



On $\eta G_{\mathcal{S}}$ -Compactness

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Abstract

Open sets may be viewed as an extension of semi-open sets by applying the notions of semi-open sets and grill nano to nG s-open sets, with the following four goals in mind: The objective is to characterize nG s-open sets by examining and proving numerous of its attributes and comments. And investigate and define new kinds of functions based on the concept of nG s-open sets, using sets of nG s-open sets, we will define a new type of Compact type and call it nG s-open compact then we will find the relationship between these new types of Compact type with nano compact. We will also talk about the relationship between nano grill semi-open sets and continuous functions and the relationship between nano grill semi-open sets and irresolute function as well we give some examples, proofs and observations about the relationship between nano grill semi-open sets and functions and their relation to nano compact.

Keywords: Nano Grill semi-open compact space, $\eta G_{\mathcal{S}}$ \mathcal{O} -semi-open compact., nano compact.

1. Introduction

The concept for grill topological spaces rests on the use of two operators: and. The pioneer of this concept was Choquet (1). Some parallels between the Choquet idea and ideas, nets and filters have been discovered. Several hypotheses and characteristics have been discussed in (2–5). It allows for the growth of the topological assembly utilized to account for intangibles like love, intelligence, beauty, instructional quality, etc. Additionally, it broadens the frontiers of Nano topological spaces by employing the concept of grill modifications in the lower approximation, the upper approximation, and the boundary region. First proposed in 1970 by Levine (6), the concept of enlarging closed sets is widely credited as a breakthrough in the field. Lower, higher,



and boundary estimates of a subset of a cosmic set with an important basis to it are the foundation on which the idea of nano topological assemblage rests.

Also, the concept of nano is used to introduce the definitions of the closed set, the interior set, and the closure set. Lellis (7) first developed this concept in 2013. The primary objective of this study is to incorporate a grill into a space containing a generalized closed nano topology. We've established contact with some very important people.

2. Preliminaries

Definition 2.1:(4), (8)

A Grill is a nonempty collection of nonempty subsets of a topological space \square

- i. $A \in \mathbb{G}$ and $A \subseteq B \subseteq \square$ then $B \in \mathbb{G}$
- ii. $A, B \subseteq \square$ and $A \cup B \in \mathbb{G}$ then $A \in \mathbb{G}$ or $B \in \mathbb{G}$. (9), (10)

Assuming that \square is a non-empty set, the following sets are grills on \square .(11), (12)

- \emptyset & $\square(\square) \setminus \{\emptyset\}$ are examples of trivial grills on \square .
- \mathbb{G}_∞ is the grill of all infinite subset of \square .
- \mathbb{G}_{\aleph_0} is the grill of all uncountable subsets of \square .
- $\mathbb{G}_\square = \{A: A \in \square(\square), \square \in A\}$ is a certain point grill on \square .
- $\mathbb{G}_A = \{B: B \in \square(\square), B \cap A^c \neq \emptyset\}$.

* If (\square, τ) is a topological space, and so the set of the all dense subset that does not already exist here is known as $\mathbb{G} = \{A: \text{int}(\text{Cl}(A)) \neq \emptyset\}$ is one kind of grill on \square (4).

* Suppose that \mathbb{G} a grill on (\square, τ) . A mapping $\mathbb{F}: \square(\square) \rightarrow \square(\square)$ is referred to as $\mathbb{F}(A) = \{\square \in \square: A \cap \hat{u} \in \mathbb{G} \text{ for every } \hat{u} \in \tau; \square \in \hat{u}\}$ for every $A \in \square(\square)$. A mapping $\psi: \square(\square) \rightarrow \square(\square)$ is referred to as $\psi(A) = A \cup \mathbb{F}(A)$ for every $A \in \square(\square)$.(13)

Kuratowski's Axioms of Closure for the map ψ are verified: (13), (14), (15)

- i. $\psi(\emptyset) = \emptyset$,
- ii. when $A \subseteq B$, then $\psi(A) \subseteq \psi(B)$,
- iii. when $A \subseteq \square$, then $\psi(\psi(A)) = \psi(A)$,
- iv. when $A, B \subseteq \square$, then $\psi(A \cup B) = \psi(A) \cup \psi(B)$.

Definition 2.2: (13)

There exists a special topology $\tau_G = \{\hat{u} \subseteq \square: \psi(\square - \hat{u}) = (\square - \hat{u})\}$, when for any $A \subseteq \square$ that corresponds inside the topological space, to a grill $\mathbb{G}(\square, \tau)$.

