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Fully Fuzzy Visible Modules with other Related Concepts

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Abstract

In previous our research, the concepts of visible submodules and fully visible modules were introduced, and then these two concepts were fuzzified to fuzzy visible submodules and fully fuzzy. The main goal of this paper is to study the relationships between fully fuzzy visible modules and some types of fuzzy modules such as semiprime, prime, quasi, divisible, F-regular, quasi injective, and duo fuzzy modules, where under certain conditions it has been proven that each fully fuzzy visible module is fuzzy duo. In addition, there are many various properties and important results obtained through this research, which have been illustrated. Also, fuzzy Artinian modules and fuzzy fully stable modules have been introduced, and we study the relationships between these kinds of modules and fully fuzzy visible modules. Many other intersecting results we found.

Keywords: Fully fuzzy visible modules , fully fuzzy stable modules, fuzzy Artiain modules, fuzzy regular rings, fuzzy quasi injective modules.

1. Introduction.

The notion of fuzzy set has been presented by Zadeh in 1965[1].The fuzzy groups have been introduced by Rosenfeld in 1971 [2]. Then, many other researchers studied different applications of fuzzy sets of algebra. The concept of fuzzy modules was presented by Negoita and Relescn in 1975 [3]. The notion of fuzzy visible submodules was introduced by Sajda K.M. and Buthyna N. S. 2021 [4]. The concept of fully fuzzy visible modules has been introduced by Sajda K.M. and Buthyna N.S. in 2022 [5]. In this paper, the relationships between fully fuzzy visible modules and other different modules have been explained, like fuzzy (prime, semiprime, semisimple, and F-egular). Also, the notions of stable submodules, fully stable modules, and quasi-injective modules are fuzzyified. Many important properties and results between these above concepts and fully fuzzy visible modules have been proven. Through this paper will be a unitary module over a commutative ring with identity and for each fuzzy ideal Ʉ of $\mathbb{F}$, *Ʉ (0)=1*.

2. Preliminaries. This section contains some definitions and properties that will be needed in our work.

2.1 Definition: Let $\mathbb{F} \neq \emptyset$ be a set and Ɨ = [0, 1] of the real line (real numbers). Let ɦ: $\mathbb{F} \rightarrow$ Ɨ is a function. Then, ɦ is known as a fuzzy set in $\mathbb{F}$ (a fuzzy subset of, [1]. Let *FSF(*$\mathbb{F}$) = {ɦ: ɦ: $\mathbb{F} \rightarrow$ Ɨ is a function }.

2.2 Definition. Let $\mathbb{Y} \neq \emptyset$ and $\mathfrak{C} \in FUS(\mathbb{Y})$. A level set of $\mathbb{Y}$ with respect to $\mathfrak{C}$ denoted by the set $\mathfrak{C}_t$, where $\mathfrak{C}_t=\{x\in \mathbb{Y} : \mathfrak{C}(x)\geq t\}$ $\forall t \in [0,1]$, [6- 8] .

2.3 Definition: Suppose that $\mathbb{M}$ be an $\mathbb{F}$ –module and Ŧ $\in FSE(\mathbb{M})$. $\mathbb{P}$ is known as a fuzzy module of an $\mathbb{F}$ -module if,

1- Ŧ *(ẕ - ƫ)*≥min { Ŧ (ẕ), Ŧ (ƫ) }, for all ẕ ,ƫ∈ $\mathbb{M}$,

2- Ŧ *(rẕ)*≥ Ŧ (ẕ), for all ẕ∈ $\mathbb{M}$, *r*∈ $\mathbb{F}$,

3- Ŧ (0)=1 [9-10] .

Let $FUM(\mathbb{M}) = \{$Ŧ: Ŧ is a fuzzy modules of an $\mathbb{F}$ − module $\mathbb{M}$ }.

2.4 Definition: Let $\mathfrak{C}$, $\mathbb{P} \in FUM(\mathbb{M})$. $\mathfrak{C}$ is known as a fuzzy submodule of $\mathbb{P}$ if $\mathfrak{C} \subseteq \mathbb{P}$ [11- 13] .

Let *FUS(*$\mathbb{P}$) = {$\mathfrak{C}$: $\mathfrak{C} \in FUM(\mathbb{M})$ and $\mathfrak{C} \subseteq \mathbb{P}$ }.

2.5 Definition: Let $\mathbb{F}$ be a ring and Ί $\in FSE(\mathbb{F})$. Ί is referred to as a fuzzy ideal of $\mathbb{F}$, if ∀ ẕ, ƫ∈ $\mathbb{F}$:

1-*Ί (ẕ -y)≥ min {Ί (ẕ), Ί (ƫ)}*

2-*Ί (ẕ.y)≥ max {Ί (ẕ), Ί (ƫ)},* [14] .

FUI($\mathbb{R}$ *)* will be used to represent the set of all fuzzy ideals of $\mathbb{F}$.

2.6 Definition: Let $\mathbb{P} \in FUM(\mathbb{M})$,$\mathfrak{C} \in FUS(\mathbb{P})$ and Ί $\in FUI(\mathbb{F})$.The product (Ί $\mathfrak{C}$)*(x)* =

$$\begin{cases} sup\{inf\{Ί(r_1), \dots, Ί(r_n), \mathfrak{C}(x_1), \dots, \mathfrak{C}(x_n) for\ some\ r_i \in \mathbb{F}, x_i \in \mathbb{M}, n \in N, & x = \sum_{i=1}^{n} r_i x_i \\ 0 & o.w. \end{cases}$$

,[9] .

Note that Ί $\mathfrak{C} \in FUS(\mathbb{P})$ if Ί *(0)=1*, [9]. And $(Ί\,\mathfrak{C})_t = Ί_t \mathfrak{C}_t$ for each $t \in [0,1]$, [15].

