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Solution of Population Growth Rate Linear Differential Model via Two Parametric SEE Transformation

Ali Moazzam
University of Agriculture Faisalabad, Pakistan.
alimoazzam7309723@gmail.com

Hira Aslam
University of Agriculture, Faisalabad, Pakistan.
hiraaslam053@gmail.com

Naila Tabassum
University of Agriculture Faisalabad, Pakistan.
nailatabassumgp@gmail.com

Emad A. Kuffi
College of Engineering, Al-Qadisiyah University, , Iraq.
emad.abbas@qu.edu.iq

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Abstract

The integral transformations is a complicated function from a function space into a simple function in transformed space. Where the function being characterized easily and manipulated through integration in transformed function space. The two parametric form of SEE transformation and its basic characteristics have been demonstrated in this study. The transformed function of a few fundamental functions along with its time derivative rule is shown. It has been demonstrated how two parametric SEE transformations can be used to solve linear differential equations. This research provides a solution to population growth rate equation. One can contrast these outcomes with different Laplace type transformations.

Keywords: Two parametric SEE transformation, Population growth rate equation, linear differential equations, time derivative rule.

1.Introduction

As The Transformations Have numerous applications in fields of engineering and other sciences. Numerous mathematicians presented the Fourier and Laplace transformations theory. In [1] authors used this technique on delay differential equations. The modified Laplace series findings from the Adomian polynomial decomposition method were used to solve the differential-integral equations can be seen in [2]. The non-linearity was circumvented by using standard Adomian polynomials. The Laplace decomposition approach and the pade approximation were put to practise in order to determine the numerically approximated results of the system of partial on linear differential equations in [3]. The Laplace decomposition approach, which was improved utilised in [4] for differential equalities of the Emden-Lane type. The author used an analytical approach to resolve the first and second types of non-linear Volterra-Fredholm integro differential equations in [5] by combining the modified Laplace Adomian substitution and Laplace Adomian polynomial decomposition methods (LADM). Laplace integral techniques are frequently utilised in the engineering and scientific fields. The use of integral transformations to correct erroneous integrals that contained error functions has been covered by a number of authors [6–9]. In [10] authors obtained the solutions to Abel's equation using the Kamal integral method. Solution to the error function based on the Kamal integral approach has shown in [11]. To solve linear partial differential equations with more than two independent variables, writers in [12] described the substitution approach of the Laplace transformation. In [13] authors used the Matlab programme to illustrate how the effects of bio-convection and activation energy on the Maxwell equation on nano-fluid. With the use of the Laplace technique of transformation, an analytical solution to the advection diffusion equation in a one-dimensional, semi-infinite media has been developed by authors of [14]. In [15 and 16] new transformations to solve new type differential equations with trigonometric coefficients was introduced in 2021 byMoazzam *et. al.* The solution to the temperature problem and some of the applications of the Kamal transformation in thermal engineering are shown in [17]. The SEE transformation, a new transformation that is very useful in the solution of differential equations and systems of differential equations, has been demonstrated in [18]. Some others transformations having different kernels then Laplace transformation are used to find the solutions of linear differential equations as Al-Zughair transformation is used to demonstrate the solution of moment Pareto differential equations with logarithmic coefficients in [19]. In paper [20], a new logarithmic kernel transformation was demonstrated which is useful for solving both ordinary differential equations and differential equations with a logarithmic coefficient.

2.Definitions and Preliminaries

In this paper, we have introduced the two parametric SEE transformation, which is a generalised version of the SEE transformation and is used in this study to solve linear differential equations. The definition of two parametric SEE transformations is given below.

Definition 1: Two parametric SEE integral transform of the function $f(t)$ for all $t \geq 0$ is defined as:

$$SEE_{\alpha,\beta}\{f(t)\} = \frac{e^{-\beta v}}{(\alpha v)^n}\int_0^\infty f(t)e^{-(\alpha v)t}dt = F_{\alpha,\beta}(v) \tag{1}$$

Where α, β are real numbers, n is the integer and v is transformed variable. By choosing the values of parameters α, β real numbers and integer n, we can get the different transformations as Laplace, Kamal, Mohand, SEE, Sumudu, Aboodh, and many other Laplace type transformations.