$\psi(A) = A \cup \mathbb{F}(A) = \tau_G\text{-Cl}(A)$ and $\tau \subseteq \tau_G$.

Remark 2.3: (4)

If $\mathbb{G} = \square(\square)/\{\emptyset\}$, then $\tau_G = \tau$.

Remark 2.4: (2)

We can find τ_G by using the base as follows $\mathcal{B}(\tau_G, \tau) = \{\square - A; \square \in \tau, A \notin \mathbb{G}\}$

Definition 2.5:(13)

Let $\square \neq \emptyset$ and \mathbb{R} be an equivalence relation on \square , $A \subseteq \square$.

- i. The upper approximation of A for \mathbb{R} is denoted by $\overline{\mathbb{R}}(A)$, where

$$\overline{\mathbb{R}}(A) = \cup_{\square \in \square} \{\mathbb{R}(\square): \mathbb{R}(\square) \cap A \neq \emptyset\}.$$

ii. The lower approximation of \mathcal{A} for \mathcal{R} is denoted by $\underline{\mathcal{R}}_{\square}(\mathcal{A})$, where

$$\underline{\mathcal{R}}_{\square}(\mathcal{A}) = \cup_{\square \in \square} \{\mathcal{R}(\square) : \mathcal{R}(\square) \subseteq \mathcal{A}\}.$$

iii. The boundary region of \mathcal{A} for \mathcal{R} is denoted by $\mathcal{B}_{\square}(\mathcal{A})$, where

$$\mathcal{B}_{\square}(\mathcal{A}) = \overline{\mathcal{R}}_{\square}(\mathcal{A}) - \underline{\mathcal{R}}_{\square}(\mathcal{A}).$$

Definition 2.6:(7), (16)

Let $\square \neq \emptyset$ and \mathcal{R} be an equivalence relation on \square and $\mathcal{t}_{\square}(\mathcal{A}) = \{\square, \emptyset, \overline{\mathcal{R}}_{\square}(\mathcal{A}), \underline{\mathcal{R}}_{\square}(\mathcal{A}), \mathcal{B}_{\square}(\mathcal{A})\}$, where $\mathcal{A} \subseteq \square$. Then $\mathcal{t}_{\square}(\mathcal{A})$ is a topology on \square named nano topology for \mathcal{A} ($\square, \mathcal{t}_{\square}(\mathcal{A})$ space is known as nano topological. The components of $\mathcal{t}_{\square}(\mathcal{A})$ are named nano-open sets denoted by η – open sets. The complement of a η – open sets is named a nano-closed set denoted by η – closed sets.

Definition 2.7:(7)

Let $(\square, \mathcal{t}_{\square})$ be N.T.S (nano topological space) and $\mathcal{A} \subseteq \square$. The nano closure (respectively, nano interior) of \mathcal{A} which is short $\eta\mathcal{C}l_{\square}(\mathcal{A})$ (respectively, $\eta\text{int}_{\square}(\mathcal{A})$) is defined by; $\eta\mathcal{C}l_{\square}(\mathcal{A}) = \cap \{\mathcal{F}, \mathcal{F}^c \in \mathcal{t}_{\square}, \mathcal{A} \subseteq \mathcal{F}\}$, (resp., $\eta\text{int}_{\square}(\mathcal{A}) = \eta\text{int}_{\square}(\mathcal{A}) = \{\hat{u}; \hat{u} \in \mathcal{t}_{\square}, \hat{u} \in \mathcal{A}\}$).

Note 8:

We said the triple $(\square, \mathcal{t}_{\square}, \mathcal{G})$ G.N.T.S (Grill nano topological space)..

Definition 2.9:(7)

Let $(\square, \mathcal{t}_{\square})$ be an N.T.S, a subset \mathcal{A} of \square is named N.S.O (nano semi-open set) $\mathcal{A} \subseteq \eta\mathcal{C}l_{\square}(\eta\text{int}_{\square}(\mathcal{A})) \iff \exists \hat{u} \in \mathcal{t}_{\square}; \hat{u} \subseteq \mathcal{A} \subseteq \eta\mathcal{C}l_{\square}(\hat{u})$.

A subset \mathcal{B} of \square is called nano semi-closed if $(\square - \mathcal{B})$ is N.S.O set The collection of all N.S.O (respectively, N.S.C) sets in a nano topological space $(\square, \mathcal{t}_{\square})$.

will be symbolized by $\eta\mathcal{S}\hat{O}(\square)$ (respectively, $\eta\mathcal{G}_s\mathcal{C}(\square)$).