2.7 Definition Let ғ be a mapping from a set $\mathbb{M}$ into a set N, Let $\mathbb{A} \in$ FSE($\mathbb{M}$) and $\mathbb{B} \in FSE(N)$. The image of $\mathbb{A}$ denoted by ғ ($\mathbb{A}$) $\in FSE(N)$ defined by :

$$ғ(\mathbb{A})(e) = \begin{cases} \{sup\{\mathbb{A}(z): z \in ғ^{-1}(e), if\ ғ^{-1}(e) \neq \emptyset, for\ all\ e \in N \\ 0 \quad o.w. \end{cases},$$

and the inverse image of $\mathbb{B}$ denoted by $f^{-1}(\mathbb{B}) \in FSE(\mathbb{M}) is$ defined by:

$ғ^{-1}(\mathbb{B})(x) = \mathbb{B}(ғ(x))$, for all ∈ $\mathbb{M}$ [15-16] .

2.8 Definition: Let $\mathbb{X}$ ∈*FUM(*$\mathbb{M}_1$*)* and $\mathbb{Y}$ ∈*FM(*$\mathbb{M}_2$*).* g : $\mathbb{X} \rightarrow \mathbb{Y}$ is called a fuzzy homomorphism if g: $\mathbb{M}_1 \rightarrow \mathbb{M}_2$ is $\mathbb{F}$-homomorphism and $\mathbb{Y}(g(\hbar) = \mathbb{X}(\hbar)$ for each $\hbar \in \mathbb{M}_1$ [15] .

2.9 Definition: Let $\emptyset \neq \mathfrak{C} \in FUS(\mathbb{P})$. The fuzzy annihilator of A denoted by *F-ann* $\mathfrak{C}$ is defined by: *(F-ann* $\mathfrak{C}$*)(r)=sup{t: t* $\in$*[0,1],*$r_t\ \mathfrak{C} \subseteq 0_1$ *},for all r*$\in \mathbb{F}$ [17-18] .

Note that that *F-ann* $\mathfrak{C}$ =$(0_1 : \mathfrak{C})$," [16]. "Hence $((F-ann\ \mathfrak{C}))_t \subseteq ann\ \mathfrak{C}_t$ [19].

2.10 Remark: For $A \in FUM(\mathbb{M})$.We let $\mathfrak{C}_* = \{x \in \mathbb{M}: \mathfrak{C}(x) = 1\}[20-21]$.

2.11 Proposition: Let $\mathbb{P} \in FUM(M)$ .Then $F-ann\ \mathbb{P} \in FUI(\mathbb{F})[9]$.

2.12 Definition: Let $K \in FUI(\mathbb{F})$. K is called a principle fuzzy ideal if $\exists \mathscr{k}_t \subseteq \mathrm{K}$ such that $\mathrm{K} = (\mathscr{k}_t)$ for each $\imath_s \subseteq \mathrm{K}$, ∃ a fuzzy singleton e_l of $\mathbb{F}$ such that $\imath_s = e_l \hbar_t$, where $s, l, t \in [0,1]$, which is $\mathrm{K} = (\mathscr{k}_t) = \{\imath_s \subseteq H: \imath_s = e_l\ \mathscr{k}_t$ for some fuzzy singleton a_l of $\mathbb{F}\}$ [22] .

2.13 Definition: Let Ϯ $\in FUM(\mathbb{M})$ and Ϯ≠ Ð∈FUS (Ϯ).Then Ð is known to as a fuzzy visible submodule if Ð=ꞁÐ ∀ non-empty ꞁ $\in FUI(\mathbb{F})$. ꞁ∈ $FUI(\mathbb{F})$ is defined visible if it is visible of a fuzzy $\mathbb{F}$ -module $\mathbb{F}$ [4].

2.14 Definition: Let $\mathbb{Y} \in FUM(\mathbb{M})$. Then $\mathbb{Y}$ is said to be fully fuzzy visible module if for any $\mathbb{Y} \neq \mathfrak{Z} \in FUS$ $(\mathbb{Y})$ is a fuzzy visible [5].

2.15 Proposition: Let $\mathbb{Y} \in FUM(\mathbb{M})$. Then $\mathbb{P}$ is fully fuzzy visible module if and only if $\mathbb{Y}_t$ is fully visible module,$\forall\ t \in (0,1]$ [5] .

2.16 Definition: Let $\mathbb{X} \in FUM(\mathbb{M})$. $X \neq \mathfrak{C} \in FUS(\mathbb{X})$, $\mathfrak{C}$ is termed as a semiprime fuzzy submodule if for each fuzzy singleton r_l of $\mathbb{F}$, $x_s \subseteq \mathbb{X}, {r_l}^2 x_s \subseteq \mathfrak{C}$ implies $r_l x_s \subseteq \mathfrak{C}$ [23] .

2.17 Remark: We assume that if $\mathfrak{C}_* = \mathfrak{Q}_*$. Then , $\mathfrak{C} = \mathfrak{Q}$ is called Condition (*) [20] .

3. Fully Fuzzy Visible Modules with Different Modules: In this section, the relationships between fully fuzzy visible modules and other fuzzy modules such as (semiprime, prime quasi prime, fully stable, Artiain, essential and quasi injective) have been investigated. Many interesting outcomes were identified.

3.1 proposition: Let $\mathbb{P} \in FUM(\mathbb{M})$ over principle fuzzy ideal ring. Then $\mathbb{P}$ is fully fuzzy visible if and only if every proper fuzzy submodule of $\mathbb{P}$ is fuzzy semiprime.

