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Comparison of Complex Sadik and KAJ Transforms for Ordinary Differential Equations to the Response of an Uncompressed Forced Oscillator

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Abstract

In this paper we have presented a comparison between two novel integral transformations that are of great importance in the solution of differential equations. These two transformations are the complex Sadik transform and the KAJ transform. An uncompressed forced oscillator, which is an important application, served as the basis for comparison. The application was solved and exact solutions were obtained. Therefore, in this paper, the exact solution was found based on two different integral transforms: the first integral transform complex Sadik and the second integral transform KAJ. And these exact solutions obtained from these two integral transforms were new methods with simple algebraic calculations and applied to different problems. The main purpose of this comparison is the exact solutions, and until we show the importance of the diversity and difference of the kernel of the integral transform by keeping the period t between 0 and infinity.

Keywords : Complex Sadik Transform, (KAJ) Kuffi Abbass Jawad Transform, Response of An Undamped uncompressed forced oscillator, Ordinary Differential Equations.

1. Introduction

First, one should know the importance of integral transforms in solving differential equations of all kinds, both ordinary and partial, where these differential equations are transformed into algebraic equations that are easier to compute by relying on the integral transform and then taking the inverse of the transform [2, 15, 27 and 29].

Integral transforms are currently also of great importance in applied mathematics, as they are used in the coding of images, among other things. They are of great importance for modern technical applications, such as in genetic engineering, and facilitate the solution of complex differential systems, such as colon cancer and drug concentration, regardless of whether they are partial or ordinary systems [13,14,27,30].

Due to the importance of integral transformations in many vital applications, many papers have appeared on this topic and in various fields [1,3-11,16-26,28]

Definition 1.1. [29] The Complex Sadik Transform (CST) is denoted by the operator $\mathbf{S}_a^c\{.\}$, thetransform form is as follows:

$$\mathbf{S}_a^c[g(t)] = \mathbf{F}^c(s^\alpha, \beta) = \frac{1}{s^\beta}\int_0^\infty g(t)e^{-is^\alpha t}dt.$$

Where s is a complex variable, α is any nonzero real number, and β is any real number.

Definition 1.2. [2] The "Kuffi-Abbas-Jawad"(KAJ) Transform denoted by the operator $\mathbf{S}_m\{.\}$, the transform form is as follows:

$$\mathbf{S}_m[g(t)] = K(v) = \frac{1}{v^n}\int_0^\infty g\left(\frac{t}{v}\right)e^{-t}dt.$$

Where n is any integer number, and $0 < l_1 \leq v \leq l_2$, where l_1 and l_2 are either finite or infinite.

1.1. The KAJ and Complex Sadik Integral Transforms for Some Basic Functions

In this part, we present the KAJ integral transform and the novel complex integral transform for some important basic functions in the following table:

Table 1: KAJ transform and the complex Sadik integral transform for some basic functions

Functions $g(t)$	$\mathbf{S}_m\{g(t)\} = K(v)$ "Kuffi Abbass Jawad (KAJ) Transform"	$\mathbf{S}_a^c\{g(t)\} = \mathbf{F}^c(s)$ "Complex Sadik Transform"
1	$\frac{1}{v^{n+1}}$	$\frac{-i}{s^{(\alpha+\beta)}}$
$t^r, r \in \mathbb{N}$	$\frac{r!}{v^{n+r}}$	$(-i)^{r+1}\frac{r!}{s^{r\alpha+(\alpha+\beta)}}$
e^{at}, $a$constant	$\frac{v}{v^n(v-a)}$	$\frac{-1}{s^\beta}\left[\frac{a}{(s^{2\alpha}+a^2)} + i\frac{s^\alpha}{(s^{2\alpha}+a^2)}\right]$
$\sin(at)$	$\frac{a}{v^{n-1}(v^2+a^2)}$	$\frac{-a}{s^\beta(s^{2\alpha}-a^2)}$
$\cos(at)$	$\frac{1}{v^{n-2}(v^2+a^2)}$	$\frac{-is^\alpha}{s^\beta(s^{2\alpha}-a^2)}$
$\sinh(at)$	$\frac{a}{v^{n-1}(v^2-a^2)}$	$\frac{-a}{s^\beta(s^{2\alpha}+a^2)}$
$\cosh(at)$	$\frac{1}{v^{n-2}(v^2-a^2)}$	$\frac{-is^\alpha}{s^\beta(s^{2\alpha}+a^2)}$

2. Complex Sadik Integral and KAJ Transforms of Derivatives:

Theorem 2.1. [29] Let $\mathbf{F}^c(s)$ be the complex Sadik integral transform of $f(t)(\mathbf{F}^c(s) = \mathbf{S}_a^c[f(t)])$, then

$$\mathbf{S}_a^c\left[f^{(n)}(t)\right] = (is^{\alpha})^n \mathbf{F}^c(s) - \frac{1}{s^{\beta}}\left[\sum_{k=1}^{n} (is^{\alpha})^{k-1} f^{(n-k)}(0)\right].$$

In this paper, we want to generalize KAJ transform of m^{th}-derivative and prove that by mathematical induction.

Theorem 2.2. Let $K(v)$ be the Kuffi-Abbas-Jawad (KAJ) transform of $f(t)(K(v) = \mathbf{S}_m[f(t)])$, then

$$\mathbf{S}_m\left[f^{(h)}(t)\right] = v^h K(v) - \frac{1}{v^n}\left[\sum_{k=1}^{h} v^k f^{(h-k)}(0)\right].$$

Proof. by Mathematical Induction

1 For $h = 1$,

$$\mathbf{S}_m[f'(t)] = vK(v) - \frac{vf(0)}{v^n}.$$

Thus true for $h = 1$.

2 Assume that, true for $h = r$ that means:

$$\mathbf{S}_m\left[f^{(r)}(t)\right] = v^r K(v) - \frac{1}{v^n}\left[\sum_{k=1}^{r} v^k f^{(r-k)}(0)\right].$$

3 We want to prove for $h = r + 1$

$$\begin{aligned}
\mathbf{S}_m\left[f^{(r+1)}(t)\right] = \mathbf{S}_m\left[\left(f^{(r)}(t)\right)'\right] \quad &= v\mathbf{S}_m\left[f^{(r)}(t)\right] - \frac{vf^{(r)}(0)}{v^n}, \\
&= v\left[v^r K(v) - \frac{1}{v^n}\left[\sum_{k=1}^{r} v^k f^{(r-k)}(0)\right]\right] - \frac{vf^{(r)}(0)}{v^n}, \\
&= v^{r+1}K(v) - \frac{1}{v^n}\left[\sum_{k=1}^{r} v^{k+1} f^{(r-k)}(0)\right]\Bigg] - \frac{vf^{(r)}(0)}{v^n}, \\
&= v^{r+1}K(v) - \frac{1}{v^n}\left[\left[\sum_{k=1}^{r} v^{k+1} f^{(r-k)}(0)\right] + vf^{(r)}(0)\right], \\
&= v^{r+1}K(v) - \frac{1}{v^n}\left[\left[\sum_{k=0}^{r} v^{k+1} f^{(r-k)}(0)\right]\right], \\
&= v^{r+1}K(v) - \frac{1}{v^n}\left[\left[\sum_{k=1}^{r+1} v^{(k-1)+1} f^{(r-(k-1))}(0)\right]\right], \\
&= v^{r+1}K(v) - \frac{1}{v^n}\left[\left[\sum_{k=1}^{r+1} v^{k} f^{(r+1-k)}(0)\right]\right], \\
&= \mathbf{S}_m\left[f^{(r+1)}(t)\right].
\end{aligned}$$

So theorem is true for $n \in \mathbb{N}$.

3. Main Results:

In this part, we present two real life problems, response of an undamped forced mechanical oscillator and response of an undamped forced electrical oscillator.

Example 3.1. (Response of an Undamped Forced Mechanical Oscillator)

Consider the differential equation of the forced mechanical oscillator:

$$\ddot{X}(t) + w_0^2 X(t) = \frac{F}{m}\cos(wt).$$

Where $w_0 = \sqrt{\frac{k}{m}}$,represents the natural frequency of the oscillator with initial boundary conditions, as follows:

$$X(0) = 0,$$
$$\dot{X}(0) = 0.$$

$$\mathbf{S}_{a}^{c}\left\{\ddot{X}(t)+w_{0}^{2}X(t)=\frac{F}{m}\cos(wt)\right\},$$

$$(is^{\alpha})^{2}\mathbf{X}^{c}(s)-\frac{\dot{X}(0)}{s^{B\beta}}-\frac{is^{\alpha}X(0)}{s^{\beta}}+w_{0}^{2}\mathbf{X}^{c}(s)=\frac{F}{m}\frac{-is^{\alpha}}{s^{\beta}(s^{2\alpha}-w^{2})},$$

$$(is^{\alpha})^{2}\mathbf{X}^{c}(s)+w_{0}^{2}\mathbf{X}^{c}(s)=\frac{F}{m}\frac{-is^{\alpha}}{s^{\beta}(s^{2\alpha}-w^{2})},$$

$$i^{2}(s^{\alpha})^{2}\mathbf{X}^{c}(s)+w_{0}^{2}\mathbf{X}^{c}(s)=\frac{-iFs^{\alpha}}{ms^{\beta}(s^{2\alpha}-w^{2})},$$

$$-s^{2\alpha}\mathbf{X}^{c}(s)+w_{0}^{2}\mathbf{X}^{c}(s)=\frac{-iFs^{\alpha}}{ms^{\beta}(s^{2\alpha}-w^{2})},$$

$$(w_{0}^{2}-s^{2\alpha})\mathbf{X}^{c}(s)=\frac{-iFs^{\alpha}}{ms^{\beta}(s^{2\alpha}-w^{2})},$$

$$\mathbf{X}^{c}(s)=\frac{-iFs^{\alpha}}{ms^{\beta}(s^{2\alpha}-w^{2})(w_{0}^{2}-s^{2\alpha})},$$

$$\mathbf{X}^{c}(s)=\frac{iFs^{\alpha}}{ms^{\beta}}\left[\frac{-1}{(s^{2\alpha}-w^{2})(w_{0}^{2}-s^{2\alpha})}\right],$$

$$\mathbf{X}^{c}(s)=\frac{iFs^{\alpha}}{ms^{\beta}}\left[\frac{1}{(s^{2\alpha}-w^{2})(w^{2}-w_{0}^{2})}+\frac{1}{(w_{0}^{2}-s^{2\alpha})(w^{2}-w_{0}^{2})}\right]$$

$$\mathbf{X}^{c}(s)=\frac{iFs^{\alpha}}{ms^{\beta}(w^{2}-w_{0}^{2})}\left[\frac{1}{(s^{2\alpha}-w^{2})}+\frac{1}{(w_{0}^{2}-s^{2\alpha})}\right],$$

$$\mathbf{X}^{c}(s)=\frac{F}{m(w^{2}-w_{0}^{2})}\left\{\frac{is^{\alpha}}{s^{\beta}}\left[\frac{1}{(s^{2\alpha}-w^{2})}+\frac{1}{(w_{0}^{2}-s^{2\alpha})}\right]\right\},$$

$$\mathbf{X}^{c}(s)=\frac{F}{m(w^{2}-w_{0}^{2})}\left\{\frac{is^{\alpha}}{s^{\beta}}\left[\frac{1}{(s^{2\alpha}-w^{2})}-\frac{1}{(s^{2\alpha}-w_{0}^{2})}\right]\right\},$$

$$\mathbf{X}^{c}(s)=\frac{F}{m(w^{2}-w_{0}^{2})}\left[\frac{is^{\alpha}}{s^{\beta}(s^{2\alpha}-w^{2})}-\frac{is^{\alpha}}{s^{\beta}(s^{2\alpha}-w_{0}^{2})}\right],$$

inverse , then

$$X(t)=\frac{F}{m(w^{2}-w_{0}^{2})}[-\cos(wt)+\cos(w_{0}t)],$$

$$X(t)=\frac{F}{m(w^{2}-w_{0}^{2})}[\cos(w_{0}t)-\cos(wt)],$$

$$X(t)=\frac{F}{m(w^{2}-w_{0}^{2})}\left[-2\sin\left(\frac{(w_{0}+w)t}{2}\right)\sin\left(\frac{(w_{0}-w)t}{2}\right)\right],$$

$$X(t)=\frac{2F}{m(w_{0}^{2}-w^{2})}\sin\left(\frac{(w_{0}+w)t}{2}\right)\sin\left(\frac{(w_{0}-w)t}{2}\right),$$

or because $\sin(-x)=-\sin(x)$

$$X(t)=\frac{F}{m(w^{2}-w_{0}^{2})}\left[-2\sin\left(\frac{(w_{0}+w)t}{2}\right)\sin\left(\frac{(w_{0}-w)t}{2}\right)\right],$$

$$X(t)=\frac{2F}{m(w^{2}-w_{0}^{2})}\sin\left(\frac{(w+w_{0})t}{2}\right)\sin\left(\frac{(w-w_{0})t}{2}\right).$$

$$\mathbf{S}_m\left\{\ddot{X}(t)+w_0^2X(t)=\frac{F}{m}\cos(wt)\right\},$$
$$v^2K(v)+\frac{1}{v^2}[-v^2X(0)-v\dot{X}(0)]+w_0^2K(v)=\frac{F}{m}\left[\frac{1}{v^{n-2}(v^2+w^2)}\right],$$
$$(v^2+w_0^2)K(v)=\frac{F}{m}\left[\frac{1}{v^{n-2}(v^2+w^2)}\right],$$
$$K(v)=\frac{F}{m}\left[\frac{1}{v^{n-2}(v^2+w^2)(v^2+w_0^2)}\right],$$
$$K(v)=\frac{F}{mv^{n-2}}\left[\frac{1}{(v^2+w^2)(v^2+w_0^2)}\right],$$
$$K(v)=\frac{F}{mv^{n-2}}\left[\frac{1}{(v^2+w_0^2)(w^2-w_0^2)}-\frac{1}{(v^2+w^2)(w^2-w_0^2)}\right],$$
$$K(v)=\frac{F}{mv^{n-2}(w^2-w_0^2)}\left[\frac{1}{(v^2+w_0^2)}-\frac{1}{(v^2+w^2)}\right],$$
$$\mathbf{S}_m[X(t)]=K(v)=\frac{F}{m(w^2-w_0^2)}\left[\frac{1}{v^{n-2}(v^2+w_0^2)}-\frac{1}{v^{n-2}(v^2+w^2)}\right],$$

inverse , then

$$X(t)=\frac{F}{m(w^2-w_0^2)}[\cos(w_0t)-\cos(wt)],$$
$$X(t)=\frac{F}{m(w^2-w_0^2)}\left[-2\sin\left(\frac{(w_0+w)t}{2}\right)\sin\left(\frac{(w_0-w)t}{2}\right)\right]$$
$$X(t)=\frac{2F}{m(w_0^2-w^2)}\sin\left(\frac{(w_0+w)t}{2}\right)\sin\left(\frac{(w_0-w)t}{2}\right),$$

or because $\sin(-x)=-\sin(x)$

$$X(t)=\frac{F}{m(w^2-w_0^2)}\left[-2\sin\left(\frac{(w_0+w)t}{2}\right)\sin\left(\frac{(w_0-w)t}{2}\right)\right],$$
$$X(t)=\frac{2F}{m(w^2-w_0^2)}\sin\left(\frac{(w+w_0)t}{2}\right)\sin\left(\frac{(w-w_0)t}{2}\right).$$

Example 3.2. (Response of an Undamped Forced Electrical Oscillator)

Consider the differential equation of the forced electrical oscillator:

$$L\ddot{Q}(t)+R\dot{Q}(t)+\frac{1}{C}Q(t)\quad=V\cos(wt),$$
$$\ddot{Q}(t)+\frac{R}{L}\dot{Q}(t)+w_0^2Q(t)\quad=\frac{V}{L}\cos(wt).$$

For an undamped forced electrical oscillator, resistance $R=0$

$$\ddot{Q}(t) + w_0^2 Q(t) = \frac{V}{L}\cos(wt).$$

Where $w_0 = \sqrt{\frac{1}{CL}}$, and $Q(t)$ is the instantaneous charge.

With the initial boundary conditions are:

$$Q(0) = 0,$$
$$\dot{Q}(0) = 0.$$

$$\mathbf{S}_a^c\left\{\ddot{Q}(t) + w_0^2 Q(t) = \frac{V}{L}\cos(wt)\right\},$$

$$(is^\alpha)^2\mathbf{Q}^c(s) - \frac{\dot{Q}(0)}{s^{B\beta}} - \frac{is^\alpha Q(0)}{s^\beta} + w_0^2\mathbf{Q}^c(s) = \frac{V}{L}\frac{-is^\alpha}{s^\beta(s^{2\alpha} - w^2)},$$

$$(is^\alpha)^2\mathbf{Q}^c(s) + w_0^2\mathbf{Q}^c(s) = \frac{V}{L}\frac{-is^\alpha}{s^\beta(s^{2\alpha} - w^2)},$$

$$i^2(s^\alpha)^2\mathbf{Q}^c(s) + w_0^2\mathbf{Q}^c(s) = \frac{-iVs^\alpha}{Ls^\beta(s^{2\alpha} - w^2)},$$

$$-s^{2\alpha}\mathbf{Q}^c(s) + w_0^2\mathbf{Q}^c(s) = \frac{-iVs^\alpha}{Ls^\beta(s^{2\alpha} - w^2)},$$

$$(w_0^2 - s^{2\alpha})\mathbf{Q}^c(s) = \frac{-iVs^\alpha}{Ls^\beta(s^{2\alpha} - w^2)},$$

$$\mathbf{Q}^c(s) = \frac{-iVs^\alpha}{Ls^\beta(s^{2\alpha} - w^2)(w_0^2 - s^{2\alpha})},$$

$$\mathbf{Q}^c(s) = \frac{iVs^\alpha}{Ls^\beta}\left[\frac{-1}{(s^{2\alpha} - w^2)(w_0^2 - s^{2\alpha})}\right],$$

$$\mathbf{Q}^c(s) = \frac{iVs^\alpha}{Ls^\beta}\left[\frac{1}{(s^{2\alpha} - w^2)(w^2 - w_0^2)} + \frac{1}{(w_0^2 - s^{2\alpha})(w^2 - w_0^2)}\right]$$

$$\mathbf{Q}^c(s) = \frac{iVs^\alpha}{Ls^\beta(w^2 - w_0^2)}\left[\frac{1}{(s^{2\alpha} - w^2)} + \frac{1}{(w_0^2 - s^{2\alpha})}\right],$$

$$\mathbf{Q}^c(s) = \frac{V}{L(w^2 - w_0^2)}\left\{\frac{is^\alpha}{s^\beta}\left[\frac{1}{(s^{2\alpha} - w^2)} + \frac{1}{(w_0^2 - s^{2\alpha})}\right]\right\},$$

$$\mathbf{Q}^c(s) = \frac{V}{L(w^2 - w_0^2)}\left\{\frac{is^\alpha}{s^\beta}\left[\frac{1}{(s^{2\alpha} - w^2)} - \frac{1}{(s^{2\alpha} - w_0^2)}\right]\right\},$$

$$\mathbf{Q}^c(s) = \frac{V}{L(w^2 - w_0^2)}\left[\frac{is^\alpha}{s^\beta(s^{2\alpha} - w^2)} - \frac{is^\alpha}{s^\beta(s^{2\alpha} - w_0^2)}\right],$$

inverse , then

$$Q(t)=\frac{V}{L(w^2-w_0^2)}[-\cos(wt)+\cos(w_0t)]$$
$$Q(t)=\frac{Q}{L(w^2-w_0^2)}[\cos(w_0t)-\cos(wt)]$$
$$Q(t)=\frac{V}{L(w^2-w_0^2)}\left[-2\sin\left(\frac{(w_0+w)t}{2}\right)\sin\left(\frac{(w_0-w)t}{2}\right)\right]$$
$$Q(t)=\frac{2V}{L(w_0^2-w^2)}\sin\left(\frac{(w_0+w)t}{2}\right)\sin\left(\frac{(w_0-w)t}{2}\right),$$

or because $\sin(-x)=-\sin(x)$

$$Q(t)=\frac{V}{L(w^2-w_0^2)}\left[-2\sin\left(\frac{(w_0+w)t}{2}\right)\sin\left(\frac{(w_0-w)t}{2}\right)\right]$$
$$Q(t)=\frac{2V}{L(w^2-w_0^2)}\sin\left(\frac{(w+w_0)t}{2}\right)\sin\left(\frac{(w-w_0)t}{2}\right)$$

Kuffi Abbass Jawad (KAJ) Transform

$$\mathbf{S}_m\left\{\ddot{Q}(t)+w_0^2Q(t)=\frac{V}{L}\cos(wt)\right\}$$
$$v^2K(v)+\frac{1}{v^2}\left[-v^2Q(0)-v\dot{Q}(0)\right]+w_0^2K(v)=\frac{V}{L}\left[\frac{1}{v^{n-2}(v^2+w^2)}\right]$$

$$(v^2+w_0^2)K(v)=\frac{V}{L}\left[\frac{1}{v^{n-2}(v^2+w^2)}\right]$$
$$K(v)=\frac{V}{L}\left[\frac{1}{v^{n-2}(v^2+w^2)(v^2+w_0^2)}\right]$$
$$K(v)=\frac{V}{Lv^{n-2}}\left[\frac{1}{(v^2+w^2)(v^2+w_0^2)}\right]$$
$$K(v)=\frac{V}{Lv^{n-2}}\left[\frac{1}{(v^2+w_0^2)(w^2-w_0^2)}-\frac{1}{(v^2+w^2)(w^2-w_0^2)}\right]$$
$$K(v)=\frac{V}{v^{n-2}(w^2-w_0^2)}\left[\frac{1}{(v^2+w_0^2)}-\frac{1}{(v^2+w^2)}\right]$$
$$\mathbf{S}_m[Q(t)]=K(v)=\frac{V}{L(w^2-w_0^2)}\left[\frac{1}{v^{n-2}(v^2+w_0^2)}-\frac{1}{v^{n-2}(v^2+w^2)}\right]$$

inverse , then

$$Q(t)=\frac{V}{L(w^2-w_0^2)}[\cos(w_0t)-\cos(wt)]$$
$$Q(t)=\frac{V}{L(w^2-w_0^2)}\left[-2\sin\left(\frac{(w_0+w)t}{2}\right)\sin\left(\frac{(w_0-w)t}{2}\right)\right]$$
$$Q(t)=\frac{2V}{L(w_0^2-w^2)}\sin\left(\frac{(w_0+w)t}{2}\right)\sin\left(\frac{(w_0-w)t}{2}\right)$$

or because $\sin(-x)=-\sin(x)$

$$Q(t) = \frac{V}{L(w^2 - w_0^2)}\left[-2\sin\left(\frac{(w_0 + w)t}{2}\right)\sin\left(\frac{(w_0 - w)t}{2}\right)\right]$$
$$Q(t) = \frac{2V}{L(w^2 - w_0^2)}\sin\left(\frac{(w + w_0)t}{2}\right)\sin\left(\frac{(w - w_0)t}{2}\right)$$

Example 3.3. [12]

Let a body A of mass 1 gram move on the x-axis. It is attracted towards the origin O with a force equals to $4x$. Also, assume that initially it is at rest when $x = 5$; then, determine its position by considering:

1 No other forces acting on it.

2 damping: force, or, in other words, resistance to the particle, is equal to 8 times the velocity at any instant.

Solution: From **Figure 1**, for $x > 0$, the net force towards the left is given by $4x$, while for

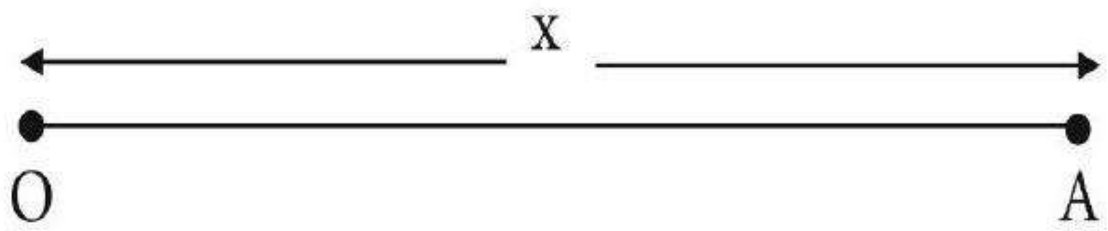

Figure 1.Body Aof mass 1

$x < 0$, the net force towards the right is given by $4x$. Thus, for both cases, the net force equals $4x$.

By Newton's second law of motion, mass × acceleration = net force,

$$\ddot{X}(t) = -4X(t)$$
$$\ddot{X}(t) + 4X(t) = 0$$

The initial conditions are

$$X(0) = 5,$$
$$\dot{X}(0) = 0.$$

Complex Sadik Transform

$$\mathbf{S}_a^c\{\ddot{X}(t)+4X(t)=0\}$$
$$(is^{\alpha})^2\mathbf{X}^c(s)-\frac{\dot{X}(0)}{s^{B\beta}}-\frac{is^{\alpha}X(0)}{s^{\beta}}+4\mathbf{X}^c(s)=0$$
$$-s^{2\alpha}\mathbf{X}^c(s)-\frac{5is^{\alpha}}{s^{\beta}}+4\mathbf{X}^c(s)=0$$
$$(4-s^{2\alpha})\mathbf{X}^c(s)=\frac{5is^{\alpha}}{s^{\beta}}$$
$$\mathbf{X}^c(s)=\frac{5is^{\alpha}}{s^{\beta}(4-s^{2\alpha})}$$
$$\mathbf{X}^c(s)=\frac{-5is^{\alpha}}{s^{\beta}(s^{2\alpha}-4)}$$

inverse

$$X(t)=5\cos(2t).$$

Kuffi Abbass Jawad (KAJ) Transform

$$\mathbf{S}_m\{\ddot{X}(t)+4X(t)=0\},$$
$$v^2K(v)+\frac{1}{v^2}\left[-v^2X(0)-v\dot{Q}(0)\right]+4K(v)=0,$$
$$v^2K(v)-\frac{5v^2}{v^n}+4K(v)=0,$$
$$K(v)=\frac{5}{v^{n-2}},$$
$$\mathbf{S}_m[X(t)]=K(v)=\frac{5}{v^{n-2}(v^2+4)}$$

inverse

$$X(t)=5\cos(2t).$$

4. Conclusions

We conclude that the complex Sadik transformation and the KAJ transform each other to an exact solution, and both are effective solutions. The two new integral transformations, the complex Sadik transformation and the Kuffi-Abbas-Jawad transformation, provide an exact solution for some mechanical and electrical theorems.

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Conflict of Interest

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