Definition 2.10:(17)

There exists a special topology $\eta\mathcal{t}_{\square\mathcal{G}} = \{\hat{u} \subseteq \square : \psi(\square - \hat{u}) = (\square - \hat{u})\}$, when for any $\mathcal{A} \subseteq \square$ that corresponds to a nano grill \mathcal{G} on the topological space $(\square, \mathcal{t}_{\square})$.

$$\psi(\mathcal{A}) = \mathcal{A} \cup \mathcal{G}(\mathcal{A}) = \eta\mathcal{t}_{\square\mathcal{G}} - \mathcal{C}l(\mathcal{A}) \text{ and } \mathcal{t}_{\square} \subseteq \eta\mathcal{t}_{\square\mathcal{G}}.$$

Definition 2.11:

Let $(\square, \mathcal{t}_{\square})$ be N.T.S and $\mathcal{A} \subseteq \square$. The nano closure (respectively, nano interior) of \mathcal{A} which is short $\eta\mathcal{C}l_{\square\mathcal{G}}(\mathcal{A})$ (respectively, $\eta\text{int}_{\square\mathcal{G}}(\mathcal{A})$) is defined by; $\eta\mathcal{C}l_{\square\mathcal{G}}(\mathcal{A}) = \cap \{\mathcal{F}, \mathcal{F}^c \in \mathcal{t}_{\square}, \mathcal{A} \subseteq \mathcal{F}\}$, (resp., $\eta\text{int}_{\square\mathcal{G}}(\mathcal{A}) = \eta\text{int}_{\square\mathcal{G}}(\mathcal{A}) = \{\hat{u}; \hat{u} \in \mathcal{t}_{\square}, \hat{u} \in \mathcal{A}\}$).

Definition 2.12: (18), (19), (20)

A topological spaces $(\square, \mathcal{t}_{\square}, \mathcal{G})$ is named nano compact space if and only if all nano open cover of \square has a finite subcover..

3. Nano Grill semi-open sets in nano compact space

Definition 3.1:

For any Grill topological space $(\square, \mathcal{t}_{\mathcal{G}})$ and $\mathcal{A} \subseteq \square$; \mathcal{A} is said to be nano Grill semi-open if there exists $\hat{u} \in \mathcal{t}_{\square}; \hat{u} - \mathcal{A} \notin \mathcal{G}$ and $\mathcal{A} - \eta\mathcal{C}l_{\square\mathcal{G}}(\hat{u}) \notin \mathcal{G}$. And \mathcal{A} denoted by $\eta\mathcal{G}_s$ -open. $\square - \mathcal{A}$ is a nano Grill semi-closed and denoted by $\eta\mathcal{G}_s$ -semi-closed and the set of all $\eta\mathcal{G}_s$ -open presently by

$\eta G_S \hat{O}(\square)$ and the set of all ηG_S -semi-closed presently by $\eta G_S \zeta(\square)$.

Example 3.2:

Let $(\square, \tau_{\square}, G)$ be a nano grill topological space to be a nano grill and

$$\begin{aligned} \square &= \{\square_1, \square_2, \square_3, \square_4\} \\ G &= \{\hat{u} \subseteq \square; \square_2 \in \hat{u}\} \\ G &= \{\{\square_2\}, \{\square_1, \square_2\}, \{\square_3, \square_2\}, \{\square_4, \square_2\}, \{\square_1, \square_2, \square_3\}, \{\square_1, \square_2, \square_4\}, \{\square_2, \square_3, \square_4\}, \square\} \\ \mathbb{R} &= \{(\square_1, \square_1), (\square_2, \square_2), (\square_3, \square_3), (\square_4, \square_4), (\square_2, \square_4), (\square_4, \square_2)\} \\ \mathbb{R} \setminus [\square] &= \{\{\square_1\}, \{\square_2, \square_4\}, \{\square_3\}\} \\ \mathbb{R} \subseteq \square, \mathbb{R} &= \{2,3\}, \\ \tau_{\mathbb{R}} &= \{\square, \emptyset, \{3\}, \{2,4\}, \{2,3,4\}\} \\ \mathcal{B} &= \{\mathbb{R} - A; \mathbb{R} \in \tau_{\mathbb{R}} \wedge A \notin G\} \\ \mathcal{B} &= \{\square, \emptyset, \{\square_1, \square_2, \square_4\}, \{\square_1, \square_2, \square_3\}, \{\square_2, \square_3, \square_4\}, \{\square_2, \square_3\}, \{\square_2, \square_4\}, \{\square_1, \square_2\}, \{\square_2\}, \{\square_3\}\} \\ &= \tau_{\mathbb{R}G} \\ \therefore \eta G_S \hat{O}(\square) &= \square(\square) . \end{aligned}$$

Proposition 3.3:

- i. Every nano open set is a ηG_S -open sets.
- ii. Every nano closed set is a ηG_S -closed.