Proof: Let $\mathbb{P}$ be a fully fuzzy visible, then $\mathbb{P}_t$ is a fully visible, where $t \in (0,1]$ by [5,proposition 3.2] and let $\mathbb{P} \neq \mathfrak{W} \in FUS(\mathbb{P})$. Then $\mathfrak{W}_t$ is a proper submodule of $\mathbb{P}_t$ and hence $\mathfrak{W}_t$ is a semprime submodule by [24 , proposition 2.6.1] . Therefore $\mathfrak{W}$ is a fuzzy semiprime by [23 , proposition 3.2.6] . Conversely, let $\mathbb{P} \neq \mathfrak{W} \in FUS(\mathbb{P})$, then $\mathfrak{W}$ is a fuzzy semprime and hence $\mathfrak{W}_t$ is semiprime by [23, proposition 3.2.6], therefore $\mathbb{P}_t$ is a fully visible by [24 , proposition 2.6.1] and hence $\mathbb{P}$ is fuzzy fully visible.

3.2 Proposition: Let $\mathbb{P} \in FUM(\mathbb{M})$ over principle fuzzy ideal ring and $\mathbb{P} \neq \mathfrak{W} \in FUS(\mathbb{P})$. Then, $\mathfrak{W}$ visible if and only if $\mathfrak{W}$ divisible.

Proof: Assume that $\mathfrak{W}$ is visible then $\forall\ r_l \neq 0_1$, we have $< r_l > \mathfrak{W} = \mathfrak{W}$, then $r_l \mathfrak{W} = \mathfrak{W}$.Therefore $\mathfrak{W}$ is divisible. Conversely, clear.

3.3 Proposition: Let $\mathbb{Y}$ be a fully fuzzy visible module over $\mathbb{F}$. Then every proper fuzzy submodule of $\mathbb{Y}$ is divisible.

Proof: Let $\mathbb{Y}$ be a fully fuzzy visible module, then every proper fuzzy submodule of $\mathbb{Y}$ is visible and hence is divisible by above proposition.

3.4 Proposition: If $\mathbb{P}$ is a fully fuzzy visible module, then $\mathbb{P}$ is F – regular fuzzy module.

Proof: By using definition(2.14) and [4, proposition 3.17], the result is true. The converse of the above proposition is not correct as we see in the following example.

Consider $\mathbb{M} = Z_6$ and $\mathbb{F} = Z$. Let $\mathbb{P}: Z_6 \to [0,1]$, as $\mathbb{P}(x) = 1\ \forall x \in Z_6$. Z_6 *as* $Z -$ module is F-regular by [24, p97] and hence $\mathbb{P}$ is F-regular by [25 , proposition 3.1.3]. But Z_6 is not fully visible by [24, p97] and hence $\mathbb{P}$ is not fully visible .

3.5 Proposition: Let $\mathbb{P}$ be a fuzzy divisible module over a field, then $\mathbb{P}$ is a fully fuzzy visible module.

Proof: From [25, proposition 3.1.8], we have $\mathbb{P}$ is an F- regular fuzzy module, then $\mathbb{P}_t$ is a F-regular module, for every $t \in (0,1]$ by [25, proposition 3.1.2]. $\mathbb{P}_t$ is divisible because $\mathbb{P}$ is a fuzzy divisible module by [23, proposition 2.2.12] and hence every proper pure submodule of $\mathbb{P}_t$ is visible by [26 , proposition 1.11]. Therefore $\mathbb{P}_t$ is fully visible, then $\mathbb{P}$ is fuzzy fully visible.

3.6 Corollary: Suppose that $\mathbb{P}$ is a non-constant fully fuzzy visible module over a fuzzy integral domain $\mathbb{F}$. Then, $\mathbb{R}$ is a fuzzy field.

Proof: Depending on proposition (3.4) and [22, remark and examples 1.3.4(4)], we get the result.

3.7 Corollary: Let $\mathbb{P} \in FUM(\mathbb{M})$. If $\mathbb{P}$ is fully visible, *then* $F - J(\mathbb{P}) = 0_1$, where $\mathbb{P}$ satisfies condition$(*)$.

Proof: $\mathbb{P}_*$ is a fully visible because $\mathbb{P}$ is a fully fuzzy visible by [5, proposition3.2] and hence$J(\mathbb{P}_*) = \{0\} = (0_1)_*$, by [24, corollary 2.6.6]. But $J(\mathbb{P}_*) = \left(F - J(\mathbb{P})\right)_* = (0_1)_*$, then $F - J(\mathbb{P}) = 0_1$.

3.8 Definition: Let $\mathbb{P} \in FUM(\mathbb{M})$. Then $\mathbb{P}$ is called fuzzy Artiain module if every descending chain fuzzy submodules of $\mathbb{P}$ is finite.

3.9 Theorem: If $\mathbb{P} \in FUM(\mathbb{M})$ Artiain $\mathbb{F}$ -module, then $\mathbb{P}_*$ is Artiain modules.

Proof: Let $\mathbb{P} \in FM(\mathbb{M})$ Artiain $\mathbb{R}$ -module and $A_1 \supseteq A_2 \supseteq \cdots$ be descending chain of submodules of $\mathbb{P}_*$. For each positive integer *i*, we define the mapping $\mathbb{P}_i: \mathbb{M} \to [0,1]$ by $\mathbb{P}_i(x) = \begin{cases} 1 & if\ x \in A_i \\ 0 & o.w \end{cases}$, clearly, $\mathbb{P}_i \in FS(\mathbb{P})$. Then $\mathbb{P}_1 \supseteq \mathbb{P}_2 \supseteq \cdots$ is descending chain of fuzzy submodules of $\mathbb{P}$, so there exist a positive integer *m* such that $\mathbb{P}_m = \mathbb{P}_{m+h}$ for every positive integer *h*. It is clear that $(\mathbb{P}_i)_* = A_i$. Now, let *h* be a positive integer and $a \in A_{m+h}$ so, $\mathbb{P}_m(a) = \mathbb{P}_{m+h}(a) = 1$, hence $a \in A_m$. Thus $A_m \supseteq A_{m+h} \supseteq A_m$. Therefore $\mathbb{P}_*$ is Artiain. The converse of above theorem is not true as in the following example.

Let $\mathbb{M} = \mathbb{F}$ *and* $\mathbb{F} = \mathbb{R}$. Express $\mathbb{P}: \mathbb{M} \to [0,1]$ by $\mathbb{P}(x) = 1\ \forall x \in \mathbb{M}$.

