The Calculation of Binding Energies for Even-Even Mg(A=20,22,28 And 30) Isotopes

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Abstract

The rotational model symmetry is a strong feature of 1d shell nuclei, where symmetry breaking spin-orbital force is rather weak. The binding energies and low-lying energy spectra of Mg (A=20,22,28 and 30) even-even isotopes have been calculated. The interaction used contains the monopole-monopole, quadrupole-quadrupole and isospin dependent terms. Interaction parameters are fixed so as to reproduce the binding of 8 nucleons in N=8 orbit for Z=12 isotope.

Key words: Binding Energy, Even-Even Isotopes, rotational model symmetry, shell nuclei, spin.

Introduction

The spin and parity for the ground state for even-even Mg (A=20, 22, 28 and 30) isotopes are verified experimentally. The values of spin and parity for the ground states of these isotopes predicted by the shell model is $\frac{1}{2}^-$ and $\frac{3}{2}^-$ to be compared with experimental values of $\frac{1}{2}^+$ and $\frac{1}{2}^-$ respectively [1]. In the region of N=1 the single particle orbits, gives an explanation of experiment values of spin and parity. As a result, the intruder states are expected to be found in low lying spectra for Mg (A=20, 22, 28 and 30) isotopes [2,3]. The rotational model was proposed and used by Elliott to describe rotational bands in light nuclei [2,4]. The quantum members $\lambda\mu$ are the appropriate repress notations of symmetry group are used in this model, to create many basis states for Mg nucleus. The four parameter residual interaction with monopole-monopole, quadrupole-quadrupole and isospin dependent interaction terms with strength parameters (p_o, p_1), χ and β respectively are used in the calculations of binding energies and law-laying spectra for ^{20}Mg, ^{22}Mg, ^{28}Mg and ^{30}Mg even-even isotopes.

The ground state band is described by the quantum number $\lambda\mu$ that extended the ground state binding energy for a given strength χ of quadrupole-quadrupole interaction. Interaction parameters χ and p_o are fitted to experimental ground state binding energy and excitation energy of 2_1^+ state in Mg (A=20, 22, 28 and 30) even-even isotopes. Interaction parameters p_1 and β are adjusted to give a best fit to experimental ground state binding energies of Mg isotopes with A=20,22,28 and 30. The calculated energy spectra are compared with available experimental data. The new mass formula [5], is also used to calculate the residual interaction. A comparison of our calculations and experimental data shows the reasonable agreement.

Theory

In this article we will write down the main equations of rotational model of Elliott [2], explaining the generators, subgroups and Casimir operators. The representation of rotational bands is characterized by quantum numbers ($\lambda\mu$), that which determine the eigenvalue of Casimir operator for symmetry group of rotational bands.

In spherical basis, three components of orbital angular momentum operator L, and five components quadrupole moment operator Q constitute the eight generators of system group of rotational bands. A components of quadrupole moment, is defined as;

$$Q_q = \sqrt{\frac{4\pi}{5}}\,\alpha^2\left(r^2 Y_{2,q}(\hat{r}) + \frac{p^2}{\alpha^2\hbar} Y_{2,q}(\hat{p})\right) \qquad \ldots(1)$$

Where $q = 0, \pm 1, \pm 2$, $\alpha^2 = \frac{m\omega}{\hbar}$ and $Y_{2,q}$ are the spherical harmonics.

The eight generators are satisfy the following commutation relations.

$$\left.\begin{aligned} [L_q, L'_q] &= -\sqrt{2}\,(11qq'|1q+q')L_{q+q'}) \\ [Q_q, L'_q] &= -\sqrt{6}\,(21qq'|2q+q')Q_{q+q'}) \\ [Q_q, Q'_q] &= 3\sqrt{10}\,(22qq'|1q+q')L_{q+q'}) \end{aligned}\right\} \qquad \ldots(2)$$

The Casimir operator, Casimir operator eigenvalues and corresponding quantum numbers for symmetry group of rotational states are listed in the following table.