Example 3.2 demonstrates that the converse of Remark 3.3(i)(ii) is not true.

Definition 3.4:

Let $(\square, \tau_{\square}, G)$ be a nano grill topological space. By a ηG_S – open cover of \square we mean a subfamily of $\eta G_S \hat{O}(\square)$ wich cover \square

Definition 3.5:

A nano grill topological space $(\square, \tau_{\square}, G)$ is said to be ηG_S – compact space if every ηG_S – open cover for \square has a finite subcover.

Theorem 3.6:

A nano grill topological space $(\square, \tau_{\square}, G)$ is be ηG_S – compact space if and only if every family of ηG_S – closed subsets of \square with finite intersection property has a non-empty intersection.

Proof:

Suppose that \square is ηG_S – compact space and let $\{F_i: i \in \Lambda\}$ be a family of ηG_S – closed subsets of \square with (F.I.P). Assume that $\bigcap_{i \in \Lambda} F_i = \emptyset$, then $\bigcup_{i \in \Lambda} F_i^c = \square$, where $\{F_i^c: i \in \Lambda\}$ is a ηG_S – open cover of \square which is a ηG_S – compact space, It follows that there exists a finite subcover $\{F_i^c\}_{i=1}^n$ such that $\square = \bigcup_{i=1}^n F_i^c$, then $\bigcup_{i=1}^n F_i = \emptyset$ which is a contradiction. Since $\{F_i: i \in \Lambda\}$ has a F.I.P. Now, suppose that every family of ηG_S – closed subsets of \square with (f.i.p) has a non-empty intersection. Assume that \square is not ηG_S – compact space, let $\{u_{\alpha}: \alpha \in \Lambda\}$ be a ηG_S – open cover of \square and suppose if possible, $\{u_{\alpha}: \alpha \in \Lambda\}$ has no finite subcover. The

collection $\{u_\alpha^c : \alpha \in \Lambda\}$ has the F.I.P, if but $\{u_\alpha^c : \alpha \in \Lambda\}$ is a family of ηG_ζ – closed sets, so $\bigcap_{\alpha \in \Lambda} u_\alpha^c \neq \emptyset$, it follows that $\bigcup_{\alpha \in \Lambda} u_\alpha \neq \square$ which is contradiction since $\{u_\alpha : \alpha \in \Lambda\}$ is a ηG_ζ – open cover of \square .

Theorem 3.7:

Every ηG_ζ – compact space is a nano compact space.

Proof:

Let $U = \{u_i, i \in \Lambda; u_i \in \tau_\square \forall i\}$ is an open cover for \square such that $\square = \bigcup_{i \in \Lambda} u_i$ and since every open set is a ηG_ζ – open sets .So, U is a ηG_ζ – open cover for \square , and since X is a not ηG_ζ – compact set .So, there exist a finite subcover say $U = \{u_1, u_2, \dots, u_n\}$ such that $\square = \bigcup_{i=1}^n u_i$. Therefore, \square is a nano compact space.

Definition 3.8:

- Let $\mathcal{F}: (\square, \tau_\square, \mathbb{G}) \rightarrow (Y, \tau_{\square'}, \mathbb{G}')$ be a function then \mathcal{F} believed to be;
1. ηG semi –continuous function, denoted by ηG_ζ -continuous function if $\mathcal{F}^{-1}(u) \in \eta G_\zeta \mathring{O}(\square)$ for all $u \in \tau_{\square'}$.
 2. Strongly ηG semi –continuous function, denoted by "Strongly ηG_ζ -continuous function" if $\mathcal{F}^{-1}(u) \in \tau_\square$, fore $u \in \eta G_\zeta \mathring{O}(Y)$.
 3. ηG semi-irresolute function, denoted by ηG_ζ -irresolute function if $\mathcal{F}^{-1}(u) \in \eta G_\zeta \mathring{O}(\square)$, for all $u \in \eta G_\zeta \mathring{O}(Y)$.

