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On ηG_Ş-Compactness

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Abstract

Open sets may be viewed as an extension of semi-open sets by applying the notions of semi-open sets and grill nano to nGs-open sets, with the following four goals in mind: The objective is to characterize nGs-open sets by examining and proving numerous of its attributes and comments. And investigate and define new kinds of functions based on the concept of nGs-open sets, using sets of nGs-open sets, we will define a new type of Compact type and call it nGs-open compact then we will find the relationship between these new types of Compact type with nano compact. We will also talk about the relationship between nano grill semi-open sets and continuous functions and the relationship between nano grill semi-open sets and irresolute function as well we give some examples, proofs and observations about the relationship between nano grill semi-open sets and functions and their relation to nano compact.

Keywords: Nano Grill semi-open compact space, ηG_Ş Ỏ-semi-open compact., nano compact.

1. Introduction

The concept for grill topological spaces rests on the use of two operators: and. The pioneer of this concept was Choquet (1). Some parallels between the Choquat idea and ideas, nets and filters have been discovered. Several hypotheses and characteristics have been discussed in (2–5). It allows for the growth of the topological assembly utilized to account for intangibles like love, intelligence, beauty, instructional quality, etc. Additionally, it broadens the frontiers of Nano topological spaces by employing the concept of grill modifications in the lower approximation, the upper approximation, and the boundary region. First proposed in 1970 by Levine (6), the concept of enlarging closed sets is widely credited as a breakthrough in the field. Lower, higher,

and boundary estimates of a subset of a cosmic set with an important basis to it are the foundation on which the idea of nano topological assemblage rests.
Also, the concept of nano is used to introduce the definitions of the closed set, the interior set, and the closure set. Lellis (7) first developed this concept in 2013. The primary objective of this study is to incorporate a grill into a space containing a generalized closed nano topology. We've established contact with some very important people.

2. Preliminaries

Definition 2.1:(4), (8)

Ⱥ Grill is a nonempty collection of nonempty subsets of a topological space χ

i. $Ⱥ \in Ǥ$ *and* $Ⱥ \subseteq Ɓ \subseteq \chi$ *then* $Ɓ \in Ǥ$
ii. $Ⱥ, Ɓ \subseteq \chi$ *and* $Ⱥ \cup Ɓ \in Ǥ$ *then* $Ⱥ \in Ǥ$ *or* $Ɓ \in Ǥ$. (9), (10)

Assuming that χ is a non-empty set, the following sets are grills on χ.(11), (12)

- $\emptyset$ & $Ƥ(\chi) \setminus \{\emptyset\}$ are examples of trivial grills on χ.
- $Ǥ_{\infty}$ is the grill of all infinite subset of χ.
- $Ǥ_{ÇÔ}$ is the grill of all uncountable subsets of χ.
- $Ǥ_{Ƥ} = \{Ⱥ : Ⱥ \in Ƥ(\chi), Ƥ \in Ⱥ\}$ is a certain point grill on χ.
- $Ǥ_{Ⱥ} = \{Ɓ : Ɓ \in Ƥ(\chi), Ɓ \cap Ⱥ^{Ç} \neq \emptyset\}$.

∗ If $(\chi, ʈ)$ is a topological space, and so the set of the all dense subset that does not already exist here is known as $Ǥ = \{Ⱥ : iηʈ(Çl(Ⱥ)) \neq \acute{\emptyset}\}$ is one kind of grill on χ (4).

∗ Suppose that Ǥ a grill on $(\chi, ʈ)$. A mapping $∯ : Ƥ(\chi) \rightarrow Ƥ(\chi)$ is referred to as $∯(Ⱥ) = \{\chi \in \chi : Ⱥ \cap û \in Ǥ$ for every $û \in ʈ; \chi \in û\}$ for every $Ⱥ \in Ƥ(\chi)$. A mapping $\psi : Ƥ(\chi) \rightarrow Ƥ(\chi)$ is referred to as $\psi(Ⱥ) = Ⱥ \cup ∯(Ⱥ)$ for every $Ⱥ \in Ƥ(\chi)$.(13)

Kuratowski's Axioms of Closure for the map ψ are verified: (13), (14), (15)

i. $\psi(\emptyset) = \emptyset$,
ii. when $Ⱥ \subseteq Ɓ$, then $\psi(Ⱥ) \subseteq \psi(Ɓ)$,
iii. when $Ⱥ \subseteq \chi$, then $\psi(\psi(Ⱥ)) = \psi(Ⱥ)$,
iv. when $Ⱥ, Ɓ \subseteq \chi$, then $\psi(Ⱥ \cup Ɓ) = \psi(Ⱥ) \cup \psi(Ɓ)$.