SU(3)	
Casimir operators	$Q\cdot Q+3L\cdot L$
Eigenvalues	$\frac{2}{3}(\lambda^2+\mu^2+\lambda\mu+3\lambda+3\mu)$
Quantum numbers	$(\lambda\mu)$

The quantum numbers $(\lambda\mu)$ characterize the permutation symmetry between harmonic oscillator quanta and Λ is proportional to the number of quanta in xy plane for case $\mu=0$ [6].

The definition of the binding energy of isotope A in residual interaction zero state I(A) as:

$$I(A)=B(A,Z)-B(20,8)-2S_p(21,9)-A-12S_n(21,8) \quad \ldots(3)$$

Where $B(A,Z)$ is the binding energy of the nucleus with Z protons and $A-Z$ neutrons, $S_p(21,9)$ is one proton separation energy of ^{21}Ne and $S_n(21,8)$ is one neutron separation energy of ^{21}Al.

An interaction Hamiltonian used here containing monopole-monopole, quadrupole-quadrupole and isospin dependent interaction terms that is;

$$H=-F_o(n)-\chi Q\cdot Q+\beta T(T+1) \quad \ldots(4)$$

Where n is the number of active nucleons (n=A-p).

The isospin dependent interaction is repulsive while the monopole-monopole and quadrupole-quadrupole interaction are attractive. The strength of monopole-monopole interaction for nucleons in the same shell is p_o and for nucleons in different shells is p_1. We write the monopole-monopole interaction for n_1 and n_2 $(n=n_1+n_2)$ nucleons in oscillator shells N=1 and N=2, respectively,[7] as:

$$F_o(n)=p_o\frac{n_1(n_1-1)}{2}+p_o\frac{n_2(n_2-1)}{2}+p_1n_1n_2$$

The quadrupole-quadrupole interaction operator may be expressed as:

$$-\chi Q\cdot Q=-\chi 3(2C_{SU(3)}-L\cdot L)$$

Where $C_{SU(3)}$ is the Casimir operator of the rotational group. The ground state isospin is defined as $T=\left|\frac{n-p}{2}\right|$, where n and p are the number of neutrons and protons respectively. A Fortran routine has been used in the calculations of C's and matrix elements of Q.Q operator between relevant good L states.

To compare the mass formula of Bethe-Weizsacker [8] with the new mass formula, which has a new parameter $\Delta(N,Z)$ and has redefinition of the pairing term δ_{new}, as:

$$B(A,Z)_{new}=a_\upsilon A-a_sA^{\frac{2}{3}}-a_c\frac{Z(Z-1)}{A^{\frac{1}{3}}}-a_{sym}\frac{(A-2Z)^2}{A}+\delta_{new}+\Delta(N,Z) \quad \ldots(5)$$

Where

$a_\upsilon=15.85$ MeV

$a_s = 18.34\ \text{MeV}$

$a_c = 0.71\ \text{MeV}$

$a_{sym} = 23.21\ \text{MeV}$

$$\delta_{new} = (1 - \exp(-\frac{Z}{c}))\delta$$

$$c = 6/\ln 2$$

$\delta = 12A^{\frac{-1}{2}}$ for even – even nuclei,

$= -12A^{\frac{-1}{2}}$ for odd – odd nuclei

$= 0$ for odd A nuclei

$$\Delta(N,Z) = \left|N - \frac{4}{3}Z\right| N^k Z \exp(-\frac{Z}{3}) \qquad \text{with} \quad k = 0.45$$

Using the new mass formula to calculate the binding energy we obtain the residual interaction $I_{new}(A)$ for Mg isotopes as:

$$I_{new}(A) = B_{new}(A,p) - B(p,n) - 2S_p(A-1,p) - (A-n)S_n(A+1,n) \qquad \ldots(6)$$

Results and Discussion

Table 1, listed Mg(A=20,22,28 and 30) even-even isotopes, besides the isotopes of A-1 (Na) and A+1 (Al) nuclei. Half-life time, Q-value, spin (π) and decay mode for Mg isotopes are listed in the table also. The Mg(A=24,25 and 26) are stable.