Proposition 3.9:

Let $\mathcal{F}: (\square, \tau_\square, \mathbb{G}) \rightarrow (Y, \tau_{\square'}, \mathbb{G}')$ be a function.

1. \mathcal{F} is ηG_ζ -irresolute function whenever \mathcal{F} is strongly ηG_ζ -continuous function.
2. If \mathcal{F} is a strongly ηG_ζ -continuous function then \mathcal{F} is a continuous function.
3. " When \mathcal{F} is a continuous function " then \mathcal{F} is ηG_ζ -continuous function.
4. \mathcal{F} is ηG_ζ -continuous function whenever \mathcal{F} is a ηG_ζ -irresolute function.

In general, the opposite of (proposition 3.9) is not supported by the following examples.

Example 3.10:

Let $\mathcal{F}: (\square, \tau_\square, \mathbb{G}) \rightarrow (\square, \tau_\square, \mathbb{G}^{\sim})$ be a function such that $\mathcal{F}(\square) = \square$ for each $\square \in \square$

Where

$$\begin{aligned} \square &= \{x_1, x_2, x_3\}, \mathbb{G} = \square(\square) \setminus \{\emptyset\} \\ \mathbb{R} &= \{(\square_1, \square_1), (\square_2, \square_2), (\square_3, \square_3), (\square_2, \square_3), (\square_3, \square_2)\} \\ \mathbb{R} \setminus [\square] &= \{(\square_2, \square_3), (\square_3, \square_2)\}, \mathbb{R} = \{(\square_1, \square_1)\} \\ \tau_\square &= \{\square, \emptyset, \{(\square_1, \square_1)\}\} \end{aligned}$$

$$\mathbb{G}^{\sim} = \{u; \square_1 \in u\}, \eta G_\zeta \mathring{O}(\square) = \{u; \square_1 \in u\} \cup \{\emptyset\}, \eta G_\zeta^{\sim} \mathring{O}(\square) = \square(\square).$$

So that, \mathcal{F} is ηG_ζ -continuous function and continuous function but it's not ηG_ζ -irresolute function and it's not strongly ηG_ζ -continuous function.

Example 3.11:

The function $\mathcal{F}: (\square, \tau_\square, \mathbb{G}) \rightarrow (\square, \tau_\square, \mathbb{G}^{\sim})$ such that

$$\mathcal{F}(\{(\square_2)\}) = \{(\square_1)\}, \mathcal{F}(\{(\square_1)\}) = \{(\square_2)\}, \mathcal{F}(\{(\square_3)\}) = \{(\square_3)\},$$

Where

$$\begin{aligned} \square &= \{\square_1, \square_2, \square_3\}, \mathcal{G} \sim = \square(\square) \setminus \{\emptyset\} \\ \mathbb{R} &= \{(\square_1, \square_1), (\square_2, \square_2), (\square_3, \square_3), (\square_2, \square_3), (\square_3, \square_2)\} \\ \mathbb{R} \setminus [\square] &= \{\{\square_2, \square_3\}, \{\square_1\}\}, \mathbb{I} = \{\square_1\} \\ \mathfrak{t}_{\mathbb{I}} &= \{\square, \emptyset, \{\square_1\}\}, \end{aligned}$$

$$\mathcal{G} = \{u; \square_1 \in u\}, \eta_{\mathcal{G}_S} \mathcal{O}(\square) = \square(\square) \setminus \{\emptyset\},$$

$\eta_{\mathcal{G}_S} \mathcal{O}(\square) = \{u; \square_1 \in u\} \cup \{\emptyset\}$, \mathcal{F} is $\eta_{\mathcal{G}_S} \mathcal{O}(\square)$ continuous function and $\eta_{\mathcal{G}_S}$ -irresolute function but it isn't continuous function and not strongly $\eta_{\mathcal{G}_S}$ -continuous function it's not since

$$\mathcal{F}^{-1}(\square_1) = \{\square_2\} \notin \mathfrak{t}_{\mathbb{I}}.$$