Definition 2.2: (13)

There exists a special topology $ʈ_{Ǥ} = \{û \subseteq \chi : \psi(\chi - û) = (\chi - û)\}$, when for any $Ⱥ \subseteq \chi$ that corresponds inside the topological space, to a grill Ǥ $(\chi, ʈ)$.

$\psi(Ⱥ) = Ⱥ \cup ∯(Ⱥ) = ʈ_{Ǥ}$- $Çl(Ⱥ)$ and $ʈ \subseteq ʈǤ$.

Remark 2.3: (4)

If $Ǥ = Ƥ(\chi)/\{\acute{\emptyset}\}$, then $ʈǤ. = ʈ$.

Remark 2.4: (2)

We can find $ʈ_{Ǥ}$ by using the base as follows $\mathcal{B}(ʈ_{Ǥ}, ʈ) = \{v - Ⱥ ; v \in ʈ, Ⱥ \notin Ǥ\}$

Definition 2.5:(13)

Let $\chi \neq \acute{\emptyset}$ and Ɽ be an equivalence relation on χ, $Ⱥ \subseteq \chi$.

i. The upper approximation of Ⱥ for Ɽ is denoted by $\overline{Ɽ_w}(Ⱥ)$, where

$$\overline{Ɽ_w}(Ⱥ) = \cup_{\chi \in \chi} \{Ɽ(\chi) : Ɽ(\chi) \cap Ⱥ \neq \acute{\emptyset}\}.$$

ii. The lower approximation of $\mathbb{A}$ for R is denoted by $\underline{\mathrm{R}_w}(\mathbb{A})$, where

$$\underline{\mathrm{R}_w}(\mathbb{A}) = \cup_{\chi\in\chi}\{\mathrm{R}(\chi): \mathrm{R}(\chi) \subseteq \mathbb{A}\}.$$

iii. The boundary region of $\mathbb{A}$ for R is denoted by $\mathrm{B}_w(\mathbb{A})$, where

$$\mathrm{B}_w(\mathbb{A}) = \overline{\mathrm{R}_w}(\mathbb{A}) - \underline{\mathrm{R}_w}(\mathbb{A}).$$

Definition 2.6:(7), (16)

Let $\chi \neq \acute{\emptyset}$ and R be an equivalence relation on χ and $\mathrm{t}_w(\mathbb{A}) = \left\{\chi, \acute{\emptyset}, \overline{\mathrm{R}_w}(\mathbb{A}), \underline{\mathrm{R}_w}(\mathbb{A}), \mathrm{B}_w(\mathbb{A})\right\}$, where $\mathbb{A} \subseteq \chi$. Then $\mathrm{t}_w(\mathbb{A})$ is a topology on χ named nano topology for $\mathbb{A}$ $(\chi, \mathrm{t}_w(\mathbb{A})$ space is known as nano topological. The components of $\mathrm{t}_w(\mathbb{A})$ are named nano-open sets denoted by $\eta - open\ sets$. The complement of a $\eta - open\ sets$ is named a nano-closed set denoted by $\eta - closed\ sets$.

Definition 2.7:(7)

Let (χ, t_w) be N.T.S (nano topological space) and $\mathbb{A} \subseteq \chi$. The nano closure (respectively, nano interior) of $\mathbb{A}$ which is short $\eta Çl_w(\mathbb{A})$ (respectively, $\eta int_w(\mathbb{A})$) is defined by; $\eta Çl_w(\mathbb{A}) = \cap \{\mathcal{F}, \mathcal{F}^Ç \in \mathrm{t}_w, \mathbb{A} \subseteq \mathcal{F}\}$, (resp., $\eta int_w(\mathbb{A})) = \eta int_w(\mathbb{A}) = \{\hat{u}; \hat{u} \in \mathrm{t}_w, \hat{u} \in \mathbb{A}\}$.

Note 8:

We said the triple $(\chi, \mathrm{t}_w, \mathrm{G})$ G.N.T.S (Grill nano topological space)..

Definition 2.9:(7)

Let (χ, t_w) be an N.T.S, a subset $\mathbb{A}$ of χ is named N.S.O (nano semi-open set) $\mathbb{A} \subseteq \eta Çl_w(\eta int_w(\mathbb{A})) \Leftrightarrow \exists \hat{u} \in \mathrm{t}_w; \hat{u} \subseteq \mathbb{A} \subseteq \eta Çl_w(\hat{u})$.