Clearly, $\mathbb{P} \in FUM(\mathbb{M})$. Describe $\mathbb{P}: \mathbb{M} \to [0,1]$ by

$\mathbb{P}_m(x) = \begin{cases} 1 & if\ x = 0 \\ \frac{1}{2^m} & if\ x \neq 0 \end{cases}$, $m \in Z^+$. Clear that $\mathbb{P}_m \in FUS(\mathbb{P})$ for each *m*. Then $\mathbb{P}_1 \supseteq \mathbb{P}_2 \supseteq \cdots$ is an infinite strictily descending chain of fuzzy submodules of $\mathbb{P}$. Thus $\mathbb{P}$ is not fuzzy Artiain. But, $\mathbb{P}_* = \mathbb{M}$, is Artiain $\mathbb{F}$ −module.

3.10 Corollary: Let $\mathbb{P}$ be a fully fuzzy visible Artiain module on $\mathbb{F}$. Then $\mathbb{P}$ is fuzzy semisimple module provided that $\mathbb{P}$ satisfies condition $(*)$.

Proof: $\mathbb{P}_*$ is a fully visible module since $\mathbb{P}$ is fully fuzzy visible and $\mathbb{P}_*$ is Artiain by theorem 3.9. Therefore $\mathbb{P}_*$ is semisimple by [24, proposition 2.6.7]. Then, $\mathbb{P}$ is fuzzy semisimple by [20, proposition 2.8(1)].

3.11 Definition: A fuzzy ring $\mathbb{F}$ is called regular if and only if every fuzzy singleton in $\mathbb{F}$ is fuzzy regular, i.e. $\forall$ fuzzy singleton x_t of $\mathbb{F}$, $t \in (0,1]$, there exist a fuzzy singleton y_k of $\mathbb{F}$ such that $x_t = x_t y_k x_t$.

3.12 Proposition: A fuzzy ring $\mathbb{F}$ is regular if and only if $\mathbb{F}$ is regular ring.

Proof: Suppose that $\mathbb{F}$ is regular, then $\forall x \in \mathbb{F}, \exists y \in \mathbb{F}$, s.t. $x = xyx$, hence $x_t = (xyx)_t = x_t y_t x_t$, where x_t, y_t fuzzy singleton of $\mathbb{F}$, $t \in (0,1]$. Therefore, a fuzzy ring $\mathbb{F}$ is regular. Conversely, let $M_{\mathbb{F}}$ be a fuzzy ring over $\mathbb{F}$, then $\forall$ fuzzy singleton $\mathrm{x_t}$ of $\mathbb{F}$, $\exists y_k$ fuzzy singleton of $\mathbb{F}$ such that $x_t = x_t y_k x_t = (xyx)_{min\{t,k\}}$, hence $x = xyx$ and $t = min\{t,k\}$. Therefore $\mathbb{F}$ is a regular.

3.13 Corollary: Let $\mathbb{P}$ be a fully fuzzy visible and Artiain module on $\mathbb{F}$, satisfied condition(*), then $M_{\mathbb{F}}/(F - annx_t)_*$ is fuzzy regular $\forall x_t \subseteq \mathbb{P}$.

Proof: $\mathbb{P}_*$ is fully visible and Artiain module as in corollary (3.10). Then $\mathbb{F}/(F - annx_t)_*$ is regular by [24, corollary 2.6.10], then $\mathrm{M}_{\mathbb{F}}/(\mathrm{F} - annx_t)_*$ is fuzzy regular by [27, theorem 3.2.10] and proposition (3.12) .

3.14 Proposition: Let $\mathbb{P} \in FUM(M)$ over principle fuzzy ideal ring. If every $\mathbb{P} \neq \mathfrak{W} \in FUS(\mathbb{P})$ over $\mathbb{F}$ is divisible, then $\mathbb{P}$ is fuzzy fully visible and hence $\mathbb{P}$ is F- regular fuzzy.

Proof: By using proposition 3.2, we have $\mathbb{P}$ a is fuzzy fully visible, then $\mathbb{P}$ is F-regular by proposition (3. 4).

3.15 Proposition: Let $\mathbb{F}$ be a principle fuzzy ideal ring and $\mathbb{P}$ is a fuzzy divisible, then the following is equivalent:

1. $\mathbb{P}$ is a fully fuzzy visible module.
2. Each $\mathbb{P} \neq \mathfrak{z} \in FUS(\mathbb{P})$ is pure.
3. Each $\mathbb{P} \neq \mathfrak{z} \in FUS(\mathbb{P})$ is divisible.
4. Each $\mathbb{P} \neq \mathfrak{z} \in FUS(\mathbb{P})$ is visible.

Proof: $1 \Longrightarrow 2$ directly from [4, proposition 3.17].

$2 \Longrightarrow 3$ Since $\mathbb{F}$ be a principle fuzzy ideal ring and $\mathbb{P}$ is a fuzzy divisible, then by[4, proposition 3.19] we have each proper submodule of $\mathbb{P}$ is visible and hence divisible.

$3 \Longrightarrow 4$ Directly from proposition 3.2.

$4 \Longrightarrow 1$ Clear.

3.16 Proposition: Let $\mathbb{P}$ be a fully fuzzy visible module over a local ring, then $\mathbb{P}$ is a fuzzy semisimple module such that $\mathbb{P}$ satisfied condition $(*)$.

Proof: $\mathbb{P}_*$ is a fully visible module by [5, proposition 3.2], then $\mathbb{P}_*$ is semisimple by [24 , proposition 2.6.13], hence $\mathbb{P}$ is a semisimple by [20, proposition. 2.8(1)].

3.17 Proposition: Let $\mathbb{P}$ be a fuzzy divisible over principle fuzzy ideal ring, then $\mathbb{P}$ is fully visible.

