# Energy Methods For Initial –Boundary String Problem

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## Abstract

We study one example of hyperbolic problems it's Initial-boundary string problem with two ends. In fact we look for the solution in weak sense in some sobolev spaces. Also we use energy technic with Galerkin's method to study some properties for our problem as existence and uniqueness.

**Keyword:** Initial- Boundary String Problem, Weak Solution, Sobolev Spaces, Galerkin's Method, Energy Estimate.

# 1.Introduction

Throughout this paper we consider energy estimate for string equation with initial –boundary conditions. We gave a little introduction for weak derivatives, weak solution in some sobolev space, for example $H_0^1(u)$, $H^{-1}(u)$ and others. In fact energy estimate has many results for example a speed of propagation [1], in our paper it leads us to straight forward existence, uniqueness [2], results and other properties like regularity. It yields also to well-posed for hyperbolic equations.

In 2007, Tarama [3], introduced wave equations with coefficients satisfying Besov type conditions and obtained the energy estimate, also he gave an example of a wave equation with continuous and differentiable coefficients for which the $L^2$ estimate holds.

Jaipong Kasemsuwan [4], concerned with the energy decay of the global solution for initial-boundary value problem to a nonlinear damped equation of suspended string with uniform density to which a nonlinear outer force works.

The string equation with initial-boundary conditions is

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2} + f(x,t) \text{ in } Q \qquad \ldots (1.1)[5, p.30]$$

Where $Q = \{(x,t); x \in (0,l), t \in (0,T)\}$,

Initial conditions:

$$\left.\begin{aligned} u(x,0) &= \varphi(x) \\ u_t(x,0) &= \psi(x) \end{aligned}\right\} \qquad \ldots\ (1.2)$$

Boundary conditions:

$$\left.\begin{aligned} u(0,t) &= 0 \\ u(l,t) &= 0 \end{aligned}\right\} \qquad \ldots (1.3)$$

This problem describes the vibration of the string in a finite interval $[0,l]$ and the constant $a$ denotes the speed of the string.

Rewrite the initial-boundary problem with the second order partial differential operator $L$ as:

$$\left.\begin{aligned} \frac{\partial^2 u}{\partial t^2} &= a^2 Lu + f(x,t) \ in\ U \times (0,T] \\ u(x,t) &= 0 \ on\ \ \partial U \times [0,T] \\ u(x,t) = \varphi(x),\ &\frac{\partial}{\partial t}u(x,t) = \psi(x) on\ U \times \{t=0\} \end{aligned}\right\} \qquad \ldots(1.4)$$

*Let us denote for* $U = (0,l)$, $U_T = U \times (0,T]$, $f: U \times (0,T] \to \mathcal{R}$ and $\varphi, \psi: U \to \mathcal{R}$ and these *functions are given.*

The function $u: \overline{U_T} \to \mathcal{R}$ is the unknown function u=u(x, t). The symbol $L$ denotes for each time $t$ a second–order partial differential operator, having the divergence form

$$Lu = -\sum_{i,j=1}^{n} (a^{ij}(x,t)\, u_{xi})_{xj} + \sum_{i=1}^{n} b^i(x,t) u_{xi} + c(x,t)u \qquad \ldots (1.5)\ [6, p.309]$$

For given coefficients $a^{ij}, b^i, c \in C^1(\overline{U} \times [0,T])$ $(i,j = 1,2,\ldots,n)$, $f \in L^2(U \times [0,T])$, $\varphi \in H_0^1(U)$, $\psi \in L^2(U)$, And always assume that $a^{ij} = a^{ji}$ $(i,j = 1,2,\ldots,n)$.

## Definition 1[6, p.378]

The partial differential operator $\frac{\partial^2}{\partial t^2} + L$ *is* said *to be (*uniformly*)* hyperbolic if there exists a constant $\theta > 0$ such that $\sum_{i,j=1}^{n} a^{ij}(x,t)\varepsilon_i\varepsilon_j \geq \theta|\varepsilon|^2$ for all (x, *t*) $\in U \times [0,T], \varepsilon \in R^n$.

## Definition 2 [6, p. 285]

Let $u \in L^1(0,T,X)$. we say $v \in L^1(0,T,X)$ is **weak derivative** of u, written: $u' = v$ provided

$$\int_0^T \varphi'(t)\, u(t)dt = -\int_0^T \varphi(t)\, v(t)dt$$

For all scalar test functions $\varphi \in C_c^\infty(0,T)$.

Let us introduce the time-dependent bilinear form:

$$B[u,v;t] = \int_U \Big(\sum_{i,j=1}^n a^{ij}\, u_{xi} v_{xj}\Big)dx + \int_U \sum_{i=1}^n \big(b^i(x,t)\, u_x v + c(x,t) u\, v\big)dx$$

for all $u, v \in H_0^1(U)$ and $0 \le t \le T$.

First we suppose u is smooth solution and defined the mapping:

$$u: [0,T] \to H_0^1(U), [u(t)](x) = u(x,t), (x \in U, 0 \le t \le T), (\,' = \tfrac{d}{dt})$$

$$f: [0,T] \to L^2\ (U),\ \ [f(t)](x) = f(x,t)$$

Now we fix any function $v \in H_0^1(U)$, multiply by v the PDE $(u_{tt} + Lu = f)v$

$$(u'', v) + B[u,v,t] = (f,v)$$

The pairing ( , ) inner product in $L^2(U)$.

We see the form: $u_{tt} = \varphi^0 + \sum_{j=1}^n \varphi^j_{xj}$

For $\varphi^0 = f - \sum_{i=1}^n b^i u_{xi} - c\,u$, $\varphi^j = f - \sum_{i=1}^n a^{ij} u_{xi}$ (j=1,…,n)

This suggests that we should look for weak solution u with u" $\in H^{-1}(U)$ for almost every where $0 \le t \le T$, $\langle u'', v\rangle$ means $u'' \in H^{-1}(U), v \in H_0^1(U)$

## Definition 3 [6, p.379]

We say a function $u \in L^2\big(0,T,H_0^1(U)\big)$, with $u' \in L^2\big(0,T,L^2(U)\big)$, $u'' \in L^2(0,T,H^{-1}(U))$ is **weak solution** of the hyperbolic initial( boundary ) problem and

i. $\langle u'', v\rangle + B[u,v,t] = (f,v)$ $for\ each\ v \in H_0^1(U)$

ii. $u(0)=\varphi$, $u'(0) = \psi$.

As known sobolev spaces are designed to contain less smooth functions that means the functions have some but not great, smoothness properties.

So the idea is to make approximation for the function u(x, t) such that u(x, t)=0 in the boundary of U by the

$$u_m(t) = \sum_{k=1}^m d_m^k(t)\ w_k(x) \qquad [6, p.353]$$

In the right side hand we have separated variables, we note that U is a domain of the variable x only, and this is the reason to consider the variable t in respect to the sobolev space $H_0^1(U)$ as a parameter. where $w_k = w_k(x)$ ( k = 1,2, ... ) are smooth functions such that $\{w_k\}_{k=1}^\infty$ is an orthogonal basis of $H_0^1(U)$, $\{w_k\}_{k=1}^\infty$ is an orthonormal basis of $L^2(U)$.

*We are* going to consider $u$ not as a function of $x$ and $t$ together, but rather as a mapping $u$ of t into the space $H_0^1(U)$ of functions of $x$.

We will look for a function $u_m$:[0, T] $\to H_0^1(U)$ of the form $u_m(t) = \sum_{k=1}^m d_m^k(t)\, w_k$ *such that*

$$(u_m'', w_k) + B[u_m, w_k; t] = (f, w_k)\ ,\ (t \in [0,T], k = 1,2,\dots,m) \qquad \dots (1.6)$$

$$\left.\begin{array}{l} d_m^k(0) = (\varphi, w_k) \\ {d_m^k}'(0) = (\psi, w_k) \end{array}\right\} \qquad \dots (1.7)$$

*where*

$$B[u_m, w_k; t] = \int_U \Big(\sum_{i,j=1}^n a^{ij}\, u_{m,xi} w_{k,xj}\Big)dx + \int_U \sum_{i=1}^n \big(b^i\, u_{m,xi} w_k + c u_m w_k\big)\, dx$$

In other words, we construct solutions of certain finite-dimensional approximations to our problem and then passing to limits, it's called Galerkins method [6, p.380].
Indeed we need some estimates to send m to infinity and to show subsequence of our solutions $u_m$ of the approximate problem .

## 2.Main Result

**Theorem (Energy estimates)(2.1):** *There exists a constant C, depending only on U,T and the coefficients on L such that:*

$$\max_{0\le t\le T}\left(\|u_m(t)\|_{H_0^1(U)}+\|u_m'(t)\|_{L^2(U)}\right)+\ \|u_m''(t)\|_{L^2\left(0,T;H^{-1}(U)\right)}$$
$$\le C\left(\|f\|_{L^2(0,T;L^2(U))}+\|\varphi\|_{H_0^1(U)}+\ \|\psi\|_{L^2(U)}\right)\quad \text{for m=1,2,…}$$

**proof** We take equation (1.6) and write it in details as:

$$\int_U (u_m'' w_k)dx+\int_U\sum_{i,j=1}^{n}(a^{ij}u_{m,xi}w_{k,xj})dx+\int_U\sum_{i=1}^{n}(b^i u_{m,xi}w_k+cu_m w_k)\,dx$$
$$=\int_U f\,w_k\,dx$$

*Multiply by* $d_m^{k\,\prime}(t)$ *so:*

$$\int_U [u_m'' w_k\ d_m^{k\,\prime}(t)]dx+\int_U(\sum_{i,j=1}^{n}a^{ij}u_{m,xi}w_{k,xj}\ d_m^{k\,\prime}(t)+\sum_{i=1}^{n}b^i u_{m,xi}w_k\ d_m^{k\,\prime}(t)$$
$$+cu_m w_k\ d_m^{k\,\prime}(t))dx\ =\int_U f\,w_k\,d_m^{k\,\prime}(t)\ dx.$$

$$\int_U(\ u_m''u_m')dx+\int_U(\sum_{i,j=1}^{n}a^{ij}u_{m,xi}u_m'+\sum_{i=1}^{n}b^i u_{m,xi}u_m'+cu_m\ u_m')\,dx=\int_U f u_m'\ dx$$

$$(u_m'',u_m')+B[u_m,u_m',;t]=(f,u_m').$$

And we have

$$(u_m'',u_m')=\frac{d}{dt}\left(\frac{1}{2}\|u_m'\|_{L^2(U)}^2\right).$$

We write :

$$\int_U\sum_{i,j=1}^{n}(a^{ij}u_{m,xi}u'_{m,xj)})dx+\int_U\sum_{i=1}^{n}(b^i u_{m,xi}u'_m+cu_m u'_m)\,dx=\ B_1+B_2$$

where

$$B_1=\int_U(\sum_{i,j=1}^{n}a^{ij}u_{m,xi}u_{mxj}'\quad and\ \ B_2=\int_U(\sum_{i=1}^{n}b^i u_{m,xi}u_m'\ +cu_m\ u_m')\,dx$$

Now we define the symmetric bilinear form: $A[u,v;t]\coloneqq\int_U(\sum_{i,j=1}^{n}a^{ij}u_{xi}v_{xj})\,dx$ ,
$u,v\in H_0^1(U)$

Then

$$\frac{d}{dt}\left(\frac{1}{2}A[u_m,u_m;t]\right)-\frac{1}{2}\int_U\ (\sum_{i,j=1}^{n}a_t^{ij}u_{m_{xi}}u_{m_{xj}})dx$$
$$=\frac{1}{2}(\int_U\frac{d}{dt}[\sum_{i,j=1}^{n}a^{ij}u_{m,xi}u_{m,xj}]\,dx-\frac{1}{2}\int_U\left(\sum_{i,j=1}^{n}a_t^{ij}u_{m_{xi}}u_{m_{xj}}\right)dx$$

$$=\frac{1}{2}\int_v \sum_{i,j=1}^{n} a_t^{ij} u_{m_{xi}} u_{m_{xj}}\,dx+\frac{1}{2}\int_v \sum_{i,j=1}^{n} a^{ij}\, u'_{m_{xi}} u_{m_{xj}}\,dx+\frac{1}{2}\int_v \sum_{i,j=1}^{n} a^{ij}\, u'_{m_{xi}} u_{m_{xj}}\,dx$$

$$-\frac{1}{2}\int_U \sum_{i,j=1}^{n} a_t^{ij} u_{m_{xi}} u_{m_{xj}}\,dx=\int_U \sum_{i,j=1}^{n} a^{ij} u_{m_{xi}}\, u'_{m_{xj}}\,dx=B_1$$

$(since\ a^{ij}=a^{ji})$

*Then*

$$B_1=\frac{d}{dt}\left(\frac{1}{2}A[u_m,u_m;t]\right)-\frac{1}{2}\int_U \sum_{i,j=1}^{n} a_t^{ij} u_{m_{xi}} u_{m_{xj}}\,dx$$

So

$$\frac{d}{dt}\left(\frac{1}{2}A[u_m,u_m;t]\right)=B_1+\frac{1}{2}\int_U \left(\sum_{i,j=1}^{n} a_t^{ij} u_{m_{xi}} u_{m_{xj}}\right)dx$$

$a_t^{ij}\in C^1(\bar{U}\times[0,T])$, so there exists $\mathrm{Max}(a_t^{ij})$ *such that*

$$B_1+\frac{1}{2}\int_U\left(\sum_{i,j=1}^{n} a_t^{ij} u_{m_{xi}} u_{m_{xj}}\right)dx\ \le B_1+C\int_U\left(\sum_{i,j=1}^{n} u_{m_{xi}} u_{m_{xj}}\right)dx$$

$$\le B_1+C\quad\int_U \sum_{j=1}^{n} {u_{m_{xj}}}^2\,dx$$

$$\le B_1+C\quad B_1+C\|u_m\|_{H_0^1}^2$$

Or we can write it as:

$$\frac{d}{dt}(A[u_m,u_m;t])=2B_1+C\|u_m\|_{H_0^1}^2 \qquad \dots(2.1)$$

We had $(u''_m,u'_m)=\frac{d}{dt}\left(\frac{1}{2}\|u'_m\|_{L^2(U)}^2\right)$

So

$$\frac{d}{dt}\left(\|u'_m\|_{L^2(U)}^2\right)=\ 2(u''_m,u'_m)=\ 2(f,u'_m)-2(B_1+B_2)$$
$$\le C(\|f\|_{L^2}^2+C\|u'_m\|_{L^2}^2-2(B_1+B_2)$$

Now we obtain

$$\frac{d}{dt}\left(\|u'_m\|_{L^2(U)}^2\right)\le\|f\|_{L^2}^2+C\|u'_m\|_{L^2}^2-2(B_1+B_2) \qquad \dots(2.2)$$

*But*

$$B_2=\int_U\left(\sum_{i=1}^{n} b^i\, u_{m,xi}u'_m+cu_m\, u'_m\right)dx\ \rightarrow |B_2|=\left|\int_U\left(\sum_{i=1}^{n} b^i\, u_{m,xi}u'_m+cu_m\, u'_m\right)dx\right|$$

$b^i\in C^1(\bar{U}\times[0,T])$ *then there exists a Max* $(b^i)$ *such that*

$$\left|\int_U\sum_{i=1}^{n}(b^i\, u_{m,xi}u'_m+cu_m\, u'_m)\,dx\right|\le C\int_U\sum_{i=1}^{n}(|u_{m,xi}u'_m|+|u_m\, u'_m|)dx$$

$$\le C\int_U\sum_{i=1}^{n}(u_{mxi}^2+u_m^2+2{u'}_m^2)dx$$

$$\leq \mathrm{C}\,(\|u_m\|^2_{H_0^1} + \|u'_m\|^2_{L^2(U)})$$

Or

$$|B_2| \leq \mathrm{C}\,(\|u_m\|^2_{H_0^1(U)} + \|u'_m\|^2_{L^2(U)})$$

From (2.1) and (2.2) we obtain :

$$\frac{d}{dt}(A[u_m, u_m; t] + \|u'_m\|^2_{L^2(U)})$$
$$\leq 2B_1 + \mathrm{C}\|u_m\|^2_{H_0^1(U)} + \mathrm{C}\|f\|^2_{L^2(U)} + \mathrm{C}\|u'_m\|^2_{L^2(U)} - 2(B_1 + B_2)$$
$$\leq \mathrm{C}\|u_m\|^2_{H_0^1(U)} + \mathrm{C}\|f\|^2_{L^2(U)} + \mathrm{C}\|u'_m\|^2_{L^2(U)} + \mathrm{C}|B_2|$$
$$\leq \mathrm{C}\|u_m\|^2_{H_0^1(U)} + \mathrm{C}\|u'_m\|^2_{L^2(U)} + \mathrm{C}\|f\|^2_{L^2(U)} + \mathrm{C}\|u_m\|^2_{L^2}$$
$$\leq \mathrm{C}\|u_m\|^2_{H_0^1(U)} + \mathrm{C}\|u'_m\|^2_{L^2(U)} + \mathrm{C}\|f\|^2_{L^2(U)}$$

So we have

$$\frac{d}{dt}(A[u_m, u_m; t] + \|u'_m\|^2_{L^2(U)}) \leq \mathrm{C}\|u_m\|^2_{H_0^1(U)} + \mathrm{C}\|u'_m\|^2_{L^2(U)} + \mathrm{C}\|f\|^2_{L^2(U)}$$

But $L$ *is hyperbolic operator and by the definition of hyperbolic operator we have*

$$\theta \int_U \sum_{i=1}^{n} u_{m_{xi}}\, dx \leq \int_U \sum_{i,j=1}^{n} a^{ij} u_{m_{xi}} u_{m_{xj}}\, dx = A[u_m, u_m; t].$$
$$\theta \int_U \sum_{i=1}^{n} u_{m_{xi}}\, dx \leq A[u_m, u_m; t].$$

And

$$\mathrm{C}\|u_m\|^2_{H_0^1(U)} \leq \mathrm{C}\int_U u_m^2\, dx + \mathrm{C}\int_U \sum_i^n u^2_{mxi} dx \;\leq\; A[u_m, u_m; t]$$

$$\frac{d}{dt}(A[u_m, u_m; t] + \|u'_m\|^2_{L^2(U)}) \leq \mathrm{C}\|u'_m\|^2_{L^2(U)} + \mathrm{C}\|f\|^2_{L^2(U)} + \mathrm{C}\,A[u_m, u_m; t]$$

We denote

$$\eta(t) = \|u'_m(t)\|^2_{L^2(U)} + A[u_m(t), u_m(t); t]$$
$$\xi(t) = \|f\|^2_{L^2(U)}$$

*So we have* $\eta'(t) \leq \mathrm{C}_1\eta(t) + \mathrm{C}_2\xi(t)$

Now we use Gronwall Inequality

$$\eta(t) \leq e^{c_1 t}(\eta(0) + c_2 \int_0^t \xi(s)\, ds) \qquad ...(2.3)$$

*But*

$$\eta(0) = \|u'_m(0)\|^2_{L^2(U)} + A[u_m(0), u_m(0); 0]$$
$$\leq \|u'_m(0)\|^2_{L^2(U)} + \int_U \sum_{i,j=1}^{n} a^{ij} u_m(0)_{xi} u_m(0)_{xj}\, dx$$
$$\leq \|u'_m(0)\|^2_{L^2(U)} + \int_U (\sum_{i,j=1}^{n} a^{ij} u_m(0)_{xi} u^2{}_m(0)_{xj})\, dx =$$
$$= \|u'_m(0)\|^2_{L^2(U)} + \|u_m(0)\|^2_{H_0^1(U)}$$
$$\leq \mathrm{C}(\|\psi\|^2_{L^2(U)} + \|\varphi\|^2_{H_0^1(U)})$$

*so*

$$\eta(0) \le C(\|\psi\|^2_{L^2(U)} + \|\varphi\|^2_{H_0^1(U)})$$

*from* (2.3)*we will get*

$$\|u_m'(t)\|^2_{L^2(U)} + A[u_m(t), u_m(t); t] \le C\left(\|\psi\|^2_{L^2(U)} + \|\varphi\|^2_{H_0^1(U)} + \int_0^t \|f\|^2{}_{L^2(0,T;L^2(U))} dt\right)$$

*so*

$$\|u_m'(t)\|^2_{L^2(U)} + A[u_m(t), u_m(t); t] \le C\left(\|\psi\|^2_{L^2(U)} + \|\varphi\|^2_{H_0^1(U)} + \|f\|_{L^2(0,T;L^2(U))}\right).$$

*But again L is hyperbolic* (2.3)

$$so\ \|u_m'(t)\|^2_{L^2(U)} + \|u_m(t)\|_{H_0^1(U)} \le C\left(\|\psi\|^2_{L^2(U)} + \|\varphi\|^2_{H_0^1(U)} + \|f\|_{L^2(0,T;L^2(U))}\right)$$

*but t is an arbitrary*
*so*

$$\max_{0\le t\le T}(\|u_m'(t)\|^2_{L^2(U)} + \|u_m(t)\|_{H_0^1(U)}) \le$$

$$C\left(\|\psi\|^2_{L^2(U)} + \|\varphi\|^2_{H_0^1(U)} + \|f\|_{L^2(0,T;L^2(U))}\right) \ldots (2.4)$$

*Now Let* $v \in H_0^1(U)$ *and* $\|v\|_{H_0^1(U)} \le 1.$

*We write* $v = v_1 + v_2$ , *where* $v_1 \in span\{w_k\}_{k=1}^m$ *and* $(v_2, w_k) = 0.$

*We have*

$$\langle u_m'', v\rangle = (u_m'', v) = \left(\sum_{k=1}^m w_k\ d_m^k{}''(t), v_1 + v_2\right)$$

$$= \left(\sum_{k=1}^m w_k\ d_m^k{}''(t), v_1\right) + \left(\sum_{k=1}^m w_k\ d_m^k{}''(t), v_2\right)$$

$$(u_m'', v_1) = (f, v_1) - B[u_m, v, ; t]$$

*but*

$$(f, v_1) \le \|f\|_{L^2(0,T;L^2(U))} \|v_1\|^2_{L^2(U)}$$

$$\le \|f\|_{L^2(0,T;L^2(U))}$$

*So*

$$B[u_m, v; t] \le \|u_m(t)\|_{H_0^1(U)} \|v_1\|_{H_0^1(U)}$$
$$\le \|u_m(t)\|_{H_0^1(U)}.$$
$$|B| \le C\|u_m(t)\|_{H_0^1(U)}$$

and we have

$$|(f, v_1)| \le \|f\|_{L^2(0,T;L^2(U))}$$

*So*

$$| < u_m'', v > | \le |(f, v_1)| + |B|$$
$$\le C(\|f\|_{L^2(0,T;L^2(U))} + \|u_m(t)\|_{H_0^1(U)})$$

*By the definition of*

$$\|u_m'' u(t)\|_{H^{-1}(U)} = \max_{\|v\|\le 1} | < u_m'', v > |.$$

$$\|u_m''\ \|_{H^{-1}(U)} \le C(\|f\|_{L^2(0,T;L^2(U))} + \|u_m(t)\|_{H_0^1(U)}).$$

*So*

$$\int_0^T \|u_m''\ \|_{H^{-1}(U)} dt \le C\int_0^T \left(\|f\|^2{}_{L^2(0,T;L^2(U))} + \|u_m(t)\|_{H_0^1(U)}\right) dt$$

$$\le C\|f\|^2{}_{L^2(0,T;L^2(U))} + \int_0^T \|u_m(t)\|^2{}_{H_0^1(U)} dt$$

$$\leq C\|f\|_{L^2(0,\mathrm{T};L^2(\mathrm{U}))} + \max\|u_{\mathrm{m}}(\mathrm{t})\|^2_{H_0^1(U)} \int_0^T dt$$

From (2.4)

$$\leq C\left(\|\varphi\|^2_{H_0^1(U)} + \|\psi\|^2_{L^2(0,\mathrm{T};L^2(\mathrm{U}))} + \|f\|^2_{L^2(0,\mathrm{T};L^2(\mathrm{U}))}\right).$$

So

$$\|u_m''\|_{L^2(0,T,H^{-1})} \leq C\left(\|\varphi\|^2_{H_0^1(U)} + \|\psi\|^2_{L^2(0,\mathrm{T};L^2(\mathrm{U}))} + \|f\|^2_{L^2(0,\mathrm{T};L^2(\mathrm{U}))}\right) \quad \dots (2.5)$$

(2.4) + (2.5) = (2.3)

## Conclusion

In our main result, we gave a good estimate to measure the regularity by using some Soblev spaces(integral spaces) which give the property of continuity to the function u which is the solution of our problem and the continuity of first derivative, second derivative more regular of course in integral sense. Also, u=0 on the boundary of the domain U is generalized sense, this means when we deal with an integral space we say that the trace of u is equal to zero on the boundary of the domain U.

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# طرائق الطاقة لمسألة الوتر ذات القيم الأبتدائية والحدودية

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## الخلاصة

درسنا أحد أمثلة مسائل القطوع الزائدة وهي مسألة الوتر ذو النهايتين ذات القيم الابتدائية والحدودية. في الحقيقة نحن نبحث عن حل بمعنى الحل الضعيف لبعض فضاءات سوبولف واستخدمنا طرائق الطاقة مع طريقة*(Galerkin)* ايضا لدراسة بعض الخصائص لمسألتنا مثل الوجود والوحدانية.

**الكلمات المفتاحية**: مسأ؟لة الوتر ذات القيم الابتدائية والحدودية، الحل الضعيف، فضاءات سوبولف، طريقة*(Galerkin* تخمين الطاقة