A subset B of χ is called *nano* semi-closed if $(\chi$-B$)$ is N.S.O set The collection of all N.S.O (respectively,N.S.C) sets in a nano topological space (χ, t_w).

will be symbolized by $\eta \c{S}\mathring{O}(\chi)$ (respectively,$\eta G_{\c{S}}Ç(\chi)$.

Definition 2.10:(17)

There exists a special topology $\eta \mathrm{t}_{wG} = \{\hat{u} \subseteq \chi: \psi(\chi - \hat{u}) = (\chi - \hat{u})\}$, when for any $\mathbb{A} \subseteq \chi$ that corresponds to a nano grill G on the topological space (χ, t_w).

$\psi(\mathbb{A}) = \mathbb{A} \cup \oint(\mathbb{A}) = \eta \mathrm{t}_{wG}$- $Çl(\mathbb{A})$ and $\mathrm{t}_w \subseteq \eta \mathrm{t}_{wG}$.

Definition 2.11:

Let (χ, t_w) be N.T.S and $\mathbb{A} \subseteq \chi$. The nano closure (respectively, nano interior) of $\mathbb{A}$ which is short $\eta Çl_w(\mathbb{A})$ (respectively, $\eta int_w(\mathbb{A})$) is defined by; $\eta Çl_{wG}(\mathbb{A}) = \cap \{\mathcal{F}, \mathcal{F}^Ç \in \mathrm{t}_w, \mathbb{A} \subseteq \mathcal{F}\}$, (resp., $\eta int_{wG}(\mathbb{A})) = \eta int_{wG}(\mathbb{A}) = \{\hat{u}; \hat{u} \in \mathrm{t}_w, \hat{u} \in \mathbb{A}\}$.

Definition 2.12: (18), (19), (20)

A topological spaces $(\chi, \mathrm{t}_w, \mathrm{G})$ is named nano compact space if and only if all nano open cover of χ has a finite subcover..

3. Nano Grill semi-open sets in nano compact space

Definition 3.1:

For any Grill topological space (χ, t_G) and $\mathbb{A} \sqsubseteq \chi$; $\mathbb{A}$is said to be nano Grill semi-open if there exists $\hat{u} \in \mathrm{t}_w$; $\hat{u} - \mathbb{A} \notin \mathrm{G}$ and $\mathbb{A} - \eta Çl_{wG}(\hat{u}) \notin \mathrm{G}$ .And $\mathbb{A}$ denoted by $\eta G_{\c{S}}$-open. $\chi - \mathbb{A}$ is a nano Grill semi-closed and denoted by $\eta G_{\c{S}}$ -semi-closed and the set of all $\eta G_{\c{S}}$ -open presently by

$\eta G_{\c{S}} \dot{O}(\chi)$ and the set of all $\eta G_{\c{S}}$-semi-closed presently by $\eta G_{\c{S}} \c{C}(\chi)$.

Example 3.2:

Let (χ, t_w, G) be a nano grill topological space to be a nano *gr*ill and

$$\chi=\{\chi_1, \chi_2, \chi_3, \chi_4\}$$
$$G = \{\hat{u} \sqsubseteq \chi; \chi_2 \in \hat{u}\}$$
$$G = \{\{\chi_2\}, \{\chi_1, \chi_2\}, \{\chi_3, \chi_2\}, \{\chi_4, \chi_2\}, \{\chi_1, \chi_2, \chi_3\}, \{\chi_1, \chi_2, \chi_4\}, \{\chi_2, \chi_3, \chi_4\}, \chi\}$$
$$R=\{(\chi_1, \chi_1), (\chi_2, \chi_2), (\chi_3, \chi_3), (\chi_4, \chi_4), (\chi_2, \chi_4), (\chi_4, \chi_2)\}$$
$$R \setminus [\chi] = \{\{\chi_1\}, \{\chi_2, \chi_4\}, \{\chi_3\}$$
$$w \sqsubseteq \chi, w = \{2,3\},$$
$$t_w = \{\chi, \acute{\emptyset}, \{3\}, \{2,4\}, \{2,3,4\}\}$$
$$\mathcal{B} = \{v - A; v \in t_w \wedge A \notin G\}$$
$$\mathcal{B} = \left\{\chi, \acute{\emptyset}, \{\chi_1, \chi_2, \chi_4\}, \{\chi_1, \chi_2, \chi_3\}, \{\chi_2, \chi_3, \chi_4\}, \{\chi_2, \chi_3\}, \{\chi_2, \chi_4\}, \{\chi_1, \chi_2\}, \{\chi_2\}, \{\chi_3\}\right\} = t_{wG}$$
$$\therefore \eta G_{\c{S}} \dot{O}(\chi) = \mathcal{P}(\chi).$$

Proposition 3.3:

i. Every nano open set is a $\eta G_{\c{S}}$ -open sets.
ii. Every nano closed set is a $\eta G_{\c{S}}$ -closed.