Proof: Depending on [4, proposition 3.19], every proper fuzzy submodule of $\mathbb{P}$ is visible and hence $\mathbb{P}$ is fully visible.

3.18 Proposition: Let $\mathbb{P}$ be a fully fuzzy visible module over $\mathbb{F}$, then $\mathbb{P}$ is a fuzzy torsion free.

Proof: $\mathbb{P}_{\mathrm{t}}$ is fully visible $\forall\, t \in (0,1]$ by [5, proposition 3.2], then $\mathbb{P}_{\mathrm{t}}$ torsion free by [24 ,proposition 2.6.16]. Therefore $\mathbb{P}$ is fuzzy torsion free by {17, proposition 2.2.23].

3.19 Proposition: Let $\mathbb{P}$ be a fully fuzzy visible module, $\mathbb{P} \neq \mathfrak{W} \in FUS(\mathbb{P})$. Then, $\mathfrak{W}$ is a prime submodule.

Proof: Because $\mathbb{P}$ is a fully fuzzy visible module and $\mathbb{P} \neq \mathfrak{W} \in FUS(\mathbb{P})$, then $\mathfrak{W}$ is pure, hence $\mathfrak{W}$ is weakly pure fuzzy submodule of $\mathbb{P}$. Therefore $\mathfrak{W}$ is a prime fuzzy by [17, proposition 2.2.22].

3.20 Corollary: If $\mathbb{P}$ is a fully fuzzy visible module, then every proper fuzzy submodule $\mathfrak{W}$ of $\mathbb{P}$ is quasi prime.

Proof: Depending proposition 3.19 and [23, proposition 3.2.2], the result hold.

3.21 Corollary: Every fully fuzzy visible module is fuzzy T-regular.

Proof: Depending on proposition (3.4) and [22, remark and examples 2.1.5(1)].

The converse of the corollary (3.21) is not true as we see in the following example:

Let $\mathbb{M} = Z_4, \mathbb{F} = Z$. Define $P: \mathbb{M} \to Z_4$ as $\mathbb{P}(x) = \begin{cases} 1 & \text{if } x \in Z_4 \\ 0 & o.w \end{cases}$, clearly, $\mathbb{P} \in$ *FUM(*$\mathbb{M}$*),* then $\mathbb{P}_t = Z_4$, $\forall\, t \in (0,1]$ as Z-module is T-regular by [24 , p99], then $\mathbb{P}$ is fuzzy T-regular by [22, proposition 2.1.2]. But Z_4 as Z-module is not fully visible module by [24, remark and examples 2.1.2], then $\mathbb{P}$ is not fully fuzzy visible.

3.22 Definition: Let $\mathbb{P} \in$ *FUM(*$\mathbb{M}$*)* , $\mathfrak{W} \in$*FUS(*$\mathbb{P}$*)*. We say that $\mathfrak{W}$ is stable if $f(\mathfrak{W}) \subseteq \mathfrak{W}$ for each fuzzy $\mathbb{F}$ -homomorphism $f: \mathfrak{W} \to \mathbb{P}$.

3.23 Proposition: Let $\mathbb{P} \in$ *FUM(*$\mathbb{M}$*)* and $\mathfrak{W} \in FUS(\mathbb{P})$ constant on $ker\, f$. Then $\mathfrak{W}$ is fuzzy stable if and only if $\mathfrak{W}_*$ is stable and $\mathbb{P}$ satisfies Condition $(*)$.

Proof: Let $\mathfrak{W}$ be a fuzzy stable. Then, $f(\mathfrak{W}) \subseteq \mathfrak{W}$ by definition (3.21). Since $\mathfrak{W}$ is constant on $\ker f$, then $f(\mathfrak{W}_*) \subseteq \mathfrak{W}_*$ by [27, lemma 3.2.5]. Therefore $\mathfrak{W}_*$ is stable. Conversely, let $\mathfrak{W}_*$ is stable. Then $f(\mathfrak{W}_*) \subseteq \mathfrak{W}_*$ and hence $(f(\mathfrak{W}))_* \subseteq \mathfrak{W}_*$ by [27, lemma 3.2.5], since $\mathfrak{W}$ satisfies condition($*$), then $f(\mathfrak{W}) \subseteq \mathfrak{W}$. Therefore $\mathfrak{W}$ is fuzzy stable.

3.24Example: Let $\mathbb{M} = \mathbb{F}$ *and* $\mathbb{F} = \mathbb{R}$. Define $\mathcal{K}: \mathbb{F} \to [0,1]$ *by* $\mathcal{K}(x) = 1\ \forall x \in \mathbb{F}$. Consider $\mathfrak{X}: \mathbb{F} \to [0,1]$ as $\mathfrak{X}(x) = \begin{cases} 1 & if\ x \in \mathrm{Q} \\ 0 & o.w \end{cases}$. Clear that $\mathfrak{X} \in FUS(\mathcal{K})$. $\mathcal{K}_* = \mathbb{R}\,, \mathfrak{X}_* = \mathrm{Q}$. We know that Q is stable of $\mathbb{F}$, then $\mathfrak{X}$ is a fuzzy stable of $\mathcal{K}$ by proposition (3.22), where $\mathfrak{X}$is coustant on $ker\, f$.

3.25 Example: Let $\mathbb{M} = Z$ *and* $\mathbb{F} = Z$. Define $\mathbb{P}: Z \to [0,1]$ *by* $\mathbb{P}(x) = 1\ \forall x \in Z$.

Suppose that $\mathfrak{W}: Z \to [0,1]$ defined as $\mathfrak{W}(x) = \begin{cases} 1 & if\ x \in 2Z \\ 0 & o.w \end{cases}$. It is clear that $\mathfrak{W} \in FUS(\mathbb{P}), \mathbb{P}_* = Z$ and $\mathfrak{W}_* = 2Z$. Then $\mathfrak{W}_*$ is not stable by [28, example and remarks 1.2(a)], and, hence, $\mathfrak{W}$ is not fuzzy stable by proposition 3.23.