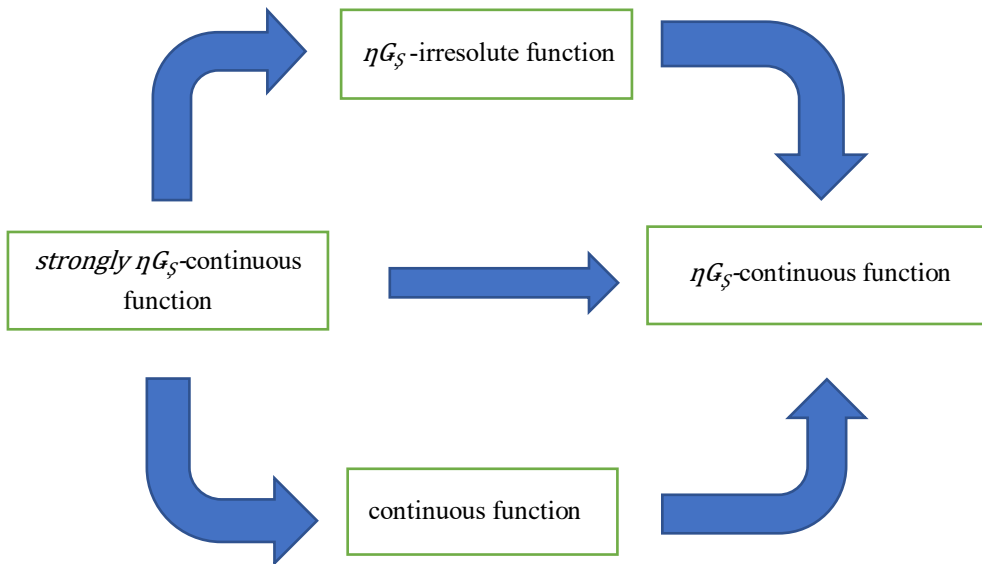


Diagram1. Continuous functions via $\eta_{\mathcal{G}_S}$ -open

Proposition 12:

- i. The $\eta_{\mathcal{G}_S}$ -irresolute image function $\eta_{\mathcal{G}_S}$ -compact space is a $\eta_{\mathcal{G}_S}$ -compact space.
- ii. In strongly $\eta_{\mathcal{G}_S}$ -continuous the image of nano compact space is a $\eta_{\mathcal{G}_S}$ -compact space.
- iii. The $\eta_{\mathcal{G}_S}$ -continuous function the image function of $\eta_{\mathcal{G}_S}$ -compact is a nano compact.

Proposition 3.13:

A $\eta_{\mathcal{G}_S}$ -closed subsets of $\eta_{\mathcal{G}_S}$ -compact space is $\eta_{\mathcal{G}_S}$ -compact.

Theorem 3.14:

If A & B are $\eta_{\mathcal{G}_S}$ -compact, then $A \cup B$ is a $\eta_{\mathcal{G}_S}$ -compact.

Proposition 3.15:

Every $\eta_{\mathcal{G}_S}$ -compact is a nano compact.

Example 3.16:

Let $(\mathbb{R}, \square, \tau_{\mathbb{R}})$ be any nano topological space such that \mathbb{R} is the set of all real numbers and $\mathbb{R} = \{(r,r), r \in \mathbb{Z}\}$ so, $\mathbb{R} \setminus [r] = \{\{r\}, r \in \mathbb{Z}\}$. Now, if $w = \{1\}$ and $G = \square[\mathbb{Z}] \setminus \emptyset$, then $\overline{\mathbb{R}}_{\mathbb{Z}} = \{1\} = \mathbb{R}_{\mathbb{Z}}$ and $\mathbb{B}_{\mathbb{Z}} = \emptyset$ so, $\tau_{\mathbb{Z}} = \eta\tau_{\mathbb{Z}G} = \{\mathbb{Z}, \emptyset, \{1\}\}$ and $\eta G_{\mathbb{Z}}\mathcal{O}(\mathbb{Z}) = \{\hat{u} \subseteq \mathbb{Z}; 1 \in \hat{u}\} \cup \emptyset$. This much is clear: $(\mathbb{R}, \tau_{\mathbb{Z}}, G)$ is a nano compact which is not $\eta G_{\mathbb{Z}}$ – compact since $L = \{\{1,r\}, r \in \mathbb{Z}\}$ is $\eta G_{\mathbb{Z}}$ – open cover has no finite subcover.

4. Conclusion

In this work, a new type of open set was studied using the concept of nano-topology, grill and nano compact which is called $\eta G_{\mathbb{Z}}$ – open sets. The properties of this set were studied. It was found that $\eta G_{\mathbb{Z}}\mathcal{O}(\square)$ represents a supra-topology space. New forms of functionality were defined by applying this notion, and the relationship between these functions was found.

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Conflict of Interest

There are no conflicts of interest.

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