Example 3.2 demonstrates that the converse of Remark 3.3(i)(ii) is not true.

Definition 3.4:

Let (χ, t_w, G) be a nano gr*i*ll topological space. By a $\eta G_{\c{S}} - open\ cover\ of\ \chi$ we mean a subfamily of $\eta G_{\c{S}} \dot{O}(\chi)$ wich cover χ

Definition 3.5:

A nano grill topological space (χ, t_w, G) is said to be $\eta G_{\c{S}} - compact\ space$if every $\eta G_{\c{S}} - open\ cover$ for χ has a finite subcover.

Theorem3.6:

A nano grill topological space (χ, t_w, G) is be $\eta G_{\c{S}} - compact\ space$ if and only if every family of $\eta G_{\c{S}} - closed\ subsets$ of χ with finite intersection property has a non-empty intersection.

Proof:

Suppose that χ is $\eta G_{\c{S}} - compact\ space$ and let $\{F_i: i \in \Lambda\}$ be a family of $\eta G_{\c{S}} - closed\ subsets$ of χ with (F.I.P). Assume that $\cap_{i\in\Lambda} F_i = \emptyset$, then $\cup_{i\in\Lambda} {F^c}_i = \chi$, where $\{{F^c}_i: i \in \Lambda\}$ is a $\eta G_{\c{S}} - open\ cover$of χ which is a $\eta G_{\c{S}} - compact\ space$, If follows that there exists a finite subcover ${\{{F^c}_i\}^n}_{i=1}$ such that $\chi = \cup_{i=1}^n {F^c}_i$, then $\cup_{i=1}^n F_i = \emptyset$ which is a contradiction. Since$F_i: i \in \Lambda\}$hasaF.I.P. Now, suppose that every family of $\eta G_{\c{S}} - closed\ subsets$ of χ with (f.i.p) has a non-empty intersection. Assume that χ is not $\eta G_{\c{S}} - compact\ space,$ let $\{u_\alpha: \alpha \in \Lambda\}$ be a $\eta G_{\c{S}} - open\ cover$of χ and suppose if possible, $\{u_\alpha: \alpha \in \Lambda\}$ has no finite subcover. The collection $\{{u^c}_\alpha: \alpha \in \Lambda\}$ has the F.I.P, if but $\{{u^c}_\alpha: \alpha \in \Lambda\}$ is a family of $\eta G_{\c{S}} - closed\ sets$, so

$\cap_{\alpha\in\Lambda} u^c{}_\alpha \neq \emptyset$, it follows that $\cup_{\alpha\in\Lambda} u_\alpha \neq \chi$ which is contradiction since $\{u_\alpha: \alpha \in \Lambda\}$ is a ηG̶Ş $-$ *open cover* of χ .

Theorem 3.7:

Every ηG̶Ş $-$ *compact space* is a nano compact space.

Proof:

Let $U = \{u_{\mathrm{i}}, \mathrm{i} \in \Lambda; u_{\mathrm{i}} \in ʈ_w \ \forall \ \mathrm{i}\}$ is an open cover for χ such that $\chi = \bigcup_{\mathrm{i}\in\Lambda} u_{\mathrm{i}}$ and since every open set is a ηG̶Ş $-$ open sets .So, U is a ηG̶Ş $-$ *open cover* for χ , and since Ӽ is a not ηG̶Ş $-$ *compact* set .So, there exist a finite subcover say $U = \{u_1, u_2, \dots, u_n\}$ such that $\chi = \bigcup_{\mathrm{i}=1}^{n} u_{\mathrm{i}}$ .Therefore, χ is a nano compact space.