Table 1: Two parametric SEE transformation of some function

Serial Numbers	*Function* $f(t)$	$SEE_{\alpha,\beta}\{f(t)\} = \frac{e^{-\beta v}}{(\alpha v)^n}\int_0^\infty f(t)e^{-(\alpha v)t}dt$
1.	k	$\frac{e^{-\beta v}k}{(\alpha v)^{n+1}}$
2.	1	$\frac{e^{-\beta v}}{(\alpha v)^{n+1}}$
3.	t	$\frac{e^{-\beta v}}{(\alpha v)^{n+2}}$
4.	t^2	$\frac{e^{-\beta v}2!}{(\alpha v)^{n+3}}$
5.	t^m	$\frac{e^{-\beta v}m!}{(\alpha v)^{n+m+1}}$
6.	e^{ut}	$\frac{e^{-\beta v}}{(\alpha v)^n(\alpha v - u)}$
7.	$sin\,(ut)$	$\frac{e^{-\beta v}}{(\alpha v)^n}\left[\frac{u}{(\alpha v)^2 + u^2}\right]$
8.	$cos(ut)$	$\frac{e^{-\beta v}}{(\alpha v)^n}\left[\frac{\alpha v}{(\alpha v)^2 + u^2}\right]$
9.	$cosh\,(ut)$	$\frac{e^{-\beta v}}{(\alpha v)^n}\left[\frac{u}{(\alpha v)^2 + u^2}\right]$
10.	$sinh\,(ut)$	$\frac{e^{-\beta v}}{(\alpha v)^n}\left[\frac{\alpha v}{(\alpha v)^2 - u^2}\right]$

Theorem 1: Two parametric SEE transform for first, second up-to nth order derivative can be described as:

1. $SEE_{\alpha,\beta}\{f'(t)\} = -\frac{e^{-\beta v}}{(\alpha v)^n}f(0) + (\alpha v)F_{\alpha,\beta}(v),$
2. $SEE_{\alpha,\beta}\{f''(t)\} = -\frac{e^{-\beta v}}{(\alpha v)^n}f'(0) - \frac{e^{-\beta v}}{(\alpha v)^{n-1}}f(0) + (\alpha v)^2F_{\alpha,\beta}(v),$
3. $SEE_{\alpha,\beta}\{f'''(t)\} = -\frac{e^{-\beta v}}{(\alpha v)^n}f''(0) - \frac{e^{-\beta v}}{(\alpha v)^{n-1}}f'(0) - \frac{e^{-\beta v}}{(\alpha v)^{n-2}}f(0) + (\alpha v)^3F_{\alpha,\beta}(v),$
4. $SEE_{\alpha,\beta}\left\{f^{(m)}(t)\right\} = -\frac{e^{-\beta v}}{(\alpha v)^n}f^{(m-1)}(0) - \frac{e^{-\beta v}}{(\alpha v)^{n-1}}f^{(m-2)}(0) - \frac{e^{-\beta v}}{(\alpha v)^{n-2}}f^{(m-3)}(0) - \cdots - \frac{e^{-\beta v}}{(\alpha v)^{n-m+1}}f(0) + (\alpha v)^m SEE_{\alpha,\beta}\{f(t)\}$

Proof : To prove 1, let double parametric SEE transformation for $f(t) = f'(t)$,

$$SEE_{\alpha,\beta}\{f'(t)\} = \frac{e^{-\beta v}}{(\alpha v)^n}\int_0^\infty f'(t)e^{-(\alpha v)t}dt$$

$$= \frac{e^{-\beta v}}{(\alpha v)^n}f(t)|_0^\infty + (\alpha v)\int_0^\infty f(t)e^{-(\alpha v)t}dt$$

$$= -\frac{e^{-\beta v}}{(\alpha v)^n}f(0) + (\alpha v)F_{\alpha,\beta}(v) \qquad (2)$$

Let double parametric SEE transformation for $f(t) = f''(t)$.

$$SEE_{\alpha,\beta}\{f''(t)\} = \frac{e^{-\beta v}}{(\alpha v)^n}\int_0^\infty f''(t)e^{-(\alpha v)t}dt$$

$$= \frac{e^{-\beta v}}{(\alpha v)^n}f'(t)|_0^\infty + (\alpha v)\int_0^\infty f'(t)e^{-(\alpha v)t}dt$$

$$= -\frac{e^{-\beta v}}{(\alpha v)^n}f'(0) + (\alpha v)SEE_{\alpha,\beta}\{f'(t)\}.$$

By the help of (2) we can get:

$$= -\frac{e^{-\beta v}}{(\alpha v)^{n}}f'(0) - \frac{e^{-\beta v}}{(\alpha v)^{n-1}}f(0) + (\alpha v)^{2}F_{\alpha,\beta}(v) \quad (3)$$

Let double parametric SEE transformation for $f(t) = f'''(t)$.

$$SEE_{\alpha,\beta}\{f'''(t)\} = \frac{e^{-\beta v}}{(\alpha v)^{n}}\int_{0}^{\infty} f'''(t)e^{-(\alpha v)t}dt$$

$$= \frac{e^{-\beta v}}{(\alpha v)^{n}}f''(t)|_{0}^{\infty} + (\alpha v)\int_{0}^{\infty} f''(t)e^{-(\alpha v)t}dt$$

$$= -\frac{e^{-\beta v}}{(\alpha v)^{n}}f''(0) + (\alpha v)SEE_{\alpha,\beta}\{f''(t)\}.$$