Figure 1, shows the variation of Q-value with Mg even-even isotopes for A=20,22,28 and 30. The valley between A=22 and A=28 isotopes, belongs to Mg(A=24 and 26), where they are stable isotopes and not including this study because they have a closed shell for neutron and proton.

Table 2: lists the representations ($\lambda\mu$) that maximize the quadrupole-quadrupole interaction and the possible L values for the ground state band.

In Figure 2, shows the plotting of the ground state binding energies in Table 3 for (A=20,22,28 and 30) isotopes, due to residual interaction active nucleons, the calculation based on the rotational band model and new mass formula (Eq. 3). The value of parameter χ is chosen to reproduce the excitation energy 2_1^+ state in Mg isotopes. The interaction parameters p_1 and β are used to find the best fit with the experimental ground states binding energies of Mg isotopes with A=22,24,28 and 30.

Table 4, lists the calculated and experimental excitation energies of 0_1^+, 2_1^+ and 0_2^+ states for comparison.

Figure 3, shows the fitting with experimental data, which is obtained with parameters $p_1 = 2.8$ MeV and $\beta = 5.8$ MeV.

The calculated binding energies show a reasonable agreement with the others.

No intruder states are found in Mg(A=20,22,28 and 30) even-even isotopes from Na(A=19,21,27 and 29) and isotopes Al(A=21,23,29 and 31) isotopes.

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Table (1): The Mg(A=20-30) even-even isotopes with their neighbor nuclei

No.	Isotopes			isotopes [7]			
	A-1	$^{even}_{even}$Mg	A+1	$T_{½}$	Q-value MeV	Spin (π)	Decay mode
1	^{19}Na	^{20}Mg	^{21}Al	95 ms	10.730	0^+	EC
2	^{21}Na	^{22}Mg	^{23}Al	3.857 s	4.725	0^+	EC
3	^{27}Na	^{28}Mg	^{29}Al	20.910 h	1.832	0^+	β^-
4	^{29}Na	^{30}Mg	^{31}Al	0.335	6.990	0^+	β^-

Table No.(2): Values of ($\lambda_1\mu_1L_1$) for active nucleons

No.	Ground state band		
	isotopes	$\lambda_1\mu_1$	L_1
1	^{20}Mg	20	0,2
2	^{22}Mg	30	0,2,4
3	^{28}Mg	22	0,2,3,4
4	^{30}Mg	22	0,2

Table No.(3): Values of Binding energies B(A,Z) and I(A) MeV

No.	isotopes	B(A,Z)	I(A) MeV
1	^{20}Mg	130.066	11.404
2	^{22}Mg	162.487	12.747
3	^{28}Mg	245.447	15.666
4	^{30}Mg	244.274	15.629

Table No.(4): Excitation energies of 0_1^+, 2_1^+ and 0_2^+ states in ^{20}Mg, ^{22}Mg, ^{28}Mg and ^{30}Mg nuclei in MeV

Spin	^{20}Mg	^{22}Mg	^{28}Mg	^{30}Mg

	Calc.	Exp.	Calc.	Exp.	Calc.	Exp.	Calc.	Exp.
0_1^+	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2_1^+	3.26	3.26	3.21	3.67	3.14	3.48	3.22	2.89
0_2^+	17.88	18.75	16.91	6.78	9.87	13.78	18.81	14.27