Definition 3.8:

Let $\mathcal{F}: (\chi, ʈ_w, G̶) \rightarrow (Y, ʈ_w{}', G̶')$ be a function then $\mathcal{F}$ believed to be;
1. ηG̶ *semi* $-$continuous function, denoted by ηG̶Ş-continuous function if $\mathcal{F}^{-1}(\mathrm{u}) \in$ ηG̶ŞỎ(χ)for all u ∈ $ʈ_w$.
2. Strongly ηG̶ *semi* $-$continuous function, denoted by". *Strongly* ηG̶Ş-continuous function " *if* $\mathcal{F}^{-1}(\mathrm{u}) \in ʈ_w$, fore all u ∈ ηG̶ŞỎ(Y) .
3. ηG̶ *semi*-irresolute function, denoted by ηG̶Ş -irresolute function if $\mathcal{F}^{-1}(\mathrm{u}) \in$ ηG̶ŞỎ(χ), for all u ∈ ηG̶ŞỎ(Y).

Proposition 3.9:

Let $\mathcal{F}: (\chi, ʈ_w, G̶) \rightarrow (Y, ʈ_w{}', G̶')$ be a function.
1. $\mathcal{F}$ is ηG̶Ş -irresolute function whenever $\mathcal{F}$ is strongly ηG̶Ş-continuous function.
2. If $\mathcal{F}$ is a strongly ηG̶Ş-continuous function then $\mathcal{F}$ is a continuous function.
3. " *When* $\mathcal{F}$ *is a continuous function* " then $\mathcal{F}$isηG̶Ş-continuous function.
4. $\mathcal{F}$ isηG̶Ş-continuous function whenever $\mathscr{F}$ is a ηG̶Ş -irresolute function.

In general, the opposite of (proposition 3.9) is not supported by the following examples.

Example 3.10:

Let $\mathcal{F}: (\chi, ʈ_w, G̶) \rightarrow (\chi, ʈ_w, G̶^{\sim})$ be a function such that $\mathcal{F}(\chi) = \chi$ for each χ ∈ χ

Where

$$\chi = \{Ӽ_1, Ӽ_2, Ӽ_3\}, G̶ = \mathcal{P}(\chi) \setminus \{\emptyset\}$$

$$Ɽ = \{(\chi_1,\chi_1),(\chi_2,\chi_2),(\chi_3,\chi_3),(\chi_2,\chi_3),(\chi_3,\chi_2)\}$$

$$Ɽ \setminus [\chi] = \{\{\chi_2,\chi_3\},\{\chi_1\}\}, w = \{\chi_1\}$$

$$ʈ_w = \{\chi, \emptyset, \{\chi_1\}\}$$

$$G̶^{\sim} = \{u; \chi_1 \in u\}, ηG̶ŞỎ(\chi) = \{u; \chi_1 \in u\} \cup \{\emptyset\}, ηG̶Ş^{\sim}Ỏ(\chi) = \mathcal{P}(\chi) .$$

So that,, $\mathcal{F}$ isηG̶Ş-continuous function and continuous function but it's not ηG̶Ş -irresolute function and it's not strongly ηG̶Ş-continuous function.

Example 3.11:

The function $\mathcal{F}: (\chi, ʈ_w, G̶) \rightarrow (\chi, ʈ_w, G̶^{\sim})$ such that

$$\mathcal{F}(\{\chi_2\}) = \{\chi_1\}, \mathcal{F}(\{\chi_1\}) = \{\chi_2\}, \mathcal{F}(\{\chi_3\}) = \{\chi_3\},$$

Where

$$\chi = \{\chi_1, \chi_2, \chi_3\}, Ꞡ^{\sim} = \mathcal{P}(\chi) \setminus \{\acute{\emptyset}\}\}$$

$$Ʀ = \{(\chi_1, \chi_1), (\chi_2, \chi_2), (\chi_3, \chi_3), (\chi_2, \chi_3), (\chi_3, \chi_2)\}$$

$$Ʀ \setminus [\chi] = \{\{\chi_2, \chi_3\}, \{\chi_1\}\}, w = \{\chi_1\}$$

$$ƫ_w = \{\chi, \acute{\emptyset}, \{\chi_1\}\},$$

$$Ꞡ = \{u; \chi_1 \in u\}, \eta Ꞡ_{Ş}\dot{O}(\chi) = \mathcal{P}(\chi) \setminus \{\acute{\emptyset}\},$$