By the help of (3) we can get:

$$= -\frac{e^{-\beta v}}{(\alpha v)^{n}}f''(0) - \frac{e^{-\beta v}}{(\alpha v)^{n-1}}f'(0) - \frac{e^{-\beta v}}{(\alpha v)^{n-2}}f(0) + (\alpha v)^{3}F_{\alpha,\beta}(v) \quad (4)$$

Similarly we can find the result form the statement 4 mentioned in theorem:

$$SEE_{\alpha,\beta}\{f^{(m)}(t)\} = -\frac{e^{-\beta v}}{(\alpha v)^{n}}f^{(m-1)}(0) - \frac{e^{-\beta v}}{(\alpha v)^{n-1}}f^{(m-2)}(0) - \frac{e^{-\beta v}}{(\alpha v)^{n-2}}f^{(m-3)}(0) - \cdots - \frac{e^{-\beta v}}{(\alpha v)^{n-m+1}}f(0) + (\alpha v)^{m}SEE_{\alpha,\beta}\{f(t)\} \quad (5)$$

Linear Property:

Let $f(t)$ and $g(t)$ be two functions then linearity property of two parametric extensions of SEE transformation define as:

$$SEE_{\alpha,\beta}\{c_1 f(t) \pm c_2 g(t)\} = c_1 SEE_{\alpha,\beta}f(t) \pm c_2 SEE_{\alpha,\beta}g(t) \quad (6)$$

Where c_1 and c_2 are constants.

Change of Scale Property:

Let $f(t)$ be a function and $SEE_{\alpha,\beta}$ of $f(t)$ is $F_{\alpha,\beta}(v)$ then change of scale property of two parametric SEE transformation can be described as:

$$SEE_{\alpha,\beta}\{f(ct)\} = \left(\frac{1}{c^{n-1}}\right)F_{\alpha,\beta}\left(\frac{v}{c}\right) \quad (7)$$

Here c is a constant value.

Shifting Property:

Let $f(t)$ be a function and $SEE_{\alpha,\beta}$ of $f(t)$ is $F_{\alpha,\beta}(v)$ then shifting property of two parametric SEE transformation can be described as:

$$SEE_{\alpha,\beta}\{e^{ct}f(t)\} = F_{\alpha,\beta}(v-c) \quad (8)$$

Theorem 2.

Let $f(t)$ and $g(t)$ be two functions and $SEE_{\alpha,\beta}$ of $f(t)$ is $F_{\alpha,\beta}(v)$, $SEE_{\alpha,\beta}$ of $h(t)$ is $H_{\alpha,\beta}(v)$ then the convolution of two parametric SEE transform can be described as:

$$SEE_{\alpha,\beta}\{f(t) * h(t)\} = -(\alpha v)^{n}F_{\alpha,\beta}(v).H_{\alpha,\beta}(v) \quad (9)$$

3.APPLICATION

Population Growth Rate Problem: Exponential growth is linked to the process of population growth rate equation decline. Authors in [21] have mathematically described the decay problems of a substance with the help of first order ordinary linear differential equation:

$$\frac{dP}{dt} = -\mu P \quad (10)$$

With the initial condition:

$$P(0) = P_0 \quad (11)$$

Where μ is a real positive number, P is the value of population at t-time and P_0 is the initial value of population at initial time t_0.

Sol: By applying the two parametric SEE transform on (10) and by using the rule mentioned in (2) we will get:

$$-\frac{e^{-\beta v}}{(\alpha v)^n}P(0)+(\alpha v)P_{\alpha,\beta}(v)=-\mu P_{\alpha,\beta}(v).$$

By using initial condition in (11) we get:

$$-\frac{e^{-\beta v}}{(\alpha v)^n}P_0+(\alpha v)P_{\alpha,\beta}(v)=-\mu P_{\alpha,\beta}(v)$$

$$[(\alpha v)+\mu]P_{\alpha,\beta}(v)=\frac{e^{-\beta v}}{(\alpha v)^n}P_0$$

$$P_{\alpha,\beta}(v)=\frac{e^{-\beta v}}{(\alpha v)^n[(\alpha v)+\mu]}P_0 \qquad (12)$$

By using the definition in (1) and table values we get:

$$P(t)=P_0e^{-\mu t} \qquad (13)$$

Equation 13 provides the solution of population growth rate problem.

4.Conclusion

SEE transformation with two parameter has been defined in this research. The solution of population growth rate equation has been described to show the efficiency of this proposed technique which can be compared with other Laplace type transformation. It can be seen that it is helpful in solving linear differential models. The proposed transformation can turned into ordinary SEE transformation by adjusting the values of parameters.

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