Bonati, C.; De Forcrand, P.; D'Elia, M.; Philipsen, O.; Sanfilippo, F.; Chiral phase transition in two-flavor QCD from an imaginary chemical potential. *Physical Review D*,**2014,** *90*(7),.074030.

9. Yi Y.; Cheng-Wei L.; Search for axion(-like) particles in heavy-ion collisions. *JhEP*,**2022**,*7*,28-33.

10. Hadi, J. M.; Al-agealy , Al Attabi H. D.; Hassan, A. A. Theoretical Calculation of the Photons Rate for the Quark-Gluon System at Compton Scattering, *International Journal of Science and Research (IJSR)* , **2016,** *5* , 8, 1770-1775,

11. Hadi, J. M.; Al-Agealy, Hyder, H. H.; Saba, M. H.; Theoretical estimation of Photons flow rate Production in quark gluon interaction at high energies, *IOP Conf. Series: Journal of Physics: Conf. Series 1003.* **2018,** *10.*1088/1742-6596/1003/1/012119.

12. Ahmed, M. A.; Saba, M. H.; Hadi, J. M.; Al-Agealy, Mohsin A. H.; Flow Production Rate of Hard Photons Probes of Quark–Anti Quark Annihilation Processes at Plasma Phase " IOP Conf. Series: Materials Science and Engineering 871. **2020**. 012089 IOP Publishing doi:10.1088/1757-899X/871/1/012089

13. Elaf, M. A.; Hadi, J. M.; Al-Agealy, Nada, F. K.; Theoretical Calculation of Photon Emission from Quark-Antiquark Annihilation Using QCD Theory. *Ibn Al-Haitham Journal for Pure and Applied Sciences, IHJPAS*. **2022,** *35*(*4*), 22-34.

14. Gale, C.; J. I.; Kapusta, Nucl. Phys. B357, 65: P. Aurenche"Photon Production in Heavy Ion Collisions"November 4, 2018, Laboratoire de Physique Th´eorique LAPTH1, BP110, F-

74941, Annecy le Vieux Cedex, France, LAPTH-Conf-893/2001, UMR 5108 du CNRS, associ´ee a` l'Universit´e de Savoie. **1991,** *23*, 45-55

15. Aurenche, P.; Gelis, F.; Zaraket, H.; Kobes, R.; Bremsstrahlung and photon production in thermal QCD. *Physical Review D*, **1998,** *58*(8), p.085003.

16. Dutta, D.; Sastry, S.V.S.; Mohanty, A.K.; Kumar, K.; Hard photon production from unsaturated quark–gluon plasma at two-loop level. *Nuclear Physics A*, **2002,** *710*(3-4), 415-438.

17. Long, J. L.; He, Z. J.; Ma, Y.G.; Liu, B.; Hard photon production from a chemically equilibrating quark-gluon plasma with finite baryon density at one loop and two loop. *Physical Review C*, **2005**, *72*(6), p.064907.

18. Al-Agealy, H. J.; Hussein, H. H.; Hussein, S. M.; Theoretical estimation of Photons flow rate Production in quark gluon interaction at high energies. In *Journal of Physics: Conference Series IOP Publishing.* . **2018,** 1003, 012119.

19. Peskin, M.E.; *An introduction to quantum field theory*. CRC press. **2018.**

20. Cottingham, W.N.; Greenwood, D.A.; *An introduction to the standard model of particle physics*. Cambridge university press. **2007.**

21. Bkmurd, R.I.; Al-Agealy, H. J.; Ashwiekh, A.M.; Investigation and Study of Photonic Current Rate in Bremsstrahlung process. In *Journal of Physics: Conference Series*. **2021,** *1879*, 3, 032094. IOP Publishing.

22. Biro, T.S.; Van Doorn, E.; Müller, B.; Thoma, M.H.; Wang, X.N.; Parton equilibration in relativistic heavy ion collisions. *Physical Review C*, **1993**,*48*(3),.1275.

23. Ahmed, E. M.; Al-Agealy, H. J.; Kadhim, N. F.; Theoretical Study of Photons Spectra around High Energy of Quark-antiquark Using QCD Theory. *NeuroQuantology*, **2022,** *20*(4), 58.

24. Saleh, M.S.; Theoretical Study of the Fugacity Effect on the Photons Emission at Annihilation Quarks Interaction(*Doctoral dissertation, Ms Thesis, Baghdad University*). **2018.**

25. Begun, V.V.; Gorenstein, M. I.; Mogilevsky, O.A.; Equation of state for the quark gluon plasma with the negative bag constant. *arXiv preprint arXiv:***2010,***1001.3139*.

26. Kasmaei, B.S.; Strickland, M.; Photon production and elliptic flow from a momentum-anisotropic quark-gluon plasma. *Physical Review D*, **2020**. *102*(1), p.014037.

27. Peitzmann, T.; Thoma, M. H.; Direct photons from relativistic heavy-ion collisions. *Physics Reports*, **2002,***364*(3), 175-246.

28. Yazdizadeh, T.; Bordbar, G.H.; The effect of a density-dependent bag constant on the structure of a hot neutron star with a quark core. *Astrophysics*, **2013,** *56*(1), 121-129.