$\eta Ꞡ_{Ş}\dot{O}(\chi) = \{u; \chi_1 \in u\} \cup \{\emptyset\}, \mathcal{F}\ is\ \eta Ꞡ_{Ş}\dot{O}(\chi)$ continuous function and $\eta Ꞡ_{Ş}$ -irresolute function but it isn't continuous function and not strongly $\eta Ꞡ_{Ş}$-continuous function it's not since

$$\mathcal{F}^{-1}(\chi_1) = \{\chi_2\} \notin ƫ_w.$$

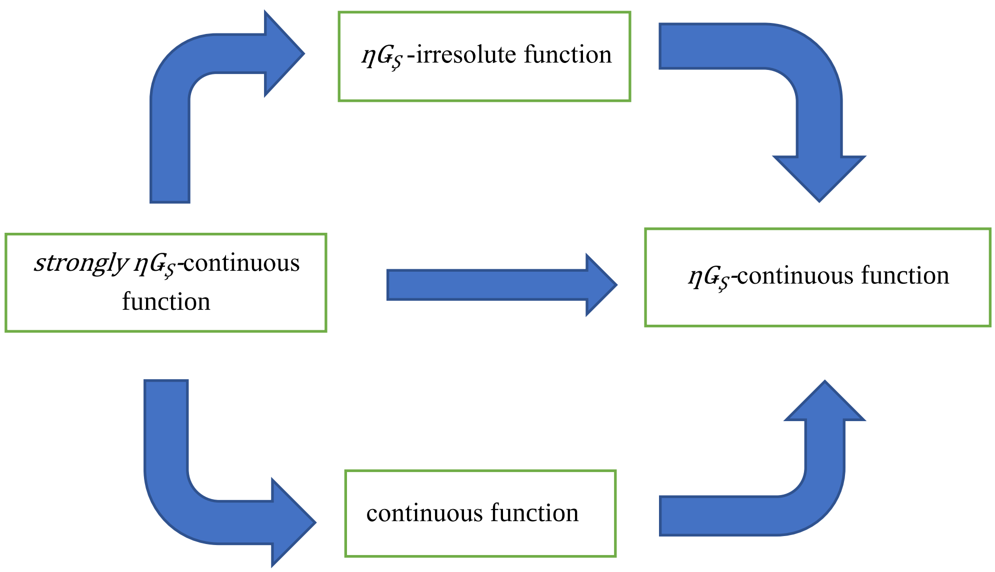

Diagram1. Continuous functions via $\eta Ꞡ_{Ş}$- open

Proposition 12:

i. The $\eta Ꞡ_{Ş}$- irresolute image function $\eta Ꞡ_{Ş} - compact\ space$ is a $\eta Ꞡ_{Ş} - compact\ space$.
ii. In strongly $\eta Ꞡ_{Ş}$ − continuous the image of nano compact space is a $\eta Ꞡ_{Ş} - compact\ space$.
iii. The $\eta Ꞡ_{Ş}$ − continuous function the image function of $\eta Ꞡ_{Ş} - compact$ is a nano compact.

Proposition 3.13:

A $\eta Ꞡ_{Ş} - closed\ subsets$ of $\eta Ꞡ_{Ş} - compact\ space$ is $\eta Ꞡ_{Ş} - compact$.

Theorem 3.14:

If A & B are $\eta Ꞡ_{Ş} - compact$, then A∪ B is a $\eta Ꞡ_{Ş} - compact$.

Proposition 3.15:

Every $\eta Ꞡ_{Ş} - compact$ is a nano compact.

Example 3.16:

Let ($\vec{Ʀ}$, χ, $ƫ_w$) be any nano topological space such that $\vec{Ʀ}$ is the set of all real numbers and Ʀ =

{(r,r), r ∈ Ṝ} so, Ʀ \ [r] = {{r}, r ∈ Ṝ}. Now, if w= {1} and Ǥ = Ƥ[Ṝ] \ Ø, then $\overline{Ʀ_w}$ = {1} = $\underline{Ʀ_w}$ *and* $Ɓ_w$ = Ø so, $ƫ_w$ = η$ƫ_{wǤ}$ = {Ṝ, Ø, {1}} and ηǤ$_Ş$Ỏ(Ṝ) = {û⊆ Ṝ; 1 ∈ û} ∪ Ø .This much is clear: (Ṝ, $ƫ_w$, Ǥ) is a nano compact which is not ηǤ$_Ş$ − *compact* since L = {{1,r}, r ∈ Ṝ} is ηǤ$_Ş$ − *open cover* has no finite subcover.

4. Conclusion

In this work, a new type of open set was studied using the concept of nano-topology ,grill and nano compact which is called ηǤ$_Ş$ − *open sets*. The properties of this set were studied. It was found that ηǤ$_Ş$Ỏ(χ) represents a supra-topology space. New forms of functionality were defined by applying this notion, and the relationship between these functions was found.

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Conflict of Interest

There are no conflicts of interest.

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