3.26 Definition: Let $\mathbb{P} \in$ *FUM(M)*. $\mathbb{P}$ is termed to be fuzzy fully stable if and only if every Ȥ $\in FUS(\mathbb{P})$ is fuzzy stable.

3.27 Proposition: Let $\mathbb{P}$ be a fuzzy $\mathbb{F}$ –module, such that every fuzzy submodule of $\mathbb{P}$ constant on $ker\, f$ and satisfies condition(*). Then $\mathbb{P}$ is a fuzzy fully stable if and only if $\mathbb{P}_*$ is fully stable.

Proof: In a similar way of proposition (3.23).

3.28 Proposition: Let $\mathbb{P}$ be a fully fuzzy visible $\mathbb{F}$ -module and $\mathrm{End}_{\mathbb{R}}(\mathbb{P}_*)$ is commutative. Then $\mathbb{P}$ is fuzzy fully stable such that $\forall\, \mathfrak{W} \subseteq \mathbb{P}$ constant on $ker\, f$ and satisfies condition (*).

Proof: $\mathbb{P}_*$ is a fully visible by[3, proposition 3.2], then $\mathbb{P}_*$ is fully stable by [24, proposition (2.6.22)], and hence each submodule of $\mathbb{P}_*$ will be stable, then $\forall\, \mathfrak{W} \in FS(\mathbb{P})$, $\mathfrak{W}_*$ will be stable, then by proposition 3.22, $\mathfrak{W}$ is a stable fuzzy submodule. Therefore, $\mathbb{P}$ is fuzzy fully stable.

3.29 Proposition: Let $\mathbb{P}$ be a fully fuzzy visible module over $\mathbb{F}$. Then each $J\mathbb{P} \in FUS(\mathbb{P})$ is an essential submodule of $\mathbb{P}$ for each non-empty $J \in FUI(\mathbb{F})$.

Proof: Let ξ be a non-trivial fuzzy submodule of $\mathbb{P}$ such that $J\mathbb{P} \cap \mathfrak{X} = 0_1$. But $\mathfrak{X}$ is visible and hence ξ is pure. Then $J\xi = J\mathbb{P} \cap \xi = 0_1$, so $J\xi = 0_1$ and hence $\xi = 0_1$. Therefore $J\mathbb{P}$ is a fuzzy essential.

3.30 Proposition: Let $\mathbb{P}$ be a fully fuzzy visible multiplication module, then every non-empty fuzzy submodule of $\mathbb{P}$ is an essential of $\mathbb{P}$.

Proof: Because $\mathbb{P}$ is fully fuzzy visible, then every fuzzy submodule $J\mathbb{P}$ is an essential for each non-empty $J \in FUI(\mathbb{F})$ by proposition (3.29). But $\mathbb{P}$ is fuzzy multiplication, then every non – empty fuzzy submodule of $\mathbb{P}$ is essential.

3.31 Proposition: Let $\mathbb{P}$ be a fully fuzzy visible module over a domain $\mathbb{F}$ and $\mathrm{End}_R(\mathbb{P}_t)$ is commutative ring. Then $\mathbb{P}$ is fuzzy divisible.

Proof : Since $\mathbb{P}_t$ is fully visible, then $\mathbb{P}_t$ is divisible by [24, proposition 2.6.42(2)]. Then $\mathbb{P}$ is divisible by [17].

Let $\mathfrak{P} \in FUM(\mathbb{M})$ and $\mathfrak{G} \in FUM(N)$. If $\mathfrak{f} : \mathbb{M} \to N$ is a $\mathbb{F}$ –module homomorphism and $\mathfrak{G}(\mathfrak{f}(x)) \geq \mathfrak{P}(x), \forall\, x \in \mathbb{M}$, then $\mathfrak{f}$ is called a fuzzy homomorphism and denoted by $\mathfrak{f}: \mathfrak{P} \to \mathfrak{G}$ [29].This definition can be used to introduce the following concept.

3.32 Definition : Let $\mathbb{P} \in FUM(\mathbb{M})$. $\mathbb{P}$ is termed to be fuzzy quasi Injective if for each submodule N of $\mathbb{M}$ and each fuzzy homomorphism $\mathfrak{f} : \mathfrak{W} \to \mathbb{P}$, where $\mathfrak{W} \in FUM(N)$ can be extended to fuzzy endomorphism $\mathfrak{g}$ such that $\mathfrak{f} = \mathfrak{g} \circ \mathrm{i}$, where $\mathrm{i}: \mathfrak{W} \to \mathbb{M}$ inclusion homomorphis.

3.33 Proposition: Let $\mathbb{V} \in FUM(\mathbb{M})$. $\mathbb{P}$ is fuzzy quasi injective iff $\mathbb{M}$ is quasi injective and $\mathbb{V}(e) = 1\ \forall e \in \mathbb{M}$.

Proof: Let $\mathbb{V}$ be a fuzzy quasi injective $\mathbb{F}$ -module and N be a submodule of $\mathbb{M}$.

Define $\mathfrak{W}: \mathrm{N} \to [0,1]$ by $\mathfrak{W}(q) = \begin{cases} 1 & \text{if } q \in \mathrm{N} \\ 0 & \text{o.w} \end{cases}$, then

$\forall\, \mathfrak{f}: \mathfrak{W} \to \mathbb{V}$ be a fuzzy homomorphism $\exists\ \mathfrak{g}: \mathbb{V} \to \mathbb{V}$ s.t. $\mathfrak{f} = \mathfrak{g} \circ \mathrm{i}$ and hence $\forall$ submodule N of $\mathbb{M}$ and $\mathfrak{f}: \mathrm{N} \to \mathbb{M}$, there exist $\mathrm{g}: \mathbb{M} \to \mathbb{M}$ such that $\mathfrak{f} = \mathfrak{g} \circ \mathrm{i}$.Now, if $\mathbb{V}(q) \neq 1$, since $\mathbb{V}(\mathfrak{g}(q)) \geq \mathbb{V}(q)$ by definition of fuzzy homomorphism, then $\mathbb{V}(\mathfrak{g}(q)) \geq \mathbb{V}(q) = \mathbb{V}(\mathrm{i}(q)) \geq \mathfrak{W}(q) = 1$, hence $\mathbb{V}(q) = 1\ \forall q \in \mathbb{M}$.