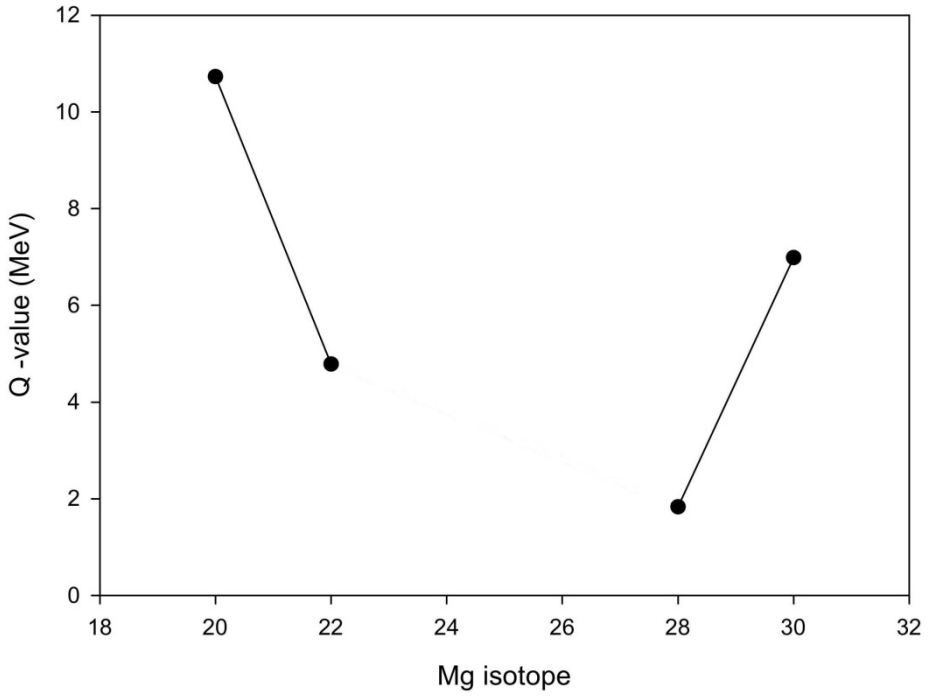

Figure No.(1): The variation of Q-value with Mg (A=20,22,28 and 30) isotopes.

