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On The Double Integral Transform (Complex EE Transform) and Their Applications

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Abstract
Due to the importance of solutions of partial differential equations, linear, nonlinear, homogeneous, and non-homogeneous, in important life applications, including engineering applications, physics and astronomy, medical sciences, and life technology, and their importance in solutions to heat transfer equations, wave, Laplace equation, telegraph, etc. In this paper, a new double integral transform has been proposed.
In this work, we have introduced a new double transform (Double Complex EE Transform). In addition, we presented the convolution theorem and proved the properties of the proposed transform, which has an effective and useful role in dealing with the solution of two-dimensional partial differential equations. Moreover, two examples of important mathematical equations are solved to illustrate method. This double integral transformation has a complex kernel.

Keywords Complex EE Transform, Double Complex EE Transform, partial differential equations.

1. Introduction
The integral transformation is one of the important and famous topics of applied mathematics [1-10]. In recent years many researchers have discovered new transformations to solve several problems in mathematics, physics, and engineering [11-27]. [28] have implemented a new transformation (Double Aboodh Transform) for solving Telegraph Equation. [29] have solved ordinary differential equations by a new transformation (Complex EE Transform). In [30], Meddahi and Jafari, introduced the new transformation (General Double Integral Transform) to solve telegraph and Klein-Gordon equations.

The relationship between this transform and the two-dimensional Laplace transform is the change in the definition of the kernel, the kernel in the proposed transform is wider than the kernel in the two-dimensional Laplace transform.

2. Definitions and Properties of Double Complex EE Transform :

Definition 2.1. [29] The Complex EE (Emad-Elaf) Transform of $f(t)$ denoted by the operator E is given by:

$$E[f(t)] = E(iv) = \int_{t=0}^{\infty} f(t)e^{-iv^n t}\ dt, n \in Z, t \geq 0, i \in C,$$

where v is a complex parameter and $Im\ (v^n) < 0$.

Now, we will introduce the definition of the double integral transform of the complex EE transform and its properties.

Definition 2.2. The Double Complex EE Transform of $f(x,t)$ denoted by the operator D_{EE} is given by:

$$D_{EE}[f(x,t)] = A(iu,iv) = \int_0^{\infty}\int_0^{\infty} f(x,t)e^{-i(u^n x+v^n t)}\, dtdx, n \in Z, t, x \geq 0, i \in C,$$

where v and u are complex parameters and every $Im\ (v^n) < 0$ and $Im\ (u^n) < 0$.

Some properties of the Double Complex EE Transform are as follows

I. If $f(x,t) = 1$, then

$$D_{EE}[1] = \int_0^{\infty}\int_0^{\infty} 1.\, e^{-i(u^n x+v^n t)}\, dtdx$$

$$= \int_0^{\infty} 1.\, e^{-iu^n x}\ dx \int_0^{\infty} 1.\, e^{-iv^n t}\ dt$$

$$= EE[1].\,EE[1]$$

$$= \frac{-i}{u^n}.\frac{-i}{v^n}$$

$$= \frac{-1}{(uv)^n}.$$

II. If $f(x,t) = x^r t^m$, then

$$D_{EE}[x^r t^m] = \int_0^{\infty}\int_0^{\infty} x^r t^m e^{-i(u^n x+v^n t)}\, dtdx$$

$$= \int_0^{\infty} x^r e^{-iu^n x}\ dx \int_0^{\infty} t^m\, e^{-iv^n t}\ dt$$

$$= EE[x^r].\,EE[t^m]$$

$$= \frac{r!}{(iu)^{(r+1)n}} \cdot \frac{m!}{(iv)^{(m+1)n}}.$$

III. If $f(x,t) = e^{ax+bt}$, then

$$D_{EE}[e^{ax+bt}] = \int_0^\infty \int_0^\infty e^{ax+bt} e^{-i(u^n x + v^n t)} \, dt dx$$

$$= \int_0^\infty e^{ax} e^{-iu^n x} \, dx \int_0^\infty e^{bt} \, e^{-iv^n t} \, dt$$

$$= EE[e^{ax}].\, EE[e^{bt}]$$

$$= (-1)\left[\frac{a + iu^n}{a^2 + u^{2n}}\right].(-1)\left[\frac{b + iv^n}{b^2 + v^{2n}}\right]$$

$$= \frac{(a + iu^n)(b + iv^n)}{(a^2 + u^{2n})(b^2 + v^{2n})}.$$

IV. If $f(x,t) = e^{-(ax+bt)}$, then

$$D_{EE}\left[e^{-(ax+bt)}\right] = \int_0^\infty \int_0^\infty e^{-(ax+bt)} e^{-i(u^n x + v^n t)} \, dt dx$$

$$= \int_0^\infty e^{-ax} e^{-iu^n x} \, dx \int_0^\infty e^{-bt} \, e^{-iv^n t} \, dt$$

$$= EE[e^{-ax}].\, EE[e^{-bt}]$$

$$= \frac{(-a + iu^n)(-b + iv^n)}{(a^2 + u^{2n})(b^2 + v^{2n})}$$

$$= \frac{(a - iu^n)(b - iv^n)}{(a^2 + u^{2n})(b^2 + v^{2n})}.$$

V. If $f(x,t) = e^{i(ax+bt)}$, then

$$D_{EE}\left[e^{i(ax+bt)}\right] = \int_0^\infty \int_0^\infty e^{i(ax+bt)} e^{-i(u^n x + v^n t)} \, dt dx$$

$$= \int_0^\infty e^{iax} e^{-iu^n x} \, dx \int_0^\infty e^{ibt} \, e^{-iv^n t} \, dt$$

$$= EE\left[e^{iax}\right].\, EE[e^{ibt}]$$

$$= \frac{1}{i(u^n - a)} \cdot \frac{1}{i(v^n - b)}$$

$$= \frac{-1}{(u^n - a)(v^n - b)}.$$

VI. If $f(x,t) = e^{-i(ax+bt)}$, then

$$D_{EE}\left[e^{-i(ax+bt)}\right] = \int_0^\infty \int_0^\infty e^{-i(ax+bt)} e^{-i(u^n x+v^n t)}\, dtdx$$

$$= \int_0^\infty e^{-iax} e^{-iu^n x}\ dx \int_0^\infty e^{-ibt}\, e^{-iv^n t}\ dt$$

$$= EE\left[e^{-iax}\right].EE\left[e^{-ibt}\right]$$

$$= \frac{1}{i(u^n+a)}.\frac{1}{i(v^n+b)}$$

$$= \frac{-1}{(u^n+a)(v^n+b)}.$$

VII. If $f(x,t) = \sin{(ax+bt)}$, then

$$D_{EE}[\sin{(ax+bt)}] = \int_0^\infty \int_0^\infty \sin{(ax+bt)} e^{-i(u^n x+v^n t)}\, dtdx$$

$$= \int_0^\infty \int_0^\infty \left(\frac{e^{i(ax+bt)} - e^{-i(ax+bt)}}{2i}\right) e^{-i(u^n x+v^n t)}\, dtdx$$

$$= \frac{1}{2i}\left[\int_0^\infty \int_0^\infty e^{i(ax+bt)} e^{-i(u^n x+v^n t)}\, dtdx\right]$$

$$- \frac{1}{2i}\left[\int_0^\infty \int_0^\infty e^{-i(ax+bt)} e^{-i(u^n x+v^n t)}\, dtdx\right]$$

$$= \frac{1}{2i}\left[D_{EE}\left[e^{i(ax+bt)}\right] - D_{EE}\left[e^{-i(ax+bt)}\right]\right]$$

$$= \frac{1}{2i}\left[\frac{-1}{(u^n-a)(v^n-b)} + \frac{1}{(u^n+a)(v^n+b)}\right]$$

$$= \frac{i(bu^n+av^n)}{(u^{2n}-a^2)(v^{2n}-b^2)}.$$

VIII. If $f(x,t) = \cos{(ax+bt)}$, then

$$D_{EE}[\cos{(ax+bt)}] = \int_0^\infty \int_0^\infty \cos{(ax+bt)} e^{-i(u^n x+v^n t)}\, dtdx$$

$$= \int_0^\infty \int_0^\infty (\frac{e^{i(ax+bt)} + e^{-i(ax+bt)}}{2}) e^{-i(u^n x + v^n t)}\, dtdx$$

$$= \frac{1}{2}\left[\int_0^\infty \int_0^\infty e^{i(ax+bt)} e^{-i(u^n x + v^n t)}\, dtdx\right]$$

$$+ \frac{1}{2}\left[\int_0^\infty \int_0^\infty e^{-i(ax+bt)} e^{-i(u^n x + v^n t)}\, dtdx\right]$$

$$= \frac{1}{2}[D_{EE}\left[e^{i(ax+bt)}\right] + D_{EE}[e^{-i(ax+bt)}]]$$

$$= \frac{1}{2}\left[\frac{-1}{(u^n - a)(v^n - b)} + \frac{-1}{(u^n + a)(v^n + b)}\right]$$

$$= \frac{-((uv)^n + ab)}{(u^{2n} - a^2)(v^{2n} - b^2)}.$$

IX. If $f(x,t) = \sinh(ax + bt)$, then

$$D_{EE}[\sinh(ax+bt)] = \int_0^\infty \int_0^\infty \sinh(ax+bt) e^{-i(u^n x + v^n t)}\, dtdx$$

$$= \int_0^\infty \int_0^\infty (\frac{e^{(ax+bt)} - e^{-(ax+bt)}}{2}) e^{-i(u^n x + v^n t)}\, dtdx$$

$$= \frac{1}{2}\left[\int_0^\infty \int_0^\infty e^{(ax+bt)} e^{-i(u^n x + v^n t)}\, dtdx\right]$$

$$- \frac{1}{2}\left[\int_0^\infty \int_0^\infty e^{-(ax+bt)} e^{-i(u^n x + v^n t)}\, dtdx\right]$$

$$= \frac{1}{2}[D_{EE}\left[e^{(ax+bt)}\right] - D_{EE}[e^{-(ax+bt)}]]$$

$$= \frac{1}{2}\left[\frac{(a + iu^n)(b + iv^n)}{(a^2 + u^{2n})(b^2 + v^{2n})} - \frac{(a - iu^n)(b - iv^n)}{(a^2 + u^{2n})(b^2 + v^{2n})}\right]$$

$$= \frac{i(av^n + bu^n)}{(a^2 + u^{2n})(b^2 + v^{2n})}.$$

X. If $f(x,t) = \cosh(ax + bt)$, then

$$D_{EE}[\cosh(ax+bt)] = \int_0^\infty \int_0^\infty \cosh(ax+bt) e^{-i(u^n x + v^n t)}\, dtdx$$

$$= \int_0^\infty \int_0^\infty (\frac{e^{(ax+bt)} + e^{-(ax+bt)}}{2}) e^{-i(u^n x + v^n t)}\, dtdx$$

$$=\frac{1}{2}\left[\int_0^{\infty}\int_0^{\infty} e^{(ax+bt)}e^{-i(u^nx+v^nt)}\,dtdx\right]$$

$$+\frac{1}{2}\left[\int_0^{\infty}\int_0^{\infty} e^{-(ax+bt)}e^{-i(u^nx+v^nt)}\,dtdx\right]$$

$$=\frac{1}{2}\left[D_{EE}\left[e^{(ax+bt)}\right]+D_{EE}\left[e^{-(ax+bt)}\right]\right]$$

$$=\frac{1}{2}\left[\frac{(a+iu^n)(b+iv^n)}{(a^2+u^{2n})(b^2+v^{2n})}+\frac{(a-iu^n)(b-iv^n)}{(a^2+u^{2n})(b^2+v^{2n})}\right]$$

$$=\frac{ab-(uv)^n}{(a^2+u^{2n})(b^2+v^{2n})}.$$

3. (Convolution Theorem) :

If $D_{EE}[f(x,t)]=A(iu,iv)$, $D_{EE}[g(x,t)]=B(iu,iv)$, and

$$D_{EE}[f**g(x,t)]=\int_0^{\infty}\int_0^{\infty} f(x-\beta,t-\alpha)g(x-\beta,t-\alpha)\,dtdx,$$

then

$$D_{EE}[f**g(x,t)]=D_{EE}[f(x,t)]\,.\,D_{EE}[g(x,t)]=A(iu,iv)\,.\,B(iu,iv).$$

4. Double EE Integral Transform for the First and Second Derivatives :

1.

$$D_{EE}\left[\frac{\partial f(x,t)}{\partial x}\right]=\int_0^{\infty}\int_0^{\infty}\frac{\partial f(x,t)}{\partial x}e^{-i(u^nx+v^nt)}\,dtdx$$

$$=\int_0^{\infty} e^{-iv^nt}\left(\int_0^{\infty}\frac{\partial f(x,t)}{\partial x}e^{-iu^nx}\,dx\right)dt$$

$$=\int_0^{\infty} e^{-iv^nt}\left(e^{-iu^nx}f(x,t)|_0^{\infty}+iu^n\int_0^{\infty} f(x,t)\,e^{-iu^nx}dx\right)dt$$

$$=\int_0^{\infty} e^{-iv^nt}\left(-f(0,t)+iu^n\int_0^{\infty} f(x,t)\,e^{-iu^nx}dx\right)dt$$

$$=-\int_0^{\infty} f(0,t)e^{-iv^nt}dt+iu^n\int_0^{\infty}\int_0^{\infty} f(x,t)e^{-i(u^nx+v^nt)}\,dtdx$$

$$=-A(0,iv)+iu^nA(iu,iv).$$

2.

$$D_{EE}\left[\frac{\partial^2 f(x,t)}{\partial x^2}\right]=\int_0^{\infty}\int_0^{\infty}\frac{\partial^2 f(x,t)}{\partial x^2}e^{-i(u^nx+v^nt)}\,dtdx$$

$$=\int_0^\infty e^{-iv^n t}(\int_0^\infty \frac{\partial^2 f(x,t)}{\partial x^2} e^{-iu^n x}\, dx)dt$$

$$=\int_0^\infty e^{-iv^n t}\left(e^{-iu^n x}\frac{\partial f(x,t)}{\partial x}|_0^\infty + iu^n\int_0^\infty \frac{\partial f(x,t)}{\partial x} e^{-iu^n x}dx\right)dt$$

$$=\int_0^\infty e^{-iv^n t}\left(-\frac{\partial f(0,t)}{\partial x} + iu^n\int_0^\infty \frac{\partial f(x,t)}{\partial x} e^{-iu^n x}dx\right)dt$$

$$=-\int_0^\infty \frac{\partial f(0,t)}{\partial x} e^{-iv^n t}dt + iu^n\int_0^\infty\int_0^\infty \frac{\partial f(x,t)}{\partial x} e^{-i(u^n x+v^n t)}\, dtdx$$

$$=-\frac{\partial A(0,iv)}{\partial x} + iu^n D_{EE}\left[\frac{\partial f(x,t)}{\partial x}\right]$$

$$=-\frac{\partial A(0,iv)}{\partial x} - iu^n A(0,iv) - u^{2n}A(iu,iv).$$

3.

$$D_{EE}\left[\frac{\partial f(x,t)}{\partial t}\right]=\int_0^\infty\int_0^\infty \frac{\partial f(x,t)}{\partial t} e^{-i(u^n x+v^n t)}\, dtdx$$

$$=\int_0^\infty e^{-iu^n x}(\int_0^\infty \frac{\partial f(x,t)}{\partial t} e^{-iv^n t}\, dt)dx$$

$$=\int_0^\infty e^{-iu^n x}\left(e^{-iv^n t}f(x,t)|_0^\infty + iv^n\int_0^\infty f(x,t)\, e^{-iv^n t}dt\right)dx$$

$$=\int_0^\infty e^{-iu^n x}\left(-f(x,0) + iv^n\int_0^\infty f(x,t)\, e^{-iv^n t}dt\right)dx$$

,

$$=-\int_0^\infty f(x,0)e^{-iu^n x}dx + iv^n\int_0^\infty\int_0^\infty f(x,t)e^{-i(u^n x+v^n t)}\, dtdx$$

$$=-A(iu,0)+iv^n A(iu,iv).$$

4.

$$D_{EE}\left[\frac{\partial^2 f(x,t)}{\partial t^2}\right]=\int_0^\infty\int_0^\infty \frac{\partial^2 f(x,t)}{\partial t^2} e^{-i(u^n x+v^n t)}\, dtdx$$

$$=\int_0^\infty e^{-iu^n x}(\int_0^\infty \frac{\partial^2 f(x,t)}{\partial t^2} e^{-iv^n t}\, dt)dx$$

$$= \int_0^\infty e^{-iu^n x}\left(e^{-iv^n t}\frac{\partial f(x,t)}{\partial t}|_0^\infty + iv^n \int_0^\infty \frac{\partial f(x,t)}{\partial t} e^{-iv^n t} dt\right)dx$$

$$= \int_0^\infty e^{-iu^n x}\left(-\frac{\partial f(x,0)}{\partial t} + iv^n \int_0^\infty \frac{\partial f(x,t)}{\partial t} e^{-iv^n t} dt\right)dx$$

$$= -\int_0^\infty \frac{\partial f(x,0)}{\partial t} e^{-iu^n x} dx + iv^n \int_0^\infty \int_0^\infty \frac{\partial f(x,t)}{\partial t} e^{-i(u^n x + v^n t)}\, dtdx$$

$$= -\frac{\partial A(iu,0)}{\partial t} + iv^n D_{EE}\left[\frac{\partial f(x,t)}{\partial t}\right]$$

$$= -\frac{\partial A(iu,0)}{\partial t} - iv^n A(iu,0) - v^{2n} A(iu,iv).$$

5.

$$D_{EE}\left[\frac{\partial^2 f(x,t)}{\partial t \partial x}\right] = \int_0^\infty \int_0^\infty \frac{\partial^2 f(x,t)}{\partial t \partial x} e^{-i(u^n x + v^n t)}\, dtdx$$

$$= \int_0^\infty e^{-iu^n x}(\int_0^\infty \frac{\partial^2 f(x,t)}{\partial t \partial x} e^{-iv^n t}\, dt)dx$$

$$= \int_0^\infty e^{-iu^n x}\left(e^{-iv^n t}\frac{\partial f(x,t)}{\partial x}|_0^\infty + iv^n \int_0^\infty \frac{\partial f(x,t)}{\partial x} e^{-iv^n t} dt\right)dx$$

$$= \int_0^\infty e^{-iu^n x}\left(-\frac{\partial f(x,0)}{\partial x} + iv^n \int_0^\infty \frac{\partial f(x,t)}{\partial x} e^{-iv^n t} dt\right)dx$$

$$= -\int_0^\infty \frac{\partial f(x,0)}{\partial x} e^{-iu^n x} dx + iv^n \int_0^\infty \int_0^\infty \frac{\partial f(x,t)}{\partial x} e^{-i(u^n x + v^n t)}\, dtdx$$

$$= -\frac{\partial A(iu,0)}{\partial x} + iv^n D_{EE}\left[\frac{\partial f(x,t)}{\partial x}\right]$$

$$= -\frac{\partial A(iu,0)}{\partial x} - iv^n A(0,iv) - (uv)^n A(iu,iv).$$

6.

$$D_{EE}\left[\frac{\partial^2 f(x,t)}{\partial x \partial t}\right] = \int_0^\infty \int_0^\infty \frac{\partial^2 f(x,t)}{\partial x \partial t} e^{-i(u^n x + v^n t)}\, dtdx$$

$$= \int_0^\infty e^{-iv^n t} (\int_0^\infty \frac{\partial^2 f(x,t)}{\partial x \partial t} e^{-iu^n x}\, dx) dt$$

$$= \int_0^\infty e^{-iv^n t} \left(e^{-iu^n x} \frac{\partial f(x,t)}{\partial t} |_0^\infty + iu^n \int_0^\infty \frac{\partial f(x,t)}{\partial t} e^{-iu^n x} dx \right) dt$$

$$= \int_0^\infty e^{-iv^n t} \left(-\frac{\partial f(0,t)}{\partial t} + iu^n \int_0^\infty \frac{\partial f(x,t)}{\partial t} e^{-iu^n x} dx \right) dt$$

$$= -\int_0^\infty \frac{\partial f(0,t)}{\partial t} e^{-iv^n t} dt + iu^n \int_0^\infty \int_0^\infty \frac{\partial f(x,t)}{\partial t} e^{-i(u^n x + v^n t)}\, dt dx$$

$$= -\frac{\partial A(0,iv)}{\partial t} + iu^n D_{EE}\left[\frac{\partial f(x,t)}{\partial t}\right]$$

$$= -\frac{\partial A(0,iv)}{\partial t} - iu^n A(iu,0) - (uv)^n A(iu,iv).$$

5. Examples of Applying The Double EE Integral Transform on Partial Differential Equations :

Example 1. consider the telegraph equation

$$u_{xx} = u_{tt} + 2u_t + u, \tag{1}$$

with ICs and BCs

$$u(0,t) = e^{-2t}, u_x(0,t) = e^{-2t}, u(x,0) = e^x, u_t(x,0) = -2e^x. \tag{2}$$

The exact solution of Eq (1) is $u(x,t) = e^{x-2t}$.

Using the Double Complex EE Transform of both sides to Eq (1), we get:

$$D_{EE}[u_{xx}] = D_{EE}[u_{tt}] + D_{EE}[2u_t] + D_{EE}[u\ ]. \tag{3}$$

Eq (3) can be written as :

$$-\frac{\partial A(0,iv)}{\partial x} - iu^n A(0,iv) - u^{2n} A(iu,iv) = -\frac{\partial A(iu,0)}{\partial t} - iv^n A(iu,0) - v^{2n} A(iu,iv)$$

$$-2A(iu,0) + 2iv^n A(iu,iv) + A(iu,iv). \tag{4}$$

Besides by using the Double Complex EE Transform of ICs and BCs, we have:

$$A(0,iv)=\frac{-(-2+iv^n)}{4+v^{2n}},A(iu,0)=\frac{-(1+iu^n)}{1+u^{2n}},$$

$$\frac{\partial A(0,iv)}{\partial x}=\frac{-(-2+iv^n)}{4+v^{2n}},\frac{\partial A(iu,0)}{\partial t}=\frac{2(1+iu^n)}{1+u^{2n}}, \tag{5}$$

set Eq (5) into Eq (4), we obtain :

$$\begin{aligned}\frac{(-2+iv^n)}{4+v^{2n}}+iu^n\frac{(-2+iv^n)}{4+v^{2n}}-u^{2n}A(iu,iv)\\ =\frac{-2(1+iu^n)}{1+u^{2n}}+iv^n\frac{(1+iu^n)}{1+u^{2n}}-v^{2n}A(iu,iv)+\frac{2(1+iu^n)}{1+u^{2n}}+2iv^nA(iu,iv)\\ +A(iu,iv),\end{aligned}$$

$$A(iu,iv)(v^{2n}-2iv^n-1-u^{2n})=iv^n\frac{(1+iu^n)}{1+u^{2n}}+\frac{u^n(v^n+2i)-i(v^n+2i)}{4+v^{2n}},$$

$$\begin{aligned}A(iu,iv)&=\frac{iv^n(1+iu^n)}{1+u^{2n}((v^n-i)^2-u^{2n})}+\frac{(v^n+2i)(u^n-i)}{(4+v^{2n})((v^n-i)^2-u^{2n})}.\\ &=\frac{(1+iu^n)}{1+u^{2n}}\frac{(-2+iv^n)}{4+v^{2n}}.\end{aligned} \tag{6}$$

To find the exact solution of Eq (1), we use the inverse Double Complex EE Transform to Eq (6), to get:

$$u(x,t)=e^{x-2t}.$$

Example 2. consider the wave equation

$$u_{xx}=u_{tt}, \tag{7}$$

with ICs and BCs

$$u(0,t)=2t,u_x(0,t)=\cos t\ ,u(x,0)=\sin x\,,u_t(x,0)=2. \tag{8}$$

The exact solution of Eq (7) is $u(x,t)=\sin x\cos t+2t$.

Using the Double Complex EE Transform of both sides to Eq (7), we get:

$$D_{EE}[u_{xx}] = D_{EE}[u_{tt}]. \tag{9}$$

Eq (9) can be written as :

$$-\frac{\partial A(0,iv)}{\partial x} - iu^n A(0,iv) - u^{2n} A(iu,iv) = -\frac{\partial A(iu,0)}{\partial t} - iv^n A(iu,0) - v^{2n} A(iu,iv). \tag{10}$$

Besides by using the Double Complex EE Transform of ICs and BCs, we have:

$$A(0,iv) = \frac{-2}{v^{2n}}, A(iu,0) = \frac{-1}{u^{2n}-1}, \frac{\partial A(0,iv)}{\partial x} = \frac{-iv^n}{v^{2n}-1}, \frac{\partial A(iu,0)}{\partial t} = \frac{-2i}{u^n}, \tag{11}$$

set Eq (11) into Eq (10), we obtain :

$$\frac{iv^n}{v^{2n}-1} + \frac{2iu^n}{v^{2n}} - u^{2n} A(iu,iv) = \frac{2i}{u^n} + \frac{iv^n}{u^{2n}-1} - v^{2n} A(iu,iv),$$

$$A(iu,iv)(v^{2n} - u^{2n}) = \frac{2i}{u^n} + \frac{iv^n}{u^{2n}-1} - \frac{iv^n}{v^{2n}-1} - \frac{2iu^n}{v^{2n}},$$

$$\begin{aligned} A(iu,iv) &= \frac{2iv^{2n} - 2iu^{2n}}{u^n v^{2n}(v^{2n} - u^{2n})} + \frac{iv^n(v^{2n}-1) - iv^n(u^{2n}-1)}{(u^{2n}-1)(v^{2n}-1)(v^{2n}-u^{2n})} \\ &= \frac{2i}{u^n v^{2n}} + \frac{iv^n}{(u^{2n}-1)(v^{2n}-1)}. \end{aligned} \tag{12}$$

To find the exact solution of Eq (7), we use the inverse Double Complex EE Transform to Eq (12), to get:

$$u(x,t) = 2t + \sin x \cos t.$$

5. Conclusion

In this paper, we have dealt with the new double transform (Double Complex EE Transform) and some important properties and definitions to it, then after that we applied this transform to solve the telegraph equation, and wave equation, which are important partial differential equations in applied mathematics.

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Conflict of Interest

The authors declare that they have no conflicts of interest.

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