Conversely, Let $\mathbb{M}$ be a quasi-injective, N be a submodule of $\mathbb{M}$ and $\mathbb{V}(q) = 1\ \forall q \in \mathbb{M}$, let $\mathfrak{W} \in FUM(N)$. Then $\forall\, \mathfrak{f}: \mathrm{N} \to \mathrm{M}\ \exists\ \mathfrak{g}: \mathbb{M} \to \mathbb{M}$s.t. $\mathfrak{f} = \mathfrak{g} \circ \mathrm{i}$, $\mathrm{i}: \mathrm{N} \to \mathbb{M}$ inclusion homomorphism, hence $\forall\, \mathfrak{f} : \mathfrak{W} \to \mathbb{V}\ \exists\ \mathfrak{g}: \mathbb{V} \to \mathbb{V}$ s.t. $\mathfrak{f} = \mathfrak{g} \circ \mathrm{i}$, where $(\mathfrak{g} \circ \mathrm{i})(q) = \mathbb{V}(\mathfrak{g} \circ \mathrm{i})(q) = \mathbb{V}(\mathfrak{g}(q)) = \mathbb{V}(q) = 1 \geq \mathbb{V}(q) = 1 \geq \mathfrak{W}(q)$.

3.34 Definition: A fuzzy ring $\mathbb{F}$ is self injective if it is fuzzy injective $\mathbb{F}$ -module.

3.35 Corollary: Suppose that $\mathbb{P}$ is a fuzzy fully visible $\mathbb{F}$ - module $\mathbb{M}$ and $\mathrm{End}_R(\mathrm{P}_*)$ is commutative, where $\mathbb{P}$ satisfies condition $(*)$. If $\mathbb{P}$ is a fuzzy quasi injective module and $\mathbb{P}$ (x)=1 $\forall$ x $\in \mathbb{M}$, then

1. Every fuzzy submodule $\mathfrak{W}$ of $\mathbb{P}$ constant on $\ker \mathfrak{f}$ is fuzzy duo module.
2. Every homomorphism image constant on $\ker \mathfrak{f}$ is duo module.
3. A fuzzy ring γ on $\mathrm{End}_{\mathbb{F}}(\mathbb{P}_*)$ is self injective.

Proof: 1. Since $\mathbb{P}$ is fully fuzzy visible and $\mathrm{End}_R(\mathbb{P}_*)$ is commutative, then $\mathbb{P}_*$ is fully visible. But $\mathbb{P}$ is fuzzy quasi injective, then $\mathbb{M}$ is a quasi injective and $\mathbb{P}(\mathrm{x}) = 1\ \forall \mathrm{x} \in \mathbb{M}$. Therefore

$\mathbb{P}_*$is duo by [5, proposition 3.8]. Let $\mathfrak{W} \in FUS(\mathbb{P})$, then $\mathfrak{W}_*$ submodule of $\mathbb{P}_*$, then $\mathfrak{W}_*$ is duo which implies that $\mathfrak{W}$ is duo by [5, proposition 3.8].

2. Since every fuzzy homomorphic image is a fuzzy submodule of $\mathbb{P}$, then by (1), we get the result.

3. Since $\mathrm{End}_{\mathbb{F}}(\mathbb{P}_*)$ is $\mathrm{End}_{\mathbb{F}}(\mathbb{P}_*)$ −injective by [24, proposition 2.6.25], then $\gamma_{\mathrm{End}_{\mathbb{F}}}(\mathbb{P}_*)$ is a fuzzy self injective [29, proposition 4.3(i)].

3.36 Corollary: Let $\mathbb{P}$ be a finitely generated fully visible module over Dedkined domain, then $\mathbb{P}$ is fuzzy duo multiplication module, where $\mathbb{P}$ satisfies condition $(*)$ and every fuzzy submodule of $\mathbb{P}$ constant on ker f.

Proof : $\mathbb{P}_*$ is finitely generated module, then $\mathbb{P}_*$ is duo multiplication module by [24, corollary 2.6.26], but $\mathbb{P}_*$ is fully visible, then $\mathbb{P}$ is duo multiplication by [25] and [5,proposition 3.8] .

3.37 Proposition: If $\mathbb{P} \in FUM(\mathbb{M})$ over a fuzzy principle ideal ring and $\mathfrak{W} \in FUM(\mathbb{P})$ be a such that $\forall x_t \subseteq \mathfrak{W}$ and a fuzzy singleton r_ℓ of $\mathbb{F}$, $x_t = r_\ell s_k x_t$ for some fuzzy singleton s_k of $\mathbb{R}$, then $\mathbb{P}$ is a fully visible fuzzy module .

Proof:

Let $x_t \subseteq \mathfrak{W}$ and fuzzy singleton r_ℓ of $\mathbb{F}$. Then $x_t \subseteq < x_t > \subseteq \mathfrak{W}$, then $x_t = s_k x_t$ for some $s_k \subseteq R$. Then for each $r_\ell \subseteq \mathbb{F}$, we get $s_k x_t = r_\ell s_k = r_\ell s_k x_t \subseteq r_\ell \mathfrak{W}$. Therefore, $x_t \subseteq r_\ell \mathfrak{W}$, but $\mathbb{F}$ is a fuzzy principle ideal ring, then $x_t \subseteq J\mathfrak{W}$ and hence $\mathfrak{W} \subseteq J\mathfrak{W}$, but $J\mathfrak{W} \subseteq \mathfrak{W}$. Therefore $\mathfrak{W} = J\mathfrak{W}$ for each nonempty fuzzy ideal J of $\mathbb{F}$and hence $\mathfrak{W}$ is a fuzzy visible submodule, thus the result holds.