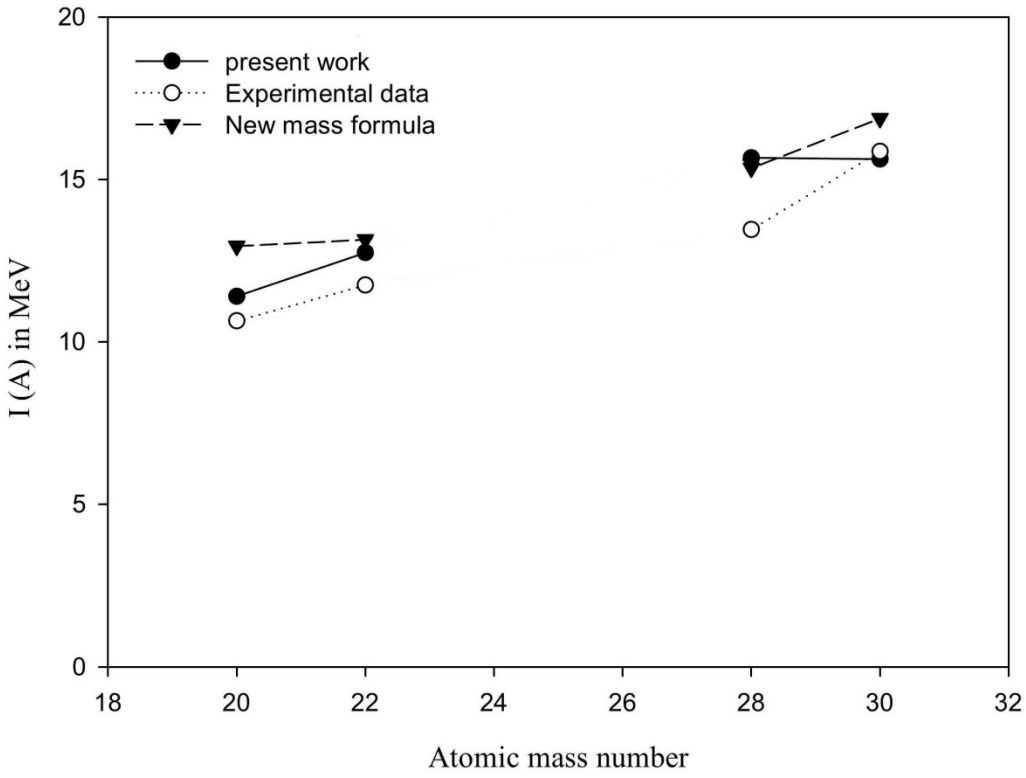

Figure No.(2): The comparison between results of the present calculations, new mass formula and experimental data

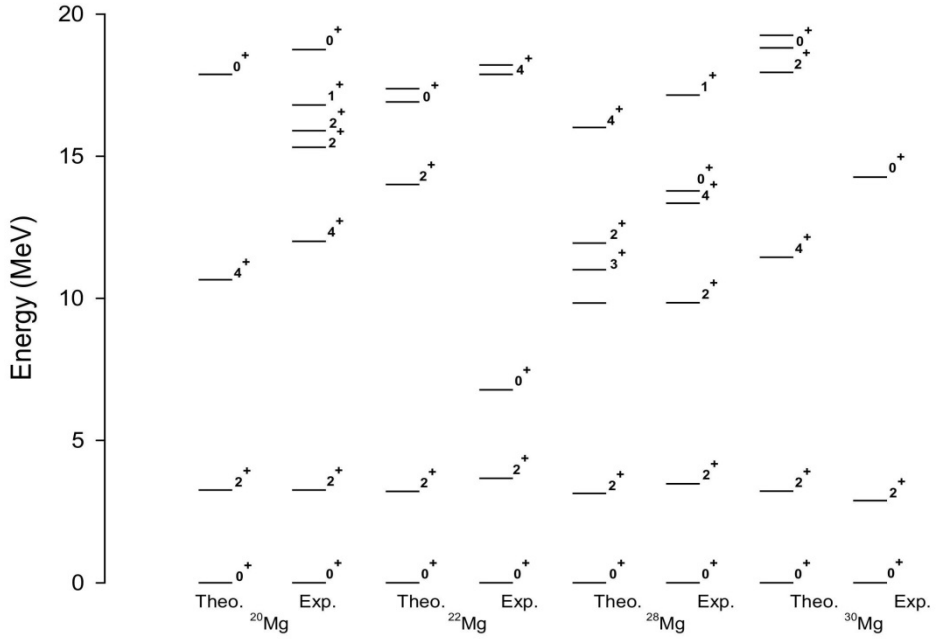

Figure No.(3): Calculated of rotational model and new mass formula and experimental ground state binding energy in Mg isotopes

حساب طاقات الربط لنظائر المغنيسيوم ذات الأعداد الكتلية 20، 22، 28 و 30 الزوجية-الزوجية

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الخلاصة

يعتبر إنموذج التناظر الدوراني وصفا جيدا للانوية ذات الغلاف 1d، إذ يكون قوة تناظر تكسر المدار – برم ضعيف، حسبت طاقات الربط واطياف طاقات المستويات الواطئة لنظائر المغنسيوم Mg (A=20,22,28 and 30) الزوجية – الزوجية تم حسابها. استخدم التفاعل المتضمن احادي القطب – احادي القطب، رباعي القطب – رباعي القطب والايزوسبن لهذه النظائر، معاملات التفاعل تم حلها لاعادة حساب طاقة الربط لثمان نيكوليونات وعدد N=8 و Z=12.

الكلمات المفتاحية : طاقة الترابط ، النظائر زوجية –زوجية ، نموذج التناظر الدوراني ،غلاف النواة ،برم.