3.38 Proposition: Let $\mathbb{F}$ be a principle ideal field and $\mathfrak{W}$ be a fuzzy submodule of a fuzzy module of an $\mathbb{F}$ -module $\mathbb{M}$. Then

1. $\mathbb{P}$ is a fully fuzzy visible.

2.$\gamma_{\mathbb{F}}/(\mathrm{F}-\mathrm{ann}x_t)_*$ is fuzzy negular ring $\forall x_t \subseteq \mathbb{P}$, where $\gamma_{\mathbb{F}}$ is a fuzzy ring of $\mathbb{F}$.

3.$\forall\, x_t \subseteq \mathbb{P}\ \exists r_\ell$ fuzzy singleton of $\mathbb{F}$, $x_t = r_\ell h_k x_t, k \in (0,1]$.

Proof: 1 ⇒ 2 P_* is fully visible by propositon (2.15) and hence $\mathbb{F}/(\mathrm{F}-\mathrm{ann}x_t)_*$ is regular ring by [24, proposition 2.6.27].Therefore $\gamma_{\mathbb{F}}/(\mathrm{F}-\mathrm{ann}x_t)_*$ is fuzzy negular ring by [27, theorem 3.2.10] and proposition 3.12.

2 ⟹ 3 Since $\gamma_{\mathbb{F}}/(\mathrm{F}-\mathrm{ann}x_t)_*$ is fuzzy regular, then $\mathbb{F}/(\mathrm{F}-\mathrm{ann}x_t)_*$ is regular ring by [24,theorem 3.2.10], hence $\forall x \in \mathbb{P}_t$, $r \in \mathbb{F}$, $x = rhx$ for some $h \in \mathbb{F}$ by [24 ,proposition 2.6.27]. Therefore, $\forall\, x_t \subseteq \mathbb{P}$, $\exists \gamma_\ell$ fuzzy singlet of $\mathbb{F}$ s.t. $x_t = r_\ell h_k x_t$, $t = \min\{\ell, k, t\}$.

2 ⟹ 3 Directly from proposition 3.37

3.39 Corollary: If $\mathbb{F}$ is a principle ideal field, then $\mathbb{M}_{\mathbb{F}}$ is fuzzy regular, if and only if $\mathbb{M}_{\mathbb{F}}$is a fuzzy $\mathbb{F}$ -module is fully visible.

Proof: Let $\mathbb{M}_{\mathbb{F}}$ be a fuzzy regular ring, then $\mathbb{F}$ is regular ring by proposition 3.12, then $\mathbb{F}$ is fully visible by [24, corollary 2.6.28], and hence $\mathrm{M}_{\mathbb{F}}$ is fully visible.

3.40 Corollary: If $\mathbb{F}$ is a field and $\mathbb{P} \in FUM(\mathbb{M})$ such that $\mathbb{M}_{\mathbb{F}}$ is a fuzzy ring on $\mathbb{F}$, then $\mathbb{P}$ is fully fuzzy visible module, if $\mathbb{M}_{\mathbb{F}}/(\mathrm{F}-\mathrm{ann}\ \mathbb{P})_t$ is a fuzzy regular ring , where $(\mathrm{F}-\mathrm{ann}\mathbb{P})_t = \mathrm{ann}\mathbb{P}_t$, $\forall\, t \in (0,1]$.

Proof: Since $\mathbb{M}_{\mathbb{F}}/\mathrm{F}-\mathrm{ann}\mathbb{P}$ is a fuzzy regular, then $\mathbb{F}/(\mathrm{F}-\mathrm{ann}\mathbb{P})_t$ is a regular ring by [27,theorem 3.2.10] and proposition 3.15, but $(\mathrm{F}-\mathrm{ann}\mathbb{P})_t = \mathrm{ann}\mathbb{P}_t$, $\forall\, t \in (0,1]$, then $\mathbb{P}_t$ is fully visible by [24]. Therefore $\mathbb{P}$ is fully fuzzy visible.

3.41 Proposition: If $\mathbb{P}$ is divisible over P.I.D. and d F-ann $\mathbb{P}$ is semimaximal ideal of $\mathbb{F}$. Then $\mathbb{P}$ is a fully fuzzy visible, where $(\mathrm{F}-\mathrm{ann}\mathbb{P})_* = \mathrm{ann}\mathbb{P}_*$. And $\mathbb{P}(x) = 1\ \forall x \in \mathbb{M}$.

Proof: P_* is divisible module by [23], since $(F - \mathrm{ann}\mathbb{P})_* = \mathrm{ann}\mathbb{P}_*$ and $F - \mathrm{ann}\mathbb{P}$ is semimaximal, then $\mathrm{ann}P_*$ is semimaximal ideal by [20, proposition (2.4)]. Then $\mathbb{P}_*$ is fully visible [24 ,proposition 2.6.38]. Therefore $\mathbb{P}$ is fuzzy fully visible.

3.42 Proposition: Let $\gamma_{\mathbb{F}}$ be a fuzzy visible ring defined as $\gamma_{\mathbb{F}}(r) = 1\ \forall r \in \mathbb{F}$, if $\mathbb{P}$ is a fuzzy $\mathbb{F}$ -module M, then $\mathbb{P}$ is flat.

Proof: Since $(\gamma_{\mathbb{F}})_t = \mathbb{F}$, then $\mathbb{F}$ is visible, then M is flat by [24, proposition 2.6.39]. Therefore $\mathbb{P}$ is fuzzy flat by [29, theorem 3.15]

4. Conclusion: In this paper, a number of characteristics and properties that explain the relationship of fully fuzzy visible modules with other modules are proved. In addition many useful examples with a collection of important results